University of Anbar

College of Science

Department of Physics

فيزياء الحالة الصلبة

Solid state Physics

المرحلة الرابعة

الكورس الاول

صلبة 1

8. The free-electron model موديل الالكترون الحر

8.1 Introduction

In the free-electron model, the conduction electrons الكترونات التوصيل are assumed افترض to be لتكون completely free حرة كليا, except ماعدا for a potential الجهد at the surface على السطح, which has الذي يملك the effect تاثير of confining حصر the electrons to the interior داخل of the specimen. According تبعا to this model لهذا الموديل, the conduction electrons move تنتقل about inside بداخل the specimen without بدون any collisions اي تصادم, except for an occasional عرضي reflection انكاس from the surface, much like مشابه جدا the molecules الجزيئات in an ideal gas الغاز المثالي. Because of this, we speak of a free-electron gas غاز الكتروني حر.

One expects الواحد يتوقع the conduction electrons to interact تتفاعل with the ions مع الايونات, and also كذلك with each other مع بعضها البعض. These interactions هذه التفاعلات are strong قوية, and hence وبالتالي the electrons ought to يجب suffer ان تعاني frequent collisions تصادمات متكررة.

The interaction التفاعل between the conduction electrons themselves نفسها, and the reason وسبب for the weakness ضعف of this interaction. There are يوجد actually بالحقيقة two reasons سببان:

First, according وفقا to the Pauli exclusion principle مبداء الاستبعاد لباولي, electrons of parallel spins البرم المتوازي tend to تميل الى stay away الابتعاد from each other عن بعضها البعض. **Second,** even حتى if their spins برمها are opposite متعاكس, electrons tend to تميل الى stay away الابتعاد from each other من بعضها البعض, in order to من اجل minimize تقليل the energy of the system. If two electrons come very close اقتربا to each other من بعضهما, the coulomb potential جهد كولوم energy becomes تصبح exceedingly large كبيرة جدا, and this violates ينهك the tendency ميل of the electron system to have الحصول على the lowest possible energy اقل طاقة ممكنة.

When these two considerations الاعتبارين are carried out تنفذ mathematically رياضيا, Each electron is surrounded محاط by a (spherical كروي) region بمنطقة which is deficient تفتقر of other electrons. This region, called a hole ثقب, has a radius of about 1 ^{o}A. As an electron moves يتحرك, its hole-sometimes known as a Fermi hole ثقب فيرمي-moves with it.

Free-electron gas in metals في المعادن differs يختلف from ordinary gas الغاز العادي in some important respects في بعض الجوانب. **First,** free-electron gas is charged مشحون (in ordinary gases the molecules are mostly neutral متعادلة). Free-electron gas is thus actually similar to a plasma البلازما. **Second,** the concentration تركيز of electrons in metals is large: N= 10^{29} electrons-m^{-3}. By contrast على العكس, the ordinary gas الغاز العادي has about 10^{25} molecules-m^{-3}. We may thus think قد نفكر of free-electron gas in a metal as a dense plasma بازما كثيفة.

8.2 Electrical Conductivity

The law of electrical conduction in metals-Ohm's law- is

$$I = V/R$$

where l is the current, V the potential difference, and R the resistance of the wire. To express this law in a form which is independent غير معتمد of the length and cross section of the wire. Suppose that L and A are, respectively, the length and cross section of the wire; then

$$J = \frac{I}{A} \,, \qquad E = \frac{V}{L} \,, \qquad R = \frac{L\,\rho}{A}$$

where **J** is the current density (current per unit area), the electric field, and $\boldsymbol{\rho}$ the electrical resistivity. The inverse مقلوب of the resistivity is called the conductivity, denoted يرمز لها by σ That is,

$$\sigma = \frac{1}{\rho}$$

Then $$J = \sigma E$$

which is the form صيغة of Ohm's law which we shall use. Since the unit of ρ is ohm-m, σ has the dimension ohm^{-1}-m^{-1}.

The current is due to the motion نتيجة حركة of the conduction electrons under the influence تاثير of the field. Because these particles الجسيمات are charged, their motion leads to تقود الى an electrical current;

Let us now treat the motion of the conduction electrons in an electric field.

Consider نعتبر one typical electron: The field exerts يأثر on the electron a force $-Ee$.

