جامعة الانبار

كلية العلوم

قسم الرياضيات

نظرية البيانات

Minimum cut & Maximum Flow

م. د. امين شامان امين

Lecture (10)

Minimum cut & Maximum Flow

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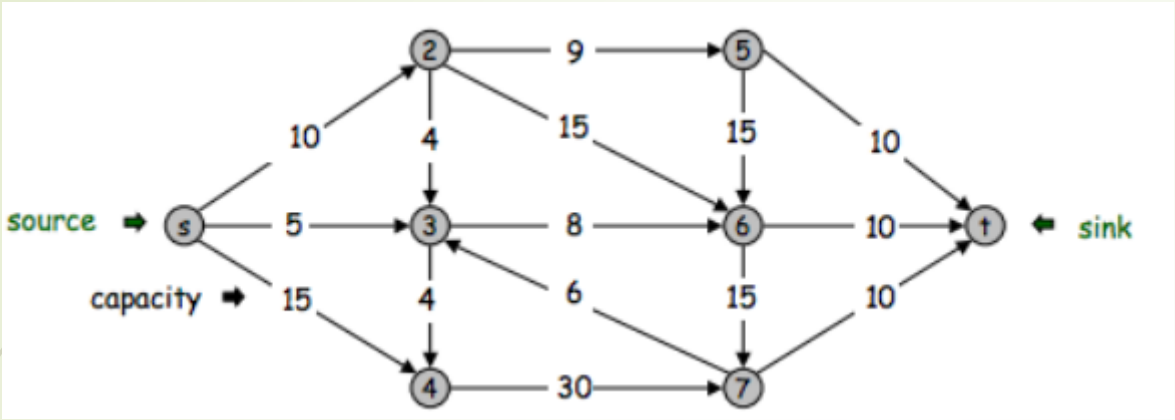
In graph theory a cut is a partition of the vertices of a graph into two disjoint subsets. In a flow network, an $s-t$ cut is a cut that requires the source and the sink to be in different subsets, and its cut-set only consists of edges going from the source's side to the sink's side.

