University of Anbar

College of Science

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Advanced Structural Geology

Title of the lecture

Rheology

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2022

Rheology

Introduction

We defined strain as the shape change that a body undergoes in the presence of a stress field. But what do we really know about the corresponding stress? And is stress independent of strain? We turn to the final and perhaps most challenging aspect of fundamental concepts: the relationship between stress and strain. Whereas it is evident that there is no strain without stress, the relationship between stress and strain is not easy to define on a physical basis. In other words, realizing that stress and strain in rocks are related is quite a different matter from physically determining their actual relationship(s). In materials science and geology, we use the term rheology to describe **the ability of stressed materials to deform or to flow**, using fundamental parameters such as **strain rate** (strain per unit of time), **elasticity**, and **viscosity**. Recalling that stress and strain are second-order tensors, their proportionality is therefore a fourth-order tensor. Up front we give a few briefs, incomplete descriptions of the most important concepts that will appear throughout these terms to help you to navigate through some of the initial material, until more complete definitions can be given.

- 1. Elasticity: Recoverable (non-permanent), instantaneous strain.
- 2. **Fracturing**: Deformation mechanism by which a rock body or mineral loses coherency by simultaneously breaking many atomic bonds.
- 3. **Nonlinear viscosity**: Permanent strain accumulation where the stress is exponentially related to the strain rate
- 4. **Plasticity:** Deformation mechanism that involves progressive breaking of atomic bonds without the material losing coherency
- 5. Strain rate: Rate of strain accumulation (typically, elongation, e, over time, t); shear strain rate, γ (gamma dot), is twice the longitudinal strain rate
- 6. **Viscosity**: Non-recoverable (permanent) strain that accumulates with time; the strain rate–stress relationship is linear

Rheology is the study of flow of matter. Rocks don't seem to change much by comparison, but remember that geologic processes take place over hundreds of thousands to millions of years. For example, yearly horizontal displacement along the San Andreas Fault (a strike-slip fault zone in California) is on the order of a few centimeters, so considerable deformation has accumulated over the last 700,000 years. Likewise, horizontal displacements on the order of tens to hundreds of kilometers have occurred in the Paleozoic Appalachian fold-and-thrust belt of eastern North America over time period of a few million years (m.y.). Geologically speaking, time is available in large supply, and given sufficient amounts of it, rocks are able to flow, not unlike syrup. The flow of window glass is an urban legend that you can refute with the information presented, when you look through the windows of an old house you may find that the glass distorts your

view. The reason is, as the story goes, that the glass has sagged under its own weight with time (driven by gravity), giving rise to a wavy image. One also finds that the top part of the glass is often thinner than the bottom part.

Strain Rate

The time interval it takes to accumulate a certain amount of strain is described by the strain rate, symbol e', which is defined as elongation (e) per time (t):

 $e' = e/t = \delta l/(l_0 t)$

You recall that elongation, length change divided by original length, $\delta l/l_o$, is a dimensionless quantity; thus, the dimension of strain rate is $[t]^{-1}$; the unit is second⁻¹. This may appear to be a strange unit at first glance, so let's use an example. If 30% finite longitudinal strain (|e| = 0.3) is achieved in an experiment that lasts one hour (3600 s), the corresponding strain rate is $0.3/3600 = 8.3 \times 10^{-5}$ /s. Now let's see what happens to the strain rate when we change the time duration of our experiment, while maintaining the same amount of finite strain.

Time interval for 30% strain

e.

1.	1 day $(86.4 \times 10^3 \text{ s})$	$3.5 imes 10^{-6}/s$
2.	1 year $(3.15 \times 10^7 \text{ s})$	$9.5 imes10^{-9}/s$
3.	1 m.y. $(3.15 \times 10^{13} \text{ s})$	$9.5 imes10^{-1}5/s$

Thus, the value of the strain rate changes as a function of the time period over which finite strain accumulates. Note that the percentage of strain did not differ for any of the time intervals. So, what is the strain rate for a fault that moves 50 km in 1 m.y.? It is not possible to answer this question unless the displacement is expressed relative to another dimension of the body, that is, as a strain. We try again: What is the strain rate of an 800-km long fault moving 50 km in 1 m.y.? We get a strain rate of $(50/800)/(3.15 \times 10^{13}) = 2 \times 10^{-15}$ /s. In many cases, commonly involving faults, geologists prefer to use the **shear strain rates** (γ ⁻). The relationship between shear strain rate and (longitudinal) strain rate is

