University of Anbar

College of Science

Department of Applied Geology

Advanced Structural Geology

Title of the lecture

Force and Stress

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Introduction

We frequently use the words force and stress in casual conversation. Stress from yet another deadline, a test, or maybe an argument with a roommate or spouse. Appropriate force is applied to reach our goal, and so on. In science, however, these terms have very specific meanings. For example, the force of gravity keeps us on the Earth's surface and the force of impact destroys our car. Like us, rocks experience the pull of gravity, and forces arising from plate interactions result in a range of geologic structures, from microfabrics to mountain ranges. The fundamentals of force and stress, followed by a look at the components of stress that eventually produce tectonic structures. We will use these concepts to examine the relationship between geologic structures and stress. To understand tectonic processes, we must be familiar with the fundamental principles of mechanics. Mechanics is concerned with the action of forces on bodies and their effect; you can say that mechanics is the science of motion. Newtonian1 (or classical) mechanics studies the action of forces on rigid bodies. In tectonic structures we commonly deal with interactions that involve not only movement, but also distortion; material displacements occur both between and within bodies.

A good number of terms and concepts will have appeared. For convenience and future reference, therefore, some of the more common terms are described:

Terminology and symbols of force and stress

Force: Mass times acceleration ($F = m \cdot a$; Newton's second law); symbol F.

Stress: Force per unit area (F/A); symbol σ.

Anisotropic stress: At least one principal stress has a magnitude unequal to the other principal stresses (describes an ellipsoid).

Deviatoric stress: Component of the stress that remains after the mean stress is removed; this component of the stress contains the six shear stresses; symbol $\sigma_{dev.}$

Differential stress: The difference between two principal stresses (e.g., $\sigma_1 - \sigma_3$), which by definition is ≥ 0; symbol $\sigma_{d.}$

Homogeneous stress: Stress at each point in a body has the same magnitude and orientation.

Hydrostatic stress/pressure: Isotropic component of the stress; strictly, the pressure at the base of a water column.

Inhomogeneous stress: Stress at each point in a body has different magnitude and/or orientation.

Isotropic stress: All three principal stresses have equal magnitude (describes a sphere) **Lithostatic stress/pressure**: Isotropic pressure at depth in the Earth arising from the overlying rock column (density × gravity × depth, $\rho \cdot g \cdot h$); symbol P_l.

Mean stress $(\sigma 1 + \sigma 2 + \sigma 3)/3$; symbol σ_{mean}

Normal stress: Stress component oriented perpendicular to a given plane; symbol σn.

Principal plane: Plane of zero shear stress; three principal planes exist.

Principal stress: The normal stress on a plane with zero shear stress; three principal stresses exist, with the convention $\sigma 1 \geq \sigma 2 \geq \sigma 3$.

Shear stress: Stress parallel to a given plane; symbol σs (sometimes the symbol τ is used).

Stress ellipsoid: Geometric representation of stress; the axes of the stress ellipsoid are the principal stresses.

Stress field: The orientation and magnitudes of stresses in a body.

FORCE

Kicking or throwing a ball show that a force changes the velocity of an object. Newton’s first law of motion, also called the Law of Inertia, says that in the absence of a force a body moves either at constant velocity or is at rest. Stated more formally: a free body moves without acceleration. Change in velocity is called acceleration [a], which is defined as velocity divided by time:

$[a]: [vt^{-1}] : [lt^{-2}]$

The unit of acceleration, therefore, is m/s^2.

Force [F], according to Newton’s Second Law of Motion, is mass multiplied by acceleration:

$[F]: [ma] : [mlt^{-2}]$

The unit of force is $kg \cdot m/s^2$, called a newton (N) in SI units. You can feel the effect of mass when you throw a tennis ball and a basketball and notice that a different force is required to move each of them.

Force, like velocity, is a vector quantity, meaning that it has both magnitude and direction. So, it can be graphically represented by a line with an arrow on one side. Manipulation of forces conforms to the rules of vector algebra. For example, a force at an angle to a given plane can be geometrically resolved into two components; say, one parallel and one perpendicular to that plane.

stress

Stress, represented by the symbol σ (sigma), is defined as the force per unit area [A], or $\sigma = F/A$. You can, therefore, consider stress as the intensity of force, or a measure of how concentrated a force is. A given force acting on a small area (the pointed hammer mentioned previously) will have a greater intensity than that same force acting on a larger area (a flat-headed hammer), because the stress associated with the smaller area is greater than that with the larger area.