Definition: (Cuts in Networks) A cut of a network D, denoted by (S,T) or K, is the set of arcs (edges) $\{st:\ s\ \epsilon S; t \in T\}$, whose removal disconnects the network into two components.

The capacity of a cut is sum of the weights of the edges beginning in S and ending in T.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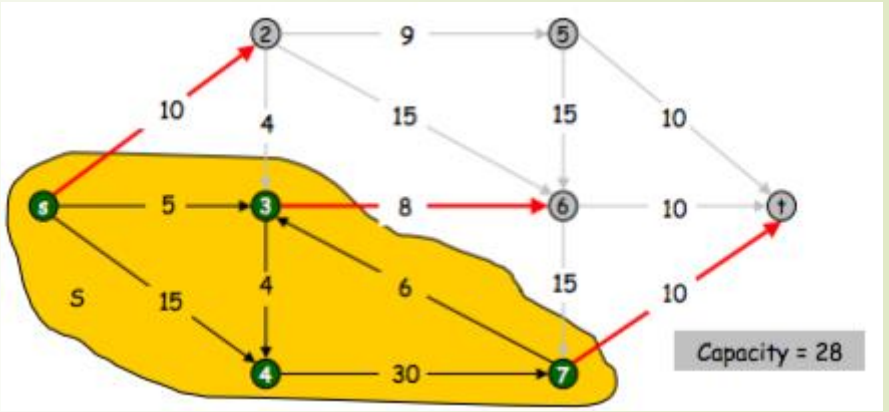
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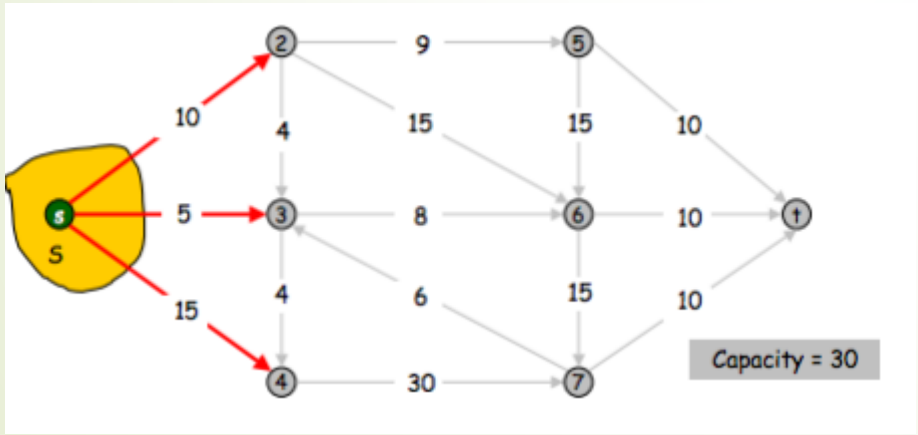
Capacity = 28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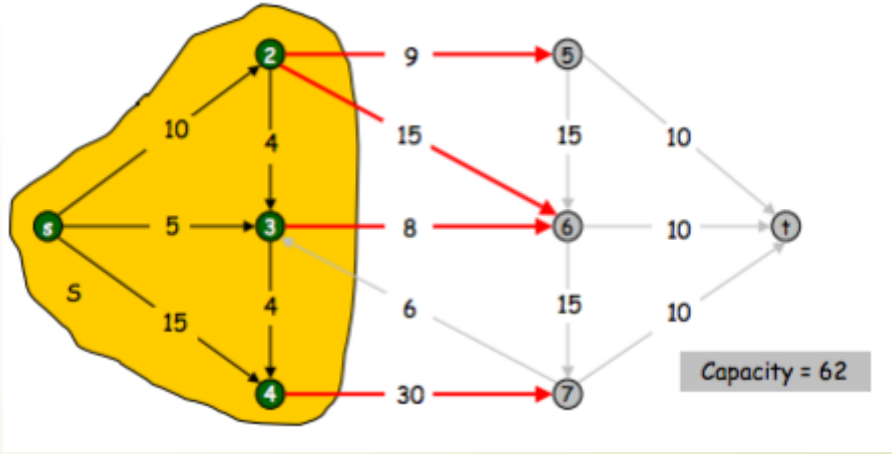
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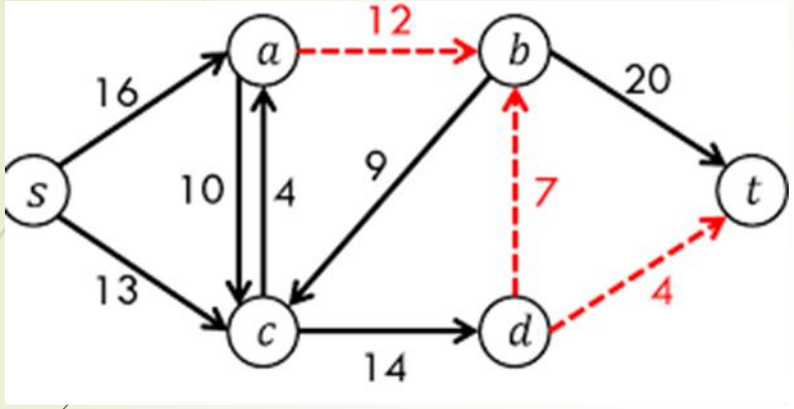
Capacity = 62



