جامعة الانبار

كلية العلوم

قسم الرياضيات

نظرية البيانات

Traversability in Graphs\ Eulerian Graphs

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Lecture (6)

Traversability in Graphs

Eulerian Graphs

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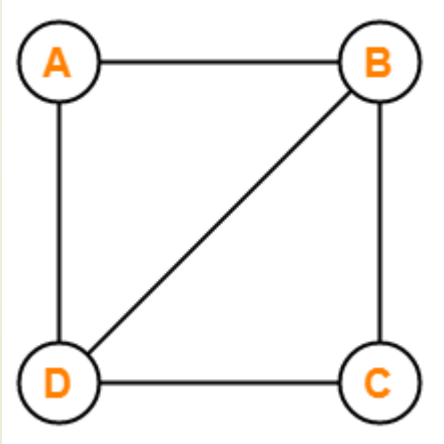
Eulerian Graphs

A trail is a walk that does not pass over the same edge twice. A trail might visit the same vertex twice, but only if it comes and goes from a different edge each time.

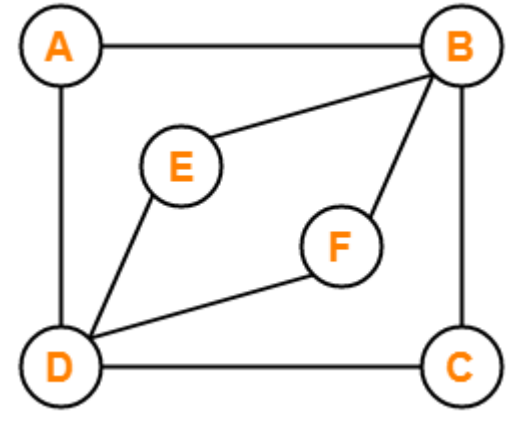
Definition: An Eulerian trail is a trail that visits every edge of the graph once and only once. It can end on a vertex different from the one on which it began. A graph of this kind is said to be traversable (semi- Eulerian) graph.

Definition: An Eulerian circuit is an Eulerian trail that is a circuit. That is, it begins and ends on the same vertex (closed Eulerian trail).

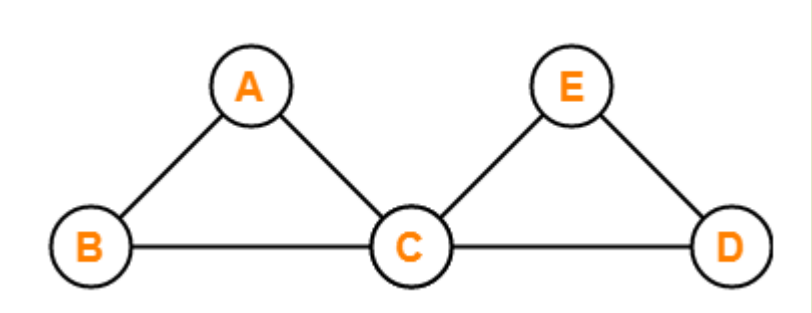
Definition: A graph is called Eulerian when it contains an Eulerian circuit.



Eulerian trail
BCDBAD



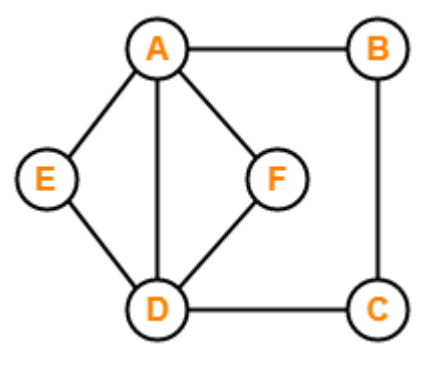
Eulerian circuit
ABCDFBEDA



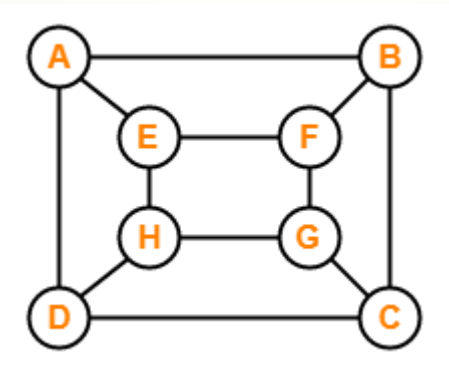
Eulerian graph
Eulerian circuit: BCDECAB

The following theorem characterizes the class of Eulerian graphs:

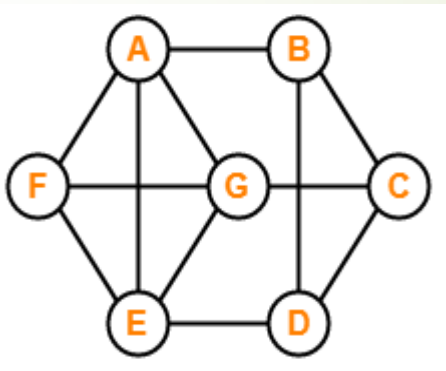
Theorem 1: (Euler Theorem) A connected graph G is Eulerian if and only if every vertex in G is of even degree.