You will see that stress is a complex topic, because its properties depend on the reference system. Stress that acts on a plane is a vector quantity, called traction, whereas stress acting on a body is described by a higher order entity, called a stress tensor. Because stress is force per unit area it is expressed in terms of the following fundamental quantities:

$[\sigma] : [mlt^{2} \cdot l^{-2}]$ or $[ml^{-1} \cdot t^{-2}]$

The corresponding unit of stress is $kg/m \cdot s^2$ (or N/m^2), which is called a pascal (Pa). Instead of this SI unit, however, many geologists continue to use the unit bar, which is approximately 1 atmosphere. These units are related as follows:

1 bar = 10^5 Pa ≈ 1 atmosphere

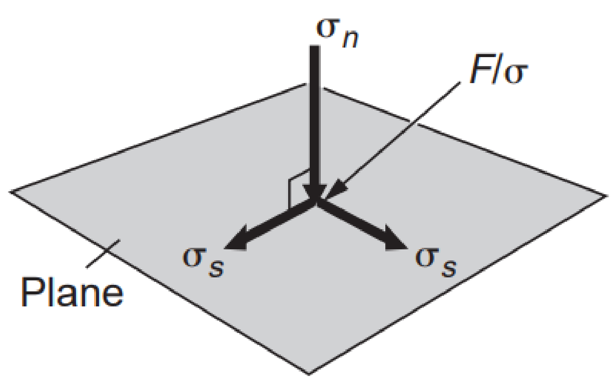
In geology you will generally encounter their larger equivalents, the kilobar (kbar) and the megapascal (MPa):

1 kbar = 1000 bar = 10^8 Pa = 100 MPa

The unit gigapascal (1 GPa = 1000 MPa = 10 kbar) is used to describe the very high pressures that occur deep in the Earth. For example, the pressure at the core-mantle boundary, located at a depth of approximately 2900 km, is ∼135 GPa, and at the center of the Earth (at a depth of 6370 km) the pressure exceeds 350 GPa.

Two-dimensional stress: Normal stress and shear stress

Stress acting on a plane is a vector quantity, meaning that it has both magnitude and direction; it is sometimes called traction. Stress on an arbitrarily oriented plane, however, is not necessarily perpendicular to that plane, but, like a vector, it can be resolved into components normal to the plane and parallel to the plane (Figure 1). The vector component normal to the plane is called the normal stress, for which we use the symbol σ_n (sometimes just the symbol σ is used); the vector component along the plane is the shear stress and has the symbol σ_s (sometimes the symbol τ (tau) is used).



σ_n is normal stress, σ_s is shear stress, F is force; σ is stress

Figure 1 The stress on a two-dimensional plane is defined by a stress acting perpendicular to the plane (the normal stress) and a stress acting along the plane (the shear stress). The normal stress and shear stress are perpendicular to one another.

In contrast to the resolution of forces, the resolution of stress into its components is not straightforward, because the area changes as a function of the orientation of the plane with respect to the stress vector.

We graphically illustrate this difference between forces and stresses on an arbitrary plane by plotting their normalized values as a function of the angle θ in Figure 2. In particular, the relationship between Fs and σ_s is instructive for gaining an appreciation of the area dependence of stress. Both the shear force and the shear stress initially increase with increasing angle θ; at 45° the shear stress reaches a maximum and then decreases, while Fs continues to increase.

Thus, the stress vector acting on a plane can be resolved into vector components perpendicular and parallel to that plane, but their magnitudes vary as a function of the orientation of the plane.