 γ = 2e[·]

variety of approaches are used to determine characteristic strain rates for geologic processes. A widely used estimate is based on the Quaternary displacement along the San Andreas Fault of California, which gives a strain rate on the order of 10^{-14} /s. Other observations (such as isostatic uplift, earthquakes, and orogenic activity) support similar estimates and typical geologic strain rates therefore lie in the range of 10^{-12} /s to 10^{-15} /s. Now consider a small tectonic plate with a

long dimension of 500 km at a divergent plate boundary. Using a geologic strain rate of 10^{-14} /s, we obtain the yearly spreading rate by multiplying this dimension of the plate by 3.15×10^{-7} /year, giving 16 cm/year, which is the order of magnitude of present-day plate velocities. On a more personal note, your 1.5-cm long fingernail grows 1 cm per year, meaning a growth rate of 0.67/year (or 2×10^{-8} /s). Your nail growth is therefore much, much faster than geologic rates, even though plates "grow" on the order of centimeters as well. We can offer many more geologic examples, but at this point we hope to leave you acquainted with the general concept of strain rate and typical values of 10^{-12} /s to 10^{-15} /s for geologic processes. Note that exceptions to this geologic range are rapid events like meteorite impacts and explosive volcanism, which are on the order of 10^{-2} /s to 10^{-4} /s.

General behavior: The creep curve

Compression tests on rock samples illustrate that the behavior of rocks to which a load is applied is not simple. Figure 1a shows what is called a creep curve, which plots strain as a function of time. In this experiment the differential stress is held constant. Three creep regimes are observed: (1) **primary** or **transient creep**, during which strain rate decreases with time following very rapid initial accumulation; (2) secondary or steady-state creep, during which strain accumulation is approximately linear with time; and (3) tertiary or accelerated creep, during which strain rate increases with time; eventually, continued loading will lead to failure. Restating these three regimes in terms of strain rate, we have regimes of (1) decreasing strain rate, (2) constant strain rate, and (3) increasing strain rate. The strain rate in each regime is the slope along the creep curve. Rather than continuing our creep experiment until the material fractures, we decide to remove the stress sometime during the interval of steady-state creep. The corresponding creep curve for this second experiment is shown in Figure 1b. We see a rapid drop in strain when the stress is removed, after which the material relaxes a little more with time. Eventually there is no more change with time but, importantly, permanent strain remains. In order to examine this behavior of natural rocks we turn to simple analogies and rheologic models.



Figure 1 Generalized strain–time or creep curve, which shows primary (I), secondary (II), and tertiary (III) creep. Under continued stress the material will fail (a); if we remove the stress, the material relaxes, but permanent strain remains (b).

Rheologic relationships

In describing the various rheologic relationships, we first divide the behavior of materials into two types, elastic behavior and viscous behavior (Figure 2). In some cases, the flow of natural rocks may be approximated by combinations of these linear rheologies, in which the ratio of stress over strain or stress over strain rate is a constant. The latter holds true for part of the mantle, but correspondence between stress and strain rate for many rocks is better represented by considering nonlinear rheologies. For each rheologic model that is illustrated in Figure 2 we show a physical analog, a creep (strain–time) curve and a stress–strain or stress–strain rate relationship, which will assist you with the descriptions below. Such equations that describe the linear and nonlinear relationships between stress, strain, and strain rate.

Elastic Behavior

What is elastic behavior and is it relevant for deformed rocks? Let's first look at the relevance. In the field of seismology, the study of earthquakes, elastic properties are very important. As you know, seismic waves from an earthquake pass through the Earth to seismic monitoring stations around the world. As they travel, these seismic waves briefly deform the rocks, but after they have passed, the rocks return to their undeformed state. To imagine how rocks are able to do so we turn to a common analog: a rubber band. When you pull a rubber band, it extends; when you remove this stress, the band returns to its original shape. The greater the stress, the farther you extend the band. Beyond a certain point, called the failure stress, the rubber band breaks and brings a painful end to the experiment. This ability of rubber to extend lies in its atomic structure. The bond lengths between atoms and the angles between bonds in a crystal structure

