جامعة الانبار

كلية العلوم

قسم الرياضيات

نظرية البيانات

Introduction to Graphs (3)

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Lecture (3)

Introduction to Graphs (3)

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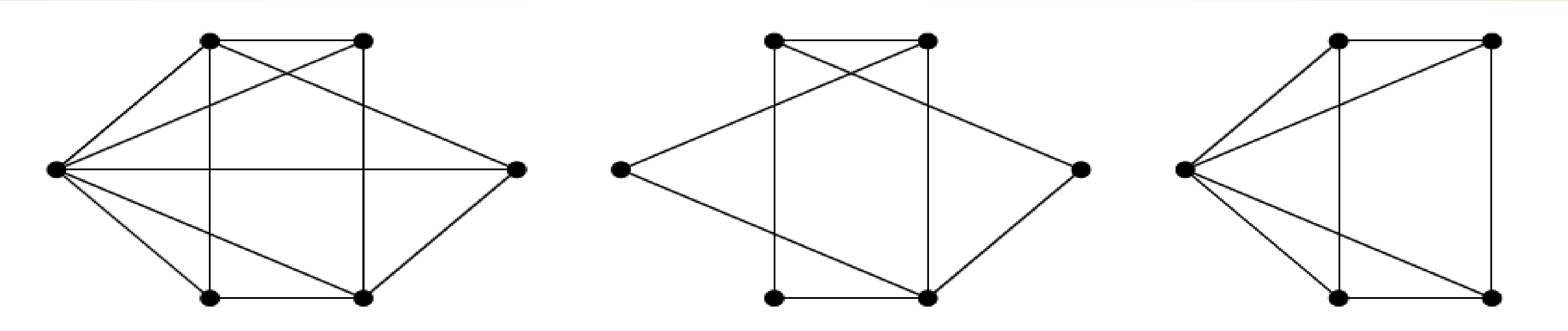
Outlines

- ✓ Subgraph of Graph.
- ✓ Spanning Subgraphs.
- ✓ Induced Subgraphs.
- ✓ Fundamental Graph Classes.

Spanning Subgraphs and Induced Subgraphs

Definition: (Subgraph of a Graph) A graph $H(V_1, E_1)$ is said to be a subgraph of a graph $G(V, E)$ if $V_1 \subseteq V$ and $E_1 \subseteq E$.

Definition: (Spanning Subgraph of a Graph) A graph $H(V_1, E_1)$ is said to be a spanning subgraph of a graph $G(V, E)$ if $V_1 = V$ and $E_1 \subseteq E$.



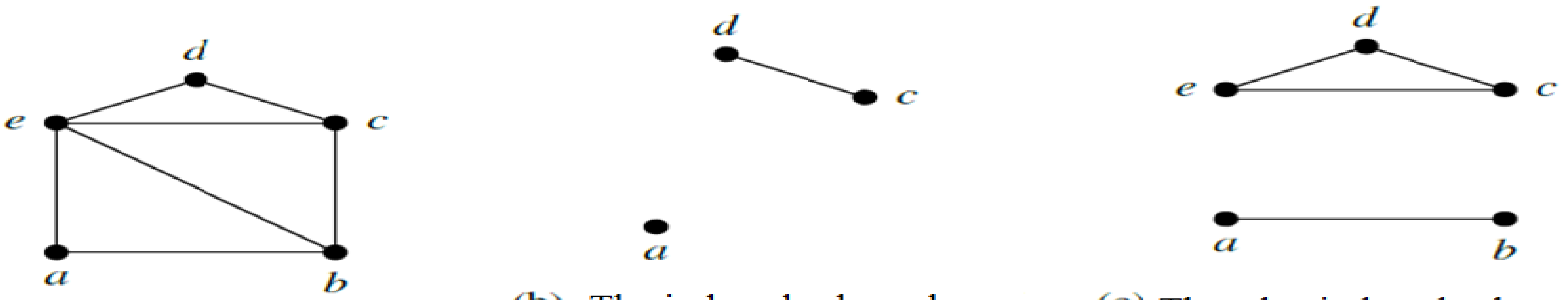
G spanning subgraph of G subgraph of G

In the above figure, the second graph is a spanning subgraph of a graph G, while the third graph is a subgraph of a graph G.

