University of Anbar

College of Science

Department of Applied Geology

Structural Geology

Title of the lecture

The failure envelope

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The failure envelope

We have examined the stresses acting on variously oriented planes. The main objective of all of this is to understand or predict the orientation and magnitude of stresses that will cause a particular rock to fracture or fail. To begin our examination of brittle failure we will imagine an experiment in which a cylinder of rock is axially compressed (Fig. 1). Suppose that the radially applied confining pressure, σ_c, while the axial load, σ_a, is gradually increased until the rock fails.

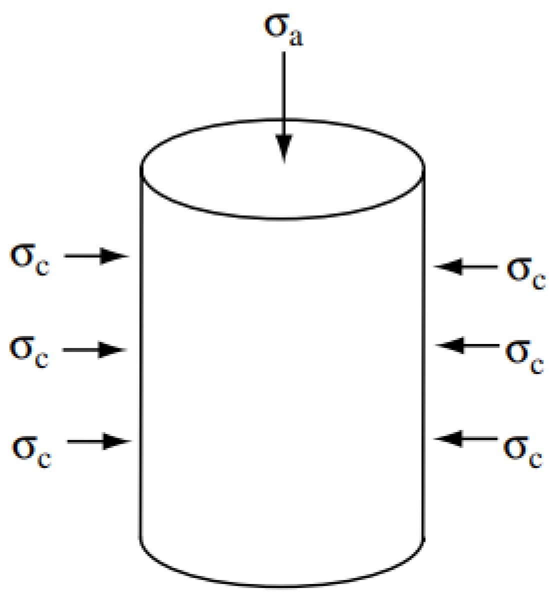

Fig. 1 Schematic diagram of a rock-fracture experiment in which a cylinder of rock is axially compressed. The axial load (σ_a) is steadily increased while the confining pressure is (σ_c).

suppose we performed a series of three experiments on identical samples, but at different confining pressures. We would find that the fracture strength of the rock increases with confining pressure. Table 1 lists the results of our hypothetical series of experiments, with Experiment 1 the confining pressure was 40MPa. In experiment 2 the confining pressure was raised to 150 MPa, and in Experiment 3 to 400 MPa. In Fig. 2 the three resulting Mohr circles are drawn. Because each experiment in this series has a higher confining pressure than the previous one, the Mohr circles at failure become progressively larger. The Mohr circles at failure under different confining pressures together define a boundary called the failure envelope for a particular rock (Fig. 2).

Table 13.2 Data from three fracture experiments on identical rock samples. The Mohr circles at failure are drawn in Fig. 2.

Experiment no.	σ_c (MPa)	σ_a at failure (MPa)	$\sigma_a - \sigma_c$ (MPa)
1	40	540	500
2	150	800	650
3	400	1400	1000

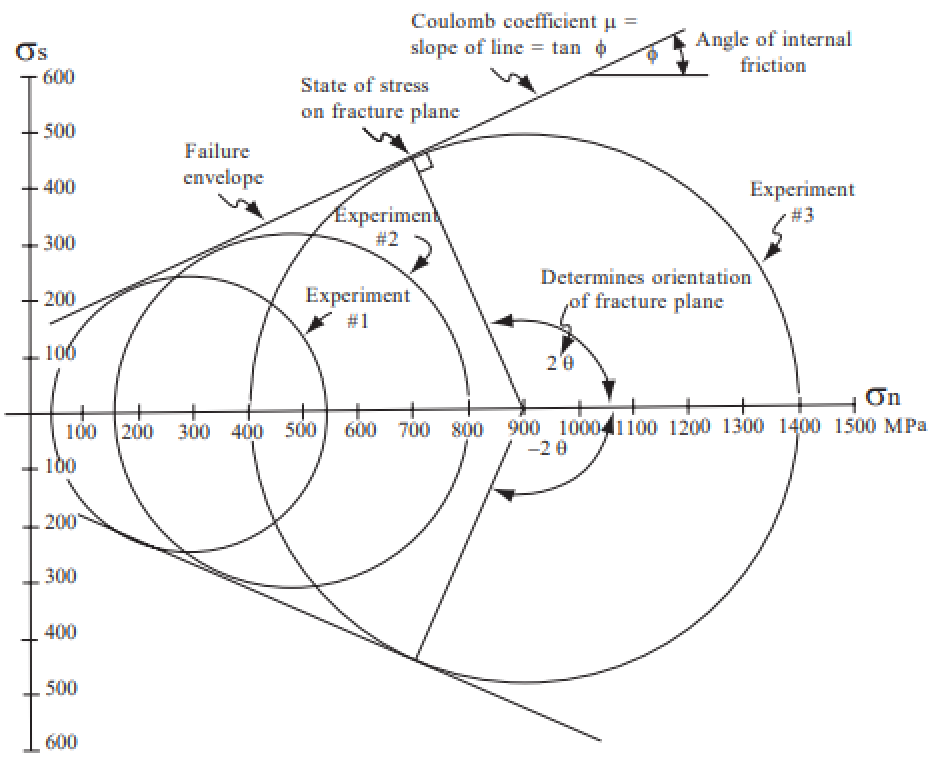

Fig.2 Main characteristics of a failure envelope. The envelope is defined by Mohr circles at failure of identical rock samples under different confining pressures. The data for these three envelopes are recorded in Table 1.

