University of Anbar
College of Science
Department of Physics

فيزياء الحالة الصلبة

Solid state Physics

المرحلة الرابعة

الكورس الاول

صلبة 1

9 The free-electron model موديل الالكترون الحر

9.1 Thermal Conductivity In Metals

When the ends of a metallic wire are at different temperatures, heat flows from the hot to the cold end. The basic experimental fact is that the heat current Q-that is the amount of thermal energy crossing a unit area per unit time, is proportional to the temperature gradient,

$$Q = -K \frac{dT}{dx}$$

where K is the thermal conductivity. In insulators, heat is carried entirely by phonons, but in metals heat may be transported by both electrons and phonons. The conductivity K is therefore equal to the sum of the two contributions,

$$K = K_e + K_{ph}$$

where K_e and K_{ph}, refer to electrons and phonons, respectively. In most metals, the contribution of the electrons greatly exceeds تفوق that of the phonons, because of the great concentration of electrons; typically $K_{ph} = 10^{-2} K_e$.

The conductivity of the phonons will be ignored تهمل in this section.

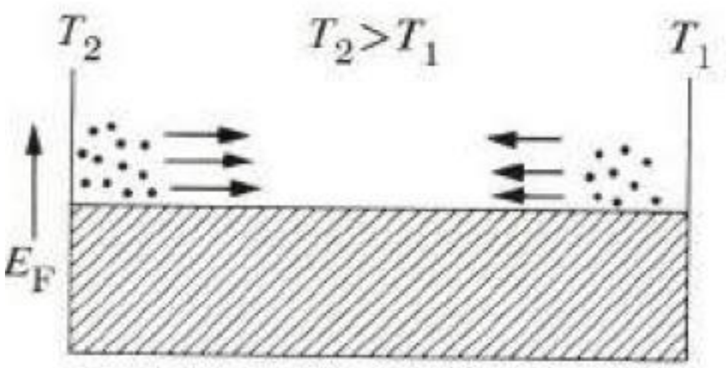


Fig. 9.1 The physical basis for thermal conductivity. Energetic electrons on the left carry net energy to the right.

The physical process by which heat conduction takes place via electrons is illustrated in Fig. 9.1. Electrons at the hot end (to the left) travel in all directions, but a certain fraction travel to the right and carry energy to the cold end. Similarly, a certain fraction of the electrons at the cold end (on the right) travel to the left and carry energy to the hot end.

These oppositely traveling electron currents are equal, but because those at the hot end are more energetic on the average than those on the right, a net energy is transported to the right, resulting in a current of heat.

To evaluate the thermal conductivity K quantitatively, we use the formula

$$k = \frac{1}{3} C_v v l$$

we recall that C_v is the specific heat per unit volume, v the speed, and l the mean free path of the particles involved.

Fermi levels are effective. Thus

$$K = \frac{1}{3}\left(\frac{\pi^2 N k^2 T}{2E_F}\right) v_F \, l_F$$

Noting that

$$E_F = \frac{1}{2} m^* v_F^2$$

and that $l_F / v_F = \tau_F$, we can simplify this expression for K to

$$K = \frac{\pi^2 N k^2 T \tau_F}{3m^*}$$

Many of the parameters appearing in the expression for K were also included in the expression for electrical conductivity σ. Recalling that