Definition: (Induced Subgraph) Suppose that V_1 be a subset of the vertex set V of a graph G. Then, the subgraph of G whose vertex set is V_1 and whose edge set is the set of edges of G that have both end vertices in V_1 is denoted by $G[V]$ called a vertex - induced subgraph (induced subgraph) of G.

Definition: (Edge-Induced Subgraph) Suppose that E_1 be a subset of the edge set E of a graph G. Then, the subgraph of G whose edge set is E_1 and whose vertex set is the set of end vertices of the edges in E_1 is denoted by $G[E]$ called an edge $-$induced subgraph of G.

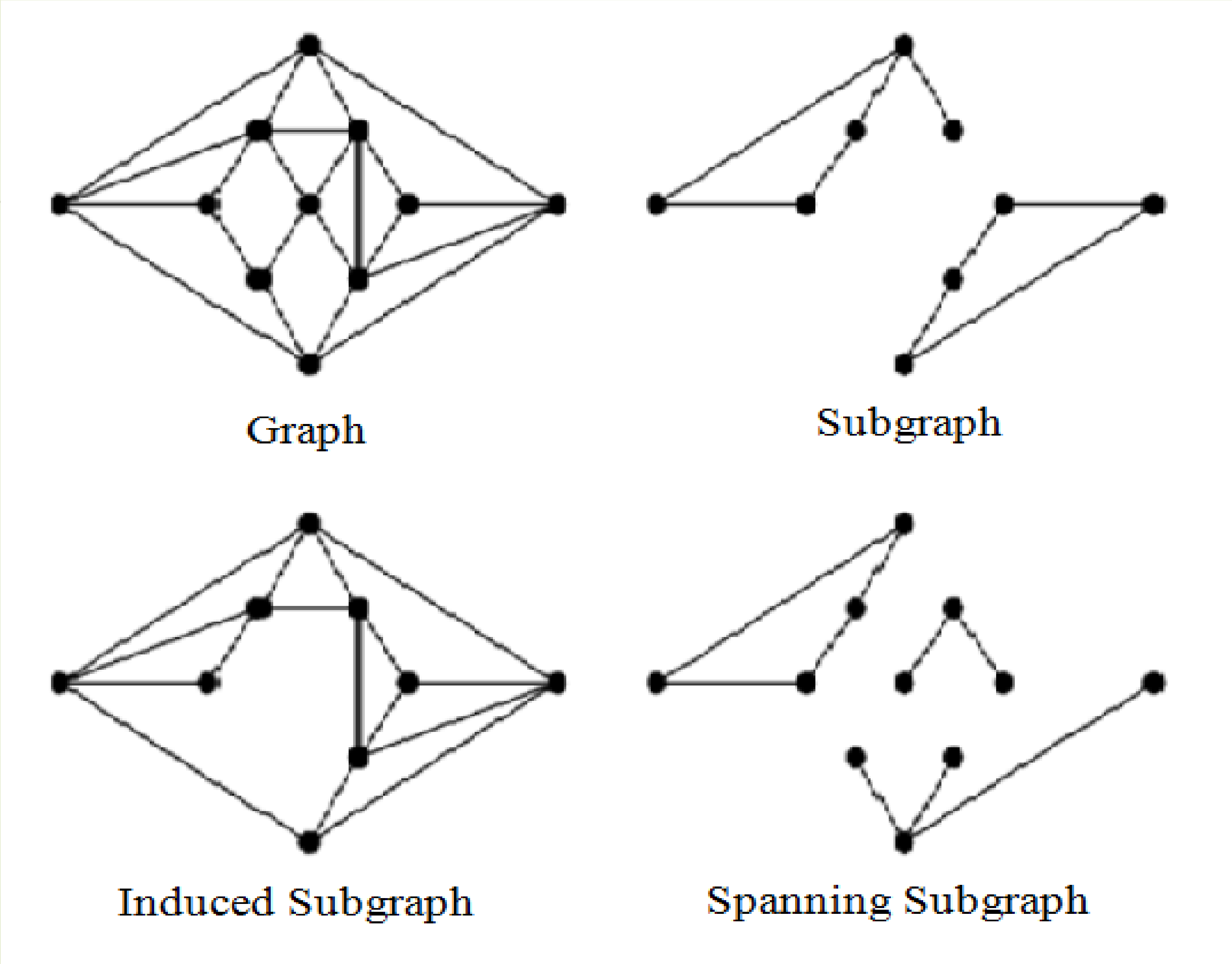
The following figure describe an induced subgraph and an edge induced subgraph of a given graph.



