University of Anbar

College of Science

Department of Applied Geology

Fourth Year

Electromagnetics

جامعة الانبار

كلية العلوم

قسم علوم الفيزياء

المرحلة الرابعة

الكهرومغناطيسية

Electrical Field

Part Three: Gauss Law

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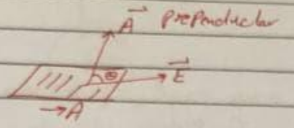
Part Three in this Chapter: Gauss Law

Gauss's Law :-

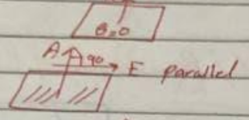
1) Flat surface



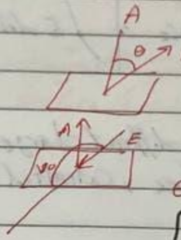
$\phi = \vec{E} \cdot \vec{A}$

$= EA\cos\theta$ ✓ Scaler quantity] القانون العام

$\phi_{max} = EA$ when $\cos\theta = 1$



$\phi = 0$



$\theta < 90$

$\cos \Rightarrow +$

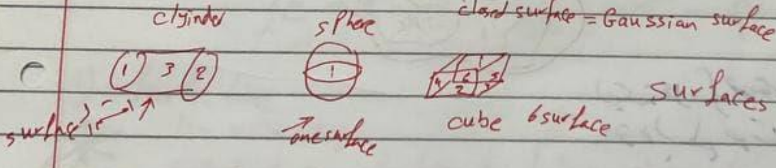
$\boxed{\phi = +}$. E outward

$\theta > 90$ $\cos\theta$ -ve

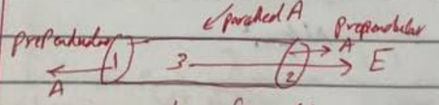
$\boxed{\phi = -}$ inward

Flux closed surface: $\phi = \oint E \cdot dA$

closed surface = Gaussian surface



$\phi = \oint E \cdot dA$



$\phi = \oint E \cdot dA$

$= \int_1 E \cdot dA + \int_2 E \cdot dA + \int_3 E \cdot dA$

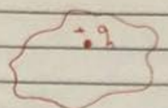
$\cos \theta = 180$; $\theta = \text{zero}$; zero

$= -\int_1 E dA + \int_2 E dA = -E\int dA + E\int dA$

$= -EA + EA = \text{zero}$

$= \phi = \text{zero}$

Gauss Law :-



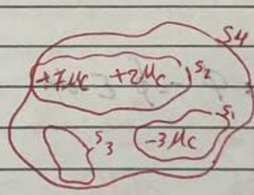
close surface (in 3 ~~dim~~ Dimension)
⟸ ArBitary
شكل السطح كيفي
3 D

$\phi = \oint E\,dA$
$= \frac{q}{\epsilon_0}$

ϵ_0 Constant / Permittivity of free space

$\therefore$ ϕ in close surface $= \boxed{\int E\,dA = \frac{q}{\epsilon_0}}$

Example :- Three diffrent charges in closed surface consist of Three 3 closed surface , Calculted ϕ_{total} ?



Note: [illegible]
[illegible]

$\therefore$

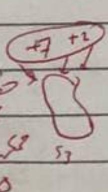
$\phi_{s_1} = \frac{q}{\epsilon_0} = \frac{-3\times10^{-6}}{8.85\times10^{-12}} =$ $\boxed{N/cm^2}$

$\phi_{s_2} = \frac{q}{\epsilon_0} = \frac{(7+2)\times10^{-6}}{\epsilon_0} =$ N/cm^2

$\phi_{s_3} =$ Zero بسبب عدم وجود شحنة

$\phi_{s_4} = \frac{q}{\epsilon_0} = \frac{(2+7-3)\times10^{-6}}{\epsilon_0}$

ملاحظة /



[illegible]

