جامعة الانبار

كلية العلوم

قسم الرياضيات

نظرية البيانات

Traversability in Graphs (II)

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Lecture (7)

Traversability in Graphs (II)

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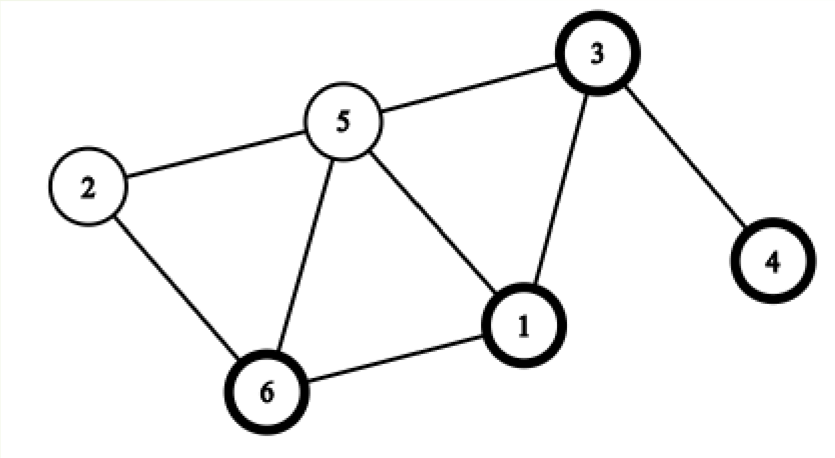
Outlines

- ✓ Hamiltonian Graphs.
- ✓ Weighted Graphs.

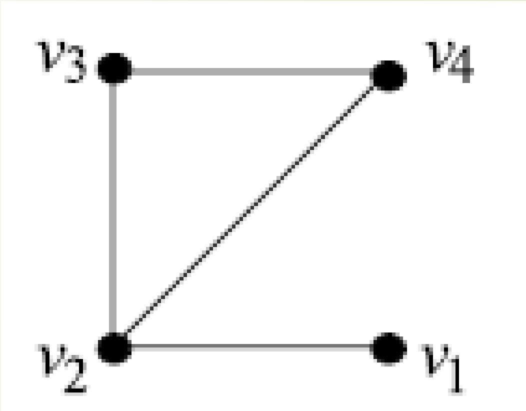
Hamiltonian Graphs

A path is a trail that does not include any vertex twice, except that its first vertex might be the same as its last.

Definition: A Hamiltonian path (Hamilton path) is a path in an undirected (or directed graph) that visits each vertex in graph. A graph that contains a Hamiltonian path is called a traceable graph

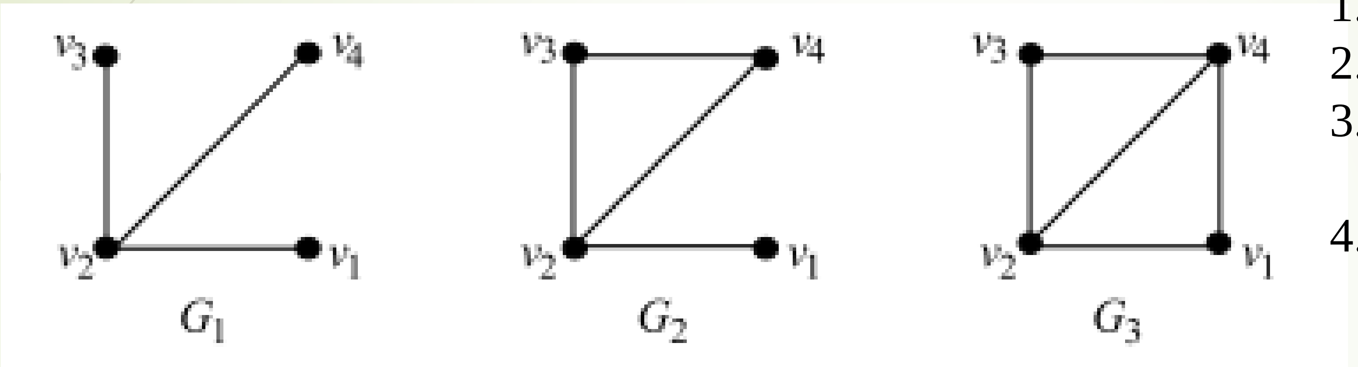


Path : 4-3-1-6.
Hamiltonian Path: 2- 6- 5- 1- 3- 4.