(a) The graph G

(b) The induced subgraph $G[a, c, d]$

(c) The edge induced subgraph $G[ab, cd, de, ce]$


Graph
Subgraph
Induced Subgraph
Spanning Subgraph

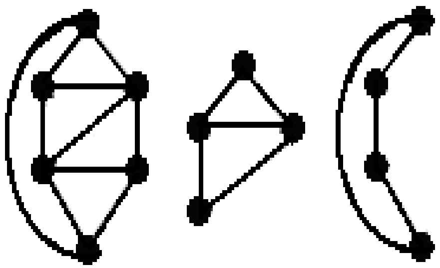
An induced subgraph is obtained by deletion of vertices only (deleting the vertex v and all edges connected to v from the graph G).

A spanning subgraph is obtained by deletion of edges only (remove the edge).

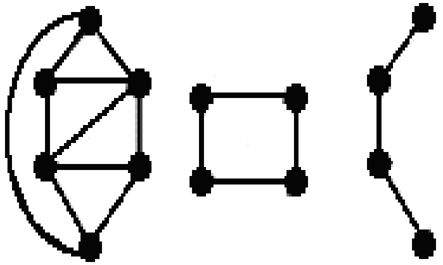
Let G have n vertices and m edges:

How many spanning subgraphs are there?
There are 2^m spanning subgraphs (all subsets of edges).

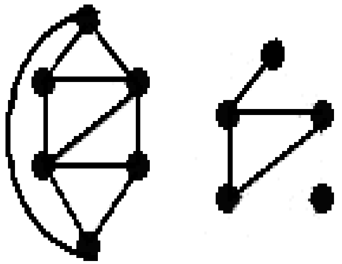
How many induced subgraphs are there?
There are 2^n induced subgraphs (all subsets of vertices).



The second two figures are vertex-induced subgraphs of the first figure.



The second two figures are edge-induced subgraphs of the first figure.



The second figure is a subgraph of the first figure, but it is neither edge-induced nor vertex-induced.

What is the difference between an induced Subgraph and a spanning Subgraph?

A spanning Subgraph contains all of the vertices from the parent graph and need not contain all of the edges. An induced subgraph contains a subset of the vertices of the parent graph along with all of the edges that connect the vertices that exist in both the parent graph and the subgraph. If a subgraph is both a spanning subgraph and an induced subgraph, it is equal to the parent graph.

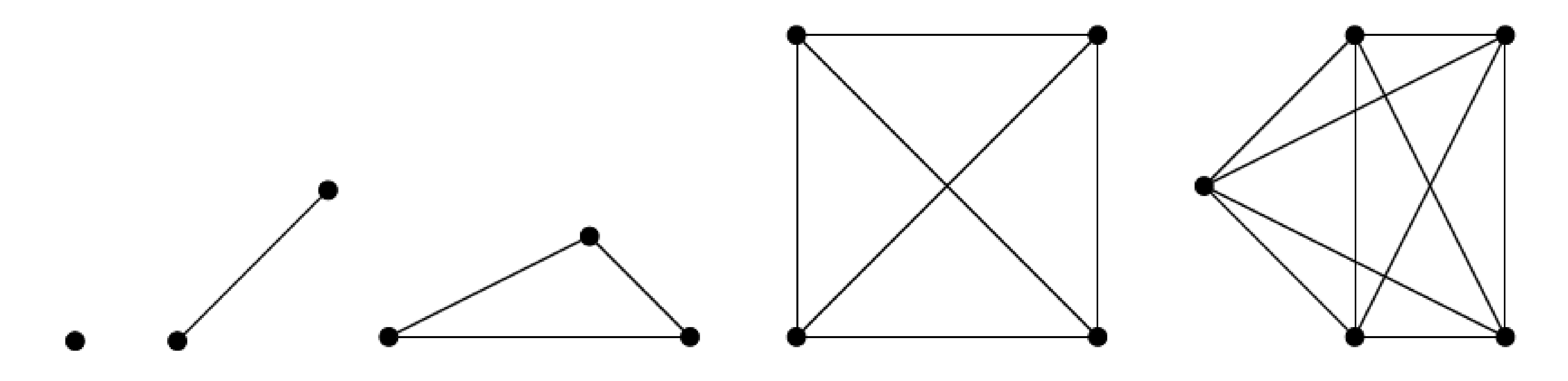
Fundamental Graph Classes:

Complete Graphs

Definition: A complete graph is a simple undirected graph (have edges that do not have a direction) in which every pair of different vertices is connected by a unique edge. A complete graph on n vertices is denoted by K_n and has $\frac{n(n-1)}{2}$ edges.