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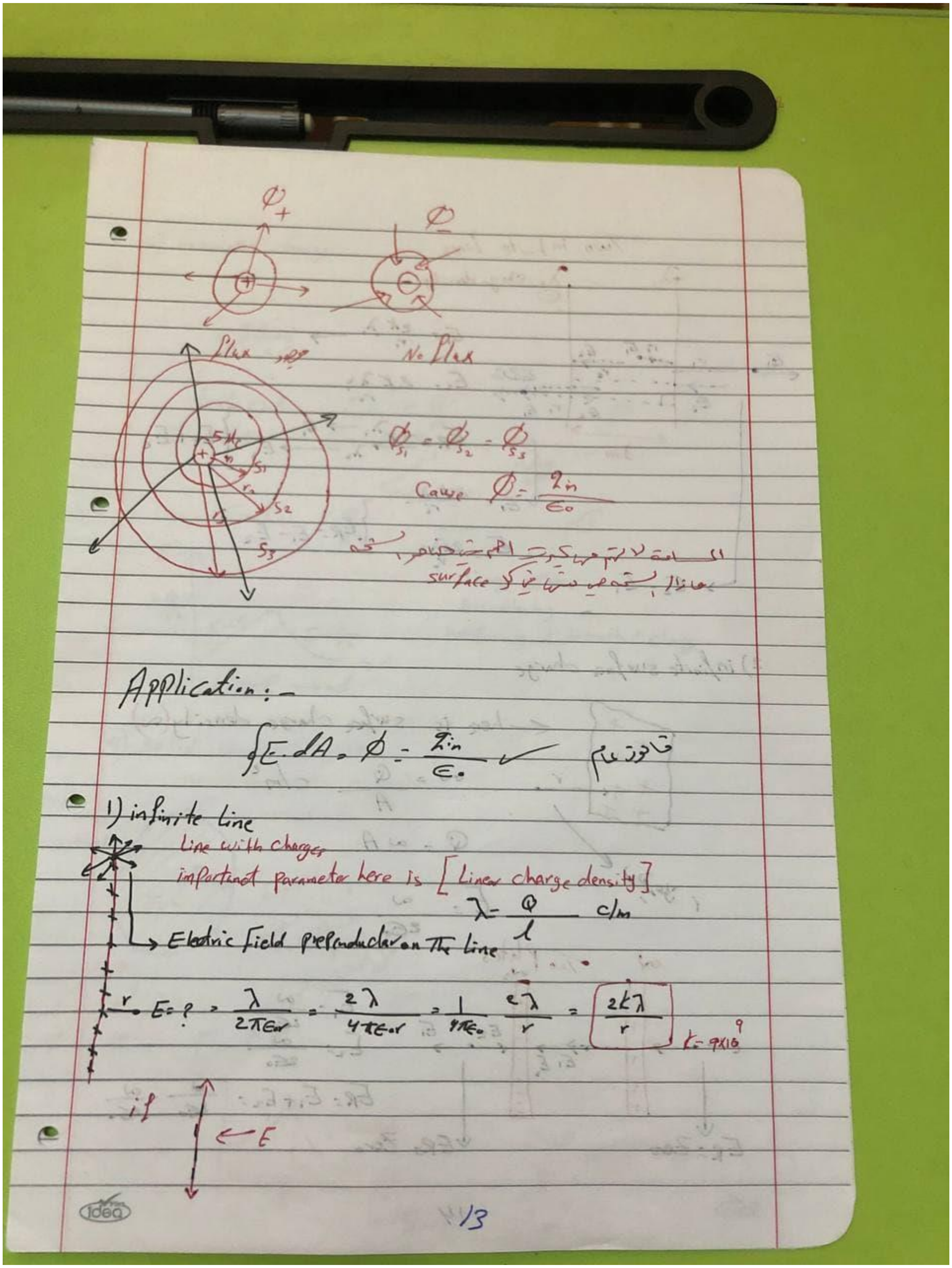

Flux برة

No Flux

$\phi_{s_1} = \phi_{s_2} = \phi_{s_3}$

Cause $\phi = \frac{q_{in}}{\epsilon_0}$

المساحة لا تهم مهما كبرت ام صغرت، لأنه
عادةً يتم قسمتها في كل surface

Application:-

$$\oint E \cdot dA = \phi = \frac{q_{in}}{\epsilon_0}$$ ✓ قانون عام

1) infinite Line

Line with charges

important parameter here is [Linear charge density]

$$\lambda = \frac{Q}{l} \quad C/m$$

→ Electric field prependuclar on The line

$$E = ? = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2\lambda}{4\pi\epsilon_0 r} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r} = \boxed{\frac{2k\lambda}{r}}$$ $k = 9\times10^9$

