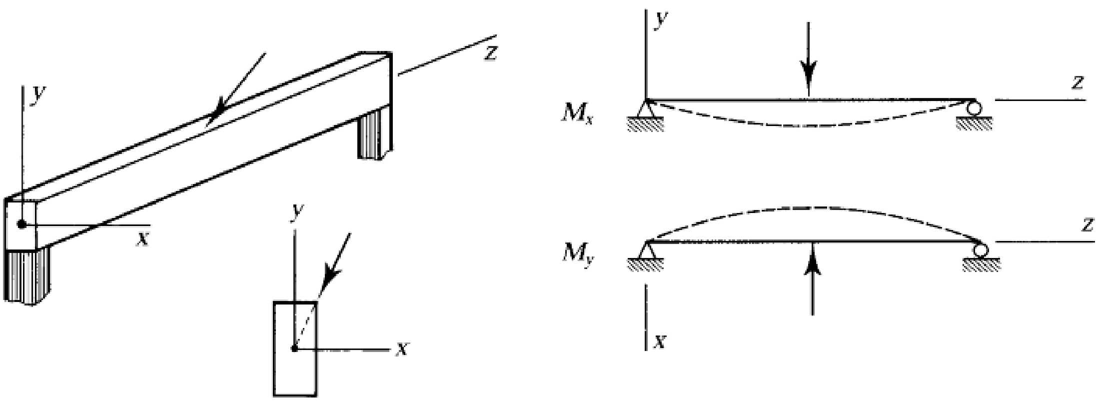
CHAPTER SIX

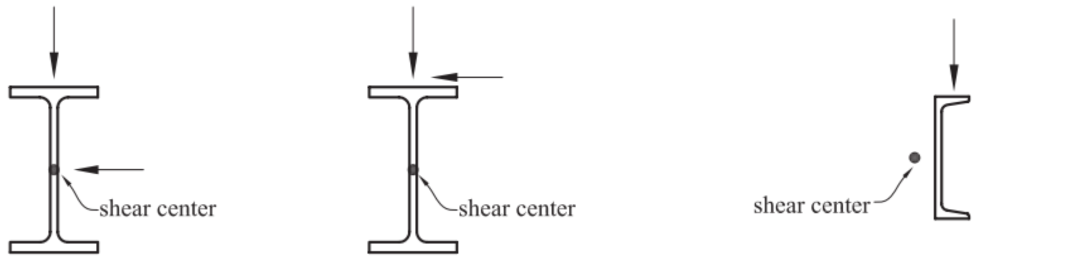
BIAXAIL BENDING

6-1 INTRODUCTION

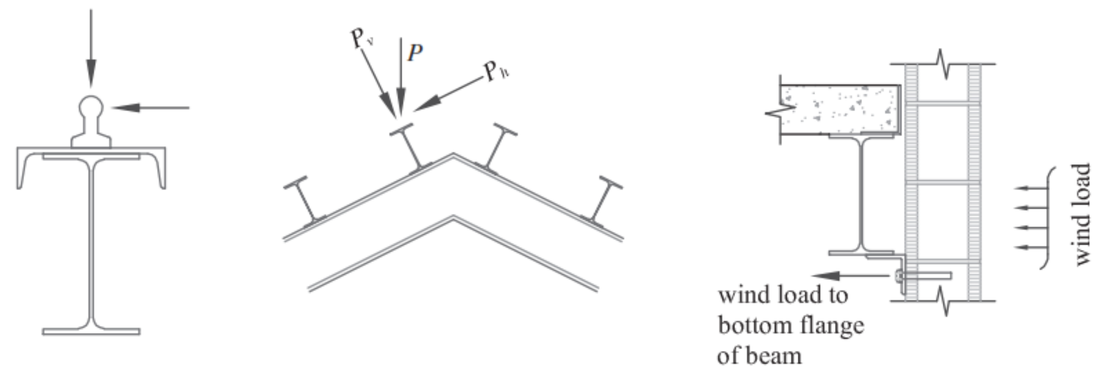
Biaxial bending is the bending of the beam about both axes (the x–x and y–y axes). Namely Biaxial bending occurs when a beam is subjected to a loading condition that produces bending about both the major (strong) axis and the minor (weak) axis.