Traceable graph
Path :$v_1 v_2 v_3 v_4$

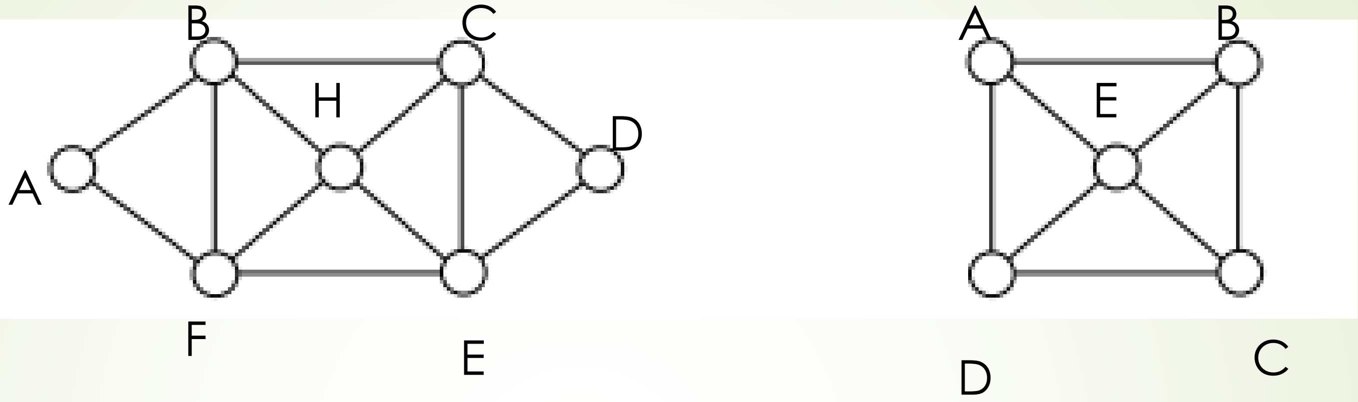
Definition: A closed Hamiltonian path in an undirected (or directed graph) is called a Hamiltonian cycle. If a Hamiltonian cycle exists in a given graph, then the graph called an Hamiltonian graph.



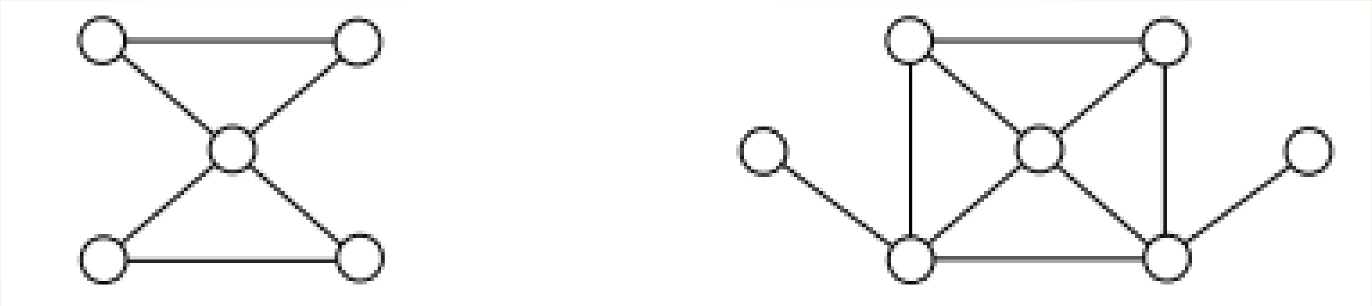
1. Every cycle graph is Hamiltonian.
2. Every wheel graph is Hamiltonian.
3. Every complete graph ($v \geq 3$) is Hamiltonian.
4. Every complete bipartite graph (except $K_{1,1}$) is Hamiltonian.

G_1 has no Hamiltonian path, and so no Hamiltonian cycle; G_2 has the Hamiltonian path $v_1 v_2 v_3 v_4$, but has no Hamiltonian cycle, while G_3 has the Hamiltonian cycle $v_1 v_2 v_3 v_4 v_1$.

Note: Any Hamiltonian cycle can be converted to a Hamiltonian path by removing one of its edges, but a Hamiltonian path can be extended to a Hamiltonian cycle only if its endpoints are adjacent.