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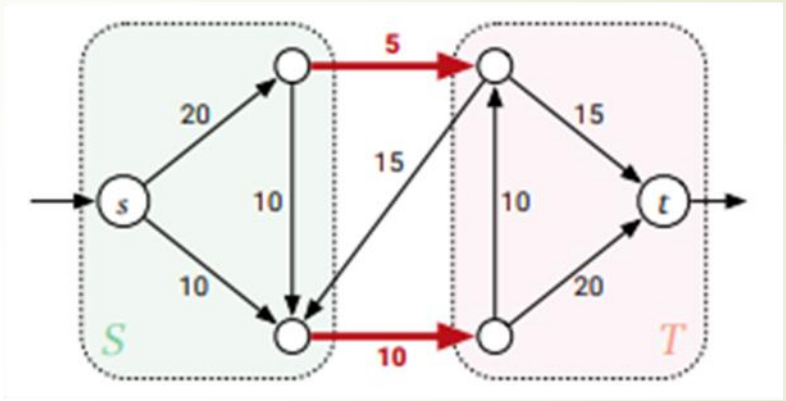

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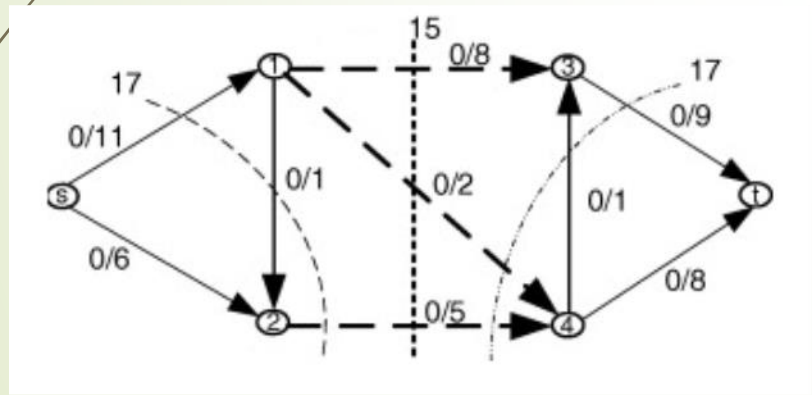