Pure biaxial bending occurs when the loads to each axis are applied directly through the shear center which is the point within a member such that when loads are applied through that point, twisting will not occur. When the applied loads do not pass through the shear center, as is often the case with singly symmetric shapes, torsion will occur. Examples of these beams are crane girders, purlins for roof framing, and unbraced beams providing lateral support to exterior cladding.

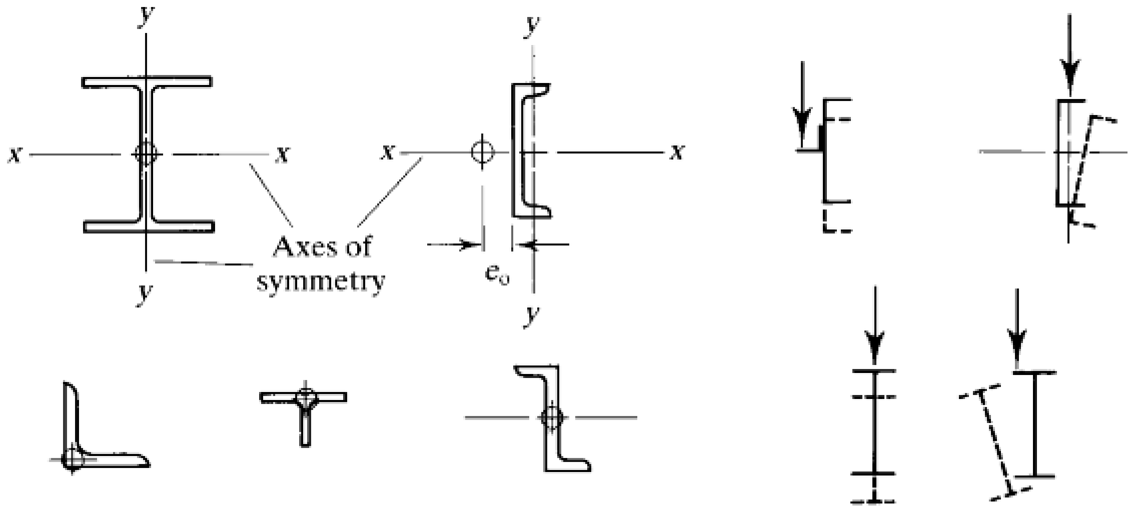


a. pure biaxial bending *b. biaxial bending with torsion* *c. torsion in a singly symmetric shape*



a. crane girder *b. roof purlins* *c. beams supporting cladding for lateral loads*

The location of the shear center for several common cross sections is shown in below, where the shear center is indicated by a circle.



6-2 WEAK-AXIS BENDING STRENGTH

The nominal bending strength for the x-axis has been discussed previously. For the y-axis, lateral-torsional buckling is not a limit state, since the member does not buckle about the strong axis when the weak axis is loaded. For shapes with **compact flanges**, the nominal bending strength about the y-axis is

$$M_{ny} = M_{py} = F_y Z_y \leq 1.6\, F_y S_y$$

The limit of 1.6 F_y S_y is to prevent excessive working load deformation and is satisfied when

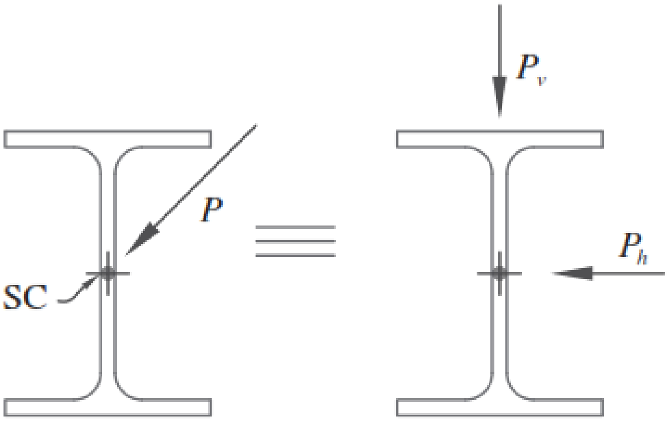
$$\frac{Z_y}{S_y} \leq 1.6$$

For shapes with **non-compact** flanges, the nominal bending strength about the y-axis is:

$$M_{ny} = M_{py} - (M_{py} - 0.7 F_y S_y)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

