Water Resources Management and Economy

University of Anbar– College of Engineering

Dams & Water Resources Engineering Department – 4th stage

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Operations Research

The first formal activities of Operations Research (OR) were initiated in England during World War II, when a team of British scientists set out to make scientifically based decisions regarding the best utilization of war materiel. After the war, the ideas advanced in military operations were adapted to improve efficiency and productivity in the civilian sector.

What Is Operations Research?

Imagine that you have a 5-week business commitment between Baghdad (BAG) and Arbil (ARB). You fly out of Baghdad on Mondays and return on Wednesdays of the same week. A regular round-trip ticket costs $400, but a 20% discount is granted if the dates of the ticket span a weekend. A one-way ticket in either direction costs 75% of the regular price. How should you buy the tickets for the 5-week period?

We can look at the situation as a decision-making problem whose solution requires answering three questions:

1. What are the decision alternatives?
2. Under what restrictions is the decision made?
3. What is an appropriate objective criterion for evaluating the alternatives?

Three alternatives are considered:

1. Buy five regular BAG - ARB - BAG for departure on Monday and return on Wednesday of the same week.
2. Buy one BAG - ARB, four ARB - BAG- ARB, that span weekends, and one ARB - BAG.
3. Buy one BAG - ARB - BAG to cover Monday of the first week and Wednesday of the last week and four ARB - BAG- ARB, to cover the remaining legs. All tickets in this alternative span at least one weekend.

The restriction on these options is that you should be able to leave **BAG** on Monday and return on Wednesday of the same week.

An obvious objective criterion for evaluating the proposed alternative is the price of the tickets. The alternative that yields the smallest cost is the best. Specifically, we have

Alternative 1 cost = 5 x 400 = $2000

Alternative 2 cost = 0.75 x 400 + 4 x (0.8 x 400) + 0.75 x 400 = $1880

Alternative 3 cost = 5 x (0.8 x 400) = $1600

Thus, you should **choose alternative 3**.

Though the preceding example illustrates the **three main components of an Operations Research model** 1- alternatives, 2- objective criterion, and 3- constraints, situations differ in the details of how each component is developed and constructed.

To illustrate this point, consider forming a maximum-area rectangle out of a piece of wire of length L inches. What should be the width and height of the rectangle?

In contrast with the tickets example, the number of alternatives in the present example is not finite; namely, the width and height of the rectangle can assume an infinite number of values. To formalize this observation, the alternatives of the problem are identified by defining the width and height as continuous (algebraic) variables.

Let

w = width of the rectangle in inches

h = height of the rectangle in inches

Based on these definitions, the constraints of the situation can be expressed verbally as

1. Width of rectangle + Height of rectangle = Half the length of the wire
2. Width and height cannot be negative

These restrictions are translated algebraically as

1. $2(w + h) = L$
2. $w \geq 0, h \geq 0$

The only remaining component now is the objective of the problem; namely, maximization of the area of the rectangle. Let Z be the area of the rectangle, then the complete model becomes:

$$\text{Maximize } Z = w*h$$

subject to

$$2(w+ h) = L \qquad w, h \geq 0$$

The optimal solution of this model is $w = h = \frac{L}{4}$, which calls for constructing a square shape.

Based on the preceding two examples, the general OR model can be organized in the following general format:

Maximize or minimize Objective Function subject to Constraints

A solution of the mode is feasible if it satisfies all the constraints. It is optimal if, in addition to being feasible, it yields the best (maximum or minimum) value of the objective function.

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Ch.2 - Modeling with Linear Programming

Chapter Guide: This chapter concentrates on model formulation and computations in linear programming (LP). It starts with the modeling and graphical solution of a two-variable problem which, though highly simplified, provides a concrete understanding of the basic concepts of LP and lays the foundation for the development of the general simplex algorithm in Chapter 3. To illustrate the use of LP in the real world, applications are formulated and solved in the areas of urban planning, currency arbitrage, investment, production planning and inventory control, gasoline blending, manpower planning, and scheduling.

Mathematical Formulation of LP model

Step 1. Study the given situation, find the key decision to be made. Hence, identify the decision variables of the problem.

Step 2. Formulate the objective function to be optimized.

Step 3. Formulate the constraints of the problem.

Step 4. Add non-negativity restrictions.

The objective function, the set of constraints, and, the non-negativity restrictions together form an LP model.

TWO-VARIABLE LP MODEL

This section deals with the graphical solution of a two-variable LP. Though two-variable problems hardly exist in practice, the treatment provides concrete foundations for the development of the general simplex algorithm.