The failure envelope is an empirically derived characteristic that expresses the combination of σ_1 and σ_3 magnitudes that will cause a particular rock (or manmade material such as concrete) to fracture. If the Mohr circle representing a particular combination of σ_1 and σ_3 intersects the material's failure envelope, then the material will fracture; if the Mohr circle does not intersect the failure envelope the material will not fracture.

The failure envelope also allows us to predict the orientation of the macroscopic fracture plane that will form when the rock fails. In an isotropic rock this will be the plane that has a state of stress represented by the point on the Mohr circle that lies on the failure envelope (Fig. 2). The angle between this plane and the σ_3 direction (angle θ) can be determined by measuring angle 2θ directly off the Mohr diagram. In the example shown in Fig. 2, angle $2\theta = 114°$, so the fracture plane will be oriented 57° from σ_3.

At intermediate confining pressures the fracture strength usually increases linearly with increasing confining pressure, producing a failure envelope with straight lines, as in Fig. 2. The angle between these lines and the horizontal axis is called the angle of internal friction, ϕ (phi), and the slope of the envelope is called the Coulomb coefficient, μ (mu):

$$\mu = \tan \phi$$

It is helpful to develop a familiarity with the Coulomb coefficient. This is a measurable property of the rock, like specific gravity, and indicates its fracture behavior at intermediate confining pressures within the earth's crust.

If the failure envelope plots as straight lines, which is typical of brittle materials at low confining pressures, then the Coulomb coefficient can be determined from a single fracture experiment, such as any of the three plotted in Fig. 2. Conversely, if the Coulomb coefficient of a rock is known, the orientation of the shear surfaces relative to σ_1 and σ_3 can be predicted. It can be seen on Fig. 2 that $2\phi = 90 + \phi$, or:

$$\Theta = 45 + \phi/2$$

Figure 3 summarizes the relationships between σ_1, σ_3, θ, ϕ, σ_n, σ_s, and the fracture plane. A material having a Coulomb coefficient m equal to zero would have an angle of internal friction ϕ equal to zero, and $\theta = 45°$. Plastic materials behave this way. As the value of μ increases, angle θ also increases. Measured values of μ for nine rock units are listed in Table 2.

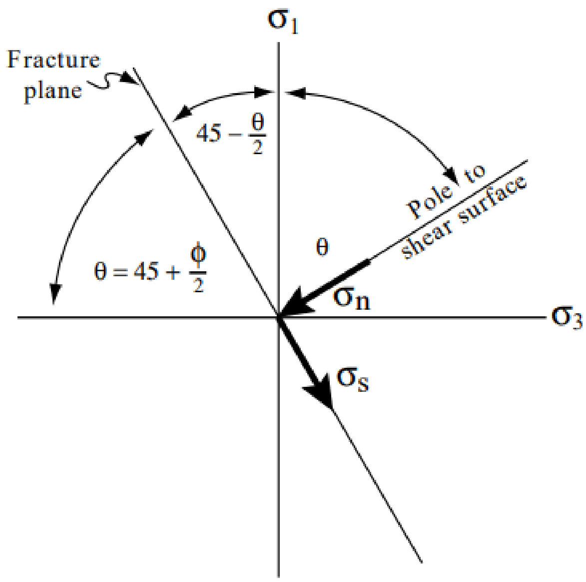

Fig. 3 Generalized relationships between the principal stresses and angles θ and ϕ.

Table 2 Coulomb coefficient m of nine rock units (from Suppe, 1985).

Formation	Coulomb coefficient
Cheshire Quartzite	0.9
Westerly Granite	1.4
Frederick Diabase	0.8
Gosford Sandstone	0.5
Carrara Marble	0.7
Blair Dolomite	0.9
Webatuck Dolomite	0.5
Bowral Trachyte	1.0
Witwatersrand Quartzite	1.0

References

Stephen M. Rowland, Las Vegas Ernest M. Duebendorfer, and Ilsa M. Schiefelbein, (2007) Structural Analysis and Synthesis, A Laboratory Course in Structural Geology. Third Edition