- **Case I: Loads Applied Through the Shear Center**

The inclined load passes through the shear center, the load is resolved into a horizontal and a vertical component, each of which passes through the shear center. This will result in no twisting of the beam and will cause simple bending about both the x- and y-axes. An interaction equation is used to determine whether the member is adequate for combined bending. This equation is given in the AISC specification as:



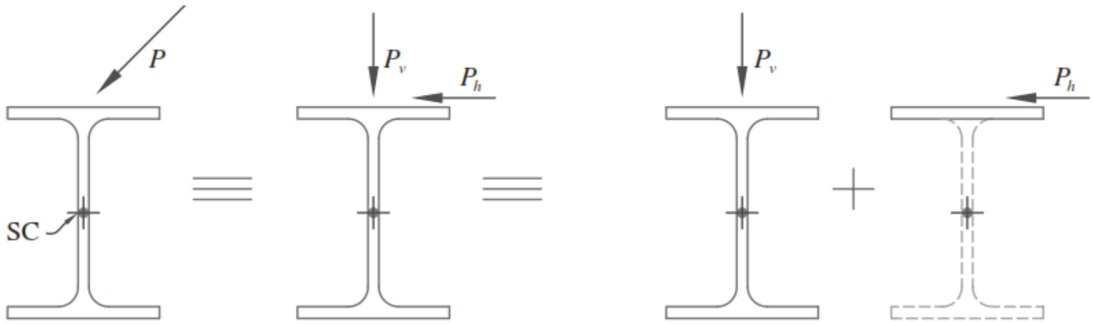
$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \leq 1.0$$

Where
M_{ux} = Factored moment about the x-axis,
M_{uy} = Factored moment about the y-axis,
M_{nx} = Nominal bending strength for the x-axis,
M_{ny} = Nominal bending strength for the y-axis,
and ϕ_b = 0.9.

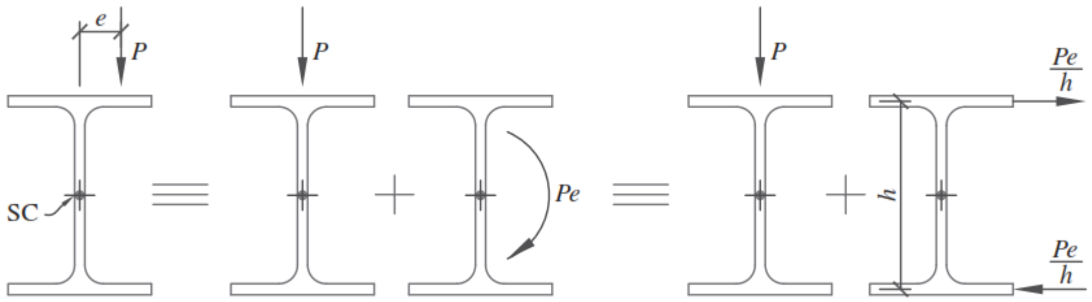
- **Case II: Loads Not Applied Through the Shear Center**

When loads are not applied through the shear center of a cross section, the result is flexure plus torsion. If possible, the structure or connection geometry should be modified to remove the eccentricity. The problem of torsion in rolled shapes is a complex one, therefore approximate conservative methods are used for dealing with it. There are two cases of biaxial bending of loads not applied through shear center:

Case 1: Beams in which the load does not pass through the shear center, but the vertical component does pass through the shear center. In which the load is resolved into a vertical component and a horizontal component located at the top flange.



Case 2: Vertical load that does not pass through the shear center. Where there is a vertical load eccentric to the shear center, the load is resolved into a vertical load coincident with the shear center and horizontal forces located at the top and bottom flanges.