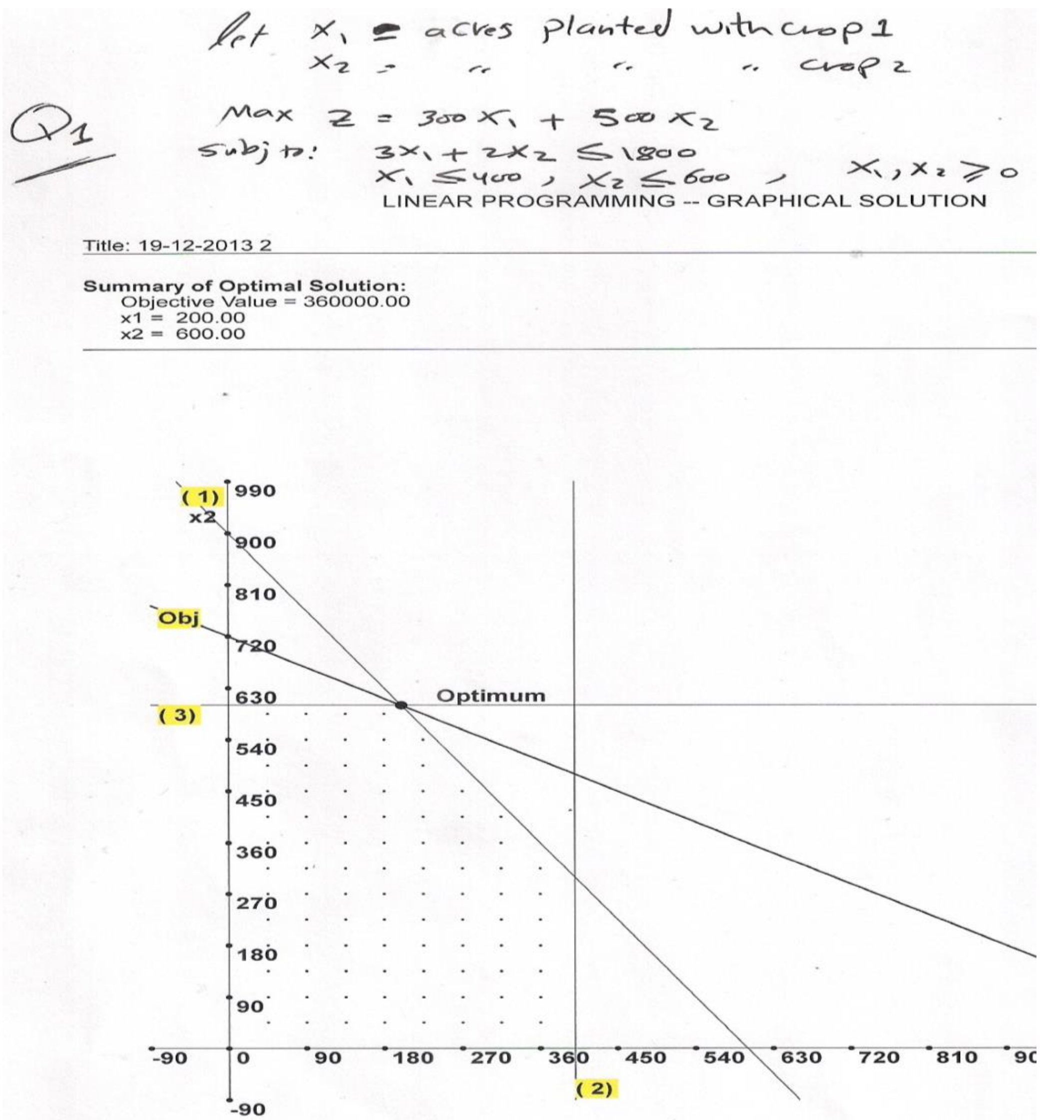
Example 1:- A farm has 1800 acre-feet of water available annually. Two crops are considered for which annual irrigation water requirements are 3 acre-feet/acre and 2 acre-feet/acre, respectively. For various reasons, no more than 400 acres can be planted in crop 1, and no more than 600 acres can be allocated to crop 2. Estimated profits are 300 $ per acre planted in crop 1, and 500 $ per acre planted in crop 2. Determine how many acres to plant in each crop to maximize profits. (Formulate and Graphically solve a linear programming model).

Sol:

The LP model, as in any OR model, has three basic components.

1. Decision variables that we seek to determine.
2. Objective (goal) that we need to optimize (maximize or minimize).
3. Constraints that the solution must satisfy.