if

$\leftarrow E$

ideа

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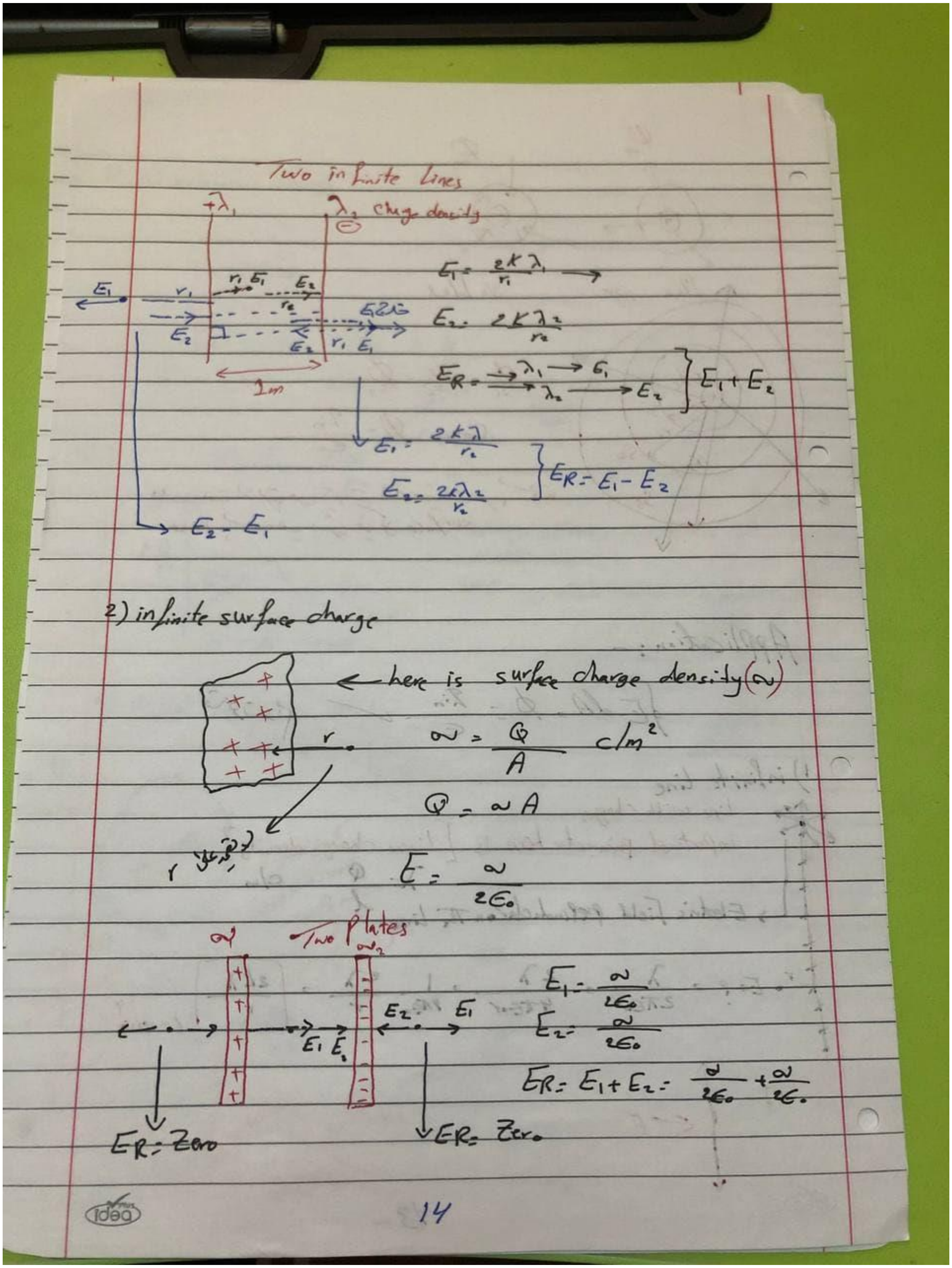


Two infinite lines

$+\lambda_1$

λ_2 charge density $\ominus$

$E_1 = \frac{2k\lambda_1}{r_1} \longrightarrow$

$E_2 = \frac{2k\lambda_2}{r_2}$

$$\left.\begin{aligned} E_R = \lambda_1 &\longrightarrow E_1 \\ \lambda_2 &\longrightarrow E_2 \end{aligned}\right\} E_1 + E_2$$

$$\left.\begin{aligned} E_1 &= \frac{2k\lambda}{r_2} \\ E_2 &= \frac{2k\lambda_2}{r_2} \end{aligned}\right\} E_R = E_1 - E_2$$

$\rightarrow E_2 - E_1$

2) infinite surface charge

← here is surface charge density (σ)

