جامعة الانبار

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Isomorphic Graphs

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Lecture (13)

Isomorphic Graphs

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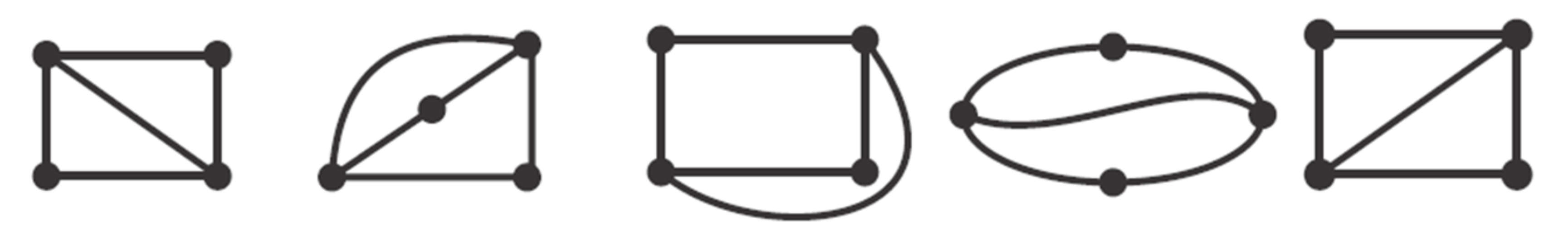
Graph isomorphism is a phenomenon of existing the same graph in more than one form. Such graphs are called an isomorphic graphs .
In graph theory, an isomorphism of graphs G and H is a bijection between the vertex sets of G and H such that any two vertices u and v of G are adjacent in G if and only if $f(u)$ and $f(v)$ are adjacent in H.

Two graphs G_1 and G_2 are said to be isomorphic ($G_1 \cong G_2$) if :
1. Their number of components (vertices & edges) is same.
2. Their edge connectivity is retained.



Necessary Conditions for Two Graphs to be Isomorphic:

For any two graphs G_1 & G_2 to be isomorphic, the following 4 conditions must be satisfied:

- $|V(G_1)| = |V(G_2)|$.
- $|E(G_1)| = |E(G_2)|$.
- Degree sequence of G_1 & G_2 are same.
- If the vertices $\{v_1, v_2, \dots, v_k\}$ form a cycle of length k in G_1, then the vertices $\{f(v_1), f(v_2), \dots, f(v_k)\}$ should also form a cycle of length k in the G_2.

Important Points: The previous 4 conditions are just the necessary conditions for any two graphs to be isomorphic.

- They are not at all sufficient to prove that the two graphs are isomorphic.
- If all the 4 conditions satisfy, even then it can't be said that the graphs are surely isomorphic.
- However, if any condition violates, then it can be said that the graphs are surely not isomorphic.

Sufficient Conditions:

The following conditions are the sufficient conditions to prove that $G_1 \cong G_2$. If any one of these conditions satisfy, then it can be said that the graphs are surely isomorphic.

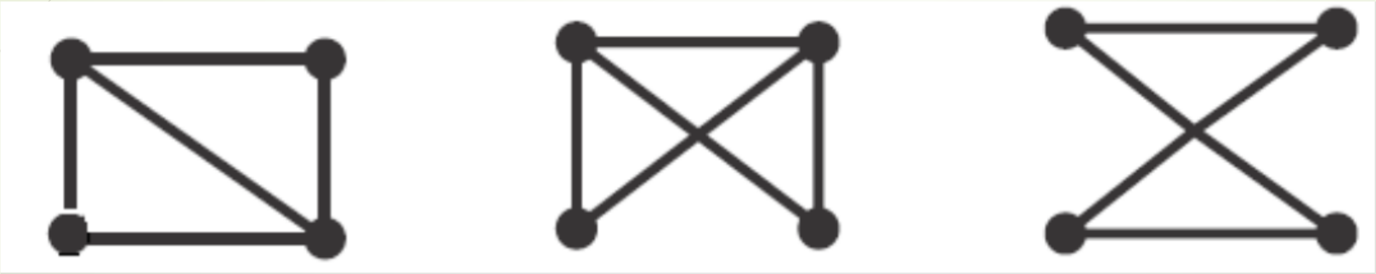
- $G_1 \cong G_2$ iff $\overline{G_1} \cong \overline{G_2}$, where G_1 and G_2 are simple graphs.
- $G_1 \cong G_2$ if their adjacency matrices are permuted equivalent. In other words, assume that $\mathrm{X}(G_1)$ & $\mathrm{X}(G_2)$ are the adjacency matrices of two isomorphic graphs G_1 & G_2, respectively, then there exist $v \times v$ permutation matrix P such that:

  $\mathrm{X}(G_1) = P^{-1}\,\mathrm{X}(G_2)P$.
- $G_1 \cong G_2$ iff their corresponding subgraphs (obtained by deleting some vertices in G_1 and their corresponding images in G_2) are isomorphic.

Example 1: Are the following two graphs isomorphic?

Solution: No, G_1 & G_2 are not isomorphic because G_1 has 5 edges and G_2 has 6 edges.

Example 2: Which of the following graphs are isomorphic?



G_1 G_2 G_3

Solution:

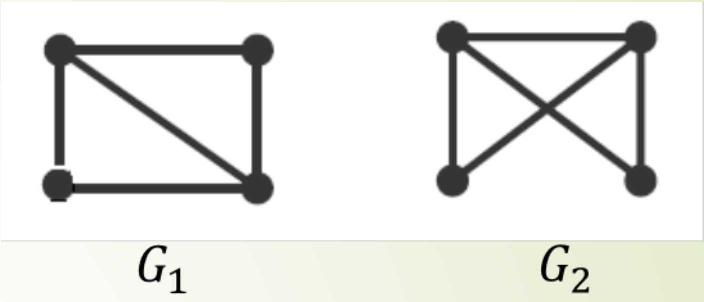
1) $|V(G_1)| = |V(G_2)| = |V(G_3)|$.

2) $|E(G_1)| = |E(G_2)| \neq |E(G_3)|$.

Then, G_3 neither isomorphic to G_1 nor isomorphic to G_2.

Now, let us check G_1 & G_2 .

3)
Degree sequence of G_1 is $\{2,2,3,3\}$.
Degree sequence of G_2 is $\{2,2,3,3\}$.



4) Cycles formed in G_1 are also formed in G_2.

Let us take the complement of G_1 and G_2.

Since , $\overline{G_1} \cong \overline{G_2}$

Thus, $G_1 \cong G_2$.

Example 3: Are the following two graphs isomorphic?

Solution:

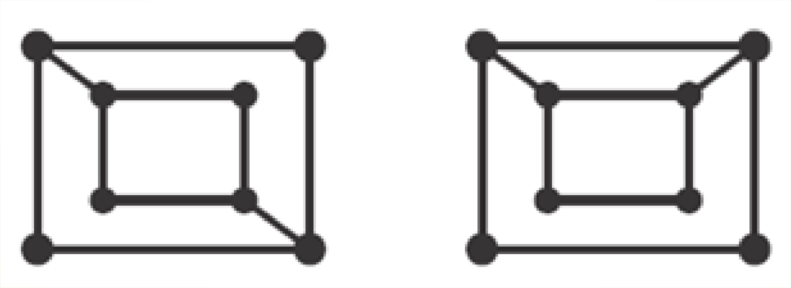
1) $|V(G_1)| = |V(G_2)|$ & $|E(G_1)| = |E(G_2)|$.

2) Degree sequence of G_1 is $\{2,2,2,2,3,3,3,3\}$

Degree sequence of G_2 is $\{2,2,2,2,3,3,3,3\}$

3) In G_2, the vertices of degree 3 form 4- cycle but in G_1 the vertices of degree 3 does not form 4- cycle.

Thus, G_1 & G_2 are not isomorphic to each other.



G_1 G_2

Example 4: Find whether the following graphs are isomorphic.

Solution:

1) $|V(G_1)| = |V(G_2)|$ &$|E(G_1)| = |E(G_2)|$.