The proper definition of the decision variables is an essential first step in the development of the model. Once done, the task of constructing the objective function and the constraints becomes more straightforward.



Example 2.

A Company produces both interior and exterior paints from two raw materials, M1 and M2. The following table provides the basic data of the problem:

	Tons of raw material per ton of		Maximum availability (tons) per day
	Exterior paint	Interior paint	
Raw material, M1	6	4	24
Raw material, M2	1	2	6
Profit per ton ($1000)	5	4	

A market survey indicates that the daily demand for interior paint cannot exceed that for exterior paint by more than 1 ton. Also, the maximum daily demand for interior paint is 2 tons. The Company wants to determine the optimum (best) product mix of interior and exterior paints that maximizes the total daily profit. (Formulate and Graphically solve a linear programming model).

Solution:

The LP model, as in any **OR** model, has three basic components.

1- Decision variables that we seek to determine.
2- Objective (goal) that we need to optimize (maximize or minimize).
3- Constraints that the solution must satisfy.

The variables of the model are defined as

xl = Tons produced daily of exterior paint
x2 = Tons produced daily of interior paint

Total profit from exterior paint = $5X_1$ (thousand) dollars
Total profit from interior paint = $4X_2$ (thousand) dollars

The objective of the company is

Maximize **$Z = 5x1 + 4x2$**

Next, we construct the constraints that restrict raw material usage and product demand. The raw material restrictions are expressed verbally as

(Usage of a raw material by both paints) ≤ (Maximum raw material availability)

$$6X1 + 4X2 \leq 24 \text{ (Raw material M1)}$$
$$Xl + 2X2 \leq 6 \text{ (Raw material M2)}$$

The first demand restriction stipulates that the excess of the daily production of interior over exterior paint, x2 - x1, should not exceed 1 ton, which translates to

$$x2 - x1 \leq 1 \quad \text{(Market limit)}$$

The second demand restriction stipulates that the maximum daily demand of interior paint is limited to 2 tons, which translates to

$$X2 \leq 2 \quad \text{(Demand limit)}$$

An implicit (or "understood-to-be") restriction is that variables x1 and x2 cannot assume negative values. The nonnegative restrictions, $x1 \geq 0$, $x2 \geq 0$, account for this requirement.

The complete Company model is:

Maximize $Z = 5X1 + 4X2$

subject to

$$6x1 + 4X2 \leq 24 \quad (1)$$
$$X1 + 2x2 \leq 6 \quad (2)$$
$$- x1 + x2 \leq 1 \quad (3)$$
$$x2 \leq 2 \quad (4)$$
$$X1, X2 \geq 0 \quad (5)$$

Any values of x1 and x2 that satisfy all five constraints constitute a feasible solution. Otherwise, the solution is infeasible.

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Step 1. Determination of the Feasible Solution Space:

First, we account for the non-negativity constraints x1≥ 0 and x2≥0. In Figure 2.1, the horizontal axis x1 and the vertical axis x2 represent the exterior- and interior-paint variables, respectively. Thus, the non-negativity of the variables restricts the solution-space area to the first quadrant that lies above the xl-axis and to the right of the x2-axis.

To account for the remaining four constraints, first replace each inequality with an equation and then graph the resulting straight line by locating two distinct points on it. For example, after replacing 6x1 + 4x2 ≤ 24 with the straight line 6x1 + 4x2 = 24, we can determine two distinct points by first setting xl = 0 to obtain x2 = 6 and then setting x2 = 0 to obtain x1 =4. Thus, the line passes through the two points (0, 6) and (4, 0), as shown by line (1) in Figure 2.1.

