CHAPTER SIX

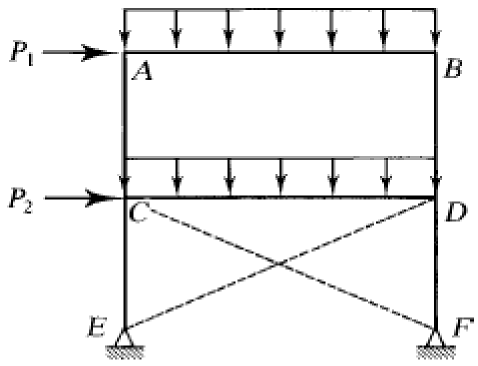
COLUMN-BEAM DESIGN

6-1 <u>Introduction</u>

While many structural members can be treated as axially loaded columns or as beams with only flexural loading, most beams and columns are subjected to some degree of both bending and axial load. This is especially true of statically indeterminate structures.

Even the roller support of a simple beam can experience friction that restrains the beam longitudinally, inducing axial tension when transverse loads are applied. In this particular case, however, the secondary effects are usually small and can be neglected. Many columns can be treated as pure compression members with negligible error. If the column is a one-story member and can be treated as pinned at both ends, the only bending will result from minor accidental eccentricity of the load.

For many structural members, however, there will be a significant amount of both effects, and such members are called *beam–columns*. Consider the rigid frame in Figure. For the given loading condition, the horizontal member *AB* must not only support the vertical uniform load but must also assist the vertical members in resisting the concentrated lateral load P_1. Member *CD* is a more critical case, because it must resist the load $P_1 + P_2$ without any assistance from the vertical members. The reason is that the x-bracing, indicated by dashed lines, prevents sideway in the lower story. For the direction of P_2 shown, member *ED* will be in tension and member *CF* will be slack, provided that the bracing elements have been designed to resist only tension. For this condition to occur, however, member *CD* must transmit the load $P_1 + P_2$ from *C* to *D*.



The vertical members of this frame must also be treated as beam–columns. In the upper story, members *AC* and *BD* will bend under the influence of *P*1. In addition, at *A* and *B*, bending moments are transmitted from the horizontal member through the rigid joints. This transmission of moments also takes place at *C* and *D* and is true in any rigid frame, although these moments are usually smaller than those resulting from lateral loads. Most columns in rigid frames are actually beam–columns, and the effects of bending should not be ignored. However, many isolated one-story columns can be realistically treated as axially loaded compression members. Another example of beam–columns can sometimes be found in roof trusses. Although the top chord is normally treated as an axially loaded compression member, if purlins are placed between the joints, their reactions will cause bending, which must be accounted for.

6-2 Interaction Formulas

The relationship between required and available strengths may be expressed as

$$\frac{\text{required strength}}{\text{available strength}} \leq 1.0$$

For compression members, the strengths are axial forces. For example, for LRFD

$$\frac{P_u}{\phi_c P_n} \leq 1.0$$

These expressions can be written in the general form: $$\frac{P_r}{P_c} \leq 1.0$$

where
P_r = required axial strength
P_c = available axial strength

If both bending and axial compression are acting, the interaction formula would be

$$\frac{P_r}{P_c} + \frac{M_r}{M_c} \leq 1.0$$

Where: M_r = required moment strength = M_u
M_c = available moment strength = $\varphi_b M_n$

For biaxial bending, there will be two moment ratios: $$\frac{P_r}{P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0$$

where the x and y subscripts refer to bending about the x and y axes.

For large axial load, the bending term is slightly reduced. The AISC requirements are given in Chapter H, "Design of Members for Combined Forces and Torsion," and are summarized as follows:

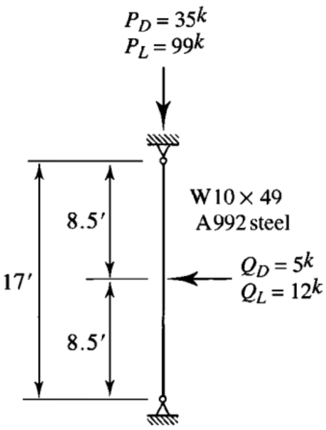
For $\frac{P_r}{P_c} \geq 0.2,$

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \textbf{(AISC Equation H1-1a)}$$

For $\frac{P_r}{P_c} < 0.2,$

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \textbf{(AISC Equation H1-1b)}$$

EXAMPLE 6-1: The beam–column shown in Figure is pinned at both ends and is subjected to the loads shown. Bending is about the strong axis. Determine whether this member satisfies the appropriate AISC Specification interaction equation.