There is also a friction force قوة احتكاك due to نتيجة the collision تصادم of the electron with the rest of the medium مع باقي الوسط. Let us assume that this friction force has the form

$$-m^* v/\tau.$$

The effective mass الكتلة الفعالة of the electron in a metal, denoted by m*, and v is the velocity of the electron and τ is a constant called the collision time زمن التصادم. Using Newton's law, we have

$$m^* \frac{dv}{dt} = -eE - m^* \frac{v}{\tau}$$

where m* is the effective mass of the electron, we see that the effect of the collision, as usual in the forces, tends to reduce the velocity to zero, that is, where $\frac{dv}{dt} = 0$. The appropriate solution

$$v = \frac{e\tau}{m^*} E$$

This, then, is the steady-state velocity سرعة الحالة المستقرة of the electron, It is opposite to E because the charge on the electron is negative

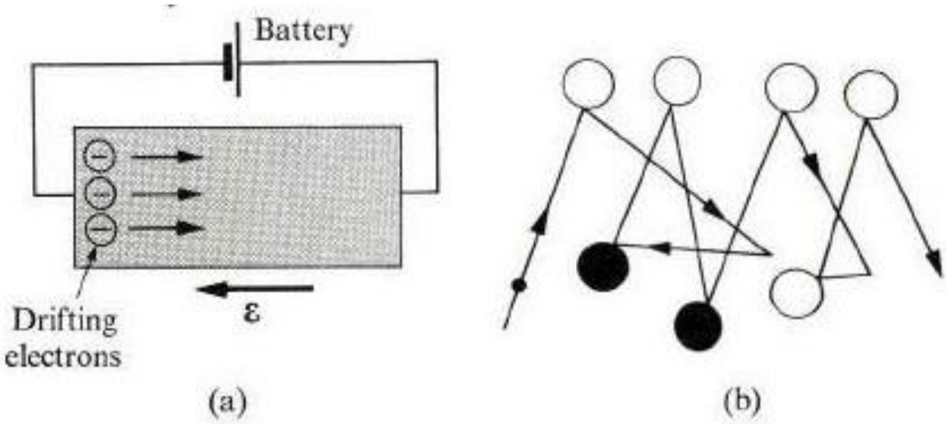

Fig. 8.1 (a) An electric field applied to a metallic wire. (b) Random versus drift motion of electrons.

We should make a distinction نفرق here between the two different velocities associated ترافق with the electron: The velocity appearing in is called the drift الانحراف velocity. This is superimposed يتم فرص on a much higher velocity or speed, known as the random velocity سرعة عشوائية, due to the random motion حركة عشوائية of the electron.

The current density J can be calculated

$$J = (-Ne)\, v_d = (-Ne)[-\frac{e\tau}{m^*}E] = \frac{Ne^2\tau}{m^*}E$$

Since there is a charge (- Ne) per unit volume, the following expression for the conductivity

$$\sigma = \frac{Ne^2\tau}{m^*}$$

we see that o increases as N increases. This is reasonable because, as N (or the concentration) increases, there are more current carriers.

8.3 Heat Capacity Of Conduction Electrons

In the free-electron model the conduction electrons are treated as free particles which obey the classical laws of mechanics, electromagnetism, and statistical mechanics, Let us calculate the heat capacity per mole for the conduction electrons onthe basis of the Drude-Lorentz model.

lt is well known from the kinetic theory of gases that a free particle in equilibrium at temperature T has an average energy of $\frac{3}{2}KT$. Therefore, the average energy per mole is

$$\langle \bar{E} \rangle = N_A(\tfrac{3}{2}kT) = \tfrac{3}{2}RT,$$

where N_A is Avogadro's number and $R = N_A k$. The electrons' heat capacity

$$C_e = \partial[\bar{E}]/\partial T$$

Therefore,

$$C_e = \tfrac{3}{2}R \simeq 3\,\text{cal/mole}\,^\circ\text{K}.$$

The total heat capacity in metals, including phonons, should then be

$$C = C_{ph} + C_e$$

The energy of the electron in a metal is quantized according to quantum mechanics. Figure 8.2 (a) shows the quantum energy levels. The energy of the highest occupied level is called the Fermi energy (or simply the Fermi) level.

