Introduction

THEORY OF ELASTIC STABILITY

Stability is not easy to define:

- Every structure is in equilibrium – static or dynamic. If it is not in equilibrium, the body will be in motion or a *mechanism*.
- A *mechanism* cannot resist loads and is of no use to the civil engineer.
- Stability qualifies the state of equilibrium of a structure. Whether it is in *stable* or *unstable* equilibrium

A stable structure is the one which remains stable for any imaginable system of loads

Therefore, we do not consider the type of loads, their numbers and their point of applications to decide whether the structure is stable or not. Normally internal and external stability of the structure is checked. If it meets the criteria of stability then the structure is checked for the total degree of indeterminacy.

The concept of the stability of various forms of equilibrium of a compressed bar is frequently explained by considering the equilibrium of a ball (rigid body) in various positions, as shown in Fig. 1:

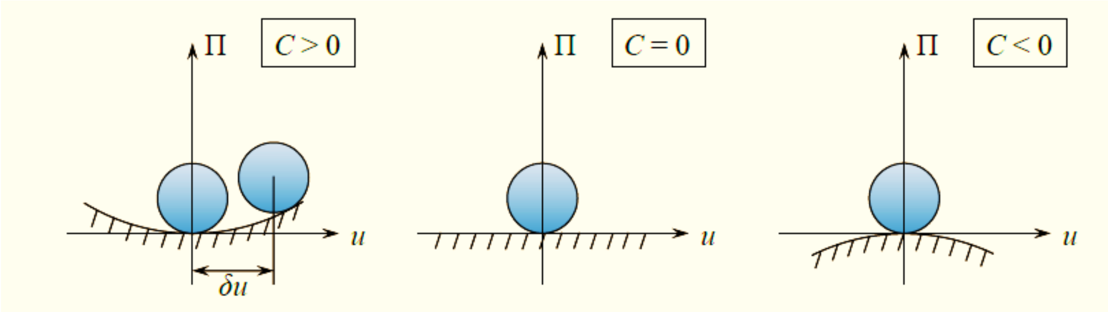

Figure 1: Illustration of stable, neutral and unstable equilibrium.

In the case of a rigid body the total potential energy is just the potential energy: $\prod = mgh = C\,u^2$

where u is the horizontal displacement of the ball from the resting position. Let's calculate the first and second variation of the function $\prod(u)$:

$$\delta\prod = 2Cu\delta u$$

$$\delta^2\prod = 2Cu\delta u\delta u$$

At the origin of the coordinate system $u = 0$, so the first variation of $\prod$ is zero no matter what the sign of the coefficient C is. In the expression for the second variation, the product $\delta u\delta u = \delta u^2$ is always non-negative. Therefore, the sign of the second variation depends on the sign of the coefficient C. From Fig.1 we infer that C > 0 corresponds to a stable configuration. The ball displaced by a small amount δu will return to the original position. By contrast, for C < 0, the ball, when displaced by a tiny amount δu, will roll down and disappear. We call this an unstable behavior. The case C = 0 corresponds to the neutral equilibrium. Then

- Structure is in stable equilibrium when small perturbations do not cause large movements like a mechanism. Structure vibrates about it equilibrium position.
- Structure is in unstable equilibrium when small perturbations produce large movements – and the structure never returns to its original equilibrium position.
- Structure is in neutral equilibrium when we can't decide whether it is in stable or unstable equilibrium. Small perturbation cause large movements – but the structure can be brought back to its original equilibrium position with no work.
- Thus, stability talks about the equilibrium state of the structure.
- A physical phenomenon of a reasonably straight, slender member (or body) bending laterally (usually abruptly) from its longitudinal position due to compression is referred to as buckling.

BUCKLING Vs. STABILITY

- Change in geometry of structure under compression – that results in its ability to resist loads – called *buckling*.
- *Buckling* is a phenomenon that can occur for structures under compressive loads.
  - The structure deforms and is in stable equilibrium in state-1.
  - As the load increases, the structure suddenly changes to deformation state-2 at some critical load P_{cr}.
  - The structure buckles from state-1 to state-2, where state-2 is orthogonal (has nothing to do, or independent) with state-1.

- What has buckling to do with stability?
  - The question is - Is the equilibrium in state-2 stable or unstable?
  - Usually, state-2 after buckling is either neutral or unstable equilibrium
  - The equilibrium state becomes unstable due to:
    - Large deformations of the structure
    - Inelasticity of the structural materials