SOLUTION:
From the column load tables, the axial compressive design strength of a W10 × 49 with $F_y = 50$ ksi and an effective length of $K_yL = 1.0 \times 17 = 17'$ is $\varphi_c P_n = 405$ kips (Table 4-1, pp. 4-20)
Since bending is about the strong axis, the design moment:
For an unbraced length $L_b = 17'$:

$L_p = 8.97'$, $L_r = 31.6'$, $\varphi_b M_p = 226.5$, BF= 3.67 (Table 3-2, pp. 3-18)

$$\phi_b M_n = C_b[\phi_b M_{px} - BF(L_b - L_p)] \le \phi_b M_{px}$$

$\varphi_b M_n = 197$ ft-kips

For the end conditions and loading of this problem, $C_b = 1.32$

For $C_b = 1.32$, the design strength is

$\varphi_b M_n = C_b \times 197 = 1.32(197) = 260$ ft-kips > $\varphi_b M_p = 226.5$

$\varphi_b M_n = 226.5$ ft-kips

Factored loads:

$P_u = 1.2PD + 1.6PL = 1.2(35) + 1.6(99) = 200.4$ kips

$Q_u = 1.2QD + 1.6QL = 1.2(5) + 1.6(12) = 25.2$ kips

The maximum bending moment occurs at midheight, so

$$M_u = \frac{25.2(17)}{4} = 107.1 \text{ ft-kips}$$

Determine which interaction equation controls:

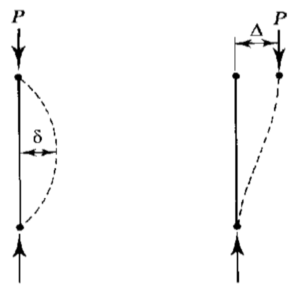
$$\frac{P_u}{\phi_c P_n} = \frac{200.4}{405} = 0.4948 > 0.2 \quad \therefore \text{ Use Equation 6.3 (AISC Eq. H1-1a).}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = \frac{200.4}{405} + \frac{8}{9}\left(\frac{107.1}{226.5} + 0\right) = 0.915 < 1.0 \quad \text{(OK)}$$

This member satisfies the AISC Specification.

6-3 <u>BRACED VERSUS UNBRACED FRAMES</u>

Figure 6.2 illustrates these two components of deflection. In Figure 6.2a, the member is restrained against sidesway, and the maximum secondary moment is $P\delta$, which is added to the maximum moment within the member. If the frame is actually unbraced, there is an additional component of the secondary



(a) (b)

moment, shown in Figure 6.2b, that is caused by sidesway. This secondary moment has a maximum value of $P\Delta$, which represents an amplification of the *end* moment. To approximate these two effects, two amplification factors, B_1 and B_2, are used for the two types of moments. The amplified moment to be used in design is computed from the loads and moments as follows (*x* and *y* subscripts are not used here; amplified moments must be computed in the following manner for each axis about which there are moments):

$$M_r = B_1 M_{nt} + B2 M_{lt} \qquad \text{(AISC Equation A-8-1)}$$

Where: Mr = required moment strength = M_u for LRFD

M_{nt} = maximum moment assuming that no sidesway occurs, whether the frame is actually braced or not (the subscript *nt* is for "no translation"). M_{nt} will be a factored load moment for LRFD

M_{lt} = maximum moment caused by sidesway (the subscript *lt* is for "lateral translation"). This moment can be caused by lateral loads or by unbalanced gravity loads. Gravity load can produce sidesway if the frame is unsymmetrical or if the gravity loads are unsymmetrically placed. M_{lt} will be zero if the frame is actually braced. For LRFD, M_{lt} will be a factored load moment

B_1 = amplification factor for the moments occurring in the member when it is braced against sidesway (P-δ moments).

B_2 = amplification factor for the moments resulting from sidesway (P-Δ moments).

In addition to the required moment strength, the required axial strength must account for second-order effects. The required axial strength is affected by the displaced geometry of the structure during loading. This is not an issue with member displacement (δ), but it is with joint displacement (Δ). The required axial compressive strength is given by