The following are first few complete graphs:

$$\binom{n}{2} = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2}$$

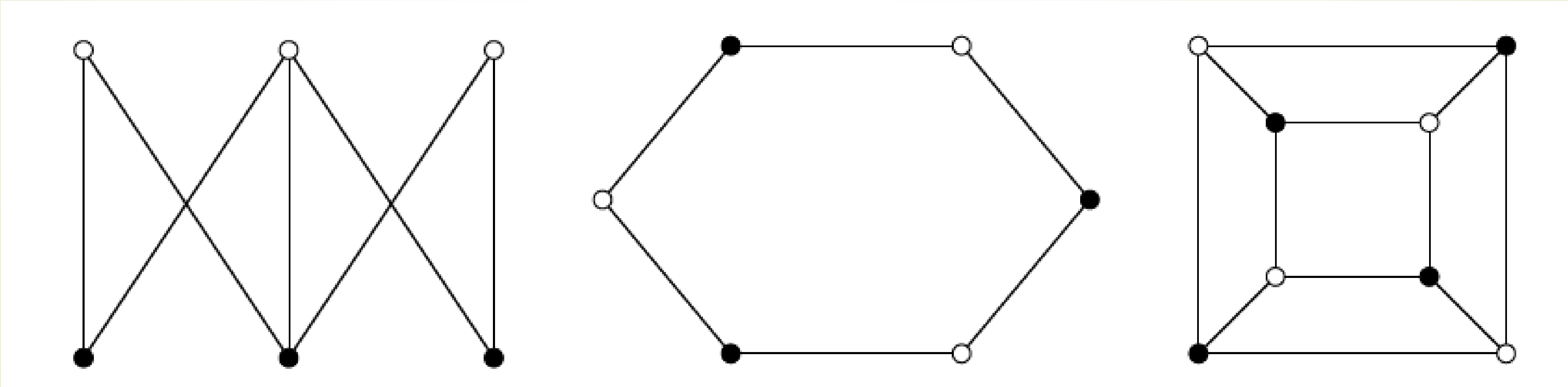


K_1 K_2 K_3 K_4 K_5

Bipartite Graphs

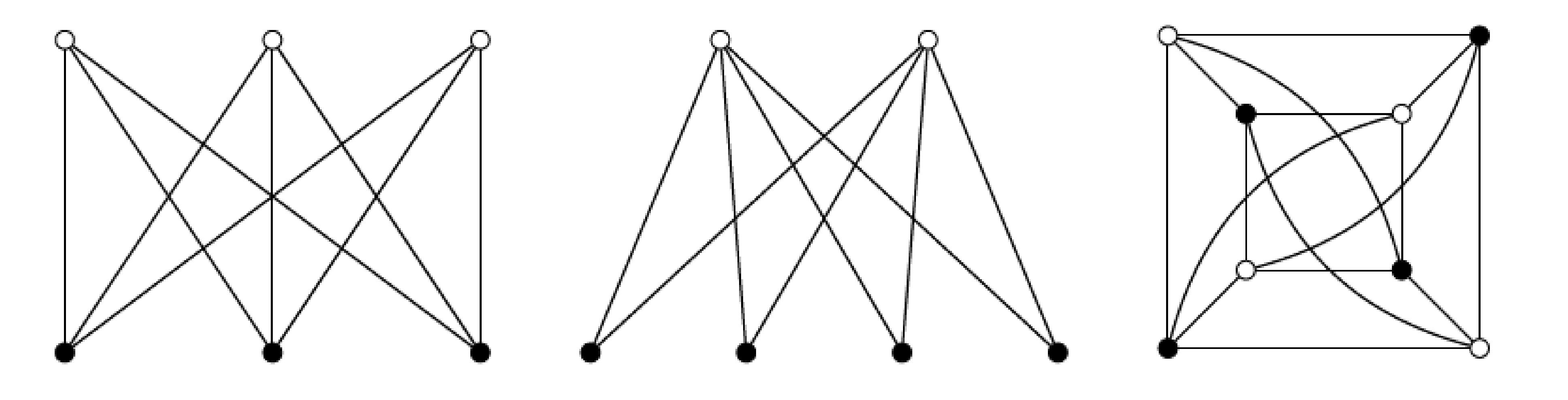
Definition: A graph G is said to be a bipartite graph if its vertex set V can be partitioned into two sets, say V_1 and V_2, such that no two vertices in the same partition can be adjacent. Here, the pair (V_1, V_2) is called the bipartition of G.

