University of Anbar

College of Science – Dept. of Physics

Lectures of Semiconductors #1

for 3th level of physics students

Lecture 9 : The Semiconductor in Equilibrium/2

by

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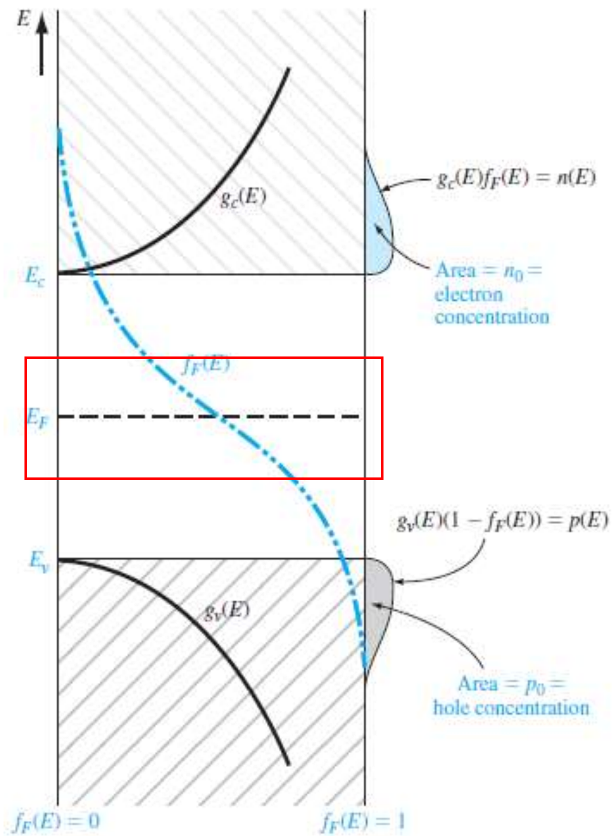
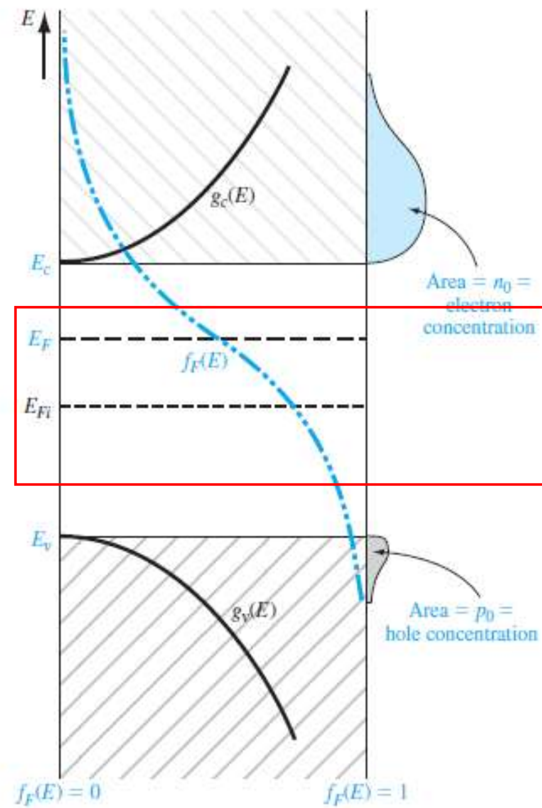
THE EXTRINSIC SEMICONDUCTOR

An extrinsic semiconductor is a semiconductor in which controlled amounts of specific dopant or impurity atoms have been added so that the thermal-equilibrium electron and hole concentrations are different from the intrinsic carrier concentration.

