University of Anbar

College of Science

Department of Applied Geology

Structural Geology

Title of the lecture

Mohr Circle

Assistant Prof. Dr. Abdulkhaleq A. Alhadithi

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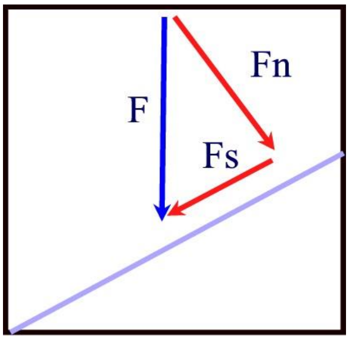
Mohr Circle

Mohr Circle developed by Otto Mohr (1835-1918), a convenient graphical means to depict states of stress.

A force applied to an area (stress) may be resolved into a normal force (Fn) perpendicular to a plane and a shear force (Fs), parallel to a plane in questions figure (1).



F= Fn + Fs

Figure (1) here, vertical force applied to inclined plane resolve into Fn perpendicular to the plane and Fs parallel to it.

Compressional stresses, the main stresses in the earth, are represented into three compressional stress axes. Two of them are horizontal and the third is vertical. σ_1 (sigma 1) is axis of maximum compressional stress. σ_2 (sigma 2) is axis of intermediate compressional stress σ_3 (sigma 3) is axis of minimum compressional stress Stress is a vector quantity that can be considered as: - σ_n (normal stress) is perpendicular to a fracture plane.

σ_s (shear stress) is parallel to a fracture plane. θ (theta) angle is formed by an inclined plane with the maximum and minimum compressional stress directions, It measures from the minimum stress axis (σ_3) to fracture plane or the angle is between σ_1 and σ_n.

There are two types of method to determine of normal stress and shear stress, mathematical and graphical, the first method is by use the following equations:

$$\sigma_n = \frac{(\sigma_1+\sigma_3)}{2} + \frac{(\sigma_1-\sigma_3)}{2} \cos 2\theta$$

$$\sigma_s = \frac{(\sigma_1 - \sigma_3)}{2} \sin 2\theta$$

The second method, graphical, is by use Mohr diagram.

Importance of Mohr Diagram:

1. For any value of maximum compressional stress value and minimum compressional stress value, one can determine the normal and shear stress for any planes that lie at an angle theta
2. Depicts the attitude of planes along which shear stress is the greatest for a given stress state.
3. The most important aspect of Mohr diagram is that it facilitates a quick, graphical determination of stresses on planes of any orientation.
4. Mohr diagrams are excellent for visualizing the state of stress but difficult for calculating stress.

Plotting Mohr's circle:

Mohr's circle is plotted on two perpendicular axes: The vertical axis (ordinate) depicts shear stress and the horizontal axis (abscissa) depicts normal stress figure (2). In geology, convention compressional stress is positive (+) and tensile stress is negative (-).

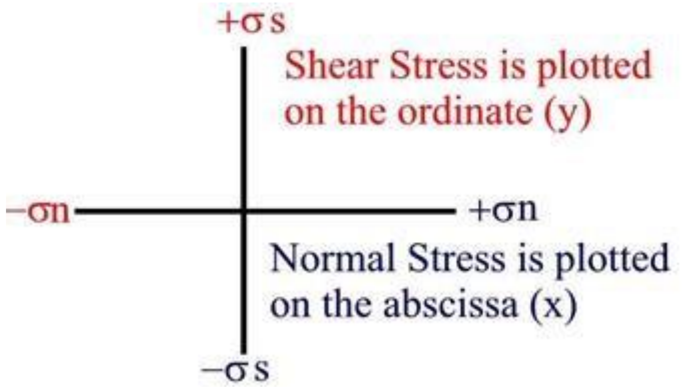


Figure (1) Normal stress plots to horizontal axis and shear stress plot to vertical axis

Maximum principal Stress sigma 1 (maximum compressional stress) and sigma 3 (minimum compressional stress) are plotted as two points on the horizontal axis. These two points define the diameter of a circle. The Circle is plotted on the abscissa figure (3).