$\sigma = \frac{Q}{A}$ c/m^2

$Q = \sigma A$

$E = \frac{\sigma}{2\epsilon_0}$

Two plates

$E_1 = \frac{\sigma}{2\epsilon_0}$

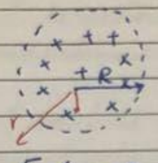
$E_2 = \frac{\sigma}{2\epsilon_0}$

$E_R = E_1 + E_2 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$

$E_R =$ Zero

$E_R =$ Zero

3) Volumetric charge

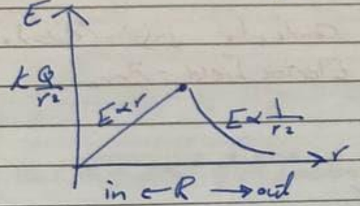


E_{out} $r > R$ $E_{out} = k\frac{Q}{r^2}$ ——— ①

E_{in} $r < R$ E_{in} $k\frac{Q}{R^3} r \rightarrow$ نق radius النقطة --- ②

← radius : الكرة

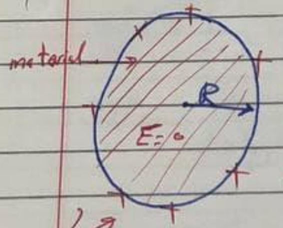
$E_{surface}$ $k\frac{Q}{R^2}$ ——— ③



charge sphere
insulating
uniformly distributed charge

4) solid conducting sphere.

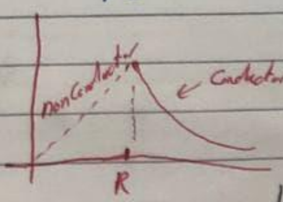
non-conducting sphere (السابقات)
(1,2,3)



كرة مصمتة
فيها مادة مصمتة
[illegible]

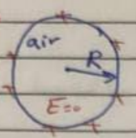
الشحنة موزعة على السطح الخارجي فقط

$E = k\frac{Q}{r^2}$

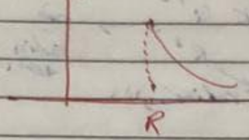


5) Conducting shell

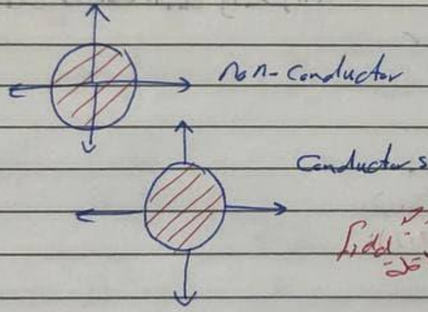
[illegible] conducting or non-conducting



$$E_{out} = \frac{kQ}{r^2}$$



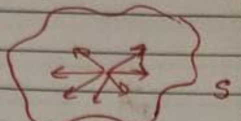
ملاحظة/ داخل ال Conductor
Electric Field = Zero



Gauss Theorem or divergence Theorem:

Let $\vec{f}$ be a vector field

S closed surface and T the region bounded by s



Flux of vector field through closed surface equal triple integral of the Divergence of vector field over the volume enclosed by closed surface

$$\underset{\text{Flux}}{\oiint_{s}} \vec{F}\cdot\vec{n}\, ds = \iiint_T \nabla\cdot\vec{F}\, dv$$

→ Normal vector to the surface

$$\nabla\cdot\vec{F} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}$$

$$\boxed{= i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}}$$

مقدار لا متجه

Ex: Find the gradient of the function $f = (x^2+y^2)^{\frac{1}{2}}$ of the point (1,1)?

$$\nabla f = i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$$

للتوضيح

$$\frac{\partial f}{\partial y} = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\cdot 2y = \frac{y}{\sqrt{x^2+y^2}}$$

$$\nabla f = i\frac{x}{\sqrt{x^2+y^2}} + j\frac{y}{\sqrt{x^2+y^2}}$$

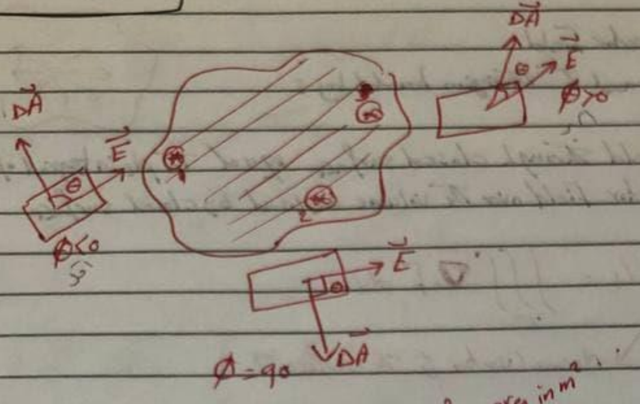
$\phi = \sum \vec{E}\cdot\Delta\vec{A}$ Approximate Flux

