

جامعة الانبار كلية العلوم قسم الرياضيات نظرية البيانات Types of Trees م. د. امين شامان امين



Lecture (12)

## Types of Trees

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If *T* is a spanning tree or spanning forest of a given graph *G*, and *e* is an edge that does not belong to *T*, then the fundamental cycle  $C_e$  defined by *e* is the simple cycle consisting of *e* together with the path in *T* connecting the endpoints of *e*.

**Definition:** A cycle formed in a graph G by adding a chord of a spanning tree T of G is called a fundamental cycle in G.



**Theorem :** A connected graph *G* is a tree if and only if adding an edge between any two vertices in *G* creates exactly one cycle.

**Theorem :** Adding a chord of a connected graph *G* to the corresponding spanning tree *T* of *G* creates a unique cycle in *G*.

**Definition:** The cyclomatic number or cycle rank of an undirected graph is the minimum number of edges that must be removed from the graph to break all its cycles, making it into a tree or forest.

Clearly, the cyclomatic number of a graph G is equal to the nullity of G.



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This graph has cyclomatic number = 2 because it can be made into a tree by removing two edges, for instance the edges 1-2 and 2-3, but removing any one edge leaves a cycle in the graph. **Theorem:** Any connected graph *G* with *n* vertices and *e* edges has e - n + 1 fundamental cycles.

**Theorem:** Any graph G with n vertices, e edges and k components has e - n + k fundamental cycles.

**Rooted Tree:** A rooted tree is a tree T with a specified vertex  $v_0$ , called the root of T. A vertex v of a rooted tree is called a vertex of level k if the length of the unique path from the root  $v_0$  to the vertex v is k. The largest level number of a rooted tree is called the height of the rooted tree.



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**Definition:** A path length of a rooted tree *T* is the sum of the levels of all pendant vertices.



The path length of the above rooted tree is 2+3+4+5+5 = 19.

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**Definition:** If every pendant vertex  $v_i$  of a tree *T* is assigned some positive real number  $w_i$ , then the weighted path length of *T* is defined as  $\sum_i w_i l_i$ , where  $l_i$  is the level of the vertex  $v_i$  from the root.



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The weighted path length of the above graph is  $2 \times 0.4 + 3 \times 0.5 + 4 \times 0.33$ + 5(0.66 + 0.25) = 8.17

## **Binary Tree:**

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A binary tree is a rooted tree in which there is only one vertex of degree 2 and all other vertices have degree 3 or 1. The vertex having degree 2 serves as the root of a binary tree.



Theorem: The number of vertices in a binary tree is odd.

**Theorem:** A binary tree on *n* vertices has  $\frac{n+1}{2}$  pendant vertices.



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**H. W:** 

- 1) Draw all unlabeled rooted trees on n vertices, where n = 1, 2, 3, 4, 5, 6.
- 2) What is the nullity of a complete graph Kn?



## **References:**

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