$$P_r = P_{nt} + B_2 P_{lt} \qquad \text{(AISC Equation A-8-2)}$$

Where: P_{nt} = axial load corresponding to the braced condition

P_{lt} = axial load corresponding to the sidesway condition

6-4 <u>MEMBERS IN BRACED FRAMES</u>

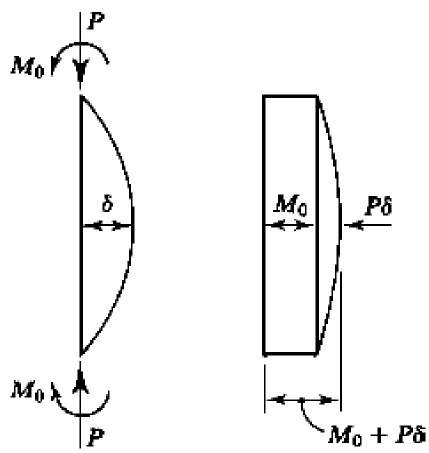
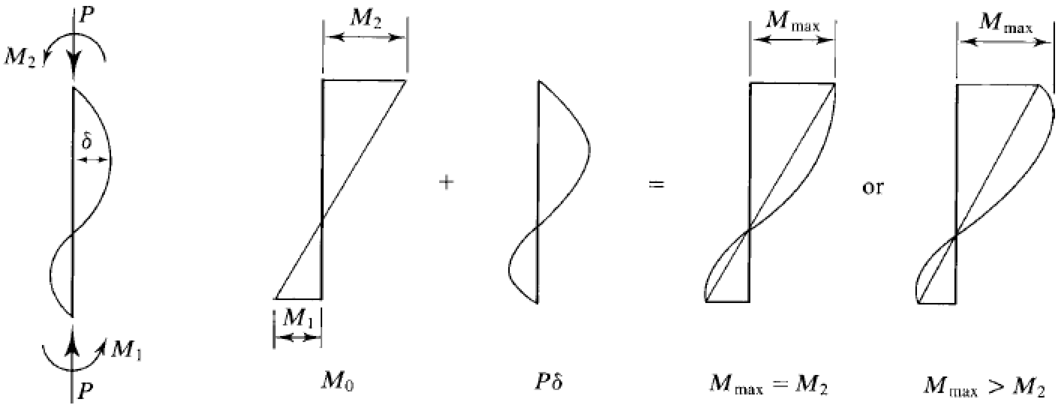


Figure 6.3 shows a member of this type subjected to equal end moments producing *single-curvature bending* (bending that produces tension or compression on one side throughout the length of the member). Maximum moment amplification occurs at the center, where the deflection is largest. For equal end moments, the moment is constant throughout the length of the member, so the maximum primary moment also occurs at the center. Thus the maximum secondary moment and maximum primary moment are additive. Even if the end moments are not equal, as

long as one is clockwise and the other is counterclockwise there will be single curvature
bending, and the maximum primary and secondary moments will occur near each other. That is not the case if applied end moments produce reverse-curvature bending as shown in Figure 6.4. Here the maximum primary moment is at one of the ends, and



maximum moment amplification occurs between the ends. Depending on the value of the axial load P, the amplified moment can be either larger or smaller than the end moment. The maximum moment in a beam–column therefore depends on the distribution of bending moment within the member. This distribution is accounted for by a factor, C_m. C_m will never be greater than 1.0. The final form of the amplification factor is

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} \geq 1 \qquad \text{(AISC Equation A-8-3)}$$

where

P_r = required unamplified axial compressive strength $(P_{nt} + P_{\ell t})$
$= P_u$ for LRFD
$= P_a$ for ASD

$\alpha = 1.00$ for LRFD
$= 1.60$ for ASD

$$P_{e1} = \frac{\pi^2 EI^*}{(K_1 L)^2} \qquad \text{(AISC Equation A-8-5)}$$

EI^* = flexural rigidity

In the direct analysis method, EI^* is a reduced stiffness obtained as

$$EI^* = 0.8\tau_b EI \qquad (6.8)$$

where

τ_b = a stiffness reduction factor

$$= 1.0 \text{ when } \frac{\alpha P_r}{P_y} \leq 0.5 \qquad \text{(AISC Equation C2-2a)}$$

$$= 4\left(\alpha\frac{P_r}{P_y}\right)\left(1-\alpha\frac{P_r}{P_y}\right) \text{ when } \frac{\alpha P_r}{P_y} > 0.5 \qquad \text{(AISC Equation C2-2b)}$$

The factor C_m applies only to the braced condition. There are two categories of members: those with transverse loads applied between the ends and those with no transverse loads. Figure 6.8b and c illustrate these two cases (member AB is the beam–column under consideration).