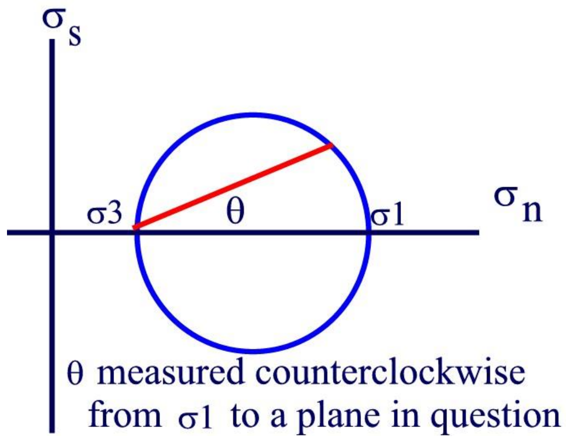


Figure (3) Values of σ_1 and σ_3 determine the diameter of Mohr circle

These points establish a radius (R) whereby:

$$R=\frac{(\sigma 1 - \sigma 3)}{2}$$

The center (C) is then plotted:

$$C=\frac{(\sigma 1 + \sigma 3)}{2}$$

We can determine the normal and shear stresses on any plane oriented at an angle theta from the abscissa, as measured counterclockwise from the minimum compressive stress direction Figure (3). Because of the properties of a circle, the angle of point P is between the center of the circle and the maximum compressional stress direction which equal 2 theta. It measures counterclockwise from the center of the circle figure (4 and 5). Experimental rock fracturing has shown the maximum shear stress acts when theta equal 45º but the maximum value of theta is 90º therefore must be use 2θ in Mohr circle as we will see that later.

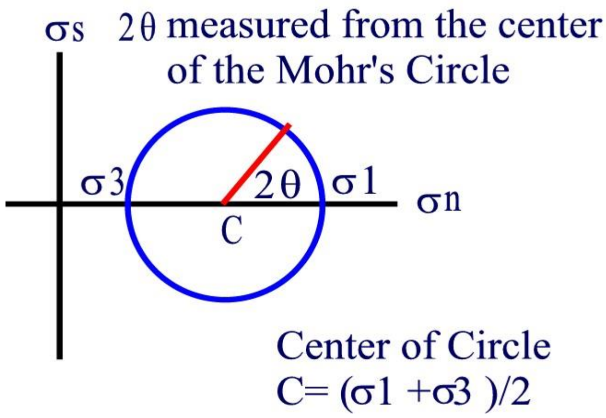


Figure (4) 2θ measures from the center of the Mohr circle.

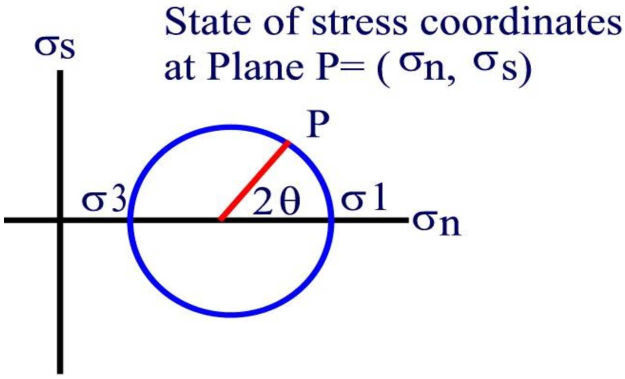


Figure (5) state of stress coordinates at plane p= (σ_n, σ_s)

Mohr circle can graphically depict stress on any plane inclined relative to the principal plane. Normal and shear stresses can be determined graphically using the circle or by using equations.