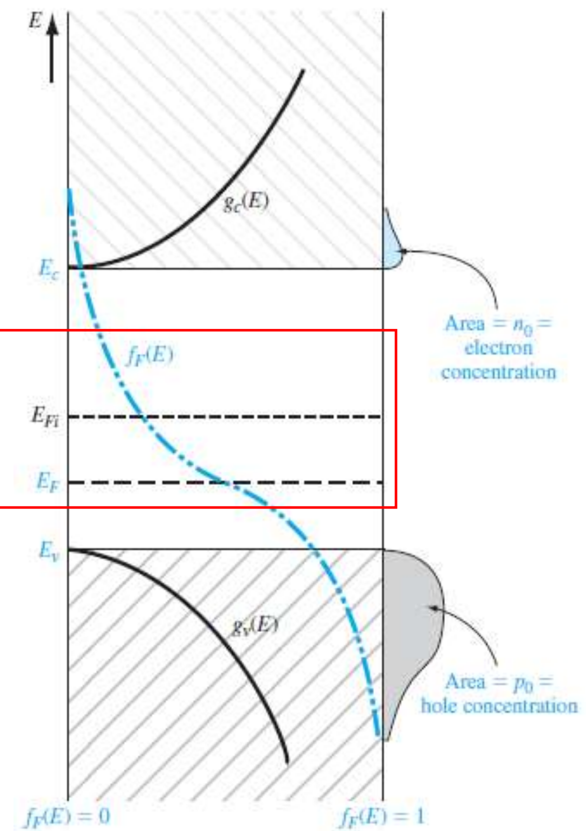
Intrinsic semiconductor



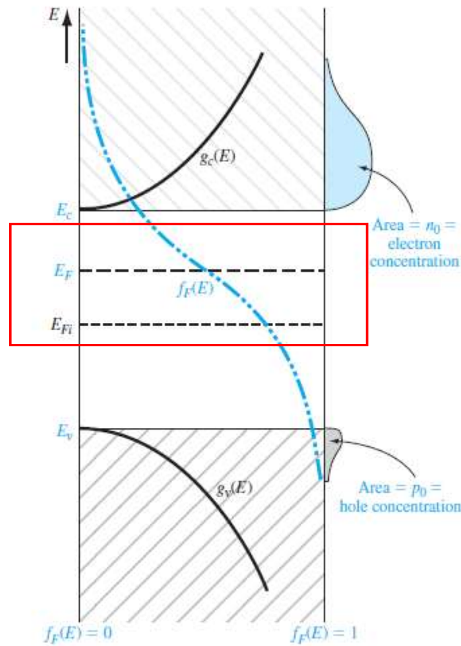
Adding the donor atoms



Adding the donor atoms

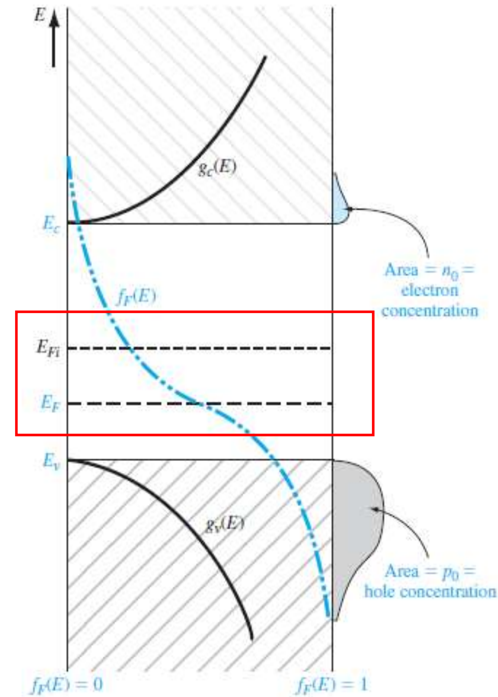


$E_F > E_{Fi}$



- The electron concentration is larger than holes concentration
- the semiconductor is n-type

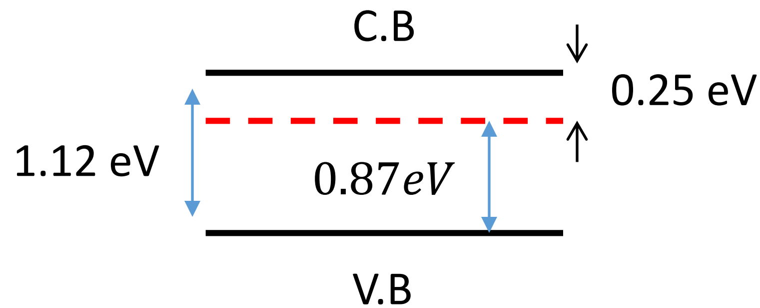
$E_F < E_{Fi}$



- The holes concentration is larger than electrons concentration
- the semiconductor is p-type

Example:

Consider silicon at $T = 300\ K$ so that $N_c = 2.8 \times 10^{19}$ cm^{-3} and $N_v = 1.04 \times 10^{19}$ cm^{-3}. If that the Fermi energy is 0.25 eV below the conduction band. If we assume that the bandgap energy of silicon is 1.12 eV. Calculate the thermal equilibrium concentrations of electrons and holes for a given Fermi energy.



$$1.12 - 0.25 = 0.87eV$$

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$n_0 = (2.8 \times 10^{19}) \exp\left(\frac{-0.25}{0.0259}\right) = 1.8 \times 10^{15}\ \text{cm}^{-3}$$

$$p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right]$$

$$p_0 = (1.04 \times 10^{19}) \exp\left(\frac{-0.87}{0.0259}\right) = 2.7 \times 10^{4}\ \text{cm}^{-3}$$

In an n-type semiconductor, electrons are referred to as ***the majority carrier*** and holes as **the minority carrier**.

While in a p-type it is in reverse order

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$ add and subtract an intrinsic Fermi energy in the exponent

$$n_0 = N_c \exp\left[\frac{-(E_c - E_{Fi}) + (E_F - E_{Fi})}{kT}\right] \rightarrow n_0 = N_c \exp\left[\frac{-(E_c - E_{Fi})}{kT}\right] \exp\left[\frac{(E_F - E_{Fi})}{kT}\right]$$

The intrinsic carrier concentration is given by

$$n_i = N_c \exp\left[\frac{-(E_c - E_{Fi})}{kT}\right]$$

so that the thermal-equilibrium electron concentration can be written as

$$n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$$

Similarly,

$$p_0 = n_i \exp\left[\frac{-(E_F - E_{Fi})}{kT}\right]$$

Here note, the Fermi level position depends on the n_o and p_o

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right]$$

$$n_0 p_0 = N_c N_v \exp\left[\frac{-(E_c - E_F)}{kT}\right] \exp\left[\frac{-(E_F - E_v)}{kT}\right]$$

n_i n_i

$$n_0 p_0 = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$$

$$n_0 p_0 = n_i^2$$

$$n_i^2 = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$$

The Fermi–Dirac Integral

$$n_0 = \int g_c(E) f_F(E)\, dE$$

$$n_0 = \frac{4\pi}{h^3}(2m_n^*)^{3/2} \int_{E_c}^{\infty} \frac{(E - E_c)^{1/2}\, dE}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

