5.4 *Classification of Cross-Section*

For the case of local buckling the slenderness is based on width/thickness ratios of the slender plate elements that make up the cross section of most steel members. The member cross sections are then classified by which of the three ranges their most slender element falls in as shown in Figure 5-4. If the most slender cross sectional element is not very slender (i.e. b/t is small) , then the cross section is said to be COMPACT. If the most slender element of the cross section falls in the transition range, then the cross section is said to be NON-COMPACT. Otherwise, when the most slender cross sectional element is very slender (i.e. b/t is large) then the cross section is said to be SLENDER. (pp. 3-5)

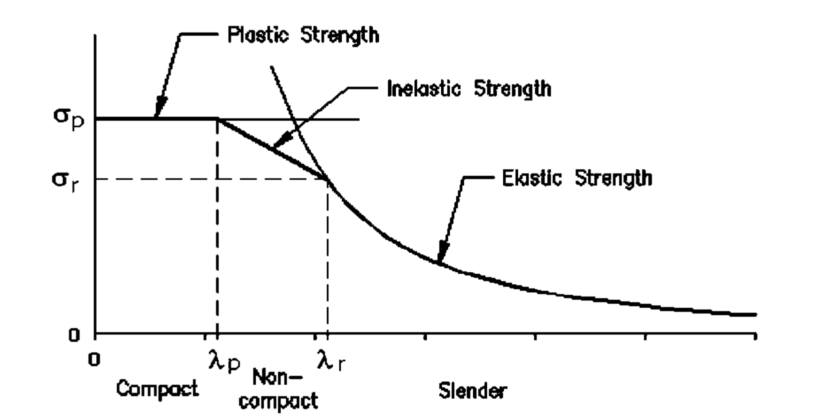


Figure 5-4 Theoretical Maximum Compressive Stress

It can be summarized as follows. Let : λ = width – thickness ratio
λ_p = upper limit for compact category
λ_r = upper limit for non-compact category
Then:

- If $\lambda \leq \lambda_p$ the shape is compact (an I-shape is compact if $\lambda_f \leq \lambda_{pf}$ and $\lambda_w \leq \lambda_{pw}$)
- If $\lambda_p < \lambda \leq \lambda_r$ the shape is noncompact
- If $\lambda > \lambda_r$ the shape is slender (an I-shape is slender if $\lambda_f \leq \lambda_{rf}$ and $\lambda_w \leq \lambda_{rw}$)

For rolled I-shape (Sec. B4, Table B 4-1, pp. 16 to18):

- $\lambda_f = b_f/2t_f$; $\lambda_{pf} = 0.38\sqrt{E/F_y}$ and $\lambda_{rf} = 1.0\sqrt{E/F_y}$
- $\lambda_w = h/t_w$; $\lambda_{pw} = 3.76\sqrt{E/F_y}$ and $\lambda_{rw} = 5.70\sqrt{E/F_y}$

These limits are also used for C-shape, except that λ for flange is: $\lambda_f = b_f/t_f$

5.5 Design Strength of Beam

5.5.1 Yielding Limit State

The specification computes the nominal moment capacity, M_n, as the maximum moment that a member can support. This maximum moment is considered to be when the cross section is fully yielded. Figure 5-5-1 Illustrates how the stress distribution changes as moment is increased on a section.

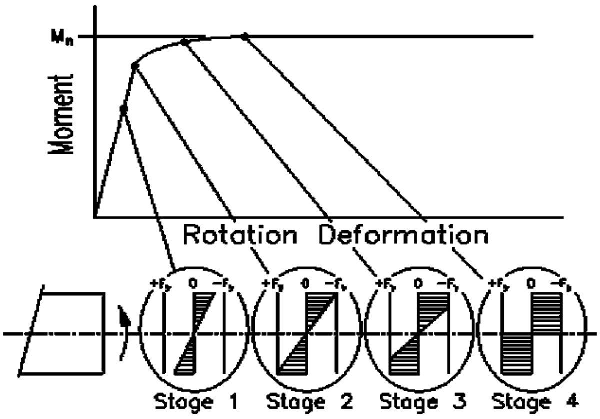


Figure 5-5-1 Flexural stress distribution

In this case M_n is the nominal flexural yielding strength of the member. For compact I-shaped members and channels bent about their major axis:

