Intensity distribution in the fringe system

Note that the edges of the bright fringes are not sharp, there is a gradual change from bright to dark.so we have knew the locations of only the centers of the bright and dark fringes on a distant screen.

Let us now direct our attention to the intensity of the light at other points between the positions of maximum constructive and destructive interference. In other words, we now calculate the distribution of light associated with the double-slit interference pattern.

Suppose that the two slits represent coherent sources of sinusoidal waves such that the two waves from the slits have the same angular frequency ɯ and a constant phase difference Ø .the total magnitude of the electric fields at point p on the screen is the superposition of the two waves.

Assuming that the two waves have the same amplitude E_0, we can write the magnitude of the electric field at point p due to each wave separately as:

$E_1=E_0\sin ɯt$, $E_2= E_0\sin(ɯt+Ø)$

Using the superposition principle, we can obtain the magnitude of the resultant electric field at point p:

$E_p=E_{1+}E_2= E_0[\sin(ɯt+Ø)+\sin ɯt$

From trigonometric identity:

$\sin A+\sin B=2\sin(A+B /2)\cos (A-B/2)$

$A=(ɯt+Ø)$,$B=ɯt$

$E_P=2E_0\sin(ɯt+Ø+ɯt /2)\cos (ɯt+Ø-ɯt /2)$

$E_P=2E_0\cos(\varnothing/2)\sin(\omega t+\varnothing/2)$

This indicates that the electric field at point p has the same frequency ω as the light at the slits. But that the amplitude of the field is multiplied by the factor $[2\cos(\varnothing/2)]$.

To check the consistency of this result note that:

If $\varnothing=0,2\pi,4\pi,\ldots\ldots\ldots$

$E_P=2E_0$

This corresponds to the condition for maximum constructive interference.

Likewise if $\varnothing=\pi,3\pi,5\pi,\ldots\ldots\ldots$, then the magnitude of the electric field at point p is zero.

This corresponds to destructive interference.

To obtain an expression for the light intensity at point p:

$I \propto E_P^2$

$$I=E_P^2=4E_0^2\cos2(\varnothing/2)\sin^2(\omega t+\varnothing/2)$$

Most light-detecting instruments measure time-averaged light intensity,

The time average of $\sin^2(\omega t+\varnothing/2)$ over one cycle $=1/2$

$I=E_P^2=4E_0^2\,(1/2)\cos^2(\varnothing/2)$

$I=2E_0^2\cos^2(\varnothing/2)$

$I=I_0\cos^2(\varnothing/2)$ (*) where I_0:is the maximum intensity on the

Screen=$2E_0^2$

For constructive int.: path difference $d=\lambda$ which corresponds to phase shift of $\varnothing=2\pi$. This implies that the ratio of path difference to wavelength is equal to the ratio of phase shift to 2π.

$d/\lambda=\varnothing/2\pi \Longrightarrow \varnothing = 2\pi d/\lambda$

$\varnothing=2\pi d\sin\Theta/\lambda$

From eq.(*) $I/I_0=\cos^2(\pi d\sin\Theta/\lambda)$

The plot of the ratio I/I_0 as a function of $d\sin\Theta/\lambda$