$\phi = \oint \vec{E}\cdot d\vec{A}$ Exact flux

$d\vec{A} = \hat{n}\, da$

$$\phi = \oint \vec{E}\cdot n\, da = \frac{Q}{\epsilon_0}$$

$$\boxed{\oint \vec{E}\cdot n\,da = \frac{Q}{\epsilon_0}}$$ The integral Form of Gauss Law



surface area in m^2

The Electric field is a vector

The amount charge in Coulombs

تكامل حول سطح مغلق →

$$\oint_S \vec{E}\cdot\hat{n}\,da = \frac{q_{enc}}{\varepsilon_0}$$

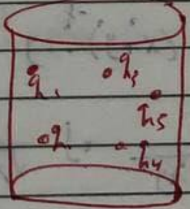
q_{enc} ⟷ in enclosed surface

ε_0 ← permittivity of Free space

Dot product to find $\vec{E}$ parallel to $\hat{n}$

Example:-

Five point charge are in enclosed in cylindrical surface S, The values of charge are $q_1 = +3nC$, $q_2 = -2nC$, $q_3 = +2nC$, $q_4 = +4nC$, $q_5 = -1nC$,
Find The Total Flux Through S



Solution:-

From Gauss law

$$\phi_E = \oint_S \vec{E}\cdot n\,d\vec{a} = \frac{q_{enc}}{\varepsilon_0}$$

$$Q_{Total} = \text{Total enclosed charge} = \sum q_i = (3-2+2+4-1)\times10^{-9}C$$
$$= 6\times10^{-9}C$$

$$\text{and } \phi = \frac{q_{enc}}{\epsilon_0} = \frac{6\times10^{-9}\,C}{8.85\times10^{-2}\,C/Vm} = 678\,V_m$$

The Differential Form of Gauss law :-

$$\oint_S \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} \qquad \text{integral Form}$$

Using The Divergence Theorem

$$\int_S \vec{F}\cdot d\vec{A} = \int_V \nabla\cdot\vec{F}\, dv$$

For $\vec{E}$

$$\oint_S \vec{E}\cdot d\vec{A} = \oint_V \nabla\cdot\vec{E}\, dv$$

$$\frac{q}{\varepsilon_0} = \oint \nabla\cdot\vec{E}\, dv \Rightarrow \frac{q}{\epsilon_0} \oint \nabla\cdot\vec{E}\, dv$$

$$q = \int_V \rho\, dv$$

$$\frac{1}{\epsilon_0}\int_V \rho\, dv = \oint \nabla\cdot\vec{E}\, dV$$

Electiric field (vector)