In each of the two cases depicted above, only about half the cross section is considered to be effective with respect to its y axis; therefore, when considering the strength of a single flange, use half the tabulated value of Z for the shape. Namely

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{0.5(\phi_b M_{ny})} \leq 1.0$$

Example 6-1: A 12 ' span simply supported beam of shape W21×68. Lateral support of the compression flange is provided only at the ends. Loads act through the shear center, producing moments about the x and y axes. The service load moments about the x axis are M_{Dx} = 48 ft-kips and M_{Lx} = 144 ft-kips. Service load moments about the y axis are M_{Dy} = 6 ft-kips and M_{Ly} = 18 ft-kips. If A992 Gr. 50 steel is used, does this beam satisfy the provisions of the AISC Specification? Assume that all moments are uniform over the length of the beam.

Solution:

1- compute the nominal flexural strength for *x*-axis bending. $L_b = 12\ ft,\ F_y = 50\,ksi$

From AISC Manual for W21 × 68 from the Z_x Table.

$L_p = 6.36\ ft\ ,\ \ L_r = 18.7\ ft\ ,\ Z_x = 160, S_x = 140$

$Z_y = 24.4\ , S_y = 15.7$

The shape is compact (no footnote to indicate otherwise)

$\text{For Compact shape and } L_p < L_b \le L_r \ \rightarrow$

$$M_{nx} = C_b\left[M_{px} - (M_{px} - 0.7F_y S_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_{px}$$

$M_{px} = F_y Z_x = 50 * 160 = 8000\ kips.in$

$C_b = 1.0$ due to bending moment is uniform

$$M_{nx} = 1.0\left[8000 - (8000 - 0.7 * 50 * 140)\left(\frac{12 - 6.36}{18.7 - 6.36}\right)\right]$$

$= 6583\ kips.in = 548.6\,kips.ft \quad < M_{px} = 8000 \ \rightarrow OK$

1- compute the nominal flexural strength for *y*-axis bending.

For the *y* axis, since the shape is compact, there is no flange local buckling and

$$M_{ny} = M_{py} = F_y Z_y = 50 * 24.4 = 1220\ k.in = 101.7\ k.ft$$

Check the upper limit

$$\frac{Z_y}{S_y} = \frac{24.4}{15.7} = 1.55 < 1.6 \rightarrow M_{ny} = M_{py} = 101.7 \text{ k.ft}$$

$$M_{ux} = 1.2M_{Dx} + 1.6M_{Lx} = 1.2*48 + 1.6*144 = 288.0 \text{ k/ft}$$

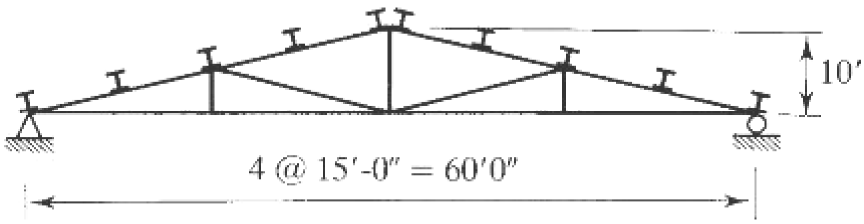
$$M_{uy} = 1.2M_{Dy} + 1.6M_{Ly} = 1.2*6 + 1.6*18 = 36.0 \text{ k/ft}$$

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}$$

$$\frac{288.0}{0.9*548.6} + \frac{36.0}{0.9*101.7} = 0.977 < 1.0 \rightarrow \text{OK}$$

The W21*68 is satisfactory

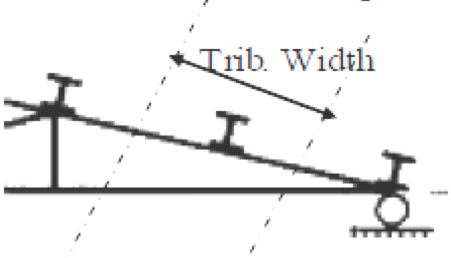
Example 6-2: A roof system consists of trusses of the type shown in Figure below spaced 15 ' apart. Purlins are to be placed at the joints and at the midpoint of each top-chord member. Sag rods will be located at the center of each purlin. The total dead load, including an estimated purlin weight, is 21 psf and live load of 21 psf. Use A36 steel and select a channel shape for the purlins.
F_y = 50 ksi.