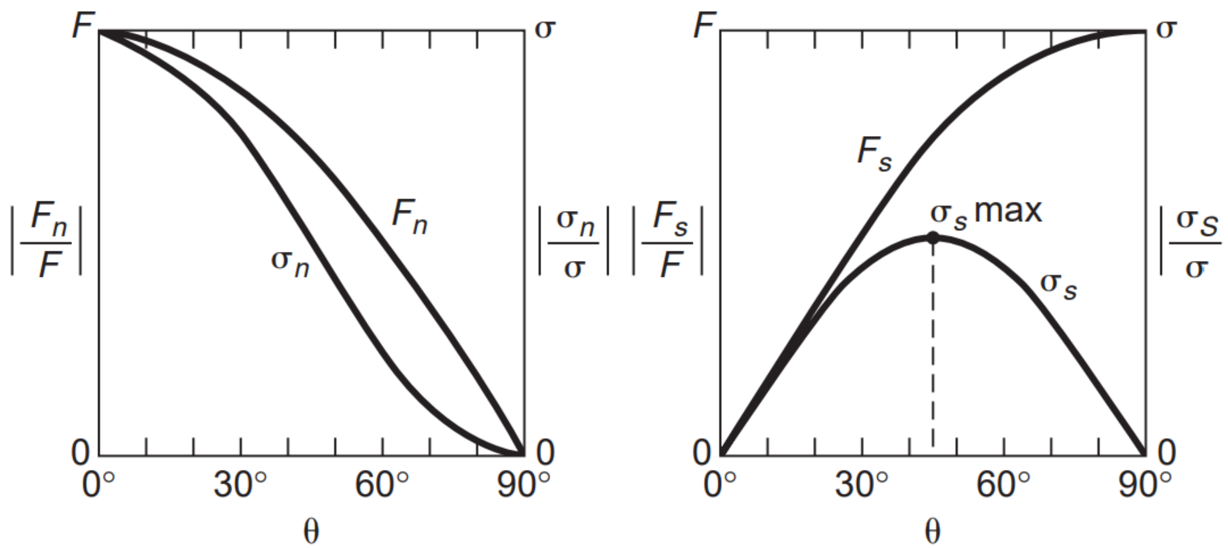


Figure 2 (left) normalized values of F_n and σ_n on plane with angle θ; (right) normalized values of F_s and σ_s on a plane with angle θ.

Three-dimensional stress: principal planes and principal stresses

Previously, we discussed stress acting on a single plane (the two-dimensional case), recognizing two vector components, the normal stress and the shear stress (Figure 1). However, to describe stress on a randomly oriented plane in space we need to consider the three-dimensional case. We limit unnecessary complications by setting the condition that the body containing the plane is at rest. So, a force applied to the body is balanced by an opposing force of equal magnitude but opposite sign; this condition is known as Newton's Third Law of Motion. Using another Newtonian sports analogy, kick a ball that rests against a wall and notice how the ball (the wall, in fact) pushes back with equal enthusiasm.

Stress at a Point

We shrink our three-dimensional body containing the plane of interest down to the size of a point for our analysis of the stress state of an object. Why this seemingly obscure transformation? Recall that two nonparallel planes have a line in common and those three or more nonparallel planes have a point in common. In other words, a point defines the intersection of an infinite number of planes with different orientations. The stress state at a point, therefore, can describe the stresses acting on all planes in a body. In Figure 3a the normal stresses (σ) acting on four planes (a–d) that intersect in a single point are drawn. For clarity, we limit our illustrations to planes that are all perpendicular to the surface of the page, allowing the use of slice through the body. You will see later that this geometry easily expands into the full three-dimensional case.

Because of Newton's Third Law of Motion, the stress on each plane must be balanced by one of opposite sign (σ = –σ). Because stress varies as a function of orientation, the magnitude of the normal stress on each plane (represented by the vector length) is different. If we draw an envelope around the end points of these stress vectors (heavy line in Figure 3a), we obtain an ellipse. Recall from geometry that an ellipse is defined by at least three nonperpendicular axes, which are shown in Figure 3a. This means that the magnitude of the stress for all possible planes is represented by a point on this stress ellipse. Now, the same can be done in three dimensions, but this is hard to illustrate on a piece of flat paper. Doing the same analysis in three dimensions, we obtain an envelope that is the three-dimensional equivalent of an ellipse, called an ellipsoid (Figure 3b). This stress ellipsoid fully describes the stress state at a point and enables us to determine the stress for any given plane. Like all ellipsoids, the stress ellipsoid is defined by three mutually perpendicular axes, which are called the principal stresses. These principal stresses have two properties: (1) they are orthogonal to each other, and (2) they are perpendicular to three planes that do not contain shear stresses; these planes are called the principal planes of stress. So, we can describe the stress state of a body simply by specifying the orientation and magnitude of three principal stresses.