If we again make a change of variable and let

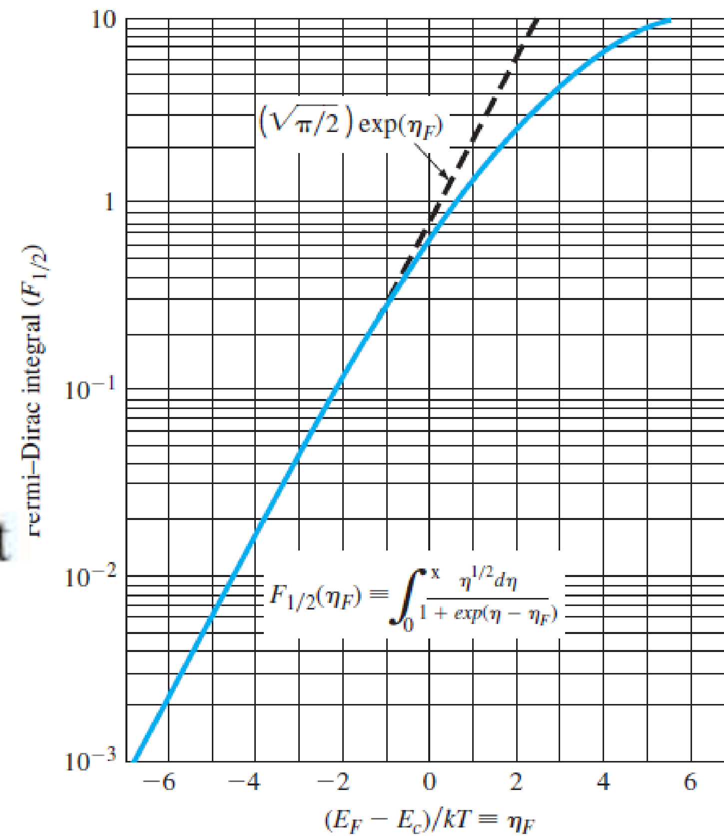
$$\eta = \frac{E - E_c}{kT} \quad \text{and also define} \quad \eta_F = \frac{E_F - E_c}{kT}$$

then n_o can be written as

$$n_0 = 4\pi\left(\frac{2m_n^* kT}{h^2}\right)^{3/2} \int_0^{\infty} \frac{\eta^{1/2}\, d\eta}{1 + \exp(\eta - \eta_F)}$$

$$F_{1/2}(\eta_F) = \int_0^{\infty} \frac{\eta^{1/2}\, d\eta}{1 + \exp(\eta - \eta_F)}$$

the Fermi–Dirac integral



$$\eta_F > 0 \quad \rightarrow \quad E_F > E_c$$

the Fermi energy is actually in the conduction band.

Example:

Let $\eta_F = 2$ so that the Fermi energy is above the conduction band by approximately $52\ meV$ at $T = 300$ K.

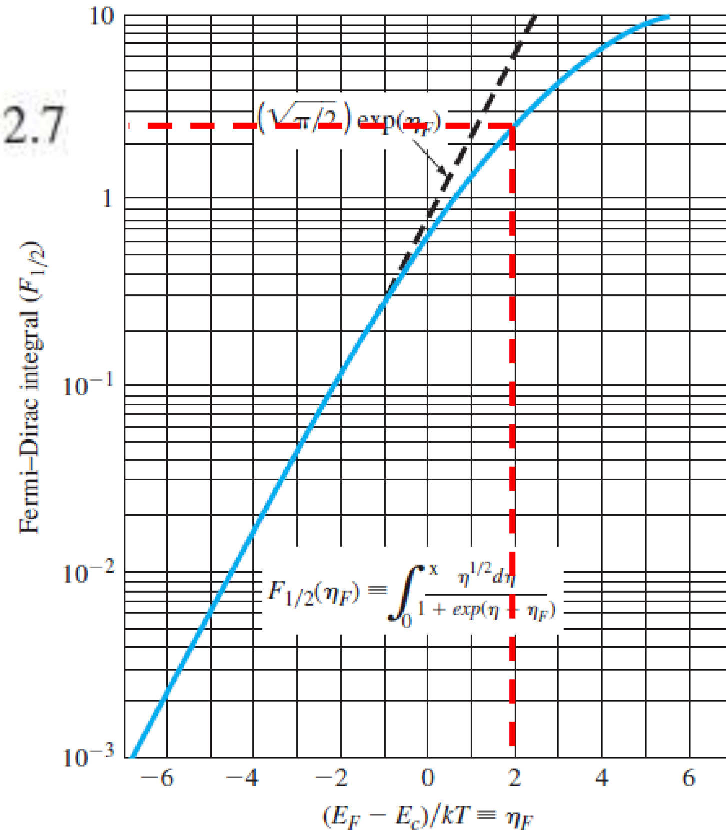
Calculate the electron concentration using the Fermi–Dirac integral.