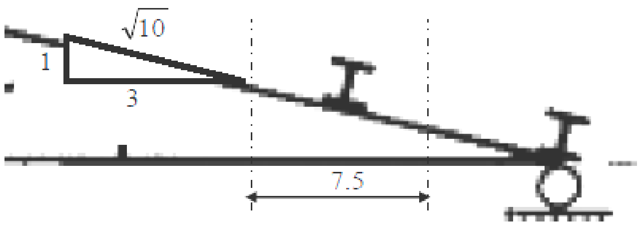


Solution: $w_u = 1.2w_D + 1.6w_L = 1.2*21 + 1.6*21 = 58.8$ psf

Trib. Width=

$$7.5 * \frac{\sqrt{10}}{3} = 7.906 \text{ ft}$$





$$\text{Purlin load} = 58.80 * 7.906 = 464.9 \text{ lb/ft}$$

$$\text{Normal Component} = \frac{3}{\sqrt{10}} * 464.9 = 441.0 \text{ lb/ft}$$

$$\text{Parallel Component} = \frac{1}{\sqrt{10}} * 464.9 = 147.0 \text{ lb/ft}$$

$$M_{ux} = \frac{w_u L^2}{8} = \frac{0.441 * 15^2}{8} = 12.40 \text{ k.ft}$$

With sag rods placed at the midpoint of each purlin, the purlins are two-span continuous beams with respect to weak axis bending. From Table 3-22c, "Continuous Beams," the maximum moment in a two-span continuous beam with equal spans is at the interior support and is given by:

$$M = 0.125 \text{ w } L^2$$

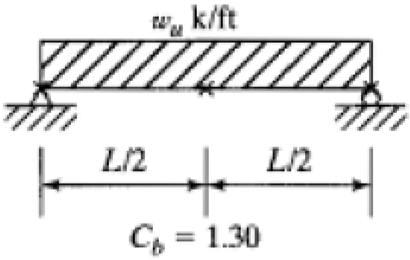
$$M_{uy} = 0.125 * 0.147 * (15/2)^2 = 1.034 \text{ k.ft}$$

Enter AISC Table 3-11 with L_b = 7.5' and Mux=12.4 and Select **C10*15.3** which is first solid line with ϕ_bMn =33.0 k.ft for C_b= 1.0.

But for C_b= 1.3

$$\phi_b M_{nx} = 1.3 * 33.0 = 42.9 \text{ k.ft}$$

$$\phi_b M_{px} = 42.9 \text{ k.ft}$$



$$\therefore \text{Use } \phi_b M_{nx} = 42.9 \text{ k.ft}$$

From AISC Manual for C10*15.3 from the Z_x Table. $Z_y = 2.34$, $S_y = 1.15$

$$\phi_b M_{ny} = \phi_b M_{py} = \phi_b F_y Z_y = 0.9 * 50 * 24.4$$
$$= 75.82 \text{ k.in} = 6.318 \text{ k.ft}$$

Check the upper limit

$$\frac{Z_y}{S_y} = \frac{2.34}{1.15} = 2.03 > 1.6 \rightarrow \phi_b M_{ny} = \phi_b (1.6 F_y S_y)$$
$$\phi_b M_{ny} = 0.9 * 1.6 * 36 * 1.15 = 59.62 \text{ k.in} = 4.968 \text{ k.ft}$$

Because the load is applied to the top flange, use only half this capacity to account for the torsional effects.

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{0.5(\phi_b M_{ny})}$$
$$\frac{12.40}{42.9} + \frac{1.034}{0.5 * 4.968} = 0.705 < 1.0 \rightarrow \text{OK}$$

Check shear

$$V_u = \frac{0.441 * 15}{2} = 3.31 \text{ kips}$$

From AISC Tables for C10*15.3

$$\phi_v V_n = 46.7 \text{ kips} > 3.31 \text{ kips} \rightarrow \text{OK}$$

Use C10*15.3