2) Degree sequence of G_1 is $\{1,2,3,3,5\}$

Degree sequence of G_2 is$\{1,2,3,3,5\}$

3) Cycle formed in G_1 are also formed in G_2.

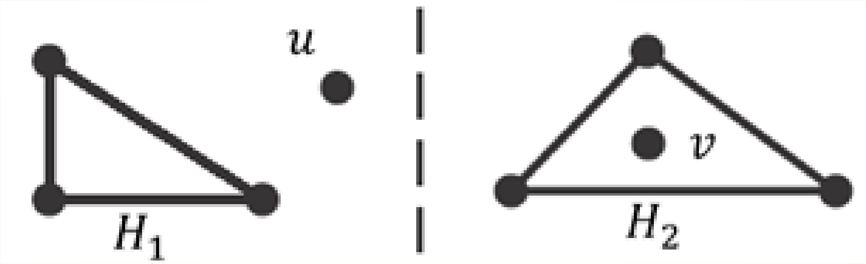
Thus, all necessary conditions are satisfied for isomorphism.

Deleting vertices u & v from corresponding graphs, we get:

Clearly, $H_1 \cong H_2$

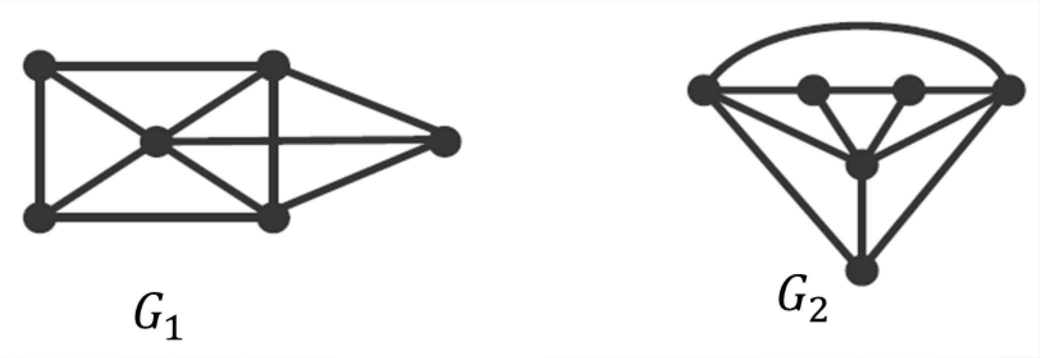
Then, $G_1 \cong G_2$.

Thus, G_1 & G_2 are isomorphic to each other.

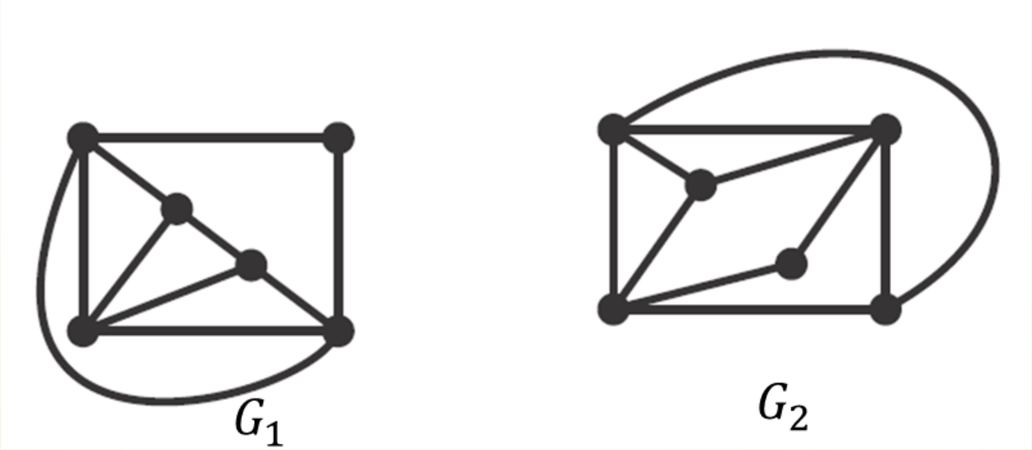


H.W: Find whether the following graphs are isomorphic

1)



2)



Thank You

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