$$M_{nx} = M_{px} = F_y Z_x \qquad \text{for strong axis bending}$$

$$M_u \leq M_d = \phi_b M_{nx}$$

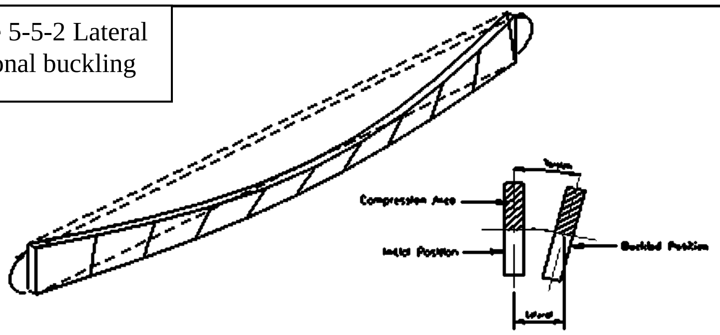
Where: $\phi_b = 0.9$

- M_p is the plastic flexural strength of the member.
- F_y is the material yield stress.
- Z is the plastic section modulus for the axis of bending being considered.

5.5.2 Lateral Torsional Buckling Limit State

5.5.2.1General: When a member is subjected to bending, one side of the member is in compression and wants to behave like a column. This means that it is subject to flexural buckling. Since the compression side is connected to the tension side (which is not prone to buckling), it cannot buckle in the plane of loading. This leaves the lateral direction as the direction of buckling. The tension side resists the buckling, resulting in the rotated cross section (i.e. the torsion). A simple experiment can be used to demonstrate this behavior, take a thin, flat bar (a typical "yard stick" works well) and apply end moments about the end with your hands. If you force bending about the strong axis, the member will buckle sideways and the section will rotate so that it is no longer vertical. This is lateral torsional buckling (LTB). The experiment is illustrated in Figure 5-5-2.

Figure 5-5-2 Lateral torsional buckling



If you bend the member about it's weak axis, this behavior is not observed. This is because the out-of-plane moment of inertia of the section is larger than in-plane moment of inertia. The out-of-plane inertia then creates a stiffness out-of-plane that is larger the in-plane, thus preventing the out-of-plane buckling. The result is that LTB is a strong axis phenomena. It need only be considered for strong axis bending. Like all buckling, the force that will cause LTB to happen (in this case, moment) is dependent on the length, or slenderness, of the "column". Figure 5-5-3 shows the general form of the curve used for LTB. For *LTB the length of the column* is length of laterally unsupported compression flange. If the length is short enough, then the member can develop it's full plastic strength. For longer lengths, there is inelastic buckling, and for long laterally unbraced lengths there is elastic buckling, following a typical buckling/plastic strength curve.

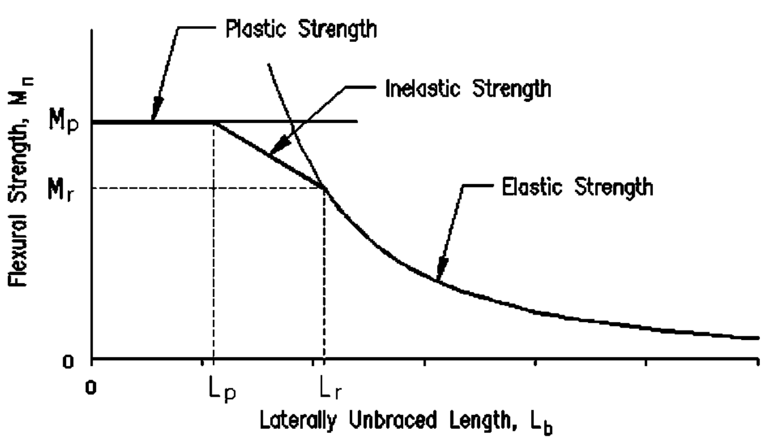


Figure 5-5-3 Strength (M_n) vs. Slenderness (L_b) for LTB

5.5.2.2 Laterally Unbraced Lengths(Classification of spans for flexure):

It is important to be able to identify laterally unbraced lengths in flexural members. The most important parameter in preventing the lateral buckling of the beam is the spacing, L_b, of the lateral bracing. There are a few criteria that must be considered.

1. The lateral support must be applied to the compression flange. Bracing at mid-height or at the tension flange is not sufficient.
2. The bracing must provide actual lateral support.