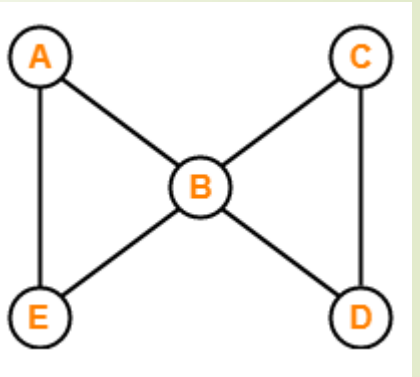
Eulerian graph
ABCDFADEA



Not Eulerian graph



Not Eulerian graph

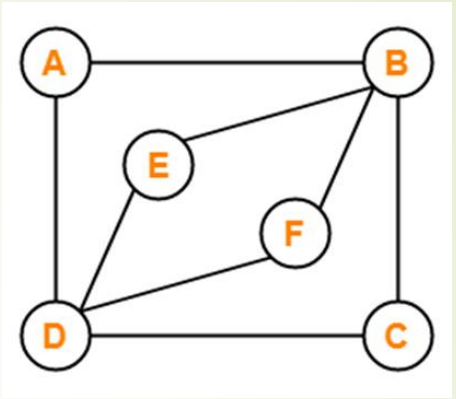


Eulerian graph
ABDCBEA

Note: i) Cycles C_n are Eulerian graphs.

ii) Paths P_n have no circuits at all $\Rightarrow$ P_n are not Eulerian graphs.

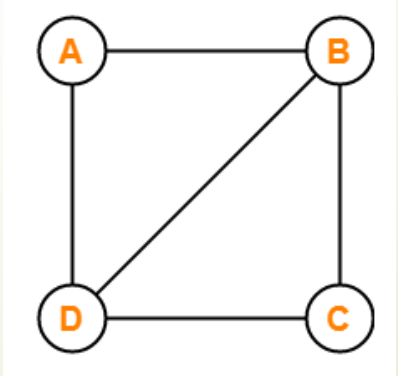
Theorem 2: A connected graph is Eulerian if and only if each of its edges lies on an odd number of cycles.



Theorem 3: A connected graph G is Eulerian if and only if it can be decomposed into edge-disjoint cycles.

Corollary: A graph is Eulerian if and only if it has an odd number of cycle decompositions.

Theorem 4: A connected graph G is semi- Eulerian if and only if it has exactly two odd degree vertices.



Important Notes

Note 1: To check whether any graph is an Eulerian graph or not, any one of the following two ways may be used:

• If the graph is connected and contains an Eulerian circuit, then it is an Eulerian graph.

• If all the vertices of the graph are of even degree, then it is an Eulerian graph.

Note 2: To check whether any graph contains an Eulerian circuit or not,

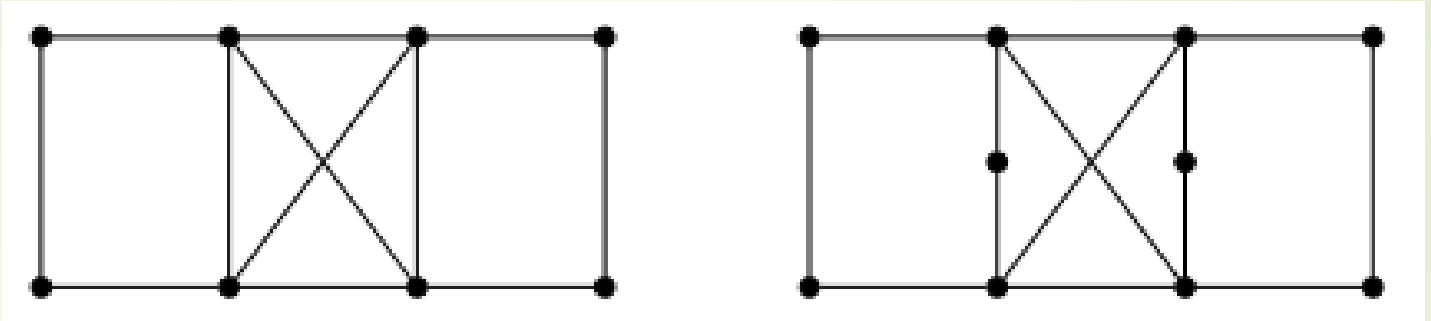
• Just make sure that all its vertices are of even degree.

• If all its vertices are of even degree, then graph contains an Eulerian circuit otherwise not.

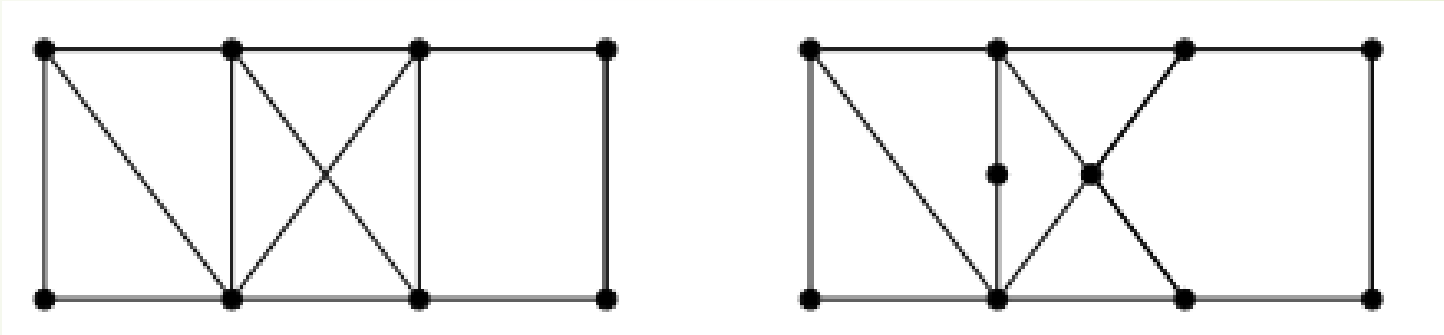
Note 3: To check whether any graph is a semi-Eulerian graph or not,

• Just make sure that it is connected and contains an Eulerian trail.

• If it has exactly two odd degree vertices, then graph is a semi-Eulerian graph otherwise not.



Eulerian Graphs



Not Eulerian Graphs

Thank You

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