$$\sigma = N e^2 \tau_F / m^*$$

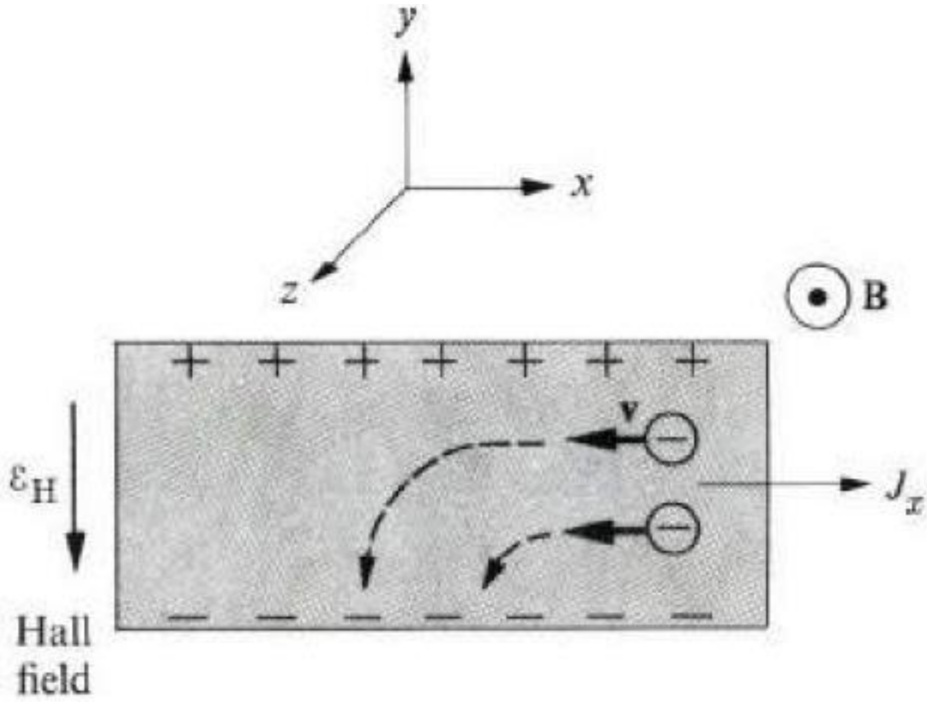
the ratio $K/\sigma T$ is given by

$$L = \frac{1}{3}\left(\frac{\pi k}{e}\right)^2$$

This Lorenz number L, because it depends only on the universal constants k and e.

9.2 The Hall effect

The physical process underlying the Hall effect is illustrated in Fig. 9.2 Suppose that an electric current J_x is flowing in a wire in the *x*-direction, and a magnetic field B_z is applied normal to the wire in the *z*-direction. We shall show that this leads to an additional electric field, normal to both J_x and B_z that is, in the *y*-direction.



let us first consider the situation before the magnetic field is applied. There is an electric current flowing in the positive x-direction, which means that the conduction electrons are drifting with a velocity *v* in the negative *x*-direction. When the magnetic field is applied, the Lorentz force F = e(*v* x B) causes the electrons to bend downward تنحرف نحو الاسفل, as shown in the figure. As a result, electrons accumulate تتجمع on the lower surface, producing تنتج a net negative charge there. A net positive charge appears on the upper surface, because of the deficiency نقص of electrons there. This combination تجمع of positive and negative surface charges creates يخلق a downward electric field, which is called the Hall field.

Let us evaluate this Hall field. The Lorentz force F_L which produces the charge accumulation تجمع in the first place is in the negative y-direction, and has the value

$$F_L = ev_x B$$

The accumulation process عملية التجمع continues تستمر until the Hall force completely cancels تلغي تماما the Lorentz force. Thus, in the steady state حالة الاستقرار, $F_H = F_L$

$$-eE_H = -ev_x B \quad or \quad E_H = v_x B$$

which is the Hall field

the velocity v_x, is expressed in terms of the current density

$$J_x = N(-e)v_x$$

This leads to

$$E_H = -\frac{1}{Ne} J_x B$$

The Hall field is thus proportional both to the current and to the magnetic field. The proportionality constant-that is, $E_H/J_x B$ is known as the Hall constant and is usually denoted by R_H. Therefore

$$R_H = -\frac{1}{Ne}$$

This result is a very useful one in practice. Since R_H is inversely proportional to the electron concentration N, it follows that we can determine N by measuring the Hall field. Another useful feature of the Hall constant is that its sign depends on the sign of the charge of the current carriers.

9.3 Failure Of The Free-Electron Model

The model is only an approximation, and as such has its limitations. Consider the following points.

a) The model suggests that, other things being equal, electrical conductivity is proportional to electron concentration.
b) The fact that some metals exhibit positive Hall constants, for example, Be, Zn, Cd. While بينما the free-electron model always predicts يتوقع a negative Hall constant.
c) Measurements of the Fermi surface indicate تشير الى that it is often non spherical غير كروي in shape. This contradicts يناقض the model, which predicts a spherical كروي FS سطح فيرمي.

Example 9.1. The atomic radius of sodium is 1.86 A° . Calculate the Fermi energy of sodium at absolute zero.

Solution: The Fermi energy at oK is given by the relation

$$E_{Fo} = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$$

where N is the number of free (valence) electrons present in volume V of the metal. We consider a unit cell of sodium which is *bcc*.

Radius of the sodium atom, $r = 1.86 \times 10^{-10}$m

Volume of the unit cell, $V = a^3 = \left(\frac{4r}{\sqrt{3}}\right)^3$

$$= \left(\frac{4 \times 1.86 \times 10^{-10}}{\sqrt{3}}\right)^3$$

$$= 7.93 \times 10^{-29}\ \text{m}^3$$

Since an atom of sodium has only one valence electron, the number of free electrons in a unit cell of sodium is 2.

$$\frac{N}{V} = \frac{2}{7.93 \times 10^{-29}} = 2.52 \times 10^{28}\ \text{electrons/m}^3.$$

$$E_{Fo} = \frac{\left(1.05 \times 10^{-34}\right)^2}{2 \times 9.1 \times 10^{-31}}\left(3\pi^2 \times 2.52 \times 10^{28}\right)^{2/3}$$

$$= 4.98 \times 10^{-19}\ \text{J}$$

$$= 3.11\ \text{eV.}$$

References المصادر المعتمدة في هذا الكورس

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2- J. S. Blakemore - Solid State Physics-Cambridge University Press (1985)

3- M. A. OMAR Elementary-solid-state-physics