جامعة الانبار

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Introduction to Graphs (2)

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Lecture (2)

Graph Theory

Introduction to Graphs (2)

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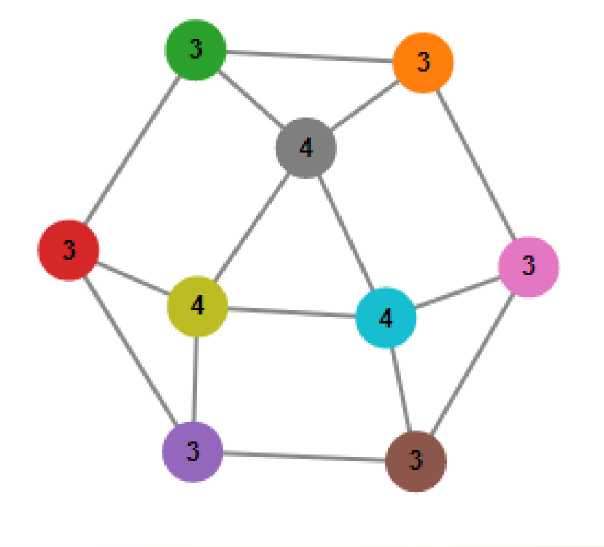
Outlines

- ✓ Degree Sequences in Graphs.
- ✓ Graphical Sequence.
- ✓ Havel Hakimi Algorithm.
- ✓ Neighborhoods.
- ✓ H. W.

Definition: (Degree Sequence) The degree sequence of a graph of order n is the n −term sequence (usually written in descending order) of the vertex degrees.

$$\text{Degree Sequence} = (4, 4, 4, 3, 3, 3, 3, 3, 3)$$



Definition: (Graphical Sequence) An integer sequence is said to be graphical if it is the degree sequence of some graphs.

Example 1: Is the sequence $S = (9,9,8,7,7,6,6,5,5)$ graphical? Justify your answer.

Solution: The sequence $S = (a_i)$ is graphical if every element of S is the degree of some vertex in a graph. For any graph, we know that $\sum_{v \epsilon V(G)} d(v) = 2E$, an even integer. Here, $\sum a_i = 62$, an even number. But note that the maximum degree that a vertex can attain in a graph of order n is $n - 1$. If S were graphical, the corresponding graph would have been a graph on 9 vertices and have $\Delta(G) = 9$. Therefore, the given sequence is not graphical.

Example 2: Is the sequence S =(9,8,7,6,6,5,5,4,3,3,2,2) graphical? Justify your answer.

Solution: The sequence $S = (a_i)$ is graphical if every element of S is the degree of some vertex in a graph. For any graph, we know that $\sum_{v \epsilon V(G)} d(v) = 2E$, an even integer. Here, we have $\sum a_i = 60$, an even number. Also, note that the all elements in the sequence are less than the number of elements in that sequence. Therefore, the given sequence is graphical.

Example 3: Is the sequence $S = (5,4,3,3,2,2,2,1,1,1,1)$ graphical? Justify your answer.

Solution: The sequence $S = (a_i)$ is graphical if every element of S is the degree of some vertex in a graph. For any graph, we know that $\sum_{v\epsilon V(G)} d(v) = 2E$, an even integer. Here, $\sum a_i = 25$, not an even number. Therefore, the given sequence is not graphical.

Havel Hakimi Theorem: The non-negative integer sequence $D = [d_i]_1^n$ is graphic if and only if $\acute{D}$ is graphic, where $\acute{D}$ is the sequence (having $n-1$ elements) obtained from D by deleting its largest element Δ and subtracting 1 from its Δ next largest elements.

Havel Hakimi Algorithm (HHA)

The Havel Hakimi algorithm gives a systematic approach to answer the question of determining whether it is possible to construct a simple graph from a given degree sequence.

Take as input a degree sequence S and determine if that sequence is graphical That is, can we produce a graph with that degree sequence?

Assume the degree sequence is S

$$S = d_1, d_2, d_3, \cdots, d_n$$

$$d_i \geq d_{i+1}$$

1. If any $d_i \geq n$ then fail
2. If there is an odd number of odd degrees then fail
3. If there is a $d_i < 0$ then fail
4. If all $d_i = 0$ then report success
5. Reorder S into non-increasing order
6. Let $k = d_1$
7. Remove d_1 from S.
8. Subtract 1 from the first k terms remaining of the new sequence
9. Go to step 3 above

Example 1:

Consider the degree sequence: S = 7, 5, 5, 4, 4, 4, 4, 3

Where n = 8 (no. of vertices)

Step 1. Degree of all vertices is less than n (no.of vertices)

Step 2. Odd number vertices are four.

Step 3. There is no degree less than zero.

Step 4. Remove '7' from the sequence and subtract 1 from the remaining new sequence and arrange again in non-increasing order to get

S = 4, 4, 3, 3, 3, 3, 2

Step 5. Now remove the first '4 ' from the sequence and subtract 1 from the remaining new sequence to get:

S = 3, 2, 2, 2, 3, 2

rearrange in non-increasing order to get:

S = 3, 3, 2, 2, 2, 2

Repeating the above step we get following degree sequences:

S = 2, 2, 2, 1, 1

S = 1, 1, 1, 1

S = 1, 1, 0

S = 0, 0

Step 6. Since all the deg remaining in the sequence is zero, the given sequence is graphical (or in other words, it is possible to construct a simple graph from the given degree sequence).

Example 2:

S = 4, 3, 3, 3, 1

Where n = 5 (no. of vertices)

Step 1. Degree of all vertices is less than n (no.of vertices)

Step 2. Odd number vertices are four.