1. If there are no transverse loads acting on the member,

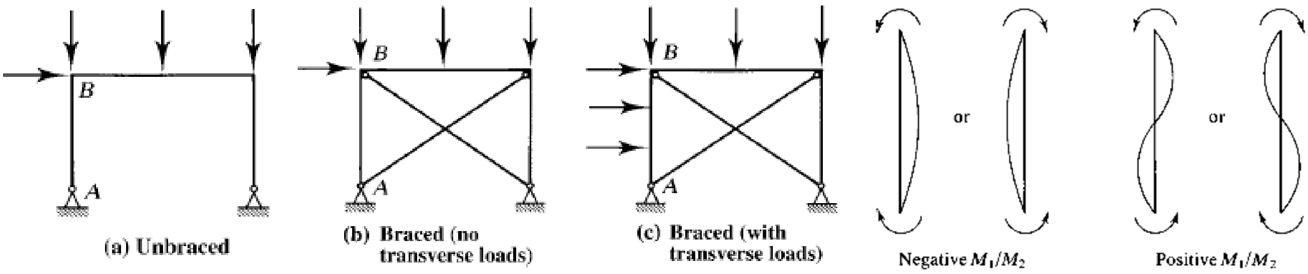
$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) \qquad \text{(AISC Equation A-8-4)}$$

   M_1/M_2 is a ratio of the bending moments at the ends of the member. M_1 is the end moment that is smaller in absolute value, M_2 is the larger, and the ratio is positive for members bent in reverse curvature and negative for single-curvature bending (Figure 6.9). Reverse curvature (a positive ratio) occurs when M_1 and M_2 are both clockwise or both counterclockwise.

2. For transversely loaded members, C_m can be taken as 1.0. A more refined procedure for transversely loaded members is provided in the Commentary to Appendix 8 of the Specification. The factor C_m is given as

$$C_m = 1 + \Psi\left(\frac{\alpha P_r}{P_{e1}}\right) \qquad \text{(AISC Equation C-A-8-2)}$$

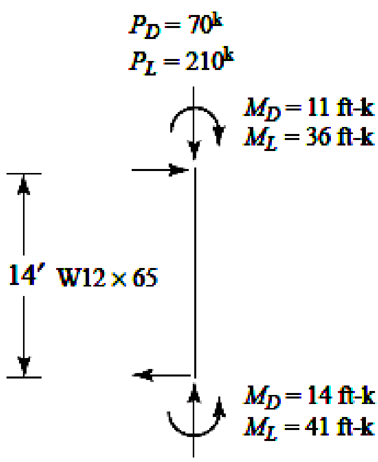
   The factor Ψ has been evaluated for several common situations and is given in Commentary Table C-A-8.1.

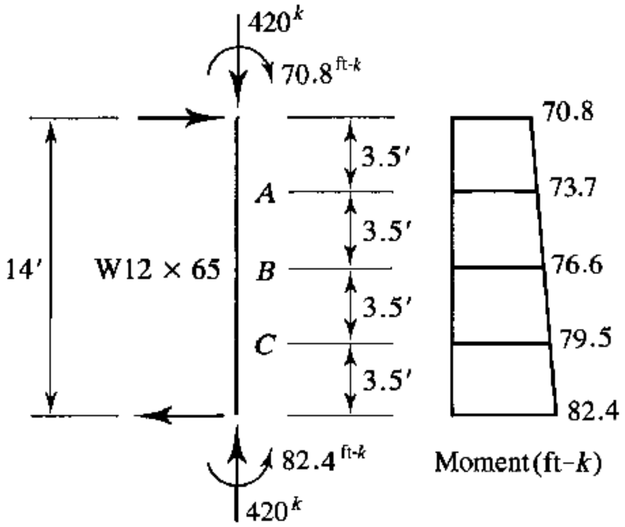


Example: The member shown in Figure is part of a braced frame. An analysis consistent with the effective length method was performed; therefore, the flexural rigidity, *EI*, was unreduced. If A572 Grade 50 steel is used, is this member adequate?

$K_x = K_y = 1.0$.

Solution: The factored loads, computed from load combination 2, are shown in Figure. Determine which interaction formula to apply. The required compressive strength is: $P_r = P_u = P_{nt} + B_2P_{lt} = 420 + 0 = 420$ kips ($B_2 = 0$ for a braced frame)





From the column load tables, for $KL = 1.0 \times 14 = 14$ feet, the axial compressive strength of a W12 × 65 is

$$\phi_c P_n = 685 \text{ kips}$$

$$\frac{P_u}{\phi_c P_n} = \frac{420}{685} = 0.6131 > 0.2 \qquad \therefore \text{ Use Equation 6.3 (AISC Equation H1-1a).}$$

In the plane of bending,

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2 (29,000)(533)}{(1.0 \times 14 \times 12)^2} = 5405 \text{ kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{70.8}{82.4}\right) = 0.9437$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r / P_{e1})} = \frac{C_m}{1 - (1.00 P_u / P_{e1})} = \frac{0.9437}{1 - (420/5405)} = 1.023$$