$$n_0 = \frac{2}{\sqrt{\pi}} N_c F_{1/2}(\eta_F)$$

For silicon at $T = 300$ K, $N_c = 2.8 \times 10^{19}\ \text{cm}^{-3}$

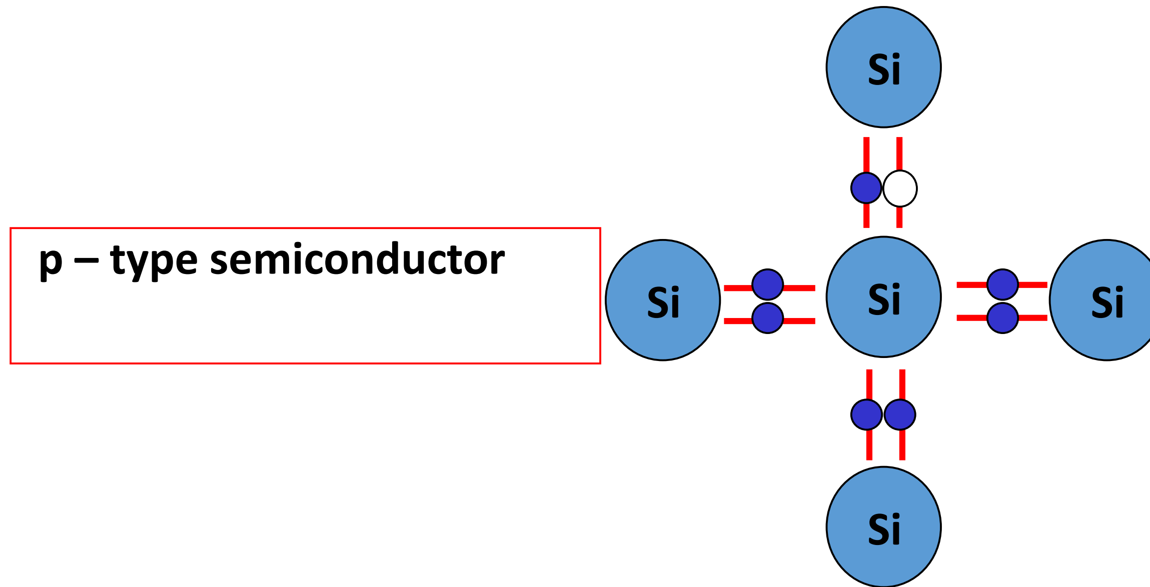
$$F_{1/2}(2) = 2.7$$

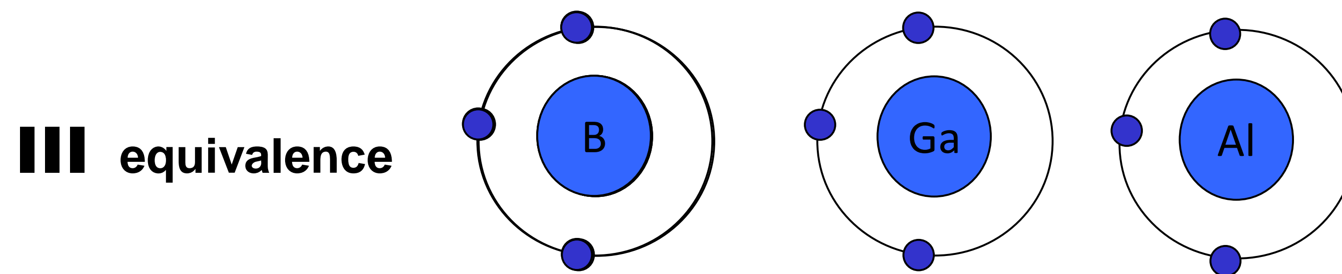
$$n_0 = \frac{2}{\sqrt{\pi}}(2.8 \times 10^{19})(2.7) = 8.53 \times 10^{19}\ \text{cm}^{-3}$$



Adding a group III elements:

The group III element has three valence electrons, which are all taken up in the covalent bonding.





These atoms is called (acceptor atoms)

Degenerate and Nondegenerate Semiconductors:

- At the concentration of dopant atoms added is small when compared to the density of host or semiconductor atoms.

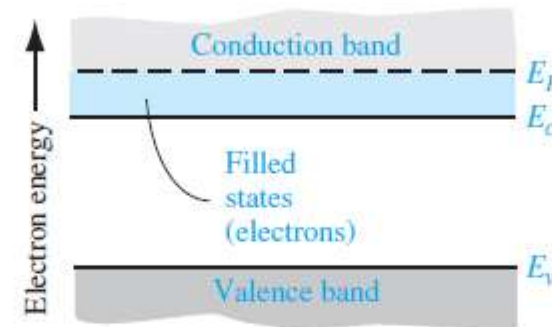
the impurities introduce

$$N_C, N_v \gg N_d, N_a$$

noninteracting donor energy states in the n-type semiconductor	noninteracting acceptor energy states in the p-type semiconductor

- If the impurity concentration increases, the distance between the impurity atoms decreases, and a point will be reached when donor electrons, will begin to interact with each other.