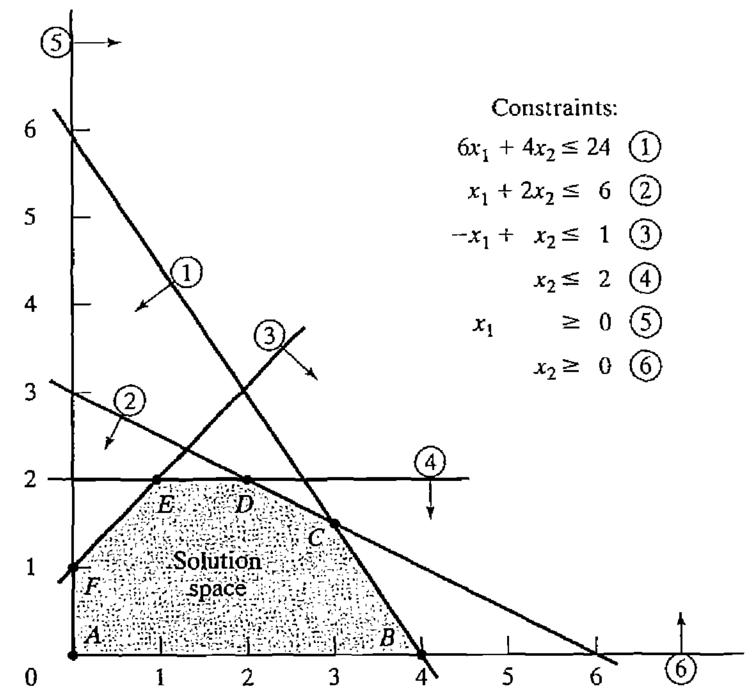


FIGURE 2.1 Feasible space of the Reddy Mikks model

Step 2. Determination of the Optimum Solution:
The feasible space in Figure 2.1 is delineated by the line segments joining the points A, B, C, D, E, and F. Any point within or on the boundary of the space ABCDEF is feasible. Because the feasible space ABCDEF consists of an infinite number of points, we need a systematic procedure to identify the optimum solution.

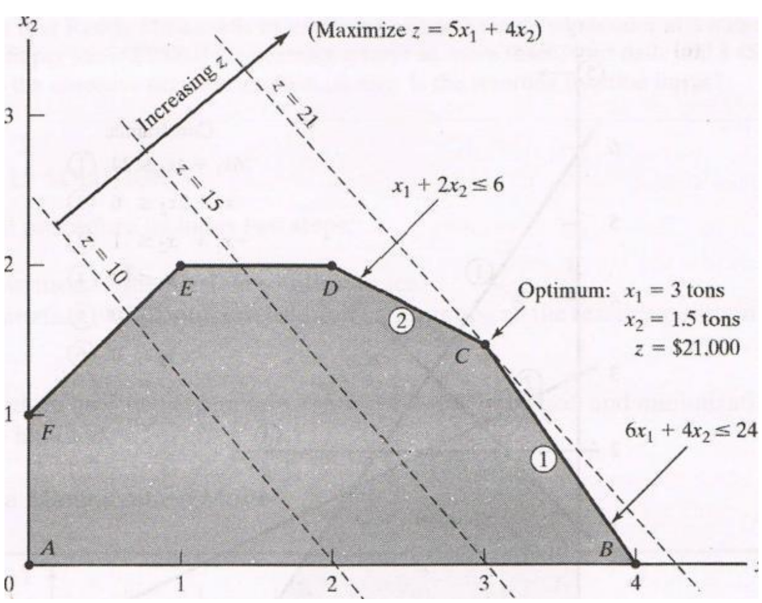
The determination of the optimum solution requires identifying the direction in which the profit function z = 5x1 + 4x2 increases (recall that we are maximizing z). We can do so by assigning arbitrary increasing values to z. For example, using z = 10 and z = 15 would be equivalent to graphing the two lines 5.x1 + 4x2 = 10 and 5x1 + 4x2 = 15. Thus, the direction of increase in z is as shown Figure 2.2. The optimum solution occurs at C, which is the point in the solution space beyond which any further increase will put z outside the boundaries of ABCDEF.

The values of x1 and x2 associated with the optimum point C are determined by solving the equations associated with lines (1) and (2)—that is,

6x1 + 4x2 = 24

x1 + 2x2 = 6

The solution is x1 = 3 and x2 = 1.5 with z = 5* 3 + 4 * 1.5 = 21. This calls for a daily product mix of 3 tons of exterior paint and 1.5 tons of interior paint. The associated daily profit is $21,000.