For the purlins to be effective as lateral supports (adequately braced beam), they must act to induce a point of inflection in the beam at the point of connection, as shown in Figure 5-5-4. In some cases, particularly cantilevered and continuous beams, the compression flange is on the bottom of the member so does not have any lateral support (.unbraced beam)

The general form of the LTB limit state follows the typical buckling curves. The slenderness parameter used is L_b, the *laterally unbraced length*. The limits of the buckling regions are specified by the terms L_p (the limit of the plastic region) and L_r (the limit of the inelastic buckling region) as shown in Figure 5-5-3. Hence:

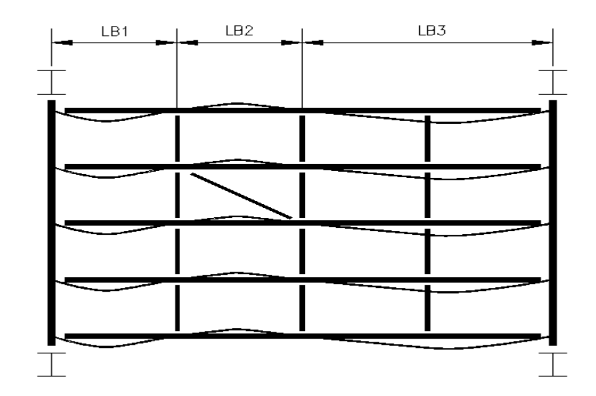


Figure 5-5-4 Lateral Bracing

- If $L_b \le L_p$ then the plastic strength, M_p, controls and LTB does not occur
- If $L_p < L_b \le L_r$ then inelastic LTB occurs
- If $L_b > L_r$ then elastic LTB occurs

Where: L_p = the limit of laterally unbraced length for plastic lateral buckling (Sec. F2, pp. 48) & (pp. 3-4 to 3-5)

$$= L_p = 1.76 r_y \sqrt{\frac{E}{F_y}}$$

L_r = the limit of laterally unbraced length for elastic lateral buckling (Sec. F2, pp. 48)

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o}} \sqrt{1 + \sqrt{1 + 6.67 (\frac{0.7 F_y S_x h_o}{EJc})^2}}$$

r_{ts} = effective radius of gyration, in (provided in AISC Table 1-1)

J = torsional constant, in^4 (AISC Table 1-1)

c = 1.0 for doubly symmetric I-shapes

h_o = distance between flange centroids, in (AISC Table 1-1)

5.5.2.3 Design moment:

- *Compact section*:

1. *Plastic Range (zone 1)*: As noted above, for a beam to be considered adequately braced, its compression flange should be either continuously braced, or the distance L_b between adjacent lateral braces should satisfy the relation (Sec. F2.1-pp. 47): $L_b \leq L_p$ (LTB does not happen) Consequently in the plastic range:

$$M_d = \phi_b M_p = \phi_b F_y Z_x \qquad \text{(I-shape bent about the major axis)}$$

2. *In-elastic Buckling Range (zone 2)*:A linear interpolating function is used to compute M_n in the in-elastic buckling range. The value resulting from the interpolation is then scaled by C_b. This value is compared with M_p to find the final M_n. Then the flexural design moment can be written as(Sec. F2.2-pp. 47):

$$M_d = \phi_b\, C_b\, (M_p - (M_p - 0.7 S_x F_y)*(L_b - L_p)/(L_r - L_p))$$

Or $$\phi_b M_n = C_b[\phi_b M_{px} - BF(L_b - L_p) \leq \phi_b M_{px}$$

C_b = a coefficient which depends on variation in moments along the span (Sec. F1.)

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$

Where:

M_{max} = largest moment in unbraced segment of a beam
M_A = moment at the ¼ point
M_B = moment at the ½ point
M_C = moment at the ¾ point

C_b = 1.0 for uniform distributed bending moment. Table 3-1(pp. 3-10) in LRDFM gives the value for C_b for simply supported beams.

3. *Elastic Buckling Range (zone 3)*: The nominal moment capacity, M_{nE}, in the elastic range is found by computing the elastic moment that creates the critical buckling stress, F_{cr}, in the compression flange(Sec. F3.2a-pp. 47).

$$M_n = F_{cr} S_x \le M_p$$

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}}\right)^2}$$

Where:

r_{ts} = effective radius of gyration, in (provided in AISC Table 1-1)
J = torsional constant, in^4 (AISC Table 1-1)
c = 1.0 for doubly symmetric I-shapes
h_o = distance between flange centroids, in (AISC Table 1-1)

- Non –compact section: if the section is non-compact because of flange or web ($\lambda_p < \lambda \le \lambda_r$)(Sec. F3.2, pp. 49):

$$M_n = \left[M_P - \left(M_p - 0.7 F_y S_x \right) \left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}} \right) \right]$$

For built-up sections with slender flanges (that is, where $\lambda > \lambda_r$) (Sec. F3.2b, pp. 49):

$$M_n = \frac{0.9 E K_c S_x}{\lambda^2}$$

$$K_c = \sqrt{\frac{h}{t_w}} \ge 0.35 \le 0.76$$

***Example Problem* 5-1:** A compact W16×45 of A992 Gr. 50 steel is used as simply supported beam of 33-ft span, as shown in Figure. Determine the max. factored, uniform load that the beam can support if lateral supports are provide: (a) at 5.5 ft interval; (b) at 11 ft interval; (c) at 33 ft interval.

