# University of Anbar 

## College of Science

# Department of Applied Geology 

Advanced Structural Geology<br>Title of the lecture<br>\section*{Deformation and Strain}

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## Deformation and Strain

## Introduction

The geologic history of most crustal rocks involves significant changes in the shape of original features like sedimentary bedding, igneous structures, rock inclusions, and grains. The formation of folds or faults springs to mind as an example of this deformation. Deformed fossils, folds, and other features document the permanent shape changes that occur in natural rocks. The study and quantification of these distortions, which occur in response to forces acting on bodies, is the subject of "Deformation and Strain." Recall the force of gravity, for example. Let us consider a more controlled experiment to analyze the response of materials to an applied force. We can change the shape of a block of clay or plasticine by the action of, say, your hands or a vise. When forces affect the spatial geometry of a body (syrup, you, plasticine, or rocks) we enter the realm of deformation. Most simply stated: deformation of a body occurs in response to forces. We will see the deformation affects stress (force acting on an area), so there is no simple stressdeformation relationship. The response of a body to forces may have many faces. In some cases, the body is merely displaced or rotated, such as when you get up from the chair and move around the room. In other cases, the body becomes distorted, as in the clay block experiment or with the flow of syrup.

## Deformation and Strain

Deformation and strain are closely related terms that are sometimes used as synonyms, but they are not the same. Deformation describes the collective displacements of points in a body; in other words, it describes the complete transformation from the initial to the final geometry of a body. This change can include a translation (movement from one place to the other), a rotation (spin around an axis), and a distortion (change in shape). Strain describes the changes of points in a body relative to each other; so, it describes the distortion of a body. This distinction between deformation and strain may not be immediately obvious from these abstract descriptions, so we use an example. In Figure 1 we change the shape and position of a square, say, a slice of the clay cube we used. We arbitrarily choose a reference frame with axes that parallel the margins of the printed page. The displacement of points within the body, represented by the four corner points of the square, are indicated by vectors. These vectors describe the displacement field of the body from the initial to the final shape. The displacement field can be subdivided into three components:

1. A distortion (Figure 1b)
2. A rotation (Figure 1c)
3. A translation (Figure 1d).


Figure 1 The components of deformation. The deformation of a square (a) is subdivided into three independent components: (b) a distortion; (c) a rotation; and (d) a translation. The displacement of each material points in the square, represented by the four corners of the initial square, describes the displacement field. The corresponding strain ellipse is also shown. The distortion occurs along the axes X and Y , which are the principal strain axes

Each component in turn can be described by a vector field (shown for point A only) and their sum gives the total displacement field. Importantly, a change in the order of addition of these vector components affects the final result. Deformation, therefore, is not a vector entity, but a second-order tensor (similar to stress). When the rotation and distortion components are zero, we only have a translation. This translation is formally called rigid-body translation (RBT), because the body undergoes no shape change while it moves. For convenience, we will simply refer to this component as translation, and the deformation is called translational. When the translation and distortion components are zero, we have only rotation of the body. By analogy to translation, we call this component rigid-body rotation (RBR), or simply spin, and the corresponding deformation is called rotational. When translation and spin are both zero, the body undergoes distortion; this component is described by strain. So, strain is a component of deformation and therefore not a synonym. In essence, we have defined deformation and strain relative to a frame
of reference. Deformation describes the complete displacement field of points in a body relative to an external reference frame, such as the edges of the paper on which Figure 1 is drawn. Strain, on the other hand, describes the displacement field of points relative to each other. This requires a reference frame within the body, an internal reference frame, like the edges of the square. Place yourself in the square and you would be unaware of any translation, just as when you are flying in an airplane or riding a train. Looking out of the window, however, makes you aware of the displacement by offering an external reference frame. One final element is missing in our description of deformation. In Figure 1 we have constrained the shape change of the square by maintaining a constant area. You recall that shape change results from the deviatoric component of the stress, meaning where the principal stresses are unequal in magnitude. The hydrostatic component of the total stress, however, contributes to deformation by changing the area (or volume, in three dimensions) of an object. Area or volume change is called dilation and is positive or negative, as the volume increases or decreases, respectively. Because dilation results in changes of line lengths it is similar to strain, except that the relative lengths of the lines remain the same. Thus, it is useful to distinguish strain from volume change. In summary, deformation is described by:

1. Rigid-body translation (or translation)
2. Rigid-body rotation (or spin)
3. Strain
4. Volume change (or dilation)

It is relatively difficult in practice to determine the translational, spin, and dilational components of deformation. Only in cases where we are certain about the original position of a body can translation and spin be determined, and only when we know the original volume of a body can dilation be quantified. On the other hand, we often do know the original shape of a body, so the quantification of strain is a common activity in structural geology.

## Homogeneous strain and the strain ellipsoid

Strain describes the distortion of a body in response to an applied force. Strain is homogeneous when any two portions of the body that were similar in form and orientation before are similar in form and orientation after strain. This can be illustrated by drawing a square and a circle on the edge of a deck of cards; homogeneous strain changes a square into a parallelogram and a circle into an ellipse (Figure 2b). We define homogeneous strain by its geometric consequences:

1. Originally straight lines remain straight.
2. Originally parallel lines remain parallel.
3. Circles become ellipses; in three dimensions, spheres become ellipsoids.

When one or more of these three restrictions does not apply, we call the strain heterogeneous (Figure c).


Figure 2 Homogeneous and heterogeneous strain. A square and a circle drawn on a stack of cards (a) transform into a parallelogram and an ellipse when each card slides the same amount, which represents homogeneous strain (b). Heterogeneous strain (c) is produced by variable slip on the cards, for example by increasing the slip-on individual cards from bottom to top.

Because conditions (1) and (2) are maintained during the deformation components of translation and rotation, deformation is homogeneous by definition if the strain is homogeneous. Conversely, heterogeneous strain implies heterogeneous deformation. Homogeneous and heterogeneous deformation should not be confused with rotational and nonrotational deformation; the latter reflect the presence of a spin component. Because heterogeneous strain is more complex to describe than homogeneous strain, we try to analyze heterogeneously strained bodies or regions by separating them into homogeneous portions. In other words, homogeneity of deformation is a matter of scale. Consider a heterogeneous deformation feature like a fold, which can be approximated by three essentially homogeneous sections: the two limbs and the hinge. The heterogeneously deformed large square of Figure 2c consists of nine smaller squares for which the strain conditions are approximately homogeneous. Given the scale dependence of homogeneity and not to complicate our explanations unnecessarily. In a homogeneously strained, two-dimensional body there will be at least two material lines that do not rotate relative to each
other, meaning that their angle remains the same before and after strain. What is a material line? A material line connects features, such as an array of grains, that are recognizable throughout a body's strain history. The behavior of four material lines is illustrated in Figure 3 for the twodimensional case, in which a circle changes into an ellipse.


Figure 3 Homogeneous strain describes the transformation of a square to a rectangle or a circle to an ellipse. Two material lines that remain perpendicular before and after strain are the principal axes of the strain ellipse (solid lines). The dashed lines are material lines that do not remain perpendicular after strain; they rotate toward the long axis of the strain ellipse.

In homogeneous strain, two orientations of material lines remain perpendicular before and after strain. These two material lines form the axes of an ellipse that is called the strain ellipse. Note that the lengths of these two material lines change from the initial to the final stage; otherwise, we would not strain our initial circle. Analogously, in three dimensions we have three material lines that remain perpendicular after strain and they define the axes of an ellipsoid, the strain ellipsoid. The lines that are perpendicular before and after strain are called the principal strain axes. Their lengths define the strain magnitude and we will use the symbols $\mathrm{X}, \mathrm{Y}$, and Z to specify them, with the convention that $\mathrm{X} \geq \mathrm{Y} \geq \mathrm{Z}$. In a more intuitive explanation, you may consider the strain ellipsoid as the modified shape of an initial sphere embedded in a body after the application of a homogeneous strain. We describe strain in two-dimensional space by the two axes of the strain ellipse and an angle describing the rotation of this ellipse. In three-dimensional space, therefore, we use the three axes of the strain ellipsoid and three rotation angles. This means that the strain ellipsoid is defined by six independent components, which is reminiscent of the stress ellipsoid. Indeed, the strain ellipsoid is a visual representation of a second rank tensor, but keep in mind that the stress and strain ellipsoids are not the same.