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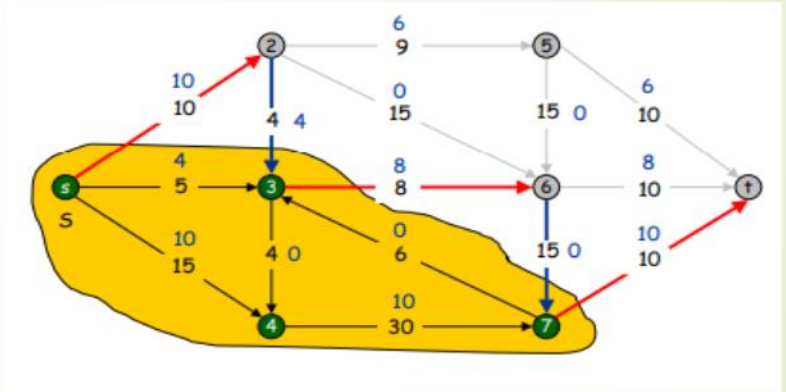
0/8



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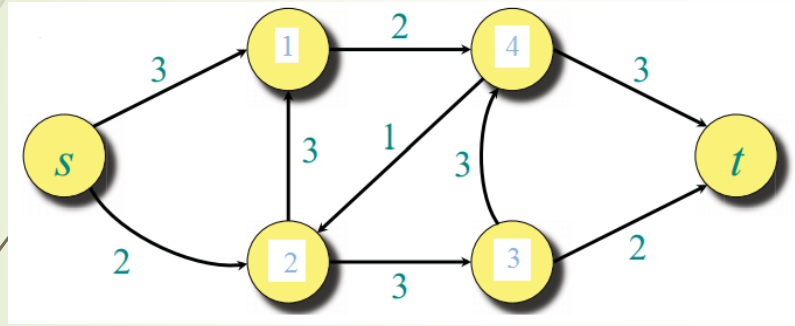
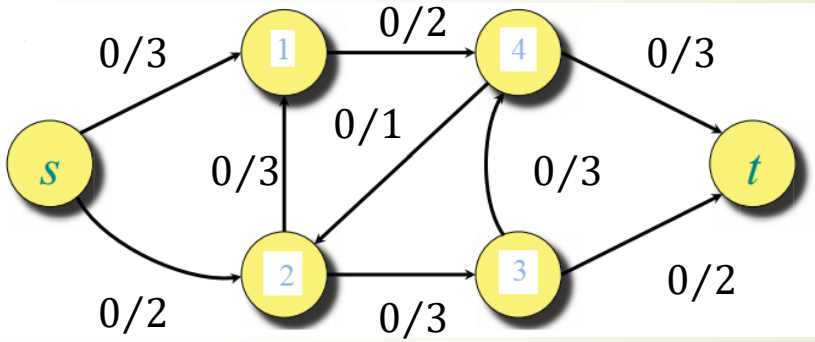
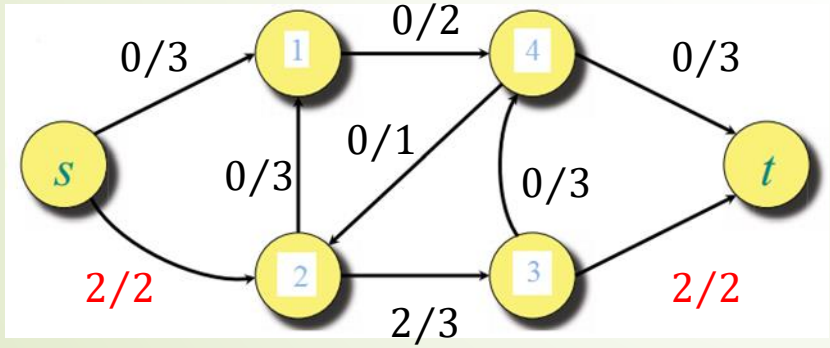
Min-cut Max-Flow Theorem: For any network D, the value of the maximum flow is equal to the capacity of the minimum cut.

Example 1: Find max. flow in the following flow network and determine the corresponding min. cut.