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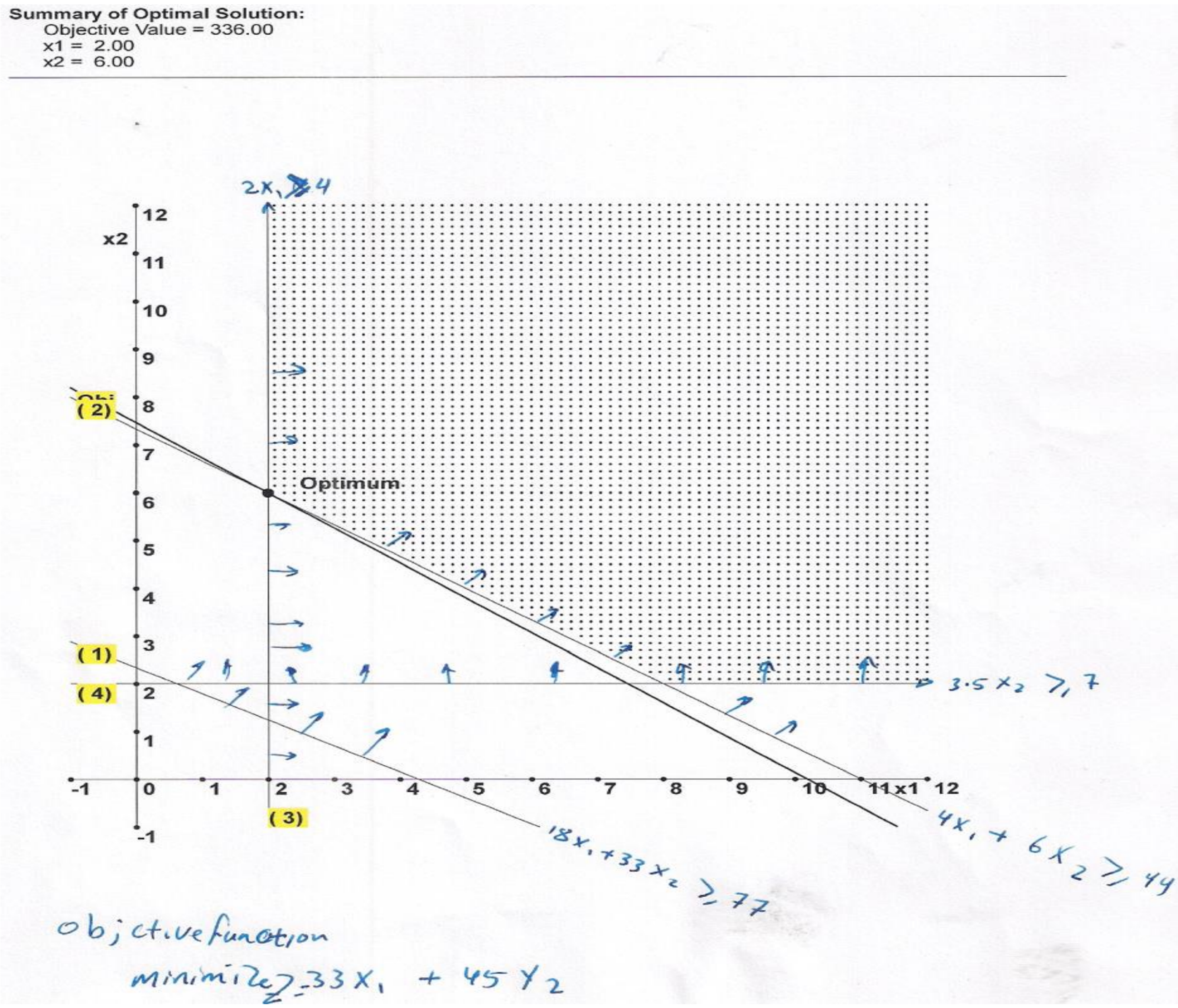
Example: Solve the following problem using Graphical method:

Minimize $Z = 33X1 + 45X2$

Subject to:

$18X1+33X2 \geq 77$ $4X1+6X2 \geq 44$ $2X1 \geq 4$ $3.5X2 \geq 7$ $X1,X2 \geq 0$



Self –Test:

A farm need at least 800 m^3 of water daily. The water is provided by two nearby wells, and have the following properties:

	TDS (ppm)	Nitrate (ppm)	Cost $/ m^3
Well 1	980	125	0.3
Well 2	300	20	0.9

The special requirements of the crop in the farm are at most 600 ppm for TDS, and at least 50 ppm for Nitrate. The farm directorate wishes to determine the daily mixture of water from the two wells to obtain daily minimum cost. (use graphical method)

Quiz 12-8-2015

Two types of crops A & B can be grown in particular irrigation area each year. Each unit quantity of two types of crops can be sold for a price and requires units of water, units of land, units of fertilizer, and units of labor as shown in table below.

(a) Structure a linear programming model for estimating the quantities of each of the two crops that should be produced in order to maximize total income.

(b) Solve the problem graphically.

	REQUIREMENTS PER UNIT OF		
Resource	Crop A	Crop B	Maximum Available Resources
Water	2	3	60
Land	5	2	80
Fertilizer	3	2	60
Labor	1	2	40
Unit Price (1000 $)	30	25	