From the Beam Design Charts in Part 3 of the *Manual* with $C_b = 1.0$ and $L_b = 14$ feet, the moment strength is

$$\phi_b M_n = 345 \text{ ft-kips}$$

For the actual value of C_b, refer to the moment diagram of Figure 6.11:

$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(82.4)}{2.5(82.4) + 3(73.7) + 4(76.6) + 3(79.5)} = 1.060$$

$$\therefore \quad \phi_b M_n = C_b(345) = 1.060(345) = 366 \text{ ft-kips}$$

But $\phi_b M_p = 356$ ft-kips (from the charts) < 366 ft-kips $\quad\therefore$ Use $\phi_b M_n = 356$ ft-kips.

(Since a W12 × 65 is noncompact for $F_y = 50$ ksi, 356 ft-kips is the design strength based on FLB rather than full yielding of the cross section.) The factored load moments are

$$M_{nt} = 82.4 \text{ ft-kips} \qquad M_{\ell t} = 0$$

From AISC Equation A-8-1, the required moment strength is

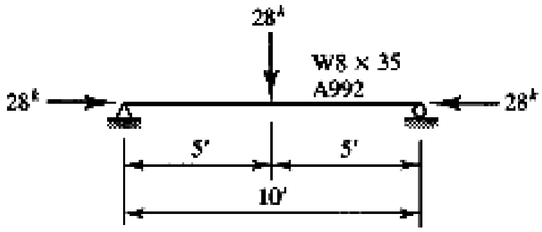
$$M_r = M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.023(82.4) + 0 = 84.30 \text{ ft-kips} = M_{ux}$$

From Equation 6.3 (AISC Equation H1-1a),

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.6131 + \frac{8}{9}\left(\frac{84.30}{356} + 0\right) = 0.824 < 1.0 \qquad \text{(OK)}$$

The member is satisfactory.

Example: The horizontal beam–column shown in Figure is subject to the service live loads shown. This member is laterally braced at its ends, and bending is about the *x*-axis. Check for compliance with the AISC Specification. $K_x = K_y = 1.0$.



Solution:
The factored axial load is: P_u = 1.6(28) = 44.8 kips
The factored transverse loads and bending moment are: Q_u = 1.6(28) = 44.8 kips
w_u = 1.2(0.035) = 0.042 kips-ft

$$M_u = \frac{44.8(10)}{4} + \frac{0.042(10)^2}{8} = 112.5 \text{ ft-kips}$$

This member is braced against sidesway, so $M_{\ell t} = 0$.

Compute the moment amplification factor. For a member braced against sidesway and transversely loaded, C_m can be taken as 1.0. A more accurate value can be found in the Commentary to AISC Appendix 8:

$$C_m = 1 + \Psi\left(\frac{\alpha P_r}{P_{e1}}\right) \qquad \text{(AISC Equation C-A-8-2)}$$

From Commentary Table C-A-8.1, $\Psi = -0.2$ for the support and loading conditions of this beam–column. For the axis of bending,

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2 (29{,}000)(127)}{(10 \times 12)^2} = 2524 \text{ kips}$$

$$C_m = 1 + \Psi\left(\frac{\alpha P_r}{P_{e1}}\right) = 1 - 0.2\left(\frac{1.00 P_u}{P_{e1}}\right) = 1 - 0.2\left(\frac{44.8}{2524}\right) = 0.9965$$

The amplification factor is

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.00P_u/P_{e1})} = \frac{0.9965}{1-(44.8/2524)} = 1.015$$

The amplified bending moment is

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.015(112.5) + 0 = 114.2 \text{ ft-kips}$$

From the beam design charts, for $L_b = 10$ ft and $C_b = 1$,

$$\phi_b M_n = 123 \text{ ft-kips}$$

Because the beam weight is very small in relation to the concentrated live load, C_b may be taken from Figure 5.15c as 1.32. This value results in a design moment of

$$\phi_b M_n = 1.32(123) = 162.4 \text{ ft-kips}$$

This moment is greater than $\phi_b M_p = 130$ ft-kips, so the design strength must be limited to this value. Therefore,