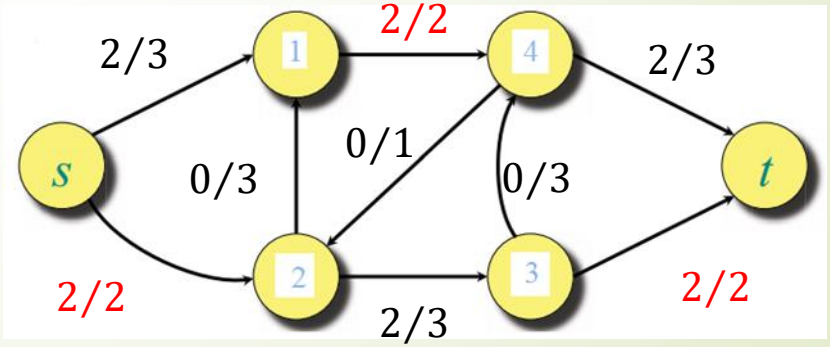




$flow = 0$



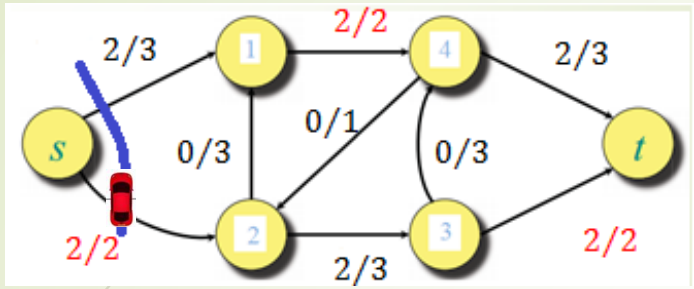
Path 1: s-2-3-t
$flow = 2$

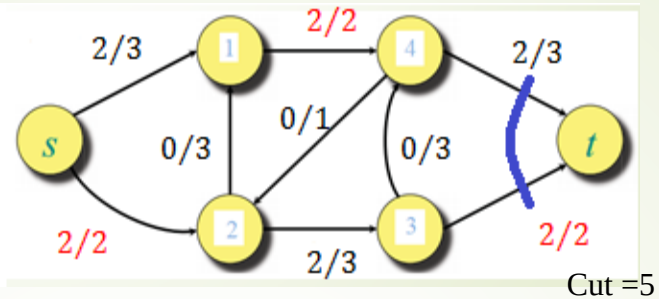


Path 2: s-1-4-t
$flow = 2$

$Max.\,flow = 4$

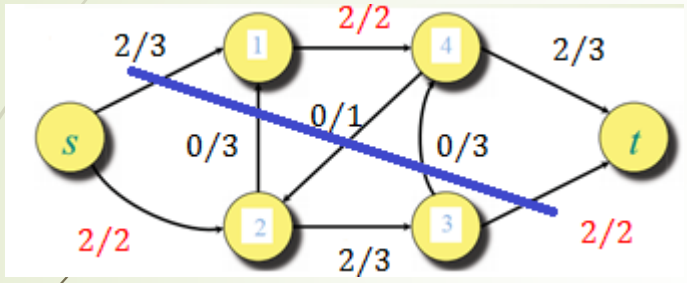
Cut =5

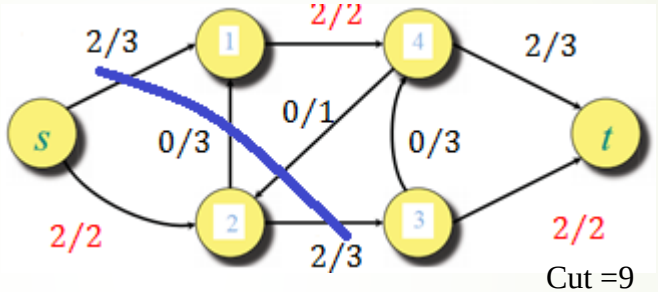




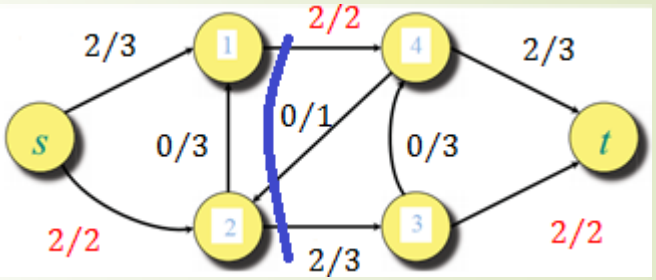
Cut =5

Cut =11



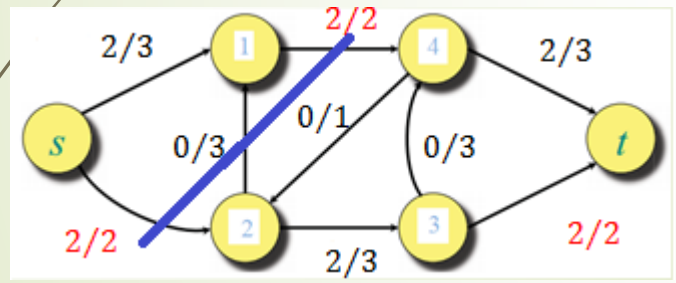