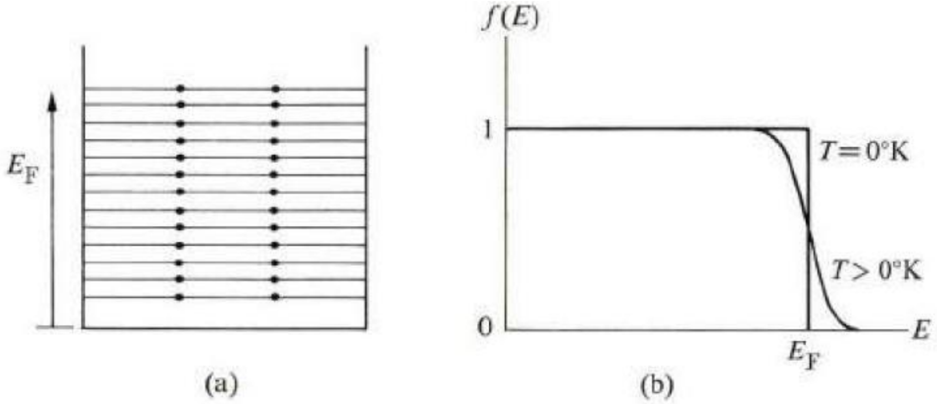

Fig. 8.2 (a) Occupation of energy levels according to the Pauli exclusion principle. (b) The distribution function f(E) versus E, at T = 0^o K and T > 0^{o}K.

The distribution of electrons among the levels is usually described by the distribution function, $f(E)$, which is defined as rhe probability that the level E is occupied by an electron. Thus if the level is certainly empty, then $f(E) = 0$, while if it is certainly full, then f (E)=1. In general, $f(E)$, has a value between zero and unity. The distribution function for electrons at T = 0 K has the form

$$f(E) = \begin{cases} 1, & E < E_F \\ 0, & E_F < E \end{cases}$$

The distribution function f (E) at temperature $T \neq 0$'K is given by

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

This is known as the **Ferrmi-Dirac distributing**,

Since only electrons within the range kT of the Fermi level are excited, we conclude that only a fraction KT/E_f of the electrons is affected. Therefore the number of electrons excited per mole is about $N_A(kT/E_f)$, and since each electron absorbs an energy kT, on the average, it follows that the thermal energy per mole is given approximately by

$$\bar{E} = \frac{N_A (kT)^2}{E_F}$$

and the specific heat $C_e = \partial \bar{E} / \partial T$ is

$$C_e = 2R \frac{kT}{E_F}$$

The so-called Fermi temperature T_f, is defined as $E_f = kT_f$, and the specific heat may now be written as

$$C_e = 2R \frac{T}{T_F}$$

8.4 The Fermi Surface

The electrons in a metal are in a continuous state of random motion. Because

these electrons are considered to be free particles, the energy of an electron is entirely kinetic, and one may therefore write

$$E = \tfrac{1}{2} m^* v^2$$

where v is the speed of the particle. Now let us introduce the concept of velocity space, whose axes are v_x, v_y, and v_z,. Each point in this space represents a unique velocity-both in magnitude and direction.

Consider the conduction electrons in this velocity space. These electrons have many different velocities, and since these velocities are random, the points representing them fill the space uniformly, as shown in figure 8.3

The radius of this sphere is the Fermi speed vf, which is related to the Fermi energy by the usual relation

$$E_F = \tfrac{1}{2} m^* v_F^2$$

The reason why all points outside the sphere are empty is that they correspond to energies greater than E., which are unoccupied at T =0° K, as discussed above. All the points inside the sphere are completely full. This sphere is known as the Fermi sphere, and its surface as the Fermi surface.

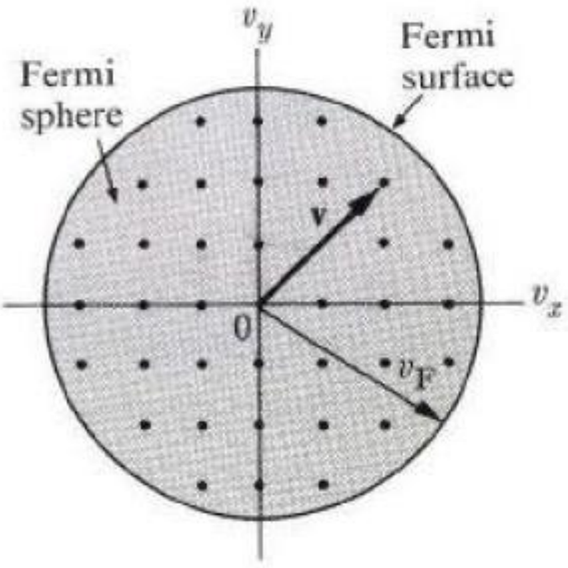

Figure 8.3 The Fermi surface and the Fermi sphere.