$$\phi_b M_n = 130 \text{ ft-kips}$$

Check the interaction formula. From the column load tables, for $KL = 10$ ft,

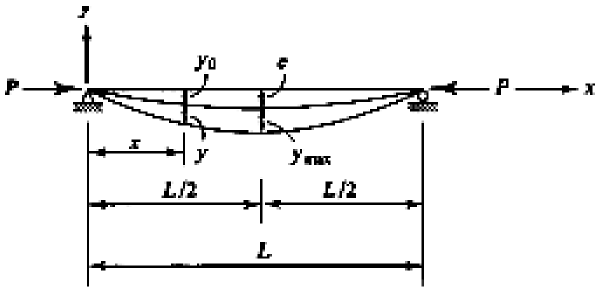
$$\phi_c P_n = 358 \text{ kips}$$

$$\frac{P_u}{\phi_c P_n} + \frac{44.8}{358} = 0.1251 < 0.2 \qquad \therefore \text{ Use Equation 6.4 (AISC Equation H1-1b).}$$

$$\frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = \frac{0.1251}{2} + \left(\frac{114.2}{130} + 0\right)$$

$$= 0.941 < 1.0 \text{ (OK)}$$

MEMBERS IN UNBRACED FRAMES



In a beam–column whose ends are free to translate, the maximum primary moment resulting from the sidesway is almost always at one end. As was illustrated in Figure 6.5, the maximum secondary moment from the sidesway is *always* at the end. As a consequence of this condition, the maximum primary and secondary moments are usually additive and there is no need for the factor *Cm*; in effect, *Cm* = 1.0. Even when there is a reduction, it will be slight and can be neglected. Consider the beam–column shown in Figure 6.6. Here the equal end moments are caused by the sidesway (from the horizontal load). The axial load, which partly results from loads not causing the sidesway, is carried along and amplifies the end moment. The amplification factor for the sidesway moments, B_2, is given by

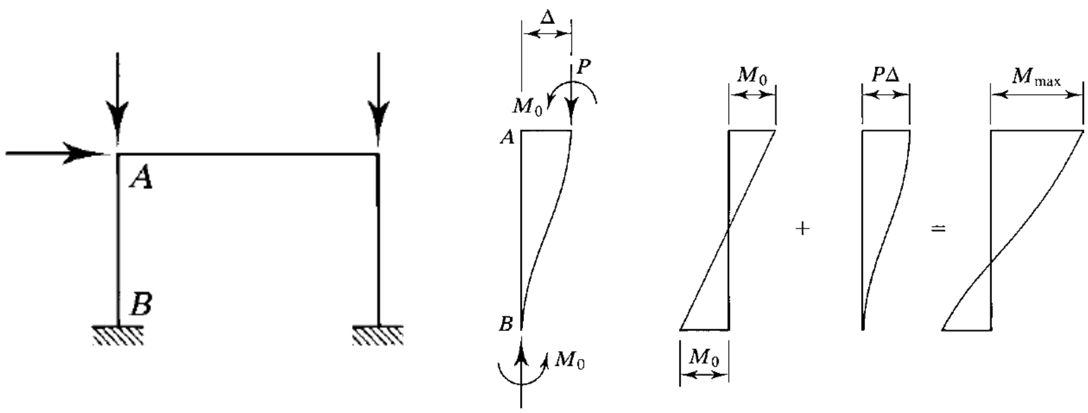
$$B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{e\,story}}} \geq 1 \qquad \text{(AISC Equation A-8-6)}$$

where

$\alpha = 1.00$ for LRFD
$= 1.60$ for ASD
P_{story} = sum of required load capacities for all columns in the story under consideration (factored for LRFD, unfactored for ASD)
$P_{e\,story}$ = total elastic buckling strength of the story under consideration

This story buckling strength may be obtained by a sidesway buckling analysis or as