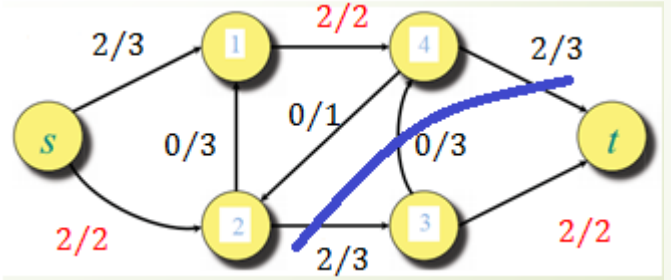
Cut =9



Cut =5

Cut = 4

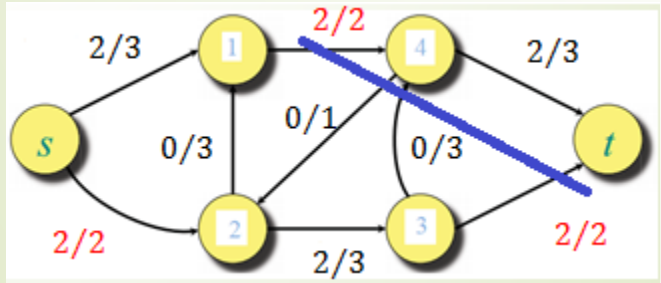


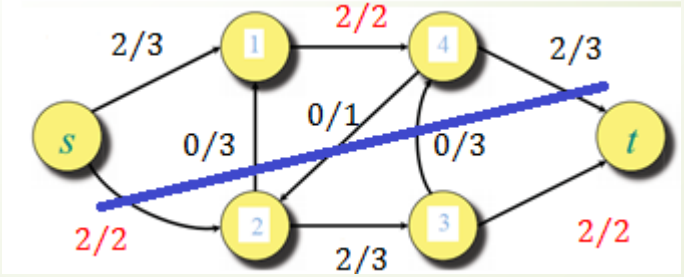


Cut =6

$$Mim.\ Cut = 4 = Max.flow$$

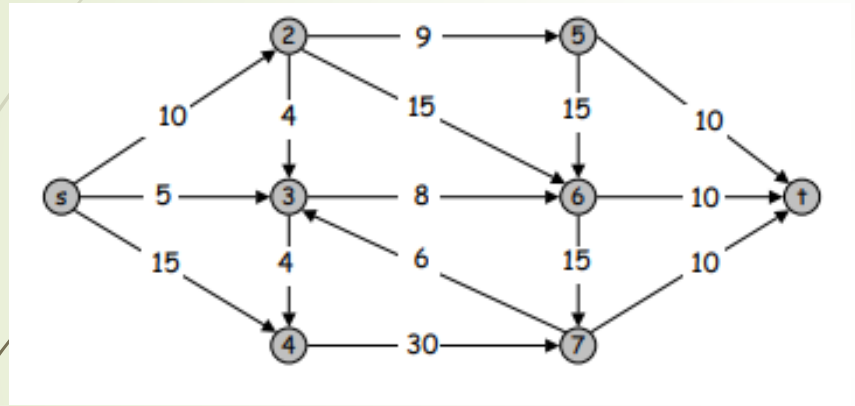
Cut =7

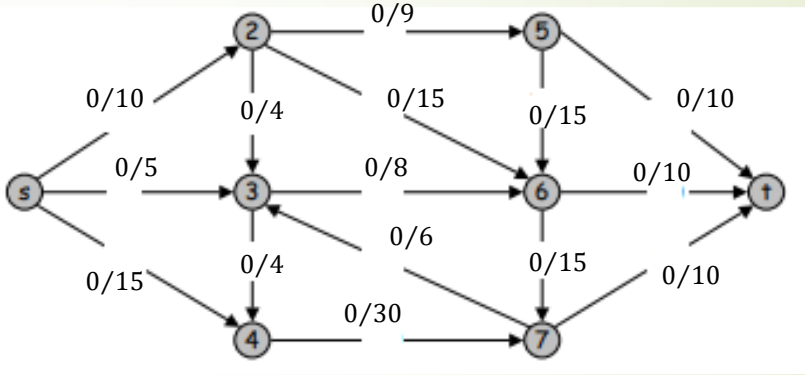




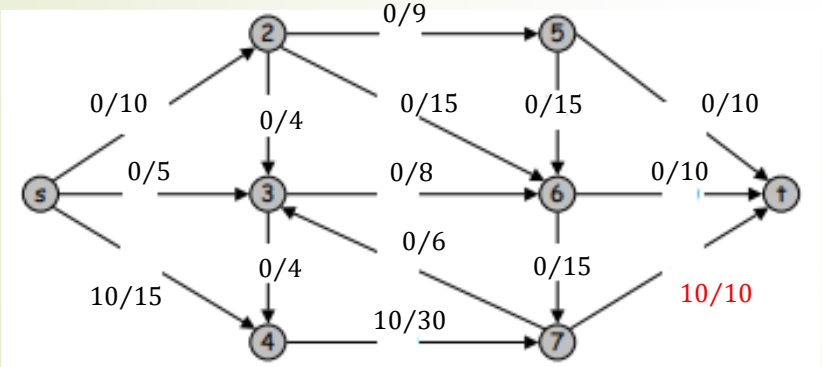
Cut =6

Example 2: Find max. flow in the following flow network and determine the corresponding min. cut.