represent a state of lowest potential energy for a crystal. These bonds are able to elongate and change their relative angles to some extent, without introducing permanent changes in the crystal structure. Rubber bands extend particularly well because rubber can accommodate large changes in the angular relationships between bonds; however, this causes a considerable increase in the potential energy, which is recovered when we let go of the band, or when it snaps. So, once the stress is released, the atomic structure returns to its energetically most stable configuration, that is, the lowest potential energy. Like the elasticity of a rubber band, the ability of rocks to deform elastically also resides in nonpermanent distortions of the crystal lattice, but unlike rubber, the magnitude of this behavior is relatively small in rocks. $\sigma = E \cdot e$

where E is a constant of proportionality called Young's modulus that describes the slope of the line in the σ -e diagram (tangent of angle θ ; Figure 2a). The unit of this elastic constant is Pascal, which is the same as that of stress (recall that strain is a dimensionless quantity). Typical values of E for crustal rocks are on the order of -10^{11} Pa. Linear Equation is also known as Hooke's Law, which describes elastic behavior. We use a spring as the physical model for this behavior (Figure 2a).



Figure 2 Models of linear rheologies. Physical models consisting of strings and dash pots, and associated straintime, stress-strain, or stress-strain rate curves are given for (a) elastic, (b) viscous, (c) viscoelastic, (d)

elasticoviscous, and (e) general linear behavior. A useful way to examine these models is to draw your own straintime curves by considering the behavior of the spring and the dash pot individually, and their interaction. Symbols used: e = elongation, $e^{\cdot} = strain$ rate, $\sigma = stress$, E = elasticity, $\eta = viscosity$, t = time, el denotes elastic component, vi denotes viscous component.

Viscous Behavior

The flow of water in a river is an example of viscous behavior in which, with time, the water travels farther downstream. With this viscous behavior, strain accumulates as a function of time, that is, strain rate. We describe this relationship between stress and strain rate as

$\sigma = \eta \cdot e^{\cdot}$

where η is a constant of proportionality called the viscosity (tan θ , Figure 2b) and e⁻ is the strain rate. This ideal type of viscous behavior is commonly referred to as Newtonian or linear viscous behavior, but do not confuse the use of "linear" in linear viscous behavior with that in linear stress–strain relationships in the elasticity. The term linear is used here to emphasize a distinction from nonlinear viscous (or non-Newtonian) behavior. To obtain the dimensional expression for viscosity, remember that strain rate has the dimension of [t⁻¹] and stress has the dimension [ml⁻¹t ⁻²]. Therefore, η has the dimension [ml⁻¹t⁻¹]. In other words, the SI unit of viscosity is the unit of stress multiplied by time, which is Pa · s (kg/m · s). In the literature we often find that the unit Poise is used, where 1 Poise = 0.1 Pa · s. The example of flowing water brings out a central characteristic of viscous behavior. Viscous flow is irreversible and produces permanent or nonrecoverable strain. The physical model for this type of behavior is the dash pot (Figure 2b), which is a leaky piston that moves inside a fluid-filled cylinder. The resistance encountered by the moving piston reflects the viscosity of the fluid. In the classroom you can model viscous behavior by using a syringe with one end open to the air.

How does the viscosity of water, which is on the order of 10^{-3} Pa · s, compare with that of rocks? Calculations that treat the mantle as a viscous medium produce viscosities on the order of 10^{20} – 10^{22} Pa · s. Obviously the mantle is much more viscous than water (>20 orders of magnitude!). You can demonstrate this graphically when calculating the slope of the lines for water and mantle material in the stress–strain rate diagram; they are 0.06° and nearly 90°, respectively. The much higher viscosity of rocks implies that motion is transferred over much larger distances. Stir water, syrup, and jelly in a jar to get a sense of this implication of viscosity. Obviously, there is an enormous difference between materials that flow in our daily experience, such as water and syrup, and the "solids" that make up the Earth. Nevertheless, we can approximate the behavior of the Earth as a viscous medium over the large amount of time available to geologic processes. Considering an average mantle viscosity of 10^{21} Pa · s and a geologic strain rate of 10^{-14} /s, the differential (or flow) stresses at mantle conditions are on the order of tens of megapascals. Using a viscosity of 10^{14} Pa · s for glass, flow at atmospheric conditions produces a strain rate that is much too slow to produce the sagging effect that is ascribed to old windows.