Step 3. There is no degree less than zero.

Step 4. Remove '4' from the sequence and subtracting 1 from the remaining new sequence and arrange again in non-increasing order we get

S = 2,2,2,0

Step 5. Again Remove '2 ' from the sequence and subtracting 1 from the remaining new sequence and arrange in non-increasing order we get

S= 1,1,0

Repeating the above step

S= 0,0

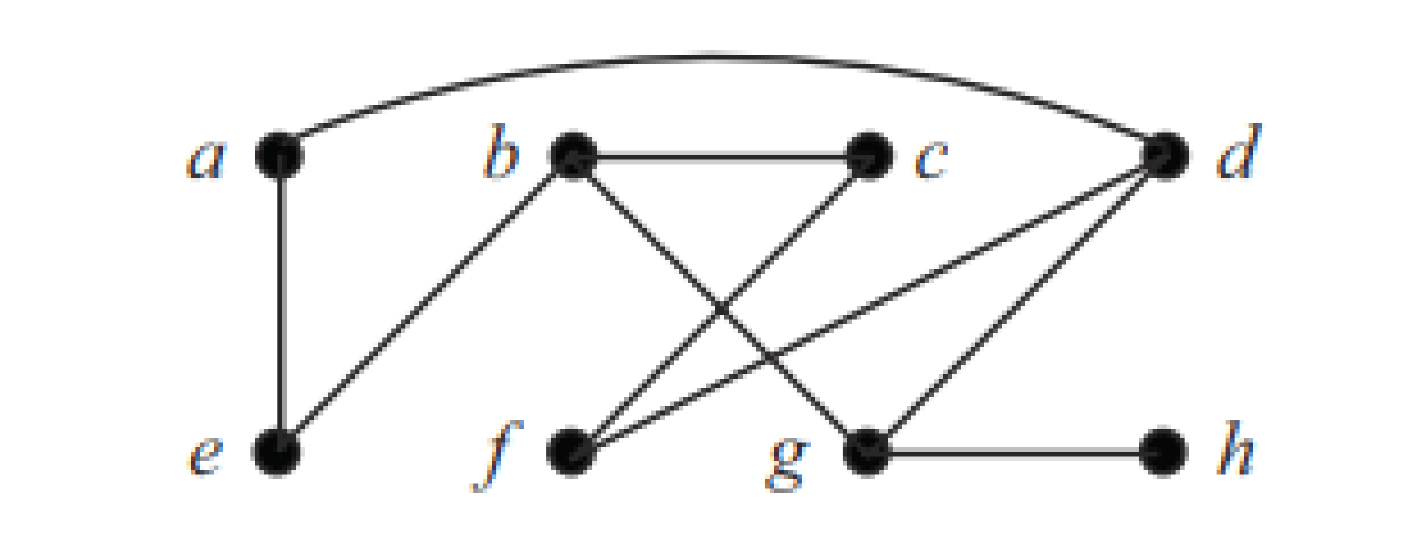
Step 6. Since all the deg remaining in the sequence is zero, the given sequence is graphical.

Neighbourhoods

Definition: (Neighbourhood of a Vertex) The neighbourhood (or open neighbourhood) of a vertex v, denoted by $N(v)$, is the set of vertices adjacent to v. That is, $N(v) = \{x \in V : vx \in E\}$. The closed neighbourhood of a vertex v, denoted by $N[v]$, is simply the set $N(v) \cup \{v\}$.

Then, for any vertex v in a graph G, we have $d(v) = |N(v)|$. A special case is a loop that connects a vertex to itself; if such an edge exists, the vertex is said to belong to its own neighbourhood.

Given a set S of vertices, we define the neighbourhood of S, denoted by $N(S)$, to be the union of the neighbourhoods of the vertices in S. Similarly, the closed neighbourhood of S, denoted by $N[S]$, is defined to be $S \cup N(S)$.



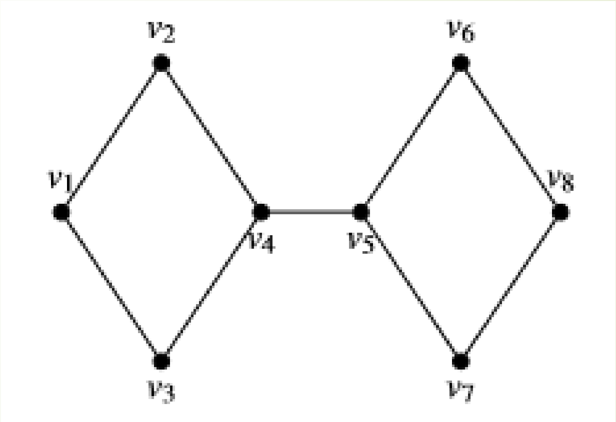
$N(f) = \{c, d\}$ & $N[f] = \{c, d, f\}$

Let $S = \{a,b,c\}, then\ N(S) = \{e,d\} \cup \{c,e,g\} \cup \{b,f\} = \{e,d,c,g,b,f\}$

$N[S] = \{e,d\} \cup \{c,e,g\} \cup \{b,f\} \cup \{a\} \cup \{b\} \cup \{c\} = \{e,d,c,g,b,f,a\}$.

H. W.

1) Verify whether the integer sequences (7,6,5,4,3,3,2) and (6,6,5,4,3,3,1) are graphical. (Hint: Use Havel Hakimi Algorithm)
2) For the following graph G , find:
$\delta(G)$, $\Delta(G)$, $N[v_5]$ and degree sequence .



Thank You

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