Inelastic Effective Length Factors

The discussion in Section 4-4-1 concerning the evaluation of effective length factors in rectangular frames was restricted to the buckling of perfectly elastic frames. However, in reality, instability of steel frames is more likely to take place after the stresses at some parts of frame have reached the yield stress.

For elastic behavior the values of coefficients G_A and G_B given in the two charts of LRFDM (p. 241 and p.242) can be used. If the elastic E still applies for the girder members, but inelastic for the columns, this can be accounted for by adjusting the G values as follows:

$$G_i = \frac{\sum E_i (I_c / L_c)}{\sum E_e (I_g / L_g)} = \frac{E_i}{E_e} G_e = \tau G_e$$

Where: G_e = elastic G factor assuming that both columns and girders behave elastically

G_i = inelastic G factor assuming that girders behave elastically while the columns behave inelastically

τ = stiffness reduction factor

Then $\tau = 1.0$ For $P_u/P_y \leq 0.39$

$\tau = 4(\alpha P_u/P_y)\,[1-(\alpha P_u/P_y)]$ For $P_u/P_y > 0.39$

Where: $P_y = F_y A_g$ and α=1 for LRFD

Vales for stiffness reduction factor τ, for different values of P_u/A_g are presented in LRFDM for steel with F_y= 35, 36, 42, 46 and 50 ksi (p. 4-317). For values of P_u/A_g smaller than those with entries in this table, the columns behaves elastically, and the reduction factor τ=1.0. Note that G = 10.0 for pin end, and G = 1.0 for fixed end the value of G at that end should not multiply by the stiffness reduction factor τ.

Note: LRFDM Tables p.(4-10) to p.(4-21) can be used for calculating design strength of column for W sections, and these values are tabulated with respect to the effective length about the minor axis K_yL_y. For buckling about major axis calculate $(KL)_{eq.}$:

$$(KL)_{eq} = \frac{K_x L_x}{r_x / r_y}$$

***Example Problem* 4-6:** Calculate the effective length for W10×60 A992 Gr.50 steel column AB in the unbraced frame shown below, which subjected to an axial factored compressive load of 450 kips.
The columns are oriented such that major axis bending occurs in the plane of frame. The columns are braced continuously along the length for out –of-plane buckling. The same column section is used for the story above. Check the column adequacy. All girders are W14×74 sections.

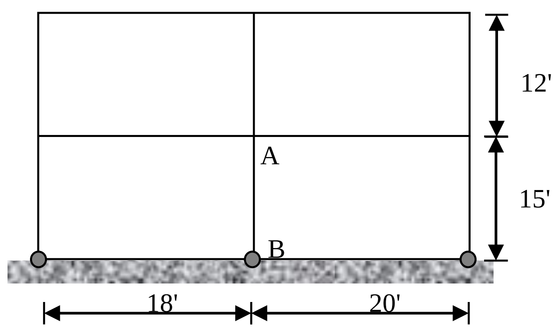

Figure 4-13: Example problem 4-6

Solution: -

- Since the columns are braced continuously along the length for out –of-plane buckling (minor axis), then $L_y = 0.0$ (No buckling occur about y-axis)
- Need to calculate K_x using alignment charts for unbraced frame:

  $I_x = 795\ in^4$ for W14×74 & $I_x = 341\ in^4$ for W10×60

$$G_A = \frac{341/12 + 341/15}{795/18 + 795/20} = 0.61 \qquad \& \qquad G_B = 10$$

- $P_y = F_y A_g = 50*17.6 = 880$ kips Then $P_u/P_y = 0.511 > 0.39$ the column partially plastifies
- Calculate $K_{x,\ inealstic}$: $P_u/A_g = 450/17.6 = 25.57$ ksi & $F_y = 50$ ksi

  Then $\tau = 0.875$

  $G_A = 0.61*0.875 = 0.53$ & $G_B = 10$ … $K_{x,\ inelastic} = 1.8$ (alignment chart)
- Design strength of the W10×60 column: $K_xL_x = 1.8*15 = 27$

  $r_x/r_y = 1.71$ Then $(KL)_{eq} = 27/1.71 = 15.79'$

  $P_{dc} = 533.67$ kips [LRFDM Table p.(4-19) - using interpolation]

  $P_{dc} = 533.67$ kips $> P_u = 450$ kips ……. OK

Example Problem 4-7: Select the lightest W12 A992 Gr. 50 for the column AB in the unbraced frame shown below, which subjected to an axial factored compressive load of 500 kips.

The columns are oriented such that major axis bending occurs in the plane of frame. The columns are braced at each story level for out –of-plane buckling. A same section is used for columns of the stories above and below. All girders are W14×68 sections.

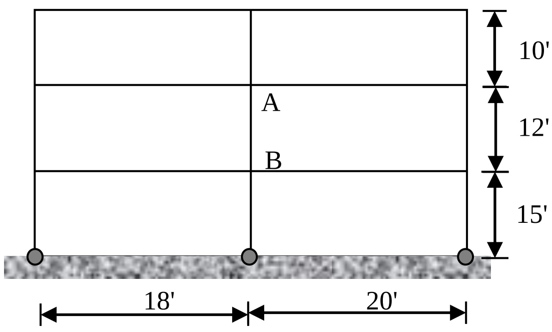

Figure 4-14: Example problem 4-7

Solution: -

- Since the columns are braced at each story level for out –of-plane buckling, then K_y = 1.0 …… K_yL_y = 1.0*12=12'
- Assume minor axis buckling governs, and F_y=50 ksi (A992 steel)

  P_{dc} = 547 kips for W12× 53 [LRFDM Table p.(4-18)]
- $P_y=F_y A_g$= 50*15.6= 780 kips Then P_u/P_y=0.641> 0.5 the column partially plastifies
- Calculate $K_{x,\ inealstic}$:
- P_u/A_g = 500/15.6 = 32.05 ksi & F_y = 50 ksi

  Then τ=0.662

$$G_A = \frac{0.662*\left(\frac{425}{10}+\frac{425}{12}\right)}{\frac{722}{18}+\frac{722}{20}} = 0.68$$

$$G_B = \frac{0.662*\left(\frac{425}{15}+\frac{425}{12}\right)}{\frac{722}{18}+\frac{722}{20}} = 0.55$$

From the chart : $K_{x\ inelastic} \approx 1.18$

- Check selected W12× 53 section for x-axis buckling:

  K_xL_x= 1.18*12 = 14.16; r_x/r_y = 2.11…Then $(KL)_{eq}$ = 14.16/2.11= 6.71'

  P_{dc} = 648.35 kips [LRFDM Table - using interpolation]

  P_{dc} = 648.35 kips > P_u= 450 kips ……. OK

- Check for local buckling:

$$\lambda_f = b_f/2t_f = 8.69 < \lambda_{rf} = 0.56\sqrt{E/F_y} = 13.5 \qquad \text{O.K.}$$

$$\lambda_w = h/t_w = 28.1 < \lambda_{rw} = 1.49\sqrt{E/F_y} = 35.9 \qquad \text{O.K.}$$

So, select a W12×53 of A992 Grade 50 steel.