This occurs when the concentration of electrons in the conduction band exceeds the density of states N_C.



a degenerate n-type semiconductor

STATISTICS OF DONORS AND ACCEPTORS

Probability Function:

Suppose we have N_i electrons and g_i quantum states, where the subscript i indicates the ith energy level.

Each donor level has two possible spin orientations for the donor electron; thus, each donor level has two quantum states

The distribution function of donor electrons in the donor energy states is then slightly different than the Fermi–Dirac function.
The probability function of electrons occupying the donor state is

It is a direct result of the spin factor

$$n_d = \frac{N_d}{1 + \frac{1}{2} exp\left(\frac{E_d - E_F}{kT}\right)} \quad \ldots\ldots (4.50) \quad \rightarrow\rightarrow = N_d + N_d^+$$

where n_d is the density of electrons occupying the donor level and E_d is the energy of the donor level.
where N_d^+ is the concentration of ionized donors.

$$n_d = \frac{N_d}{1+\frac{1}{g} exp\left(\frac{E_d-E_F}{kT}\right)} \quad \ldots\ldots (4.50) \quad \rightarrow\rightarrow = N_d + N_d^+ \ldots\ldots(4.51)$$

where N_d is the concentration of ionized donors.

$$\frac{1}{2} \rightarrow\rightarrow \frac{1}{g}$$

where g is called a degeneracy factor.

In the same way for acceptor atoms

$$p_a = \frac{N_a}{1+\frac{1}{g}exp\left(\frac{E_F-E_a}{kT}\right)} \rightarrow\rightarrow = N_a + N_a^- \text{......(4.52)}$$

where N_a is the concentration of acceptor atoms,
E_a is the acceptor energy level,
p_a is the concentration of holes in the acceptor states,
and N_a^- is the concentration of ionized acceptors.

In this case, a hole in an acceptor state corresponds to an acceptor atom that is neutrally charged and still has an “empty” bonding position

Complete Ionization and Freeze-Out

From Eq.(4.50) $n_d = \dfrac{N_d}{1+\frac{1}{g}exp\left(\frac{E_d - E_F}{kT}\right)}$, If we assume that $(E_d - E_F) \gg kT$, then

$$n_d \approx \frac{N_d}{1+\frac{1}{2}exp\left(\frac{E_d-E_F}{kT}\right)} = 2N_d exp\left[\frac{-(E_d-E_F)}{kT}\right] \text{.........(4.53)}$$

the Boltzmann approximation is also valid for the electrons in the conduction band so

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

determine the relative number of electrons in the donor state compared with the total number of electrons;

$$\frac{n_d}{n_d + n_o} = \frac{2N_d exp\left[\frac{-(E_d - E_F)}{kT}\right]}{2N_d exp\left[\frac{-(E_d - E_F)}{kT}\right] + N_c exp\left[\frac{-(E_c - E_F)}{kT}\right]} \dots\dots\dots (5.54)$$

$$\frac{n_d}{n_d + n_o} = \frac{1}{1 + \frac{N_c}{2N_d} exp\left[\frac{-(E_c - E_d)}{kT}\right]} \dots\dots\dots (5.55)$$

The factor $(E_c - E_d)$ is just the ionization energy of the donor electrons.

EXAMPLE

Determine the fraction of total electrons in the donor states at $T = 300$ K of the phosphorus-doped silicon if the concentration (N_d) of phosphorus is 10^{16} cm^{-3} at *300K*.