Maximum shear stress occurs on planes oriented 45° to the maximum and minimum compressive stress directions; thus, these points plot at the top and bottom of Mohr's Circle

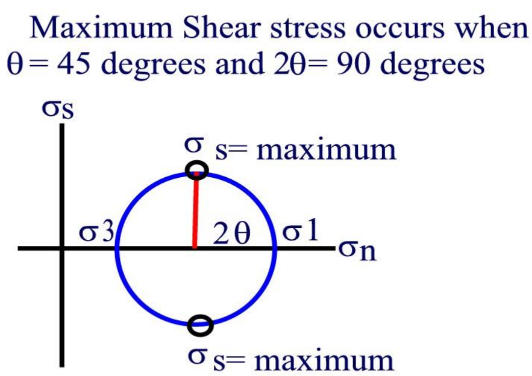


Figure (6) maximum shear stress is in two points positive and negative.

Differential stress, that is the difference between the maximum and minimum compressional stresses, is the most important factor in rock fracturing. The intermediate principal stress generally does not cause rock fracturing. On a Mohr diagram, the following senses of shear conventions apply; Sinistral (counterclockwise) shear is Positive (+) and dextral (clockwise) shear is Negative (-). Angles 2 theta associated with planes experiencing sinistral shear plot in the upper hemisphere. Angles 2 theta associated with planes experiencing dextral shear plot in the lower hemisphere. The axes of Mohr diagram do not have a geographic orientation however, prior to constructing a Mohr diagram it is useful to sketch a block diagram of the orientations of the principal stress axes and the plane in question to ascertain the relative sense of shear and orientation of principal stress axes.

Mohr Envelop of Failure:

Mohr envelop of failure is represented by a straight line with a slope equal to Coulomb coefficient. Number of Mohr circles is plotted and a line tangential to the circles is drawn. It is constructed by using a series of experiments in which the principal stresses change. Failure occurs when the Mohr circle intersects the envelope of failure.

Coulomb's coefficient $\text{U} = \tan \phi$

U(mu) Coulomb coefficient (coefficient of internal friction) slope of the line (envelop of failure) ϕ (phi) angle of internal friction

Experiments are usually constructed with an axial load (maximum compressional stress) is applied to a rock cylinder under a confining pressure figure (7).

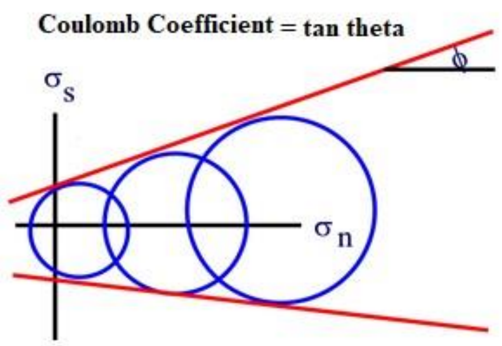


Figure (7) Coulomb failure envelop

Mohr's circle depiction of effective stress and fluid pore pressure: -

Effective Stress = normal stress minus the pore fluid pressure.

$\sigma_e = \sigma_d - P_f$

Mohr circle remains same size but is translated to the left along the horizontal axis figure (8).

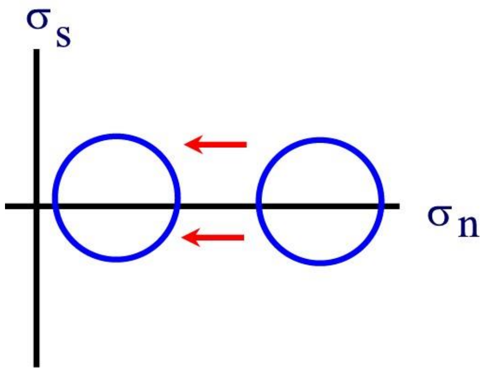


Figure (8) Pore fluid pressure reduces the normal stress

Increase in P_f results in:

1. A reduction in the strength of the rock.
2. Facilitates hydraulic fracturing.

References

Stephen M. Rowland, Las Vegas Ernest M. Duebendorfer, and Ilsa M. Schiefelbein, (2007) Structural Analysis and Synthesis, A Laboratory Course in Structural Geology. Third Edition.