charge Density in Coulombs per m^3

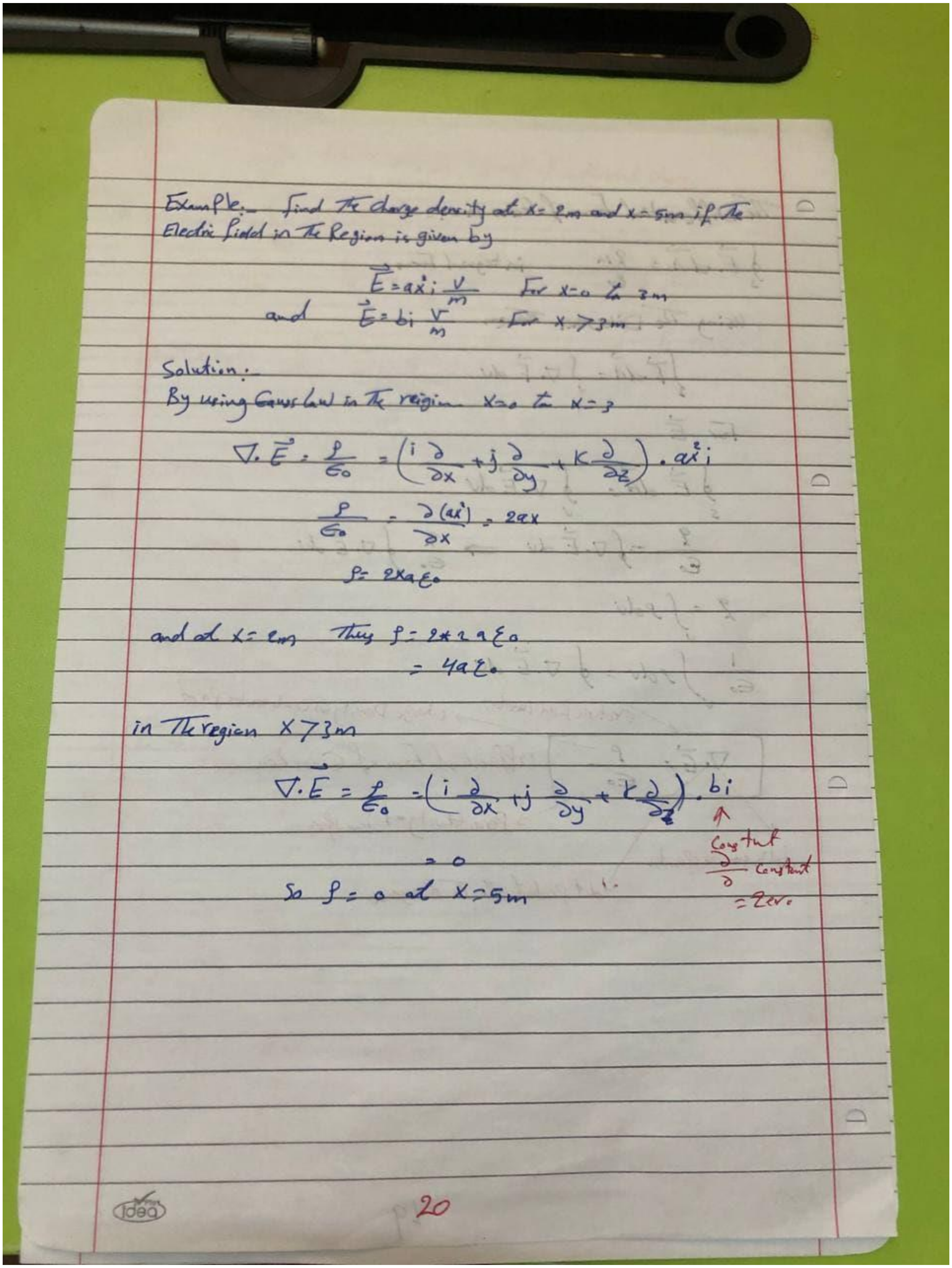
$$\boxed{\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0}}$$

Differential Form of Gauss Law

Permitivity of free space

del is a vector operator

dot product into The divergence



Example:- find the charge density at $x = 2m$ and $x = 5m$ if the Electric field in the Region is given by

$$\vec{E} = ax^2 i \ \frac{V}{m} \quad \text{For } x = 0 \text{ to } 3m$$

$$\text{and} \quad \vec{E} = b i \ \frac{V}{m} \quad \text{For } x > 3m$$

Solution:-

By using Gaus law in the reigin $x = 0$ to $x = 3$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} = \left(i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right) \cdot ax^2 i$$

$$\frac{\rho}{\epsilon_0} = \frac{\partial (ax^2)}{\partial x} = 2ax$$

$$\rho = 2xa\epsilon_0$$

and at $x = 2m$ Thus $\rho = 2 * 2 a \epsilon_0$

$$= 4a\epsilon_0$$

in The region $x > 3m$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} = \left(i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right) \cdot \underset{\uparrow}{bi}$$

Constant

$$\frac{\partial}{\partial} \text{constant} = \text{zero}$$

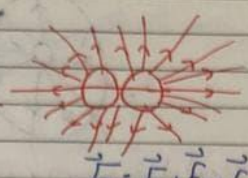
$$= 0$$

So $\rho = 0$ at $x = 5m$

Electric Field due to a group of individual charge

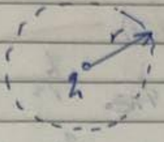
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum \frac{q_i}{r_i^2} \hat{r}$$

$$\vec{E} = \frac{F_1}{q_0} + \frac{F_2}{q_0} + \frac{F_3}{q_0} \cdots \frac{F_n}{q_0}$$



$$\vec{F_0} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_n}$$

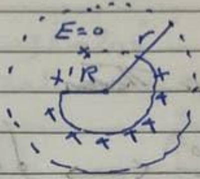
Point charge $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$



Conducting sphere كرة مشحونة charge Q

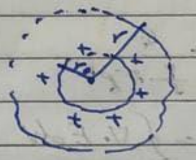
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$$

$E = 0$ inside

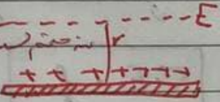


Uniform charge insulating sphere
Charge Q , Radius r_0

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r} \quad \text{outside}$$



$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Qr}{r_0^3} \hat{r} \quad \text{inside}$$