## Coaxial and non-coaxial strain accumulation

In figure 3, we saw that strain involves the rotation of material lines. Recall that a material line is made up of a series of points in a body; for example, a row of calcium atoms in a calcite crystal or an array of grains in a quartzite. There is no mechanical contrast between the material line and
the body as a whole, so that material lines behave as passive markers. All material lines in the body, except those that remain perpendicular before and after a strain increment (that is, the principal strain axes), rotate relative to each other. In the general case for strain, the principal incremental strain axes are not necessarily the same throughout the strain history. In other words, the principal incremental strain axes rotate relative to the finite strain axes, a scenario that is called non-coaxial strain accumulation. The case in which the same material lines remain the principal strain axes at each increment is called coaxial strain accumulation. These important concepts are not obvious, so we turn to a classroom experiment for further exploration.

First, we examine non-coaxial strain accumulation. Take a deck of playing cards (or a thick phone book) and draw a circle on the face perpendicular to the cards. By sliding the cards past one another by roughly equal amounts, the initial circle changes into an ellipse (Figure 4a). Draw the ellipse axes (i.e., incremental strain axes) X 1 and Y 1 on the face. Continuing to slide the cards produces a more elliptical shape. Again, mark the ellipse axes X2 and Y2 of this second step on the cards, but use another color. Continue this action a third time so that in the end you have three steps (increments) and three X-Y pairs. Note that the last ellipse represents the finite strain. Now, as you return the cards to their starting configuration, by restoring the original circle, you will notice that the pairs of strain axes of the three increments do not coincide. For each step a different set of material lines-maintained perpendicularity, and thus the incremental strain axes do not coincide with the finite strain axes. You also see that with each step the long axis of the finite strain ellipse rotated more toward the shear plane over which the cards slide. You can imagine that a very large amount of sliding will orient the long axis of the finite strain ellipse nearly parallel to (meaning a few degrees off) the shear plane.

In the case of coaxial strain accumulation, we return to our earlier experiment with clay (Figure 4b). Take a slice of clay with a circle drawn on its front surface and press down on the top and bottom. When you draw the incremental strain axes at various steps, you will notice that they coincide with one another, while the ellipticity (the X/Y ratio) increases. So, with coaxial strain accumulation there is no rotation of the incremental strain axes with respect to the finite strain axes.


Figure 4 Non-coaxial (a) and coaxial (b) strain. The incremental strain axes are different material lines during noncoaxial strain. In coaxial strain the incremental strain axes are parallel to the same material lines. Note that the magnitude of the strain axes changes with each step.

The component describing the rotation of material lines with respect to the principal strain axes is called the internal vorticity, which is a measure of the degree of non-coaxiality. If there is zero internal vorticity, the strain history is coaxial (as in Figure 4b), which is sometimes called pure shear. The non-coaxial strain history in Figure 4a describes the case in which the distance perpendicular to the shear plane (or the thickness of our stack of cards) remains constant; this is also known as simple shear. In reality, a combination of simple shear and pure shear occurs, which we call general shear (or general non-coaxial strain accumulation; Figure 5)


Figure 5 A combination of simple shear (a special case of noncoaxial strain) and pure shear (coaxial strain) is called general shear or general non-coaxial strain. Two types of general shear are transtension (a) and transpression (b), reflecting extension and shortening components.