$$P_{e\,story} = R_M \frac{HL}{\Delta_H} \qquad \text{(AISC Equation A-8-7)}$$

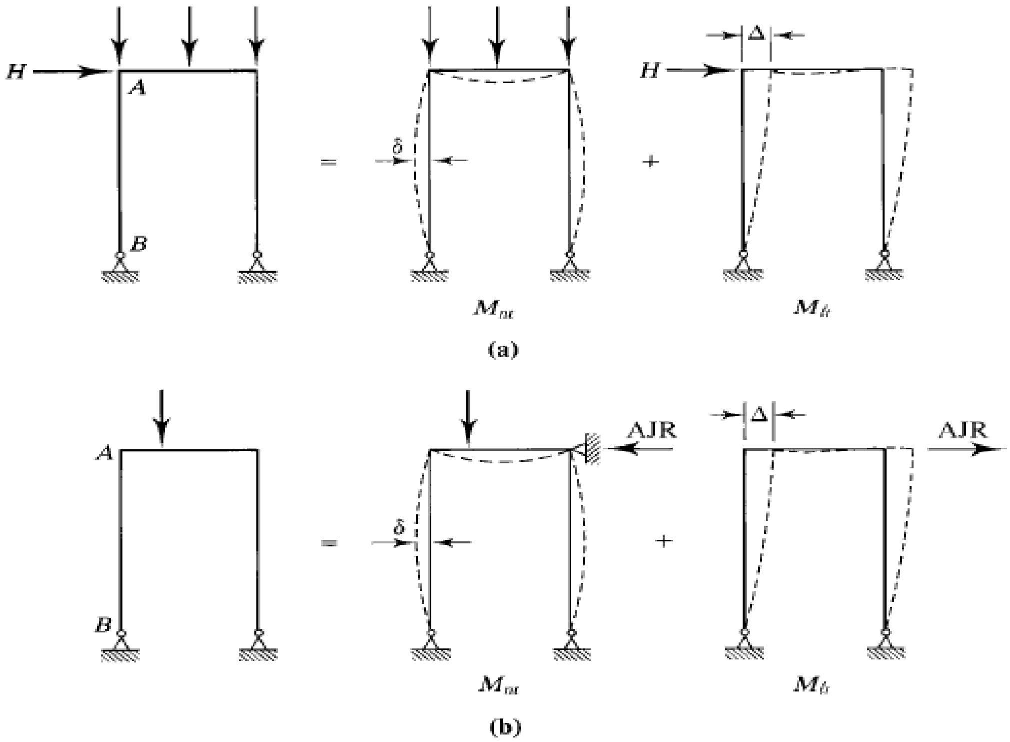


where

$$R_M = 1 - 0.15\frac{P_{mf}}{P_{story}} \qquad \text{(AISC Equation A-8-8)}$$

P_{mf} = sum of vertical loads in all columns in the story that are part of *moment frames*
L = story height
Δ_H = interstory drift = drift (sidesway displacement) of the story under consideration
H = story shear = sum of all horizontal forces causing Δ_H

Note that, if there are no moment frames in the story, $P_{mf} = 0$ and $R_M = 1.0$. If all of the columns in the story are members of moment frames, then $P_{mf} = P_{story}$ and $R_M = 0.85$. The rationale for using the total story load and strength is that *B*2 applies to unbraced frames, and if sidesway is going to occur, all columns in the story must sway simultaneously. In most cases, the structure will be made up of plane frames, so P_{story} and P_e story are for the columns within a story of the frame, and the lateral loads *H* are the lateral loads acting on the frame at and above the story. With ΔH caused by *H*, the ratio $H/\Delta H$ can be based on either factored or unfactored loads. In situations where *Mnt* and *M*□*t* act at two different points on the member, as in Figure 6.4, AISC Equation A-8-1 will produce conservative results. Figure 6.7 further illustrates the superposition concept. Figure 6.7a shows an unbraced frame subject to both gravity and lateral loads. The moment M_{nt} in member *AB* is computed by using only the gravity loads. Because of symmetry, no bracing is needed to prevent sidesway from these loads. This moment is amplified with the factor B1 to account for the Pδ effect. M_{lt}, the moment corresponding to the sway (caused by the horizontal load H), will be amplified by B_2 to account for the PΔ effect.



In Figure 6.7b, the unbraced frame supports only a vertical load. Because of the unsymmetrical placement of this load, there will be a small amount of sidesway. The moment M_{nt} is computed by considering the frame to be braced—in this case, by a fictitious horizontal support and corresponding reaction called an *artificial joint restraint* (AJR). To compute the sidesway moment, the fictitious support is removed, and a force equal to the artificial joint restraint, but opposite in direction, is applied to the frame. In cases such as this one, the secondary moment *P*Δ will be very small, and M_{lt} can usually be neglected.

6-6 DESIGN OF BEAM–COLUMNS

Because of the many variables in the interaction formulas, the design of beam–columns is essentially a trial-and-error process. The procedure can be explained as follows. If we initially assume that AISC Equation H1-1a governs, then

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0 \qquad \text{(AISC Equation H1-1a)}$$

This can be written as

$$\left(\frac{1}{P_c}\right)P_r + \left(\frac{8}{9M_{cx}}\right)M_{rx} + \left(\frac{8}{9M_{cy}}\right)M_{ry} \le 1.0$$

or

$$pP_r + b_x M_{rx} + b_y M_{ry} \le 1.0 \qquad (6.9)$$

where

$$p = \frac{1}{P_c}$$

$$b_x = \frac{8}{9M_{cx}}$$

$$b_y = \frac{8}{9M_{cy}}$$

If AISC Equation H1-1b controls (that is, $P_r/P_c < 0.2$, or equivalently, $pP_r < 0.2$), then use

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0 \qquad \text{(AISC Equation H1-1b)}$$

or

$$0.5pP_r + \frac{9}{8}(b_x M_{rx} + b_y M_{ry}) \le 1.0 \qquad (6.10)$$

$$p = \frac{1}{P_c} = \frac{1}{\phi_c P_n}$$

$$b_x = \frac{8}{9M_{cx}} = \frac{8}{9(\phi_b M_{nx})}$$

$$b_y = \frac{8}{9M_{cy}} = \frac{8}{9(\phi_b M_{ny})}$$

Table 6-1 gives values of p, b_x, and b_y for all W shapes listed in Part 1 of the *Manual*, "Dimensions and Properties," except for those smaller than W8. The values of C_b, B_1, and B_2 must be calculated independently for use in the computation of M_r (M_u for LRFD). The procedure for design is as follows:

1. Select a trial shape from Table 6-1 of the *Manual*.
2. Use the effective length KL to select p, and use the unbraced length L_b to select b_x (the constant b_y determines the weak axis bending strength, so it is independent of the unbraced length). The values of the constants are based on the assumption that weak axis buckling controls the axial compressive strength and that $Cb = 1.0$.
3. Compute pP_r. If this is greater than or equal to 0.2, use interaction Equation 6.9. If pP_r is less than 0.2, use Equation 6.10.
4. Evaluate the selected interaction equation with the values of p, b_x, and b_y for the trial shape.
5. If the result is not very close to 1.0, try another shape. By examining the value of each term in Equation 6.9 or 6.10, you can gain insight into which constants need to be larger or smaller.
6. Continue the process until a shape is found that gives an interaction equation result less than 1.0 and close to 1.0 (greater than 0.9).

Verification of assumptions:

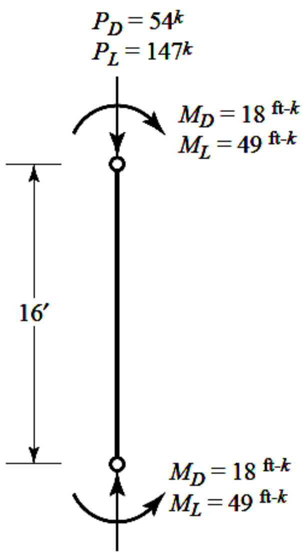
- If strong axis buckling controls the compressive strength, use an effective length of

$$KL = \frac{K_x L}{r_x / r_y}$$

to obtain p from Table 6-1.

- If C_b is not equal to 1.0, the value of b_x must be adjusted.

EXAMPLE: Select a W12 shape of A992 steel for the beam–column of Figure. This member is part of a braced frame and is subjected to the service-load axial force and bending moments shown (the end shears are not shown). Bending is about the strongaxis, and $K_x = K_y = 1.0$. Lateral support is provided only at the ends. Assume that $B_1 = 1.0$.



Solution:

The factored axial load is

$P_{nt} = P_u = 1.2PD + 1.6PL = 1.2(54) + 1.6(147) = 300$ kips

(There is no amplification of axial loads in members braced against sidesway.) The factored moment at each end is

$M_{ntx} = 1.2M_D + 1.6M_L = 1.2(18) + 1.6(49) = 100$ ft-kips

Since $B_1 = 1.0$, the factored load bending moment is

$M_{ux} = B1M_{ntx} = 1.0(100) = 100$ ft-kips

The effective length for compression and the unbraced length for bending are the same: $KL = L_b = 16$ ft

To be sure that we have found the lightest W12, try the next lighter one, a W12 × 53, with $p = 2.21 \times 10^{-3}$ and $b_x = 3.52 \times 10^{-3}$.

$$pP_u + b_x M_{ux} + b_y M_{uy} = (2.21 \times 10^{-3})(300) + (3.52 \times 10^{-3})(100) + 0$$
$$= 1.02 \qquad \text{(N.G.)}$$

Try a W12 shape. **Try a W12 × 58**, with $p = 2.01 \times 10^{-3}$ and $b_x = 3.14 \times 10^{-3}$.

$$pP_u + b_x M_{ux} + b_y M_{uy} = (2.01 \times 10^{-3})(300) + (3.14 \times 10^{-3})(100) + 0$$
$$= 0.917 < 1.0 \qquad \text{(OK)}$$

Verify that Equation 6.9 is the correct one:

$$\frac{P_u}{\phi_c P_n} = pP_u = (2.01 \times 10^{-3})(300) = 0.603 > 0.2 \ \therefore \text{ Equation 6.9 controls, as assumed.}$$

Use a W12 × 58.