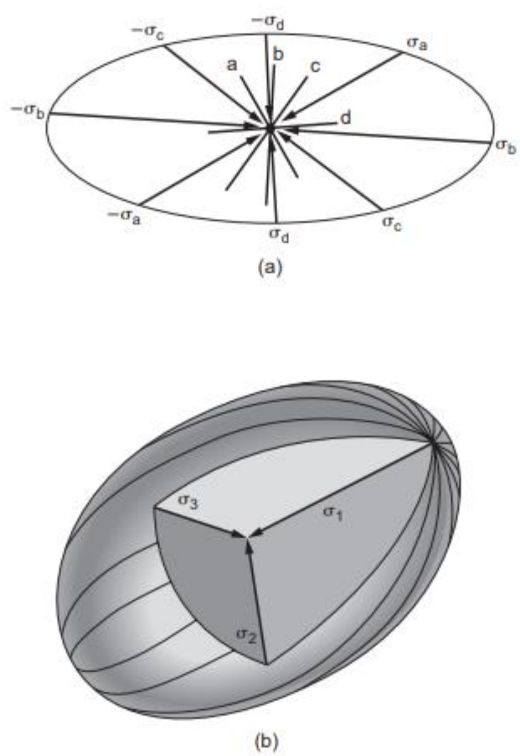


Figure 3 (a) A point represents the intersection of an infinite number of planes, and the stresses on these planes describe an ellipse in the two-dimensional case. In three dimensions this stress envelope is an ellipsoid (b), defined by three mutually perpendicular principal stress axes ($\sigma_1 \geq \sigma_2 \geq \sigma_3$). These three axes are normal to the principal planes of stress.

Stress States

If the three principal stresses are equal in magnitude, we call the stress **isotropic**. This stress state is represented by a sphere rather than an ellipsoid, because all three radii are equal. If the principal stresses are unequal in magnitude, the stress is called **anisotropic**. By convention, the maximum principal stress is given the symbol σ1, the intermediate and minimum principal stresses acting along the other two axes are given the symbols σ2 and σ3, respectively. Thus, by (geologic) convention:

$\sigma_1 \geq \sigma_2 \geq \sigma_3$

By changing the relative values of the three principal stresses we define several common stress states:

General triaxial stress:	$\sigma_1 > \sigma_2 > \sigma_3 \neq 0$
Biaxial (plane) stress:	one axis = 0 (e.g., $\sigma_1 > 0 > \sigma_3$)
Uniaxial compression:	$\sigma_1 > 0$; $\sigma_2 = \sigma_3 = 0$
Uniaxial tension:	$\sigma_1 = \sigma_2 = 0$; $\sigma_3 < 0$
Hydrostatic stress (pressure):	$\sigma_1 = \sigma_2 = \sigma_3$

Deriving some stress relationships

Now that we can express the stress state of a body by its principal stresses, we can derive several useful relationships. Let's carry out a simple classroom experiment in which we compress a block of clay between two planks (Figure 4). As the block of clay develops a fracture, we want to determine what the normal and the shear stresses on the fracture plane are. To answer this question our approach is similar to our previous one, but now we express the normal and shear stresses in terms of the principal stress axes.

The principal stresses acting on our block of clay are σ_1 (maximum stress), σ_2 (intermediate stress), and σ_3 (minimum stress). Since we carry out our experiment under atmospheric conditions, the values of σ_2 and σ_3 will be equal, and we may simplify our analysis by neglecting σ_2 and considering only the σ_1-σ_3 plane, as shown in Figure 4. The fracture plane makes an angle θ (theta) with σ_3. This plane makes the trace AB in Figure 4b, which we assign unit length (that is, 1) for convenience. We can resolve AB along AC (parallel to σ_1) and along BC (parallel to

σ_3). Then, by trigonometry, we see that the area represented by AC = sin θ, and the area represented by BC = cos θ. Note that if we assign dimension L to AB then AC = L · sin θ and BC = L · cos θ.

Next, we consider the forces acting on each of the surface elements represented by AB, BC, and AC. Since force equals stress times the area over which it acts, we obtain

force on side BC = $\sigma_1 \cdot \cos\theta$

force on side AC = $\sigma_3 \cdot \sin\theta$

The force on side AB consists of a normal force (i.e., $\sigma_n \cdot 1$) and a shear force (i.e., $\sigma_s \cdot 1$); recall that force is stress times area. For equilibrium, the forces acting in the direction of AB must balance, and so must the forces be acting perpendicular to AB (which is parallel to CD).