Solution: - From LRFDM, for W16×45:
A= 13.3 in^2; Z_x= 82.3 in^3; S_x= 72.7 in^3; I_y=32.8 in^4; r_y = 1.57 in and F_y= 50 ksi. J_c=1.11.

a) L_p = 5.55 Tables 3-2. p.(3-17)
L_b= 5.5' < L_p =5.55'
Then $M_d = \phi_b M_{px} = \phi_b F_y Z_x$ = 309ft-kips

$M_{max} = M_d = \frac{q_{u1} L^2}{8}$ ….. $q_{u1} = \frac{309 * 8}{33^2} = 2.27$ klf

Note: The max. factored, uniform load for F_y= 36 (For MC-Section) & F_y=50 ksi (For W-Section), are tabulate in LRFDM to Tables 3-6. p.(3-33) to p.(3-95) for fully braced beam or when $L_b < L_p$.

for our example enter Factored Uniform Loads

Q_u = 74.8 kips p. (3-61)

For W16×45, F_y= 50 ksi and L=33'

q_{u1} = 74.8/33 =2.27 klf

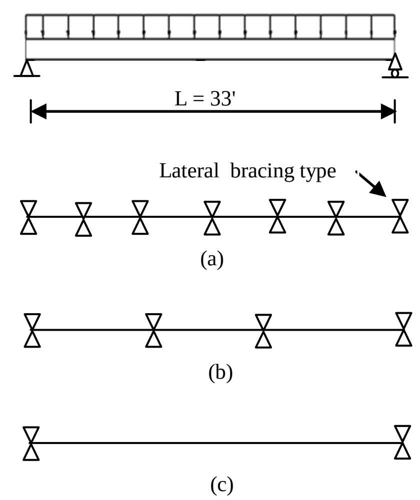


Figure: Example problem 5-1

b) L_p = 5.55 < L_b= 11' then calculate L_r

L_r = 15.2 > L_b= 11' Tables 3-2. p.(3-17)

$$M_d = \phi_b C_b (M_{px} - (M_{px} - 0.7S_xF_y)*(L_b - L_p)/(L_r - L_p))$$

Or $$\phi_b M_n = C_b[\phi_b M_{px} - BF(L_b - L_p) \le \phi_b M_{px}$$

C_b = 1.01 …. (Table 3-1, p. 3-10) , BF= 10.8 ……Tables 3-2. p.(3-17)

M_d= 252.6 ft-kip

$$M_{max} = M_d = \frac{q_{u1}L^2}{8} \quad ….. \quad q_{u1} = \frac{252.6*8}{33^2} = 1.86 \text{ klf}$$

c) L_b= 33' > L_r

$$M_n = F_{cr}S_x \le M_p \qquad F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_x h_o}\left(\frac{L_b}{r_{ts}}\right)^2}$$

C_b = 1.14 …. (Table 3-1, p. 3-10)

S_x= 72.7 in^3, h_o= 16.5 in, r_{ts}= 1.88 in, J= 1.11

M_d = 830.6 >ϕ_b M_{px}

$$M_{max} = M_d = 309 = \frac{q_{u1}L^2}{8} \quad ….. \quad q_{u1} = \frac{309*8}{33^2} = 2.27 \text{ klf}$$

Example Problem 5-2: A W12×65 of A992 Gr. 50 steel has unbraced length of 11'. Determine the design bending moment.

Solution: - From LRFDM (Table 3-2, pp. 3-17) for W12×65; Z_x= 96.8 in^3; r_y=3.02" and F_y= 50 ksi.

$\lambda_f = b_f/2t_f$ =9.92 ; $\lambda_{pf} = 0.38\sqrt{E/F_y}$ =9.15 and $\lambda_{rf} = 1.0\sqrt{E/F_r}$ = 24.08

$\lambda_w = h/t_w$ = 24.9 ; $\lambda_{pw} = 3.76\sqrt{E/F_y}$ =90.6 and $\lambda_{rw} = 5.70\sqrt{E/F_y}$ =137

As $\lambda_{pf} < \lambda_f < \lambda_{rf}$ ……the flange is noncompact, but web is compact

$M_{px} = F_y Z_x$= 50*87.9 = 4840 in-kips = 403.33 ft-kips

$M_d = \phi_b M_n = \phi_b [M_{px} - (M_{px} - M_{rx})*(\lambda_b - \lambda_p)/(\lambda_r - \lambda_p)]$ = 395.7 ft-kip.