طريقه المعادلات المتجانسه

Def/ Afunction f(x,y) is called homogeneous of order n iff

$$f(\lambda x,\lambda y) = \lambda^n f(x,y)$$

Ex/ f(x,y)=$2y^3 e^{y/x} - \frac{x^4}{x+3y}$

f(λx, λy)=$2\lambda^3 y^3 e^{\lambda y/x\lambda} - \frac{\lambda^4 x^4}{\lambda x+3\lambda y}$

f(λx, λy)=$2\lambda^3 y^3 e^{y/x} - \frac{\lambda^3 x^4}{x+3y}$

$\lambda^3\left[2y^3 - \frac{x^4}{x+3y}\right] = \lambda^3 f(x,y)$ المتجانسه من الدرجه الثالثه

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Ex/ $f(x,y) = \frac{x^4}{yx^3+y^4} + ln\frac{x^2+y^2}{x^2-y^2} + tan^{-1}\frac{x}{y}$

$$f(\lambda x,\lambda y) = \frac{\lambda^4 x^4}{\lambda y\lambda^3 x^3+\lambda^4 y^4} + \ln\frac{\lambda^2 x^2+\lambda^2 y^2}{\lambda^2 x^2-\lambda^2 y^2} + \tan^{-1}\frac{x\lambda}{y\lambda}$$

$$= \frac{\lambda^4 X^4}{\lambda^4(yX^3+y^4)} + \mathrm{Ln}\frac{\lambda^2(X^2+y^2)}{\lambda^2(X^2+y^2)} + \tanh^{-1}\frac{\lambda y}{\lambda X}$$

$$= \frac{X^4}{yX^3+y^4} + \mathrm{Ln}\frac{X^2+y^2}{X^2-y^4} + \tanh^{-1}\frac{y}{X}$$

$= \lambda^0 f(x,Y) . \Longrightarrow$ Homogeneous of ordr (0)

ويمكن القول ايضا ان المعادله التفاضليه متجانسه اذا امكن كتابتها على الشكل التالي

$$= \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

و هذا الشكل ممكن ان نحصل عليه بقسمة حدود المعادله على اكبر اس ل المتغير X و نحل المعادله نفرض ان

$$\frac{y}{x} = V \Longrightarrow y = VX \Longrightarrow \frac{dy}{dX} = V + X\frac{dV}{dX}$$

و بتعويض هذه القيم بالمعادله الاصليه نحصل على

$$V + X\frac{dV}{dx} = f(V) \Longrightarrow X\frac{dV}{dX} = f(V) - V \Longrightarrow \frac{dV}{f(v)-v} = \frac{dX}{X} \quad .... (*)$$

المعادله (*) معادله مفصولة المتغيرات يتم حلها بالطريقه السابقة (فصل المتغيرات)

Example 1 : $(X^2 + 3y^2)\,dX - 2Xy\,dy = 0$

$$\Rightarrow (X^2 + 3y^2)\,dx = 2Xy\,dy \Rightarrow \frac{dy}{dX} = \frac{X^2}{2Xy} \div X^2$$

$$\Rightarrow \frac{\frac{X^2}{X^2} + 3\frac{y^2}{X^2}}{2\frac{Xy}{X^2}} = \frac{1+3(\frac{y}{x})^2}{2(\frac{y}{x})}$$

$$Let\ \frac{y}{x} = v \Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}.$$

وبتعويض هذه القيم بالمعادلة الأصلية نحصل على :

$$v + x\frac{dv}{dx} = \frac{1+3v^2}{2v} \Rightarrow x\frac{dv}{dx} = \frac{1+3v^2}{2v} - v$$

$$= \frac{1+3v^2-2v^2}{2v} = \frac{1+v^2}{2v}.$$

$$\Rightarrow xdv = \left(\frac{1+v^2}{2v}\right)dx \Rightarrow \frac{2v\,dv}{1+v^2} = \frac{dx}{x}$$

$$\Rightarrow Ln(1+v^2) = Lnx + Lnc \Rightarrow Ln(1+v^2) - Lnx = Lnc$$

$$\Rightarrow Ln\left(\frac{1+v^2}{x}\right) = Lnc$$

$$\frac{1+v^2}{x} = c \Rightarrow 1 + v^2 = cx \Rightarrow 1 + \frac{y^2}{x^2} = cx$$

$$\Rightarrow x^2 + y^2 - cx^3 = 0 \qquad [General\ Solution]$$

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Example $\boxed{2}$:- $\boldsymbol{\frac{dy}{dx} = \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}}}$

$$Let\ \frac{y}{x} = v \Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + \frac{dv}{dx}$$

وبتعويض هذه القيم بالمعادلة الأصلية نحصل على :

$$v + x\frac{dv}{dx} = v + \sqrt{1+v^2} \implies xdv = \sqrt{1+v^2}\,dx$$

$$\implies \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}\ .$$

$$Let\ \ v = tan\theta \implies dv = sec^2\theta\ d\theta$$

$$\sqrt{1+v^2} = \sqrt{1+tan^2\theta} = \sqrt{sec^2\theta} = \sec\theta.$$

$$\implies \int \frac{dv}{\sqrt{1+v^2}} = \int \frac{sec^2\theta\ d\theta}{\sec\theta} = \int \sec\theta\ d\theta$$

$$= Ln|\sec\theta + \tan\theta| + Ln\ c$$

$$\because v = \tan\theta \implies \sec\theta = \sqrt{1+v^2}$$

$$\implies \int \frac{dv}{\sqrt{1+v^2}} = Ln\left|\sqrt{1+v^2} + v\right| + Ln\ c$$

$$\implies \left|\sqrt{1+v^2} + v\right| = x\ c \implies \sqrt{1+\frac{y^2}{x^2}} + \frac{y}{x} = x\ c\ .$$

$$\implies \sqrt{\frac{x^2+y^2}{x^2}} + \frac{y}{x} = c\ x$$

$$\implies \frac{1}{x}\sqrt{x^2+y^2} + \frac{y}{x} = c\ x \implies \sqrt{x^2+y^2} + y - c\ x^2 = 0$$