## Types of strain

1. Coaxial strain: Strain in which the incremental strain axes remain parallel to the finite strain axes during progressive strain
2. Heterogeneous strain: Strain in which any two portions of a body similar in form and orientation before strain undergo relative change in form and orientation (also: inhomogeneous strain)
3. Homogeneous strain: Strain in which any two portions of a body similar in form and orientation before strain remain similar in form and orientation after strain
4. Incremental strain: Strain state of one step in a progressive strain history
5. Instantaneous strain: Incremental strain of vanishingly small magnitude (a mathematical descriptor); also called infinitesimal incremental strain
6. Finite strain: Strain that compares the initial and final strain configurations; sometimes called total strain
7. Non-coaxial strain: Strain in which the incremental strain axes rotate relative to the finite strain axes during progressive

## Strain quantities

Having examined the necessary fundamentals of strain, we can now turn our attention to practical applications in structural analysis using the quantification of strain. How much strain does this deformation feature represent? How do we go about determining this? We will examine strain quantification using three measures: length change or longitudinal strain, volume change or volumetric strain, and angular change or angular strain. You'll find that all of these approaches are pertinent to the analysis of our little fold.

## Longitudinal Strain

Longitudinal strain is defined as a change in length divided by the original length. Longitudinal strain is expressed by the elongation, $e$, which is defined as
$\mathrm{e}=(1-\mathrm{lo}) / \mathrm{lo}$ or $\mathrm{e}=\delta 1 / \mathrm{lo}$
where $l$ is the final length, $l_{o}$ is the original length, and $\delta l$ is the length change (Figure 6a).
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\mathrm{e}=\frac{\ell-\ell_{0}}{\ell_{0}}=\frac{\delta \ell}{\ell_{0}} ; \quad \mathrm{s}=\frac{\ell}{\ell_{0}}
$$

(a)

(c)

(b)

(d)

Figure 6 Strain quantities. The elongation, e, and stretch, $s$, in (a); the angular shear, $\psi$, and the shear strain, $\gamma$, in (b). In (c) the relationships between quadratic elongation ( $\lambda$ ), stretch (s), and angular shear ( $\psi$ ) are shown for line OP that transforms into OP' (d).

Because we divide values with the same units, longitudinal strain is a dimensionless quantity. A longitudinal strain of 0.3 for a stretched rod or a continental region is independent of the original dimensions of the object. This definition of elongation implies that negative values of e reflect shortening whereas positive values of e represent extension. We label the maximum, intermediate, and minimum elongations, $e_{1}$, $e_{2}$, and $e_{3}$, respectively, with $e_{1} \geq e_{2} \geq e_{3}$. Remember the sign convention we just described! In practice, geologists commonly give the elongation in percent, using the absolute value, $|\mathrm{e}| \times 100 \%$, and the terms shortening and extension instead of a negative or positive sign; for example, $30 \%$ extension or $40 \%$ shortening.

## Volumetric Strain

A relationship similar to that for length changes holds for three-dimensional (volume) change. For volumetric strain, $\Delta$, the relationship is
$\Delta=\left(\mathrm{V}-\mathrm{V}_{\mathrm{o}}\right) / \mathrm{V}_{\mathrm{o}} \quad$ or $\quad \Delta=\delta \mathrm{V} / \mathrm{V}_{\mathrm{o}}$
where V is the final volume, $\mathrm{V}_{\mathrm{o}}$ is the original volume, and $\delta \mathrm{V}$ is the volume change. Like longitudinal strain, volumetric strain is a ratio of values with the same units, so it also is a dimensionless quantity. Positive values for $\Delta$ represent volume gain, whereas negative values represent volume loss.

## Angular Strain

Longitudinal and volumetric strain are relatively straightforward and easily defined strain parameters. Angular strains are slightly more difficult to handle as they measure the change in angle between two lines that were initially perpendicular. The change in angle is called the angular shear, $\psi$, but more commonly we use the tangent of this angle, called the shear strain, $\gamma$ (Figure 6b):
$\gamma=\tan \psi$

Like the longitudinal and volumetric strains, the shear strain is a dimensionless parameter.

## References

Ben A. van der Pluijm, and Stephen Marshak. (2004) EARTH STRUCTURE AN INTRODUCTION TO STRUCTURAL GEOLOGY AND TECTONICS. Second edition.