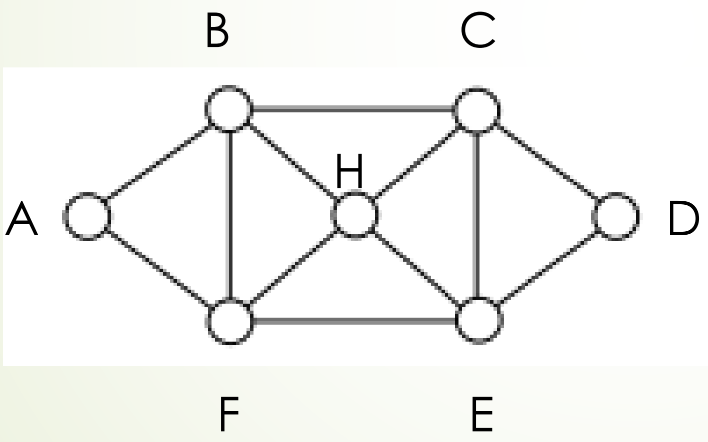
Hamiltonian graphs

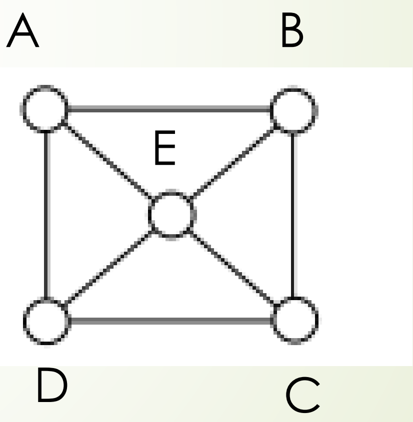


Not Hamiltonian graphs

A necessary and sufficient condition for a graph to be a Hamiltonian is still to be determined. But there are a few sufficient conditions for certain graphs to be Hamiltonian as follows:

Dirac's Theorem: A graph with n vertices $(n \geq 3)$, and every vertex having degree $\geq \frac{n}{2}$ is Hamiltonian .





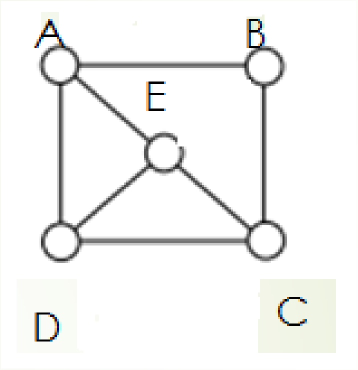
$\frac{n}{2} = \frac{7}{2} = 3.5$, $d(A) = d(D) = 2$

$d(B) = d(C) = d(E) = d(F) = d(H) = 4$

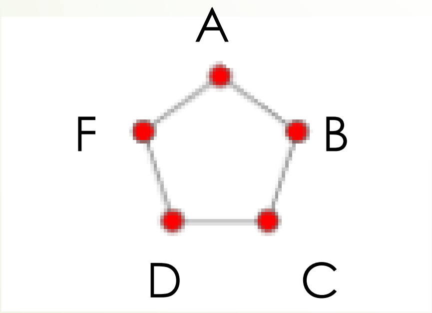
$\frac{n}{2} = \frac{5}{2} = 2.5$, $d(E) = 4$

$d(A) = d(B) = d(C) = d(D) = 3$

Ore's Theorem: Let G be a n vertices graph ($n \geq 3$), such that $d(u) + d(v) \geq n$, for every pair of non adjacent vertices u, v in G, then G is Hamiltonian.



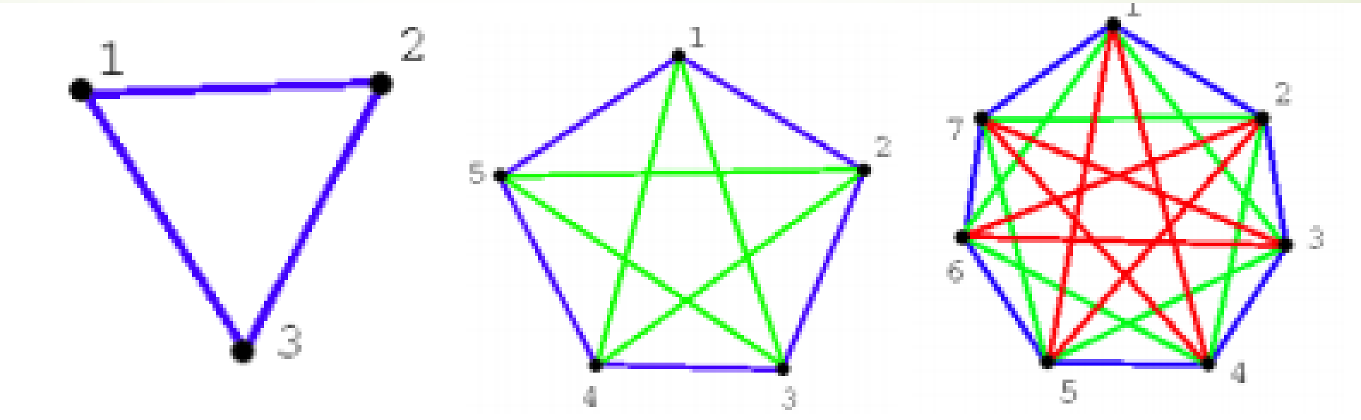
$$d(A) + d(C) = 6$$
$$d(B) + d(E) = 5$$
$$d(B) + d(D) = 5$$



$$d(A) + d(C) = 4$$
$$d(A) + d(D) = 4$$
$$d(B) + d(D) = 4$$
$$d(B) + d(F) = 4$$
$$d(C) + d(F) = 4$$

The following theorem determines the number of edge-disjoint Hamilton cycles in a complete graph K_n, where n is odd.