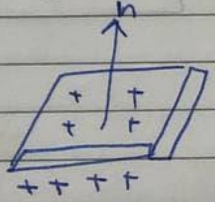
infinite line charge

$$\vec{E} = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r} \hat{r}$$

كثافة الشحنة الخطية

المسافة عن الشحنة لا نهائية الطول

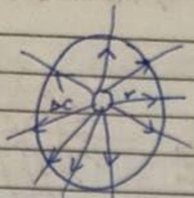
infinite Flat plane السطح المشحون



$$E = \frac{\sigma}{2\epsilon_0} \hat{n}$$

كثافة الشحنة السطحية

Ex.:



in This Fig you have a charge equal To 1C put in a center of a spherical surface, Calculate ϕ?

شحنة كهربائية مقدارها 1c موضوعة في مركز سطح كروي نصف قطره r [illegible]
[illegible]

Sol.: $\phi = EA$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

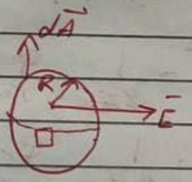
$$\therefore \phi = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,4\pi r^2$$

Divergence Theorem

$$\nabla\cdot E = \frac{\rho}{\epsilon_0} \longleftrightarrow \oint_s \vec{E}\,d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$\vec{dA} = \hat{n}\,dA$$

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3}$$

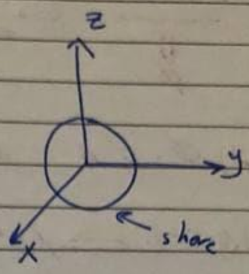


$$\oint_s \vec{E}\,d\vec{A} = \oint_V \nabla\cdot E\,dV$$

$$= \oint_V \nabla\cdot E\,dV = \frac{1}{\epsilon_0}\int_V \rho\,dV$$

$$\boxed{\nabla\cdot E = \frac{\rho}{\epsilon_0}}$$ Differential Form of Gauss Form

Ex:- find Flux of Vector Field $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ over The unit sphere $x^2 + y^2 + z^2 = 1$.



Sol:- $\nabla \cdot f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$

$1 + 1 + 1 = 3$

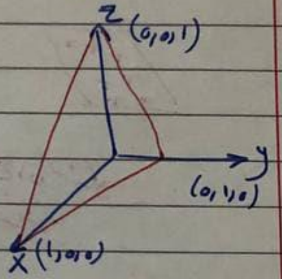
$\oiint_s \vec{F} \cdot \hat{n}\, da = \iiint_v \nabla \cdot F\, dv$

$\int_v \nabla \cdot f\, dv = V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi$

Ex:- Find Flux of The vector Field $F = x^2\hat{i} + xy\hat{j} - 2xz\hat{k}$ over The surface bounded by The plane $x + y + z = 1$ and $x, y, z =$ Zero

Sol:- $\nabla \cdot \vec{F} = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$

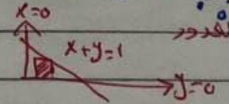


$= 2x + x - 2x \Rightarrow x$

$\oint_s F \cdot \hat{n}\, ds = \int_v \nabla \cdot f\, dV = \iiint x\, dV$

$= \iiint_{R\ 0}^{1-x-y} x\, dz\, dy\, dx$

$= \iint_R xz \Big|_0^{1-x-y} dy\, dx = \iint_R x(1-x-y)\, dy\, dz = \int_0^1\int_0^{1-x} [x - x^2 - xy]\, dy\, dx$