Solution

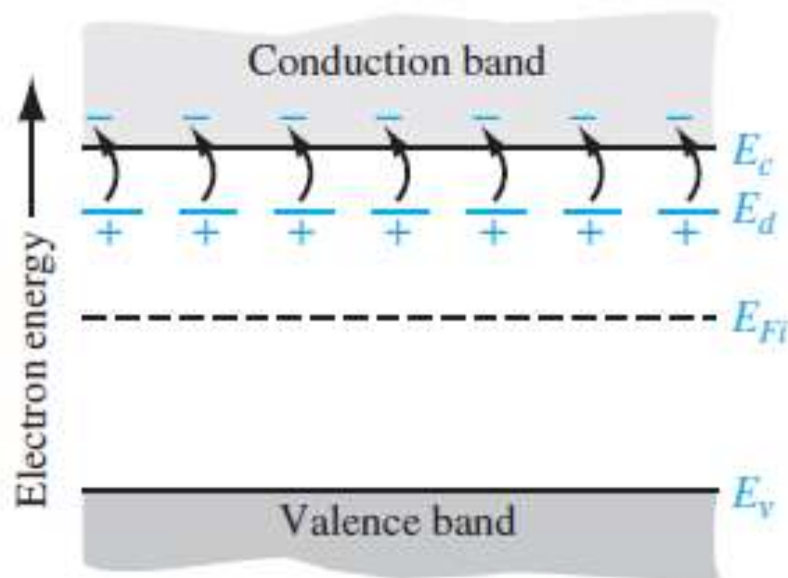
From **Table 4.1**

$$N_c = 2.8 \times 10^{19}\ cm^{-3}$$

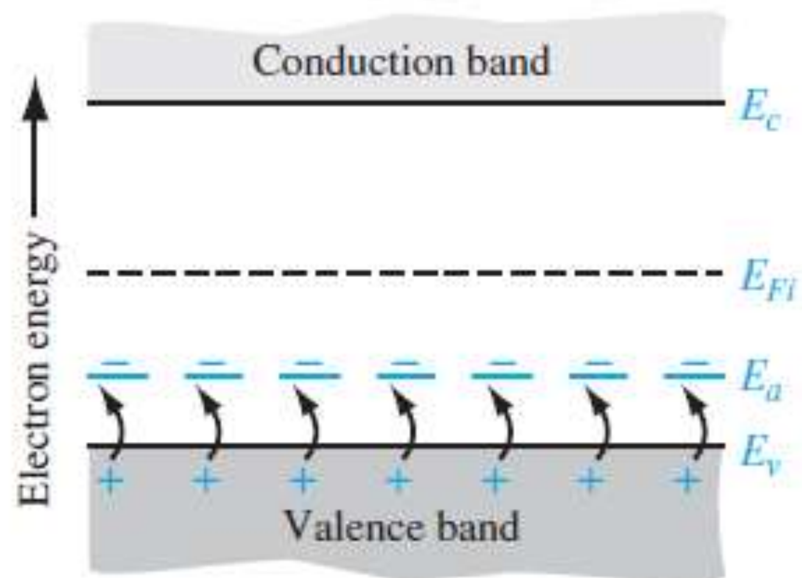
$$\frac{n_d}{n_d + n_o} = \frac{1}{1 + \frac{N_c}{2N_d} exp\left[\frac{-(E_c - E_d)}{kT}\right]} \quad \dots\dots\dots (5.55)$$

$$\frac{n_d}{n_0 + n_d} = \frac{1}{1 + \frac{2.8 \times 10^{19}}{2(10^{16})} \exp\left(\frac{-0.045}{0.0259}\right)} = 0.0041 = 0.41\%$$

At room temperature, there is also essentially complete ionization of the acceptor atoms. This means that each acceptor atom has accepted an electron from the valence band so that p_a is zero. At typical acceptor doping concentrations, a hole is created in the valence band for each acceptor atom.



(a)



(b)

At absolute zero degrees,

all electrons are in their lowest possible energy state; that is, for an n-type semiconductor, each donor state must contain an electron,

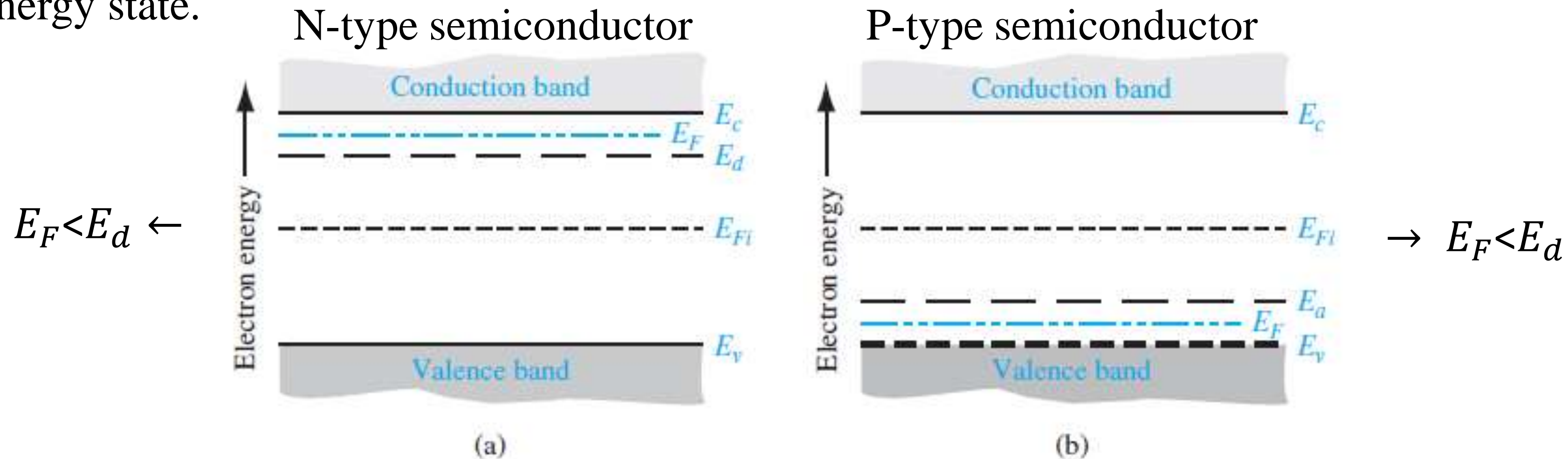
therefore, $n_d = N_d \quad or \quad N_d^+ = 0$

at $T = 0K$, $$n_d = \frac{N_d}{1 + \frac{1}{2}exp\left(\frac{E_d - E_F}{kT}\right)} \quad \dots\dots (4.50) \quad \rightarrow\rightarrow = N_d + N_d^+$$

$= 0$

$e^{-\infty} = 0 \rightarrow \therefore E_F > E_d$

In the case of a p-type semiconductor at absolute zero temperature, the impurity atoms will not contain any electrons, so that the Fermi energy level must be below the acceptor energy state.



