University of Anbar

College of Science

Department of Applied Geology

Fourth Year

Electromagnetics

جامعة الانبار

كلية العلوم

قسم علوم الفيزياء

المرحلة الرابعة

الكهرومغناطيسية

Electrostatics

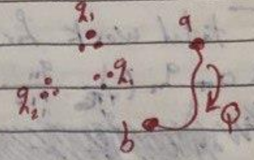
Ninth Part: Work and Energy

Dr. Israa Kamil Ahmed

د . اسراء كامل احمد

Ninth Part in this Chapter: Work and Energy

The work done to move acharge.



if we want To move acharge Q from point a to point p

The Electric force on Q is $F = QE$

The force must exert in apposition to this electrical force is $-QE$ / واحد ضدّ

The work is therfor

$$W = \int_a^b F \cdot dl = -Q \int_a^b E \cdot dl = Q\,[V(b) - V(a)]$$

$$V(b) - V(a) = \frac{W}{Q}$$

The potential difference beTween Points (a)(b) is equal to The work per unit charge required to carry a particle from (a) to (b)

$$W = Q\,(V(r) - \cancel{V(\infty)}^{\text{zero}})$$

$$W = Q\,V(r)$$

Potential Energy is potential
(work)

The Energy of a point charge distribution

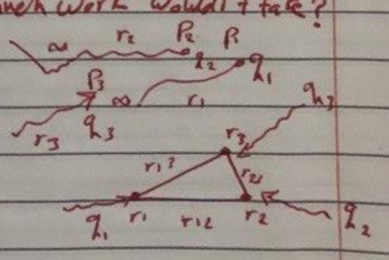
if we have a collection of point charge q, How much work would it take?

1) w_1 for $q_1 = 0$ cause No Electric field

2) $W_2 = q_2 \left[\frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_{12}}\right]$

3) $W_3 = q_3 \left[\frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right]$

4) $W_4 = q_4 \left[\frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_{14}} + \frac{q_2}{r_{24}} + \frac{q_3}{r_{34}}\right]$



The total work for The four charges

$$W = 0 + q_2\left(\frac{1}{4\pi\epsilon_0}\frac{q_1}{r_{12}}\right) + q_3\left(\frac{1}{4\pi\epsilon_0}\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right) + q_4\left(\frac{1}{4\pi\epsilon_0} \cdots\right)$$

[illegible]

$$W = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_1 q_4}{r_{14}} + \frac{q_2 q_3}{r_{23}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right)$$

number of charges

$$W = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j>i}}^{n}\frac{q_i q_j}{r_{ij}} = \frac{1}{4\pi\epsilon_0}\frac{1}{2}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{q_i q_j}{r_{ij}}$$

$$= \frac{1}{2}\sum_{i=1}^{n} q_i\left(\sum_{j}\frac{1}{4\pi\epsilon_0}\frac{q_j}{r_{ij}}\right)$$

$$= \frac{1}{2}\sum_{i=1}^{n} q_i V(r_i)$$ Dose not include The q_i Potential

$$\boxed{\frac{1}{2}\sum_{i=1}^{n} q_i V_{(r_i)}}$$

The Energy of a Continuous charge Distribution

$$\boxed{W = \frac{1}{2}\sum q_i V(r_i)}$$ The Energy of a point charge and it dose not include the point charge itself

$W = \frac{1}{2}\int \rho V d\tau$ for a volume

$W = \frac{1}{2}\int \sigma V da$ $\frac{1}{2}\int \sigma V da$ for a surface

$W = \frac{1}{2}\int \lambda V dl$ for a linear density

$\rho = -\varepsilon_0 \nabla\cdot\vec{E}$

$$W = \frac{\varepsilon_0}{2}\int \underset{\uparrow \text{scalar}}{(\nabla\cdot E)}\ \underset{\nwarrow \text{scalar}}{V}\, d\tau \longrightarrow \nabla\cdot(\underset{\text{scalar field}}{f}\underset{\text{vector field}}{\vec{A}}) = \underline{f(\nabla\cdot\vec{A})} + \vec{A}\cdot(\nabla f)$$

$$\frac{\varepsilon_0}{2}\int \nabla\cdot(V\vec{E}) - \vec{E}(\nabla V)\, d\tau$$

$\nabla V = -E$

$$\frac{\varepsilon_0}{2}\int \nabla\cdot(V\vec{E}) + E^2\, d\tau$$

if we Apply Gauss form of Divergence

$$\int_V (\nabla\cdot F)\, d\tau = \oint_{\text{surface}} F\cdot da$$

$$\frac{\varepsilon_0}{2}\cancel{\oint_{\text{surface}} V\vec{E}\cdot da} + \frac{\varepsilon_0}{2}\int E^2\, d\tau$$

0 Cause V_∞ = Zero

$$\boxed{W = \frac{\varepsilon_0}{2}\int E^2\, d\tau}$$ for all Space

65

Reference:

1) INTRODUCTION to ELECTRODYNAMICS, Third Edition, David j.Griffths