ونستمر بأكمال التكامل حتى نقدر

Cylindrical Coordinates

$x = r\cos\theta \quad y = r\sin\theta \quad x^2 + y^2 = r^2$

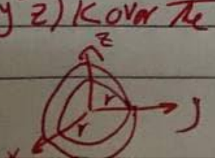
$dv = r\, dr\, d\theta\, dz$

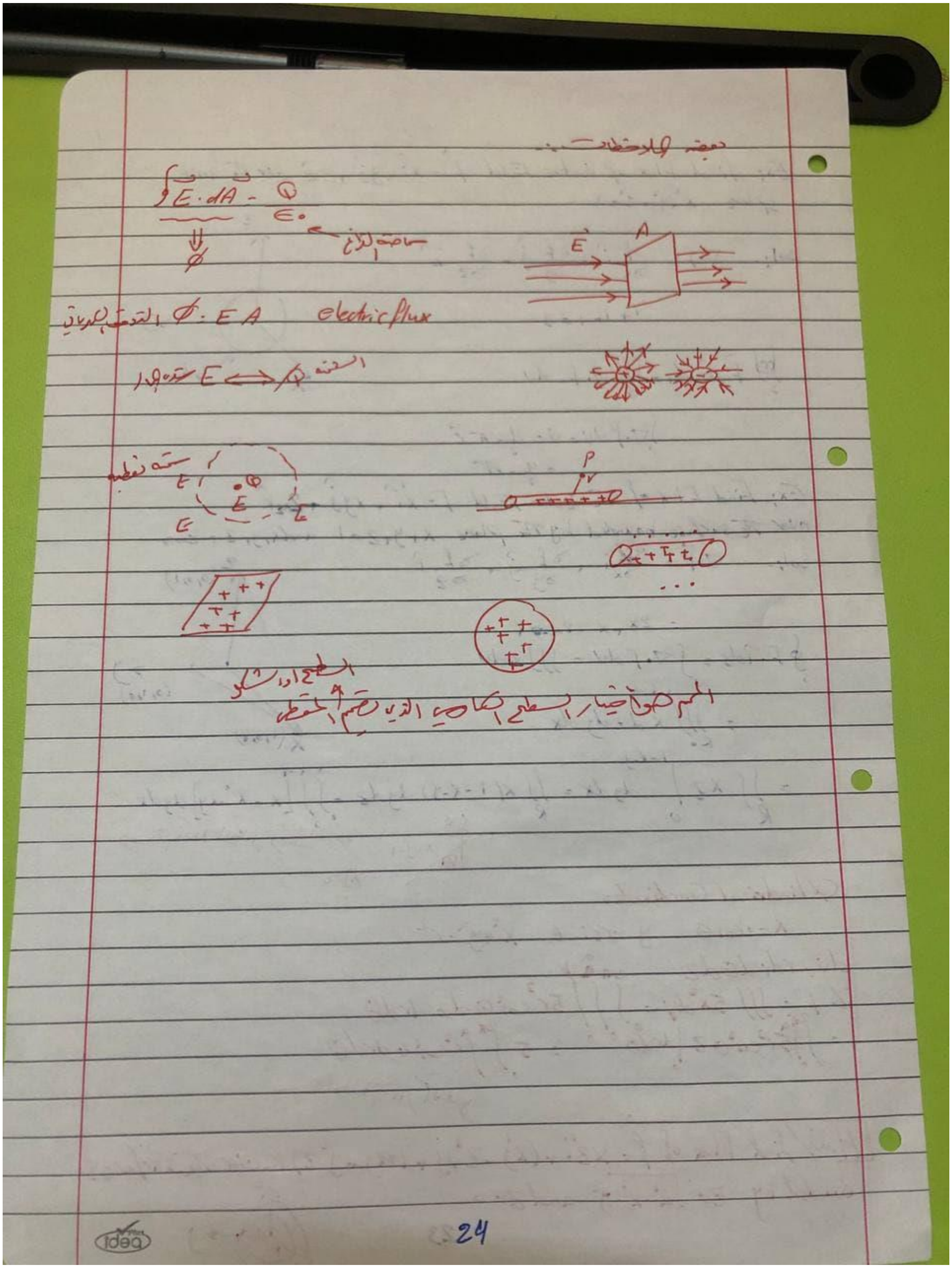
$\iiint 5x^2 dv = \int_0^{2\pi}\int_0^{a}\int_0^{b} 5r^3\cos^2\theta\, dz\, dr\, d\theta$

$= \int_0^{2\pi}\int_0^{a} 5r^3\cos^2\theta\, z\Big|_0 dr\, d\theta = 5\int_0^{2\pi}\int_0^{a} r^3\cos^2\theta\, dr\, d\theta$

ونستمر بالتكامل

$\boxed{H.W}$/ find Flux of $\vec{f} = xz^2 i + (x^2 y - z^3) j + (2xy + y^2 z) K$ over The surfaces Bounded by $z = \sqrt{a^2 - x^2 - y^2}$ and $z = 0$





بعض الملاحظات :-

$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}$ ← سماحية الفراغ

$\Downarrow$

ϕ

$\phi = EA$ التدفق الكهربائي electric flux

$E \Leftrightarrow Q$ الشحنة شدة المجال

السطح او الشكل

المهم هو اختيار السطح الخاص الذي يضم الشحنة

Reference:

1) INTRODUCTION to ELECTRODYNAMICS, Third Edition, David j.Griffths