No electrons from the donor state are thermally elevated into the conduction band or the valence band respectively; this effect is called freeze-out.

EXAMPLE

Determine the temperature at which 90 percent of acceptor atoms are ionized. Consider p-type silicon doped with boron at a concentration of $N_a = 10^{16}\ cm^{-3}$ and $g = 4$.

Solution

$$\frac{p_a}{p_0 + p_a} = \frac{1}{1 + \frac{N_v}{4N_a} \cdot \exp\left[\frac{-(E_a - E_v)}{kT}\right]}$$

$$\frac{p_a}{p_a + p_o} = \frac{1}{1 + \frac{N_v\left(\frac{T}{300}\right)^{3/2}}{4N_a} exp\left[\frac{-(E_a - E_v)}{kT}\right]}$$

From Table 4.1 $N_c = 1.04 \times 10^{19}\ cm^{-3}$ and from Table 4.3, the Ionization energy for ***Si*** with acceptor Boron atoms is 0.045 eV

$$(kT)_T = (kT)_{RT}\left(\frac{T}{300}\right) = 0.0259\left(\frac{T}{300}\right)\ eV$$

$$0.1 = \frac{1}{1 + \frac{(1.04 \times 10^{19})\left(\frac{T}{300}\right)^{3/2}}{4 \times 10^{16}} exp\left[\frac{-(0.045)}{0.0259\left(\frac{T}{300}\right)}\right]} \Rightarrow T = 193\ \mathrm{K}$$

H.Ws

Ex. 4.7, Ex 4.8 TYU 4.9, TYU 4.10

4.31: The electron and hole concentrations as a function of energy in the conduction band and valence band peak at a particular energy as shown in Figure 4.8. Consider silicon and assume $E_c - E_F = 0.20\ eV$. Determine the energy, relative to the band edges, at which the concentrations peak.

4.32: For the Boltzmann approximation to be valid for a semiconductor, the Fermi level must be at least $3kT$ below the donor level in an n-type material and at least $3kT$ above the acceptor level in a p-type material. If T = 300 K, determine the maximum electron concentration in an n-type semiconductor and the maximum hole concentration in a p-type semiconductor for the Boltzmann approximation to be valid in (a) silicon and (b) gallium arsenide.

CHARGE NEUTRALITY

In thermal equilibrium, the semiconductor crystal is electrically neutral. The electrons are distributed among the various energy states, creating negative and positive charges, but the net charge density is zero.