Theorem: In a complete graph K_n, where $n \geq 3$ is odd, there are $\frac{n-1}{2}$ edge-disjoint Hamilton cycles.

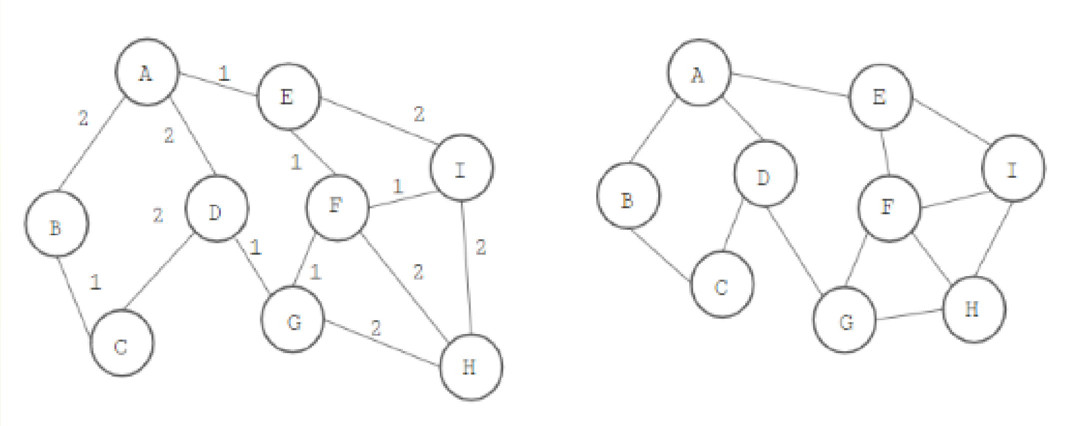


K_3 K_5 K_7

Weighted Graphs

Definition: (Weighted Graphs) A weighted graph is a graph G in which each edge e has been assigned a real number w(e), called the weight (or length) of the edge e.

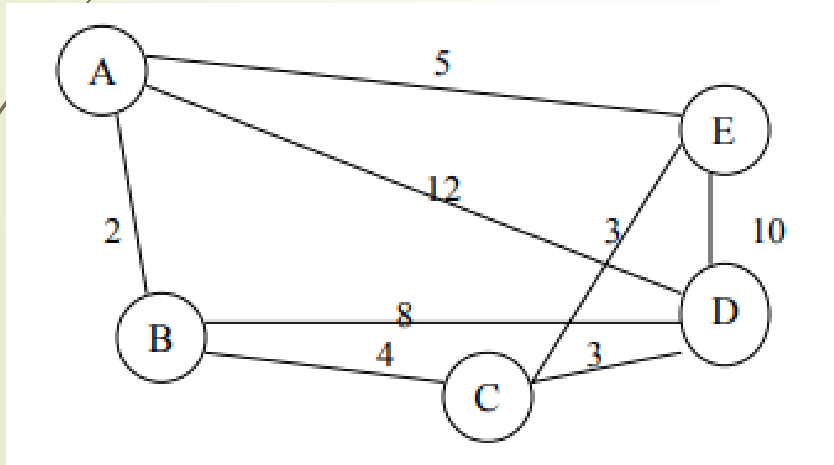
If H is a subgraph of a weighted graph, the weight $w(H)$ of H is the sum of the weights $w(e_1) + w(e_2) + \cdots + w(e_k)$ where $\{e_1, e_2, \ldots, e_k\}$ is the set of edges of H.



Weighted Graph   Unweighted Graph

The Traveling Salesman Problem:

Introduction The traveling salesman problem consists of a salesman and a set of cities. The salesman has to visit each one of the cities starting from a certain one (e.g. the hometown) and returning to the same city. The challenge of the problem is that the traveling salesman wants to minimize the total length of the trip.



The problem lies in finding a minimal path passing from all vertices once. For example the path 1 $\{A, B, C, D, E, A\}$ and the path 2 $\{A, B, C, E, D, A\}$ pass all the vertices but Path1 has a total length of 24 and Path2 has a total length of 31.

Thank You

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