Viscoelastic Behavior

Consider the situation in which the deformation process is reversible, but in which strain accumulation as well as strain recovery are delayed; this behavior is called viscoelastic behavior. A simple analog is a water-soaked sponge that is loaded on the top. The load on the soaked sponge is distributed between the water (viscous behavior) and the sponge material (elastic behavior). The water will flow out of the sponge in response to the load and eventually the sponge will support the load elastically. For a physical model we place a spring (elastic behavior) and a dash pot (viscous behavior) in parallel (Figure 2c). When stress is applied, both the spring and the dash pot move simultaneously. However, the dash pot retards the extension of the spring. When the stress is released, the spring will try to return to its original configuration, but again this movement is delayed by the dash pot.

Elastic-Viscous Behavior

Particularly instructive for understanding earth materials is elastico-viscous behavior, where a material behaves elastically at the first application of stress, but then behaves in a viscous manner. When the stress is removed the elastic portion of the strain is recovered, but the viscous component remains. We can model this behavior by placing a spring and a dash pot in series (Figure 2d). The spring deforms instantaneously when a stress is applied, after which the stress is transmitted to the dash pot. The dash pot will move at a constant rate for as long as the stress remains. When the stress is removed, the spring returns to its original state, but the dash pot remains where it stopped earlier. When the spring is extended, it stores energy that slowly relaxes as the dash pot moves, until the spring has returned to its original state. The time taken for the stress to reach 1/e times its original value is known as the **Maxwell relaxation time**, where e is the base of natural logarithm (e = 2.718). Stress relaxation in this situation decays exponentially. The Maxwell relaxation time, tM, is obtained by dividing the viscosity by the shear modulus (or rigidity): $tM = \eta/G$

In essence the Maxwell relaxation time reflects the dominance of viscosity over elasticity. If tM is high then elasticity is relatively unimportant, and vice versa. Because viscosity is temperature dependent, tM can be expressed as a function of temperature. Figure 5.4 graphs this relationship between temperature and time for appropriate rock properties and shows that mantle rocks typically behave in a viscous manner (as a fluid). The diagram also suggests that crustal rocks normally fail by fracture (elastic field), but lower crustal rocks deform by creep as well. This discrepancy reflects the detailed properties of crustal materials and their nonlinear viscosities, as discussed later. Maxwell proposed this model to describe materials that initially show elastic behavior, but given sufficient time display viscous behavior, which matches the behavior of Earth rather well. Recall that seismic waves are elastic phenomena (acting over short time intervals) and that the mantle is capable of flowing in a viscous manner over geologic time

(acting over long time intervals). Taking a mantle viscosity of 10^{21} Pa \cdot s and a rigidity of 10^{11} Pa, and assuming an olivine-dominated mantle, we get a Maxwell relaxation time for the mantle of 10^{10} s, or on the order of 1000 years. This time agrees well with the uplift that we see following the retreat of continental glaciers after the last Ice Age, which resulted in continued uplift of regions like Scandinavia over thousands of years after the ice was removed.

General Linear Behavior

So far, we have examined two fundamental and two combined models and, with some further fine-tuning, we can arrive at a physical model that fairly closely approaches reality while still using linear rheologies. Such general linear behavior is modeled by placing the elastico-viscous and viscoelastic models in series (Figure 2e). Elastic strain accumulates at the first application of stress (the elastic segment of the elastic-viscous model). Subsequent behavior displays the interaction between the elastico-viscous and viscoelastic models. When the stress is removed, the elastic strain is first recovered, followed by the viscoelastic component. However, some amount of strain (permanent strain) will remain, even after long time intervals (the viscous component of the elastico-viscous model). The creep (e–t) curve for this general linear behavior is shown in Figure 2e and closely mimics the creep curve that is observed in experiments on natural rocks (compare with Figure 1b). We will not present the lengthy equation describing general linear behavior here, but you realize that it represents some combination of viscoelastic and elastico-viscous behavior.

References

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