Hence, resolving along CD:

force ⊥ AB = force ⊥ BC resolved on CD + force ⊥ AC resolved on CD or

$1 \cdot \sigma_n = \sigma_1 \cos\theta \cdot \cos\theta + \sigma_3 \sin\theta \cdot \sin\theta$

$\sigma_n = \sigma_1 \cos2\theta + \sigma_3 \sin2\theta$

Substituting these trigonometric relationships, we obtain

$\cos2\theta = 1/2(1 + \cos 2\theta)$

$\sin2\theta = 1/2(1 - \cos 2\theta)$

Simplifying, gives

$\sigma n = 1/2(\sigma_1 + \sigma_3) + 1/2(\sigma_1 - \sigma_3)\cos 2\theta$

and, force parallel AB = force ⊥ BC resolved on AB + force ⊥ AC resolved on AB or

$1 \cdot \sigma s = \sigma_1 \cos\theta \cdot \sin\theta - \sigma_3 \sin\theta \cdot \cos\theta$

Note that the force perpendicular to AC resolved on AB acts in a direction opposite to the force perpendicular to BC resolved on AB, hence a negative sign is needed, which further simplifies to $\sigma s = (\sigma_1 - \sigma_3)\sin\theta \cdot \cos\theta$

Substituting this trigonometric relationship, we get

$\sin\theta \cdot \cos\theta = 1/2 \sin 2\theta$ which gives

$\sigma s = 1/2(\sigma_1 - \sigma_3) \sin 2\theta$ Eq. 3.10

From Equations we can determine that the planes of maximum normal stress are at an angle θ of 0° with σ3, because cos 2θ reaches its maximum value (cos 0° = 1). Secondly, planes of maximum shear stress lie at an angle θ of 45° with σ3 because sin 2θ reaches its maximum value (sin 90° = 1). Whereas faulting resulted in a shearing motion along the fault plane, we find that the fault plane in our experiment is not parallel to the plane of maximum shear stress (θ > 45°).

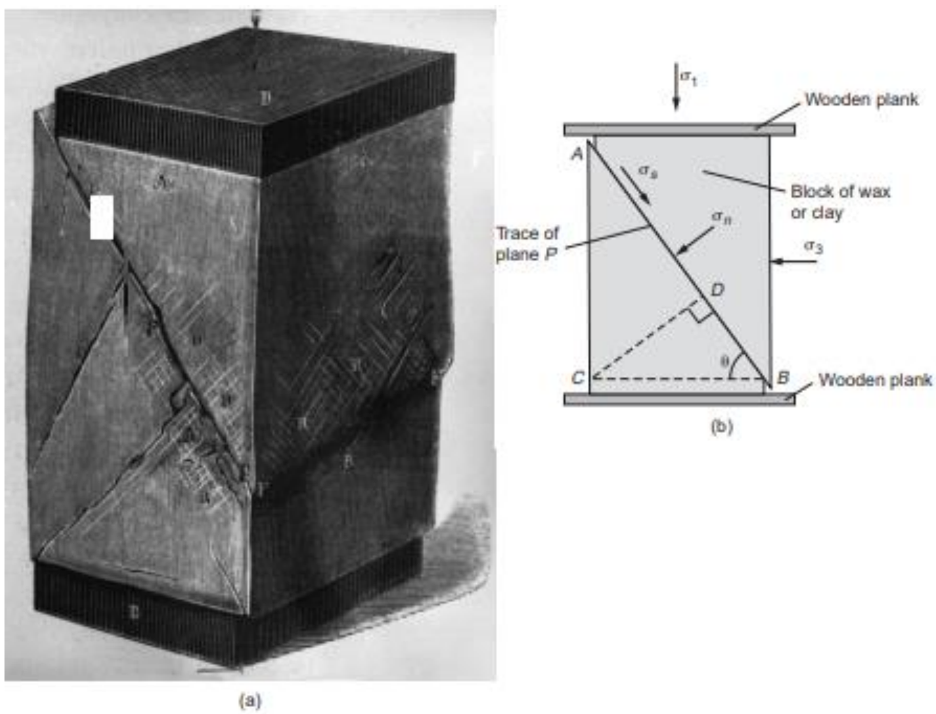


Figure 4 Determining the normal and shear stresses on a plane in a stressed body as a function of the principal stresses. (a) An illustration from the late nineteenth-century fracture experiments using wax. (b) For a classroom experiment, a block of clay is squeezed between two planks of wood. AB is the trace of fracture plane P in our body that makes an angle θ with σ_3. The two-dimensional case shown is sufficient to describe the experiment, because σ_2 equals σ_3 (atmospheric pressure).

References

Ben A. van der Pluijm, and Stephen Marshak. (2004) EARTH STRUCTURE AN INTRODUCTION TO STRUCTURAL GEOLOGY AND TECTONICS. Second edition.