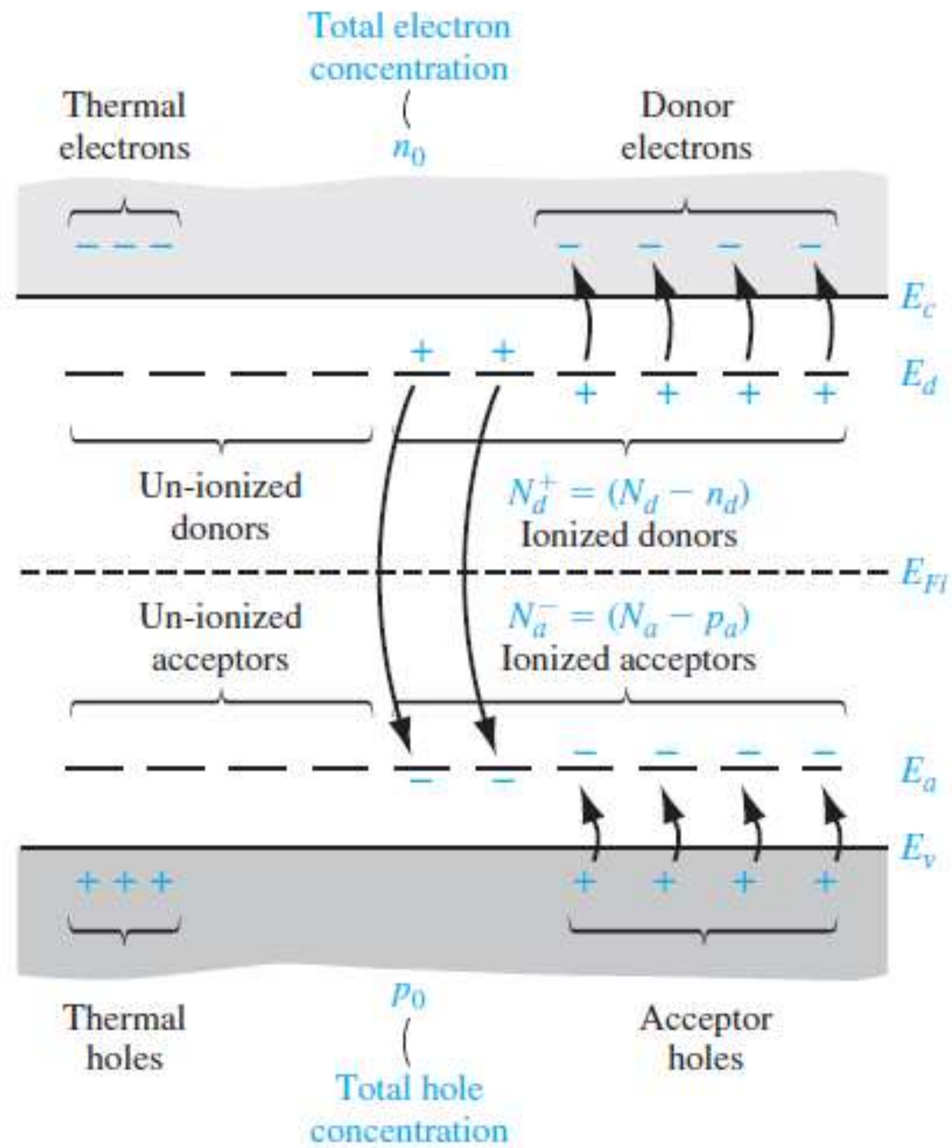
Compensated Semiconductors

A compensated semiconductor is one that contains both donor and acceptor impurity atoms in the same region.

A compensated semiconductor can be formed, for example, by diffusing acceptor impurities into an n-type material or by diffusing donor impurities into a p-type material.

Conditions:

An n-type compensated semiconductor occurs when $N_d > N_a$,
A p-type compensated semiconductor occurs when $N_d < N_a$.
If $N_d = N_a$, we have a completely compensated semiconductor



Total electron concentration
n_0
Thermal electrons
Donor electrons
E_c
E_d
Un-ionized donors
$N_d^+ = (N_d - n_d)$
Ionized donors
E_{Fi}
Un-ionized acceptors
$N_a^- = (N_a - p_a)$
Ionized acceptors
E_a
E_v
Thermal holes
p_0
Total hole concentration
Acceptor holes

The charge neutrality condition is expressed by equating the density of negative charges to the density of positive charges.

$$n_o + N_a^- = p_o + N_d^+ \quad \text{................(4.56)}$$

we have

$$n_d = N_d + N_d^+ \Rightarrow N_d^+ = N_d - n_d$$

$$p_a = N_a + N_a^- \Rightarrow N_a^- = N_a - p_a$$

then,,

$$n_o + (N_a - p_a) = p_o + (N_d - n_d) \quad \text{................(4.57)}$$

where n_o and p_o are the thermal-equilibrium concentrations of electrons and holes in the conduction band and valence band, respectively.

The parameter n_d is the concentration of electrons in the donor energy states, so N_d^+ is the concentration of positively charged donor states.

p_a is the concentration of holes in the acceptor states, so N_a^- is the concentration of negatively charged acceptor states

At Thermal-Equilibrium Electron Concentration

The charge neutrality condition is expressed by equating the density of negative charges to the density of positive charges.

$$n_o + (N_a - p_a) = p_o + (N_d - n_d) \quad \text{................(4.57)}$$

If we assume complete ionization, n_d and p_a are both zero

$$n_o + N_a = p_o + N_d \quad \text{................(4.58)}$$

we have, $n_i^2 = n_o p_o$

$$n_o + N_a = \frac{n_i^2}{n_o} + N_d \quad \text{................(4.59)}$$

The electron concentration n_o can be determined using the quadratic formula, or

$$n_o = \frac{N_d - N_a}{2} + \sqrt{\frac{N_d - N_a}{2} + n_i^2} \quad \text{...........(4.60)}$$

The positive sign in the quadratic formula must be used, since, in the limit of an intrinsic semiconductor when $N_a = N_d = 0$, the electron concentration must be a positive quantity, or $n_0 = n_i$.

Equation (4.60) is used to calculate the electron concentration in an n-type semiconductor, or when $N_d > N_a$. Although Equation (4.60) was derived for a compensated semiconductor, the equation is also valid for $N_a = 0$.

EXAMPLE

Determine the thermal-equilibrium electron and hole concentrations in silicon at T 300 K for given doping concentrations. (a) Let $N_d = 10^{16}\ cm^{-3}$ and $N_a = 0$. (b) Let $N_d = 5 \times 10^{15} cm^{-3}$ and $N_a = 1.5 \times 10^{15}$. Recall that $n_i = 1.5 \times 10^{10} cm^{-3}$.in silicon at $T = 300\ K$.

$$n_o = \frac{N_d - N_a}{2} + \sqrt{\frac{N_d - N_a}{2} + n_i^2} \quad \text{............(4.60)}$$

■ **Solution**

(*a*) From Equation (4.60), the majority carrier electron concentration is

$$n_0 = \frac{10^{16}}{2} + \sqrt{\left(\frac{10^{16}}{2}\right)^2 + (1.5 \times 10^{10})^2} \cong 10^{16}\ \text{cm}^{-3}$$

The minority carrier hole concentration is found to be

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4\ \text{cm}^{-3}$$

(*b*) Again, from Equation (4.60), the majority carrier electron concentration is

$$n_0 = \frac{5 \times 10^{15} - 2 \times 10^{15}}{2} + \sqrt{\left(\frac{5 \times 10^{15} - 2 \times 10^{15}}{2}\right)^2 + (1.5 \times 10^{10})^2} \cong 3 \times 10^{15}\ \text{cm}^{-3}$$

The minority carrier hole concentration is

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{3 \times 10^{15}} = 7.5 \times 10^4\ \text{cm}^{-3}$$