The Fermi surface (FS), which is very significant in many solid-state phenomena-for example, transport properties-is not affected appreciably by temperature. When the temperature is raised, only relatively few electrons are excited from the inside to the outside of the Fermi surface.

Furthermore, the Fermi speed, like the Fermi surface, is independent of temperature. The value of the Fermi energy is determined primarily by the electron concentration.

$$E_F = \frac{\hbar^2}{2m^*}(3\pi^2 N)^{2/3}$$

8.5 Electrical Conductivity; Effects Of The Fermi Surface

We discussed electrical conductivity in Section 8.2, in which we treated electrons on a classical basis. How are the results modified when the FS is taken into account?

Let us refer to Fig. 8.4. In the absence of an electric field, the Fejmi sphere is centered at the origin (Fig. 8.4 a). The various electrons are all moving-some at very high speeds-and they carry individual currents, But the total current of the system is zero, because, for every electron at velocity v there exists another electron with velocity - v, and the sum of their two currents is zero.

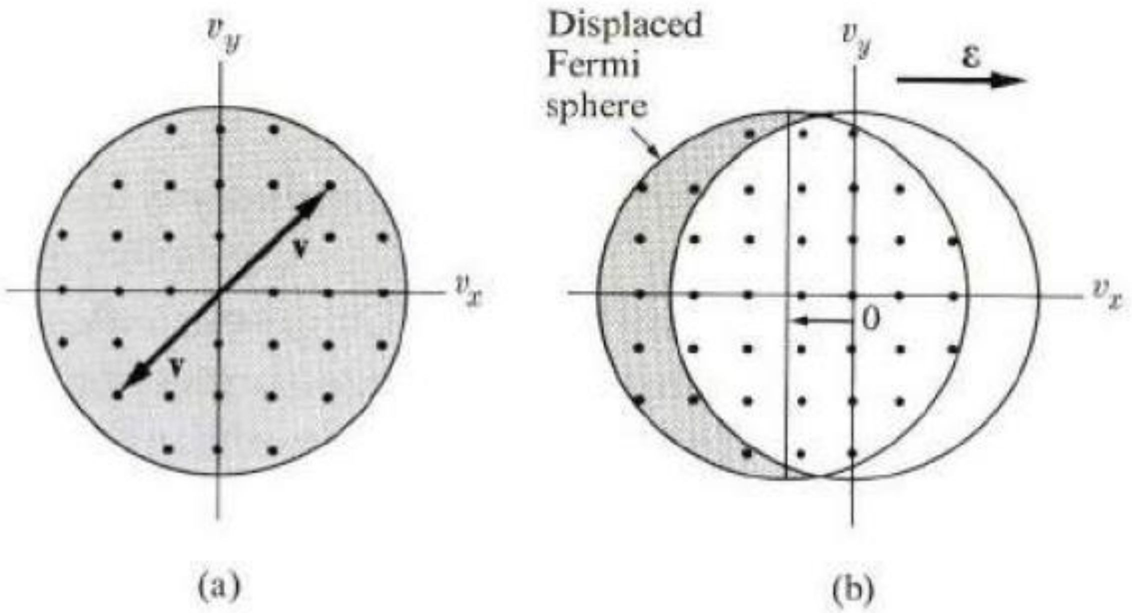

Figure 8.4 (a) The Fermi sphere at equilibrium. (b) Displacement of the Fermi sphere due to an electric field.

Let us estimate the current density

$$J \simeq -e N (v_d/v_F)(-v_F) = N e v_d$$

which, on substitution of $v_d = -(e\tau/m^*)\mathscr{E}$, yields

$$J = \frac{N e^2 \tau_F}{m^*} \mathscr{E}$$

where τ_F is the collision time of an electron at the FS. The resulting electrical conductivity is therefore

$$\sigma = \frac{N e^2 \tau_F}{m^*}$$

If we substitute $\tau_F = l_F/v_F$

$$\sigma = \frac{Ne^2 l_F}{m^* v_F}.$$

The only quantity on the right side which depends on temperature is the mean free path l_F. Since $l_F \sim 1/T$ at high temperature, , it follows that $\sigma \sim 1/T$.

References

1- Charles Kittel - Introduction to Solid State Physics-Wiley (2005)
2- J. S. Blakemore - Solid State Physics-Cambridge University Press (1985)
3- M. A. OMAR Elementary-solid-state-physics