In the following figures gives some examples of bipartite graphs. In all these graphs, the white vertices belong to the same partition, say V_1 and the black vertices belong to the other partition, say V_2.



Definition: A bipartite graph G is said to be a complete bipartite graph if every vertex of one partition is adjacent to every vertex of the other. A complete bipartite graph with bipartition (V_1, V_2) is denoted by $K_{a,b}$, where $a = |V_1|,\ b = |V_2|$ and has ab edges.

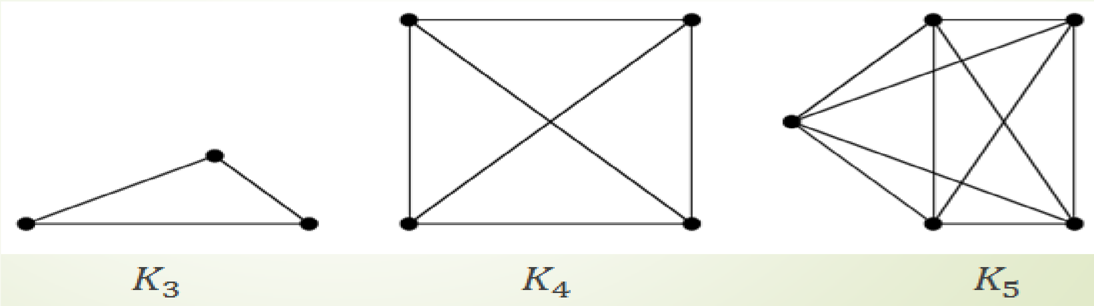
The following graphs are also some examples of complete bipartite graphs. In these examples also, the vertices in the same partition have the same colour.



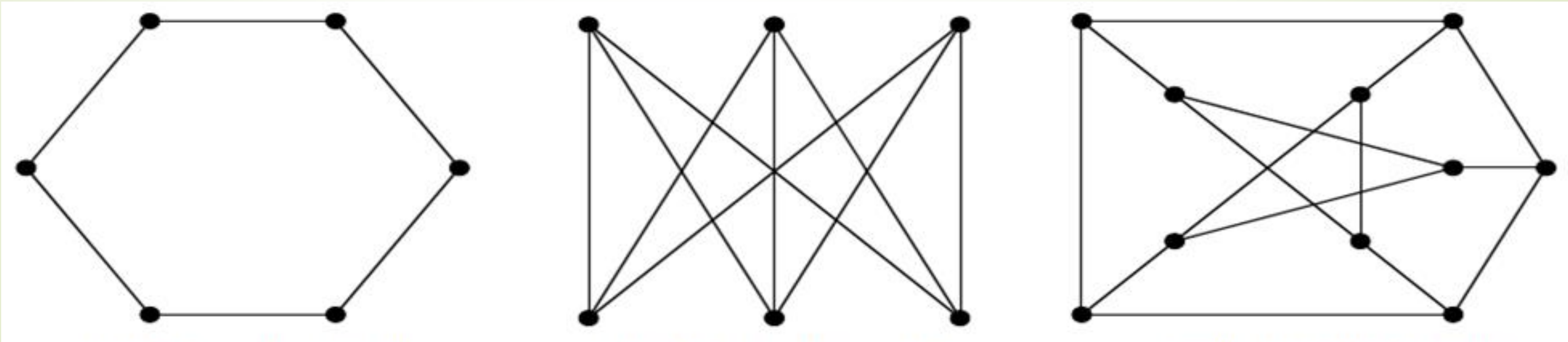
Regular Graphs

Definition: A graph G is said to be a regular graph if all its vertices have the same degree. A graph G is said to be a $k-$ regular graph if $d(v) = k, \forall\, v \in V(G)$.

Note: Every complete graph is an $(n-1)-$ regular graph.



If the degree of all vertices in each partition of a complete bipartite graph is the same. Hence, the complete bipartite graphs are also called biregular graphs.



(a) A 2-regular graph (b) A 3-regular graph (c) A 3-regular graph

Thank You

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