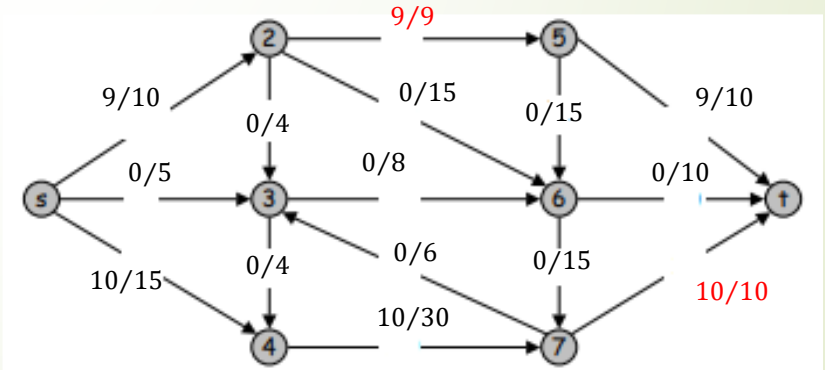




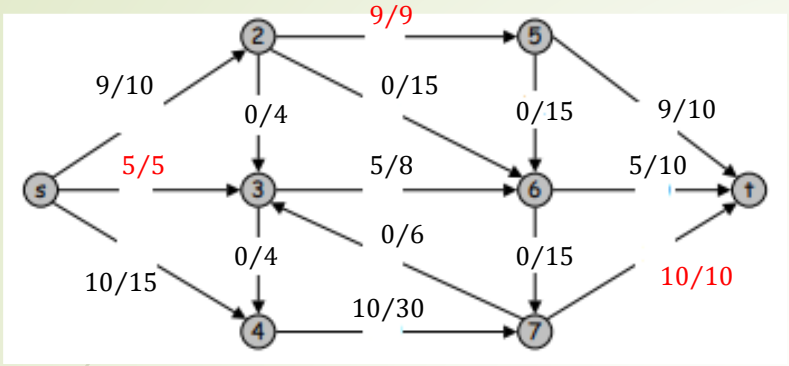
$flow = 0$



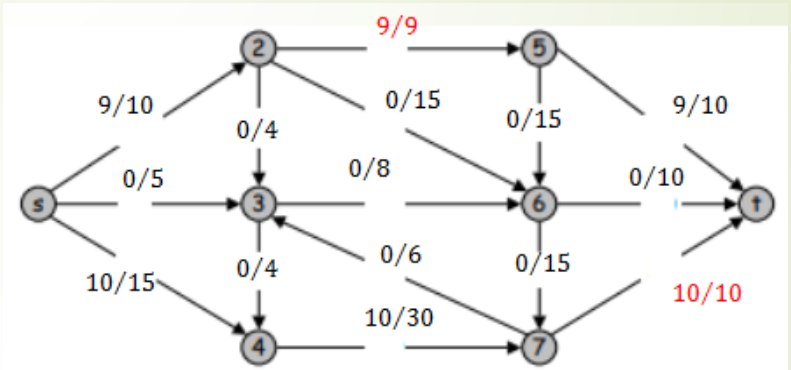
$Path\ 1: s - 4 - 7 - t$
$flow = 10$

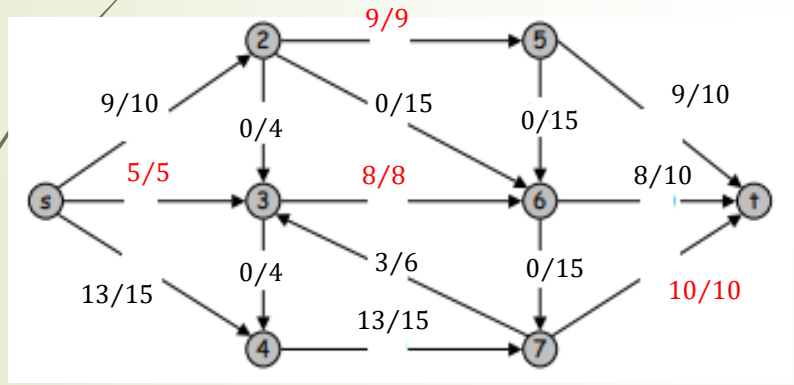


$Path\ 2: s - 2 - 5 - t$
$flow = 9$

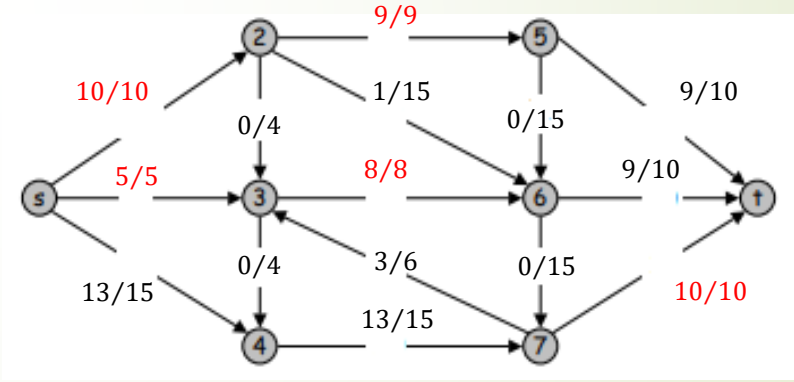


$Path\ 3: s - 3 - 6 - t$
$flow = 5$



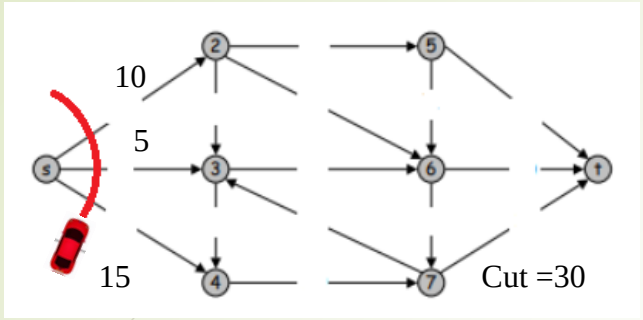


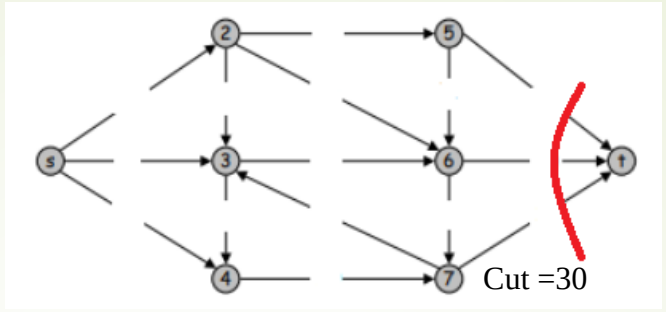
$Path\ 4: s - 4 - 7 - 3 - 6 - t$
$flow = 3$

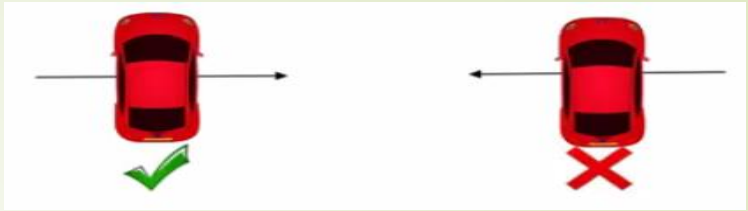


$Path\ 5: s - 2 - 6 - t$
$flow = 1$

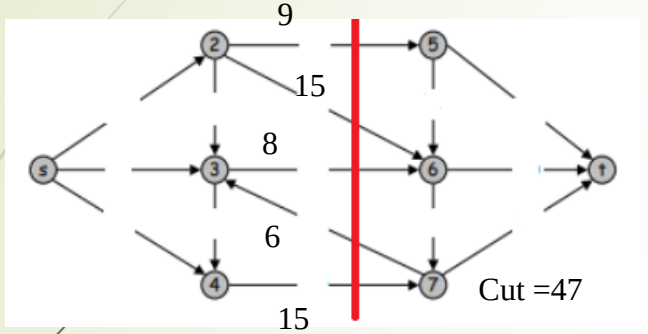
$Max.flow = 10 + 9 + 5 + 3 + 1 = 28$

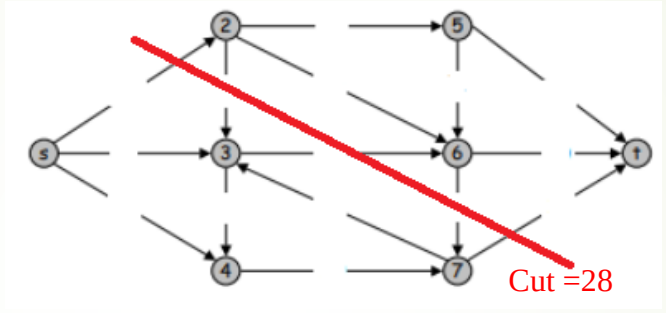


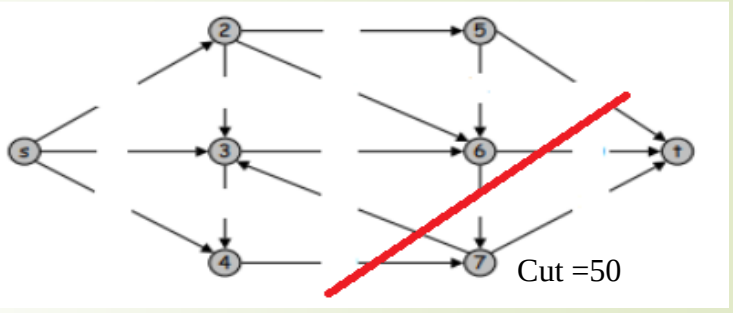


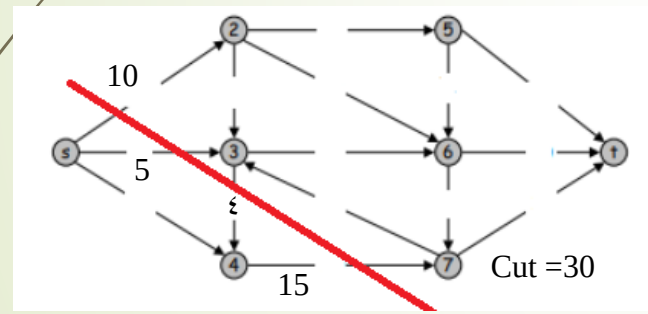


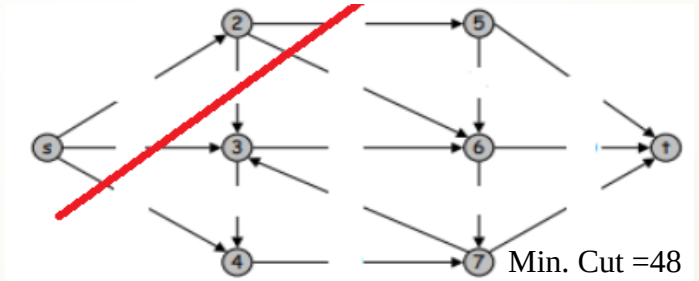
$$Mim.\ Cut = 28 = Max.flow$$

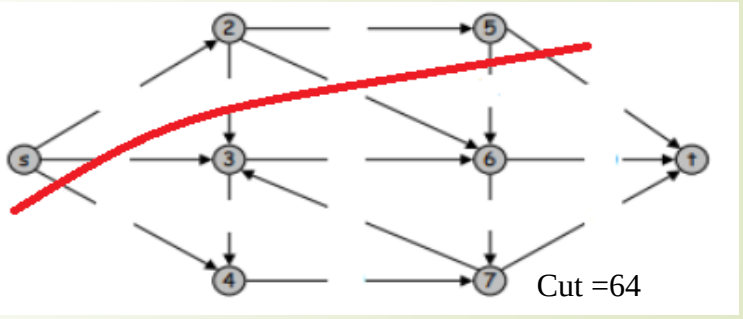


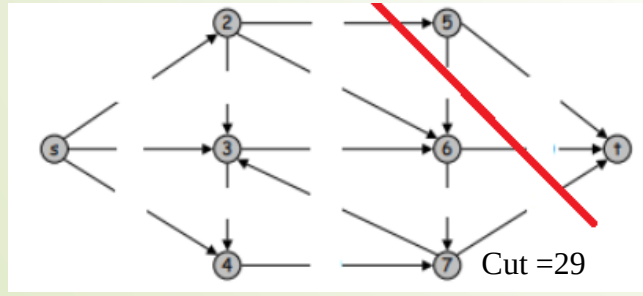


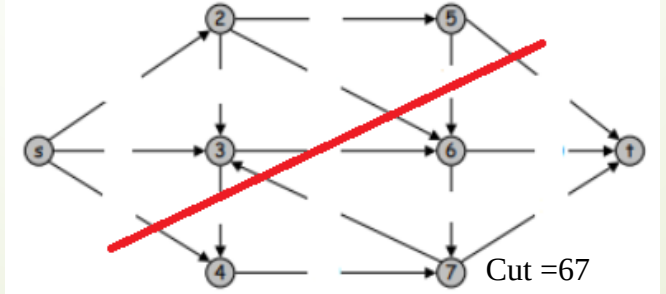


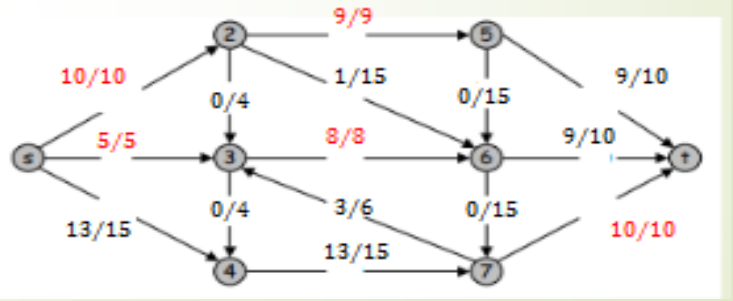












Thank You

References:

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