University of Anbar

College of Science – Dept. of Physics

Lectures of Semiconductors #1

for 3th level of physics students

Lecture 4 : Introduction to Quantum Mechanics/2

by

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Example:

calculate the penetration depth *of* a particle impinging on a potential barrier. Consider an incident electron that is traveling at a velocity of 1 x 10^5 *m/s* in region I . And the potential barrier at (*x = 0*) is twice as large as the total energy of the incident particle, or that $V_o = 2E$.

Solution:

When ***V (x)*** = 0, the total energy is also equal to the kinetic energy so that:

$$E = T = \frac{1}{2} m\upsilon^2 = \frac{1}{2} \times 9.1 \times 10^{-31} \times 10^{10} = 4.56\ 10^{-21} \text{ J}$$

$$= 2.85 \times 10^{-2} \quad \text{eV}$$

the constant K_2 is given by :

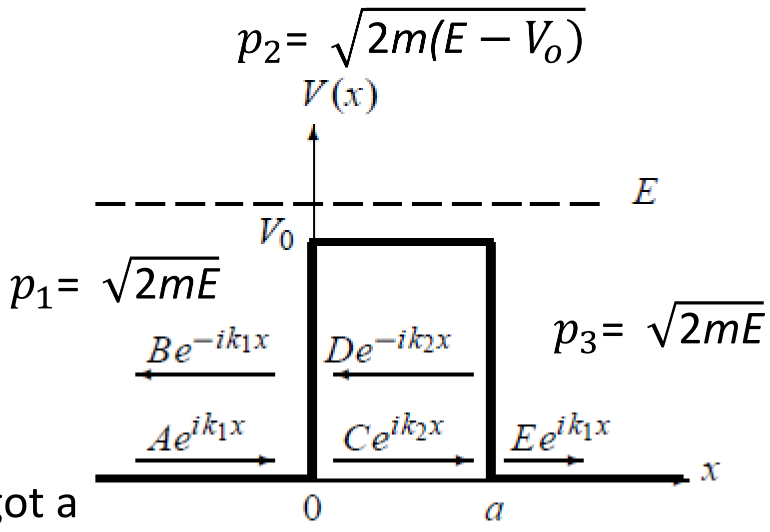
$$k_2 = \sqrt{\frac{2m\,(V_o - E)}{\hbar^2}} = \sqrt{\frac{2m\,(2E - E)}{\hbar^2}} = \sqrt{\frac{2m\,E}{\hbar^2}}$$

we want to determine the distance ***x*** = d at which the wave function magnitude has decayed to e^{-1} of its value at **x** = 0. Then, for this case, we have $\boldsymbol{K_2 d}$ = 1 or

$$1 = d\left(\sqrt{\frac{2m\,E}{\hbar^2}}\right) \Rightarrow d = \sqrt{\frac{\hbar^2}{2m\,E}} = 11.6 \times 10^{-10} \quad \text{m}$$

$$d = 11.6\ \text{Å}$$

2.3.4 : The Potential Barrier



1- The Case $E > V_0$

Schrodinger's time-independent wave equation is :

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}(E - V(x))\,\psi(x) = 0 \qquad \text{.......}(63)$$

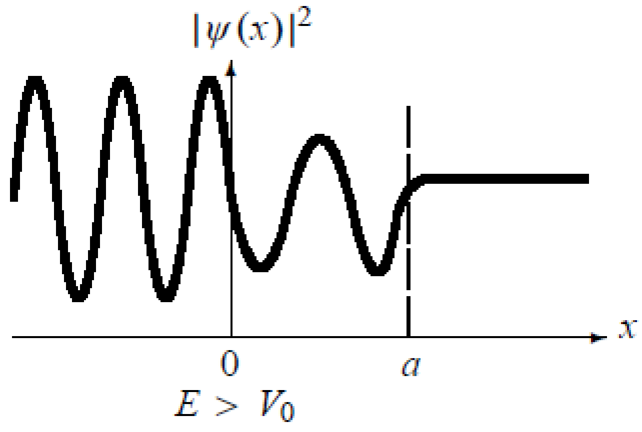
When this equation is applied in a three regions , we got a three solutions as :

$$k_1 = \sqrt{2mE/\hbar^2}$$

$$k_2 = \sqrt{2m(E - V_o)/\hbar^2}$$

$$\psi(x) = \begin{cases} \psi_1(x) = Ae^{ik_1x} + Be^{-ik_1x}, & x \le 0, \\ \psi_2(x) = Ce^{ik_2x} + De^{-ik_2x}, & 0 < x < a, \\ \psi_3(x) = Ee^{ik_1x}, & x \ge a, \end{cases}$$

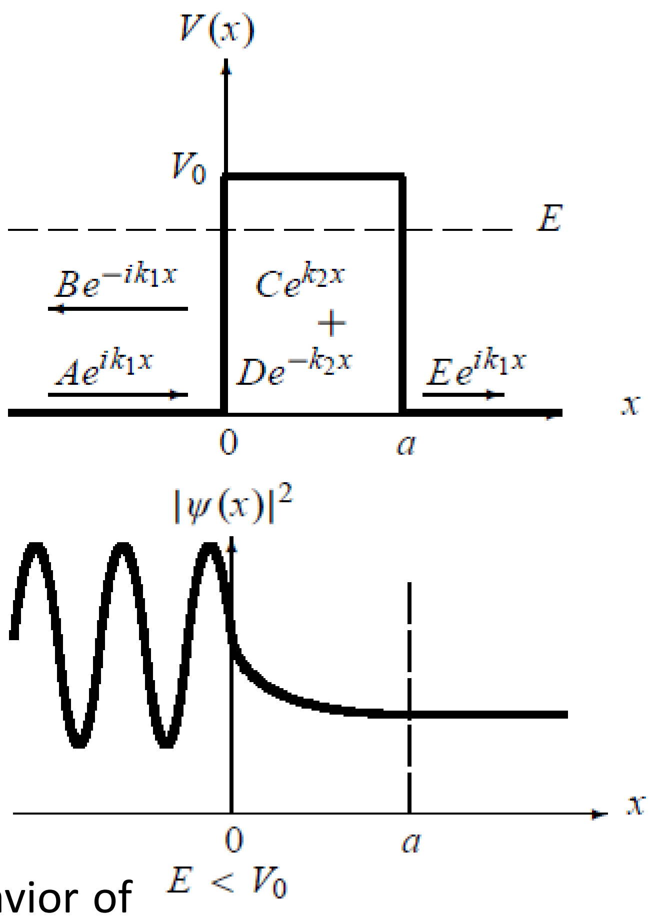
the particles in this case have enough energy to cross the barrier, none of the particles will be reflected back; all the particles will emerge on the right side of $x = a$: *total transmission*.



($H.W10$): Solve the Sch. Eq. of the three regions .

2- The Case $E < V_0$

$$\psi(x) = \begin{cases} \psi_1(x) = Ae^{ik_1x} + Be^{-ik_1x}, & x \leq 0, \\ \psi_2(x) = Ce^{k_2x} + De^{-k_2x}, & 0 < x < a, \\ \psi_3(x) = Ee^{ik_1x}, & x \geq a, \end{cases}$$



The transmission coefficient is thus given by

$$T = \frac{|E|^2}{|A|^2}$$

$$T = \left[1 + \frac{1}{4}\frac{V_0^2}{E(V_0 - E)}\sinh^2\left(\frac{a}{\hbar}\sqrt{2m(V_0 - E)}\right)\right]^{-1}$$

The **transmission coefficient** is used to describe the behavior of waves incident on a barrier. The transmission coefficient represents the probability flux of the transmitted wave relative to that of the incident wave. It is often used to describe the probability of a particle tunneling through a barrier.

A special Case of $E < V_0$ is $(E << V_0)$:

$$T \simeq \left\{ \frac{1}{4\varepsilon(1-\varepsilon)} \left[\frac{1}{2} e^{\lambda\sqrt{1-\varepsilon}} \right]^2 \right\}^{-1} = 16\varepsilon(1-\varepsilon)e^{-2\lambda\sqrt{1-\varepsilon}} \quad : \varepsilon = \frac{E}{V_o}$$

$$= \frac{16E}{V_0}\left(1 - \frac{E}{V_0}\right) e^{-(2a/\hbar)\sqrt{2m(V_0-E)}}.$$

Example :

Calculate the probability of an electron tunneling through a potential barrier.
Consider an electron with an energy of 2 eV impinging on a potential barrier with V_o = 20 eV and a width of 3 Å .

Solution:

$$p = \sqrt{2m\,(V_o - E)} \quad : \quad k_2 = \frac{p}{\hbar}$$

$$k_2 = \sqrt{\frac{2m(V_o - E)}{\hbar^2}}$$

$$k_2 = \sqrt{\frac{2(9.11 \times 10^{-31})(20-2)(1.6 \times 10^{-19})}{(1.054 \times 10^{-34})^2}}$$

$$k_2 = 2.17 \times 10^{10} \quad \text{m}^{-1}$$

$$T \approx 16 \times \frac{2}{20}\left(1 - \frac{2}{20}\right) Exp\left[-2 \times 2.17 \times 10^{10} \times 3 \times 10^{-10}\right]$$

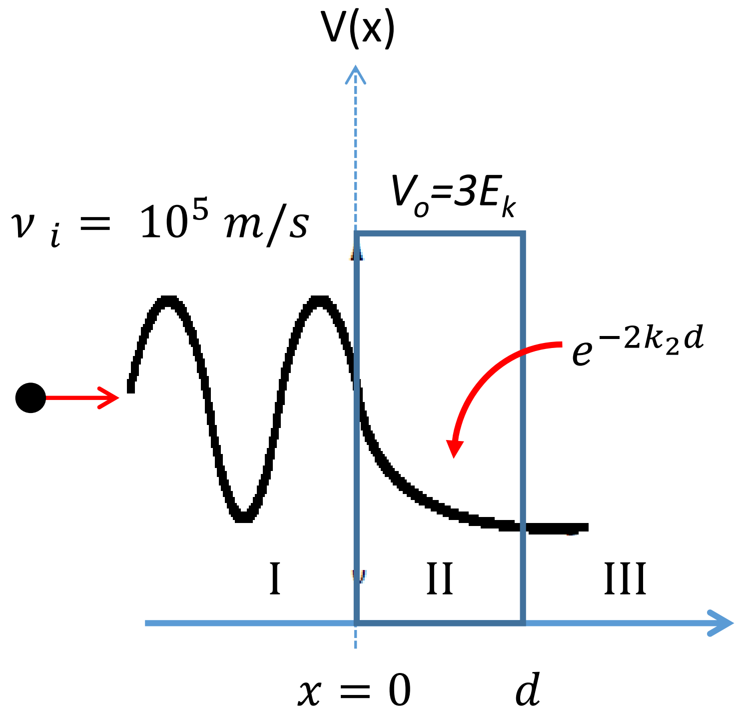
$$T = 3.17 \times 10^{-6}$$

PROBLEMS:

- **Section 2.2: Schrodinger's Wave Equation (all problems)**
- **Section 2.3: Applications of Schrodinger's Wave Equation (even problems)**

E2.7 : The probability of finding a panicle at a distance *d* in region II compared to that at x = 0 is given by $exp\ (-2K_2 d)$. Consider an electron traveling in region I at a velocity of 10^5m/s incident on a potential barrier whose height is **3** times the kinetic energy of the electron. Find the probability of finding the electron at a distance d compared to **x** = 0 where d is *(a)* 10 Å and *(b)* 100 Å into the potential barrier

According to the law of conservation of energy:



$$\sum_I E = \sum_{II} E$$

$$k.E_1 = V_o + k.E_2$$

$$k_1 = \sqrt{\frac{2m\ (K.E_1)}{\hbar^2}}$$

$$k_2 = \sqrt{\frac{2m\ (V_o - EII)}{\hbar^2}} \Rightarrow \quad E_{II} = V_o - \frac{{k_2}^2\hbar^2}{2m} = K.E_1$$

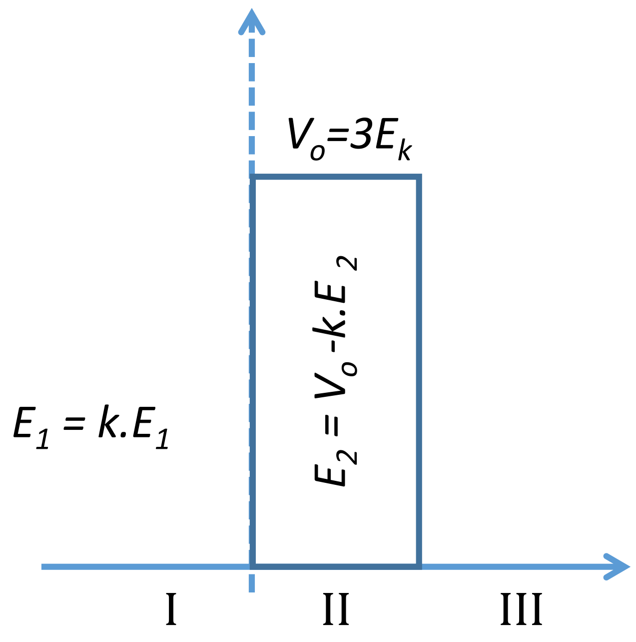
$$3K.E_1 - \frac{k_2{}^2\hbar^2}{2m} = K.E_1$$

$$2\,K.E_1 = \frac{k_2{}^2\hbar^2}{2m}$$

$$2 * \frac{1}{2}mv_i{}^2 = \frac{k_2{}^2\hbar^2}{2m}$$

$$k_2{}^2 = \frac{2mv_i{}^2}{\hbar^2} = \frac{2m^2}{\hbar^2} \times 10^{10}$$

$$k_2 = \sqrt{\frac{2\times(9.1\times10^{-31})^2\times10^{10}}{(1.054\times10^{-34})^2}} = 12.21\times10^8\ m^{-1}$$



$$\therefore [Exp\,(-2k_2d\,)]\,\% = \begin{cases} 8.7\% & \rightarrow\ d = 10\ \text{Å} \\ 2.5\times10^{-9}\% & \rightarrow\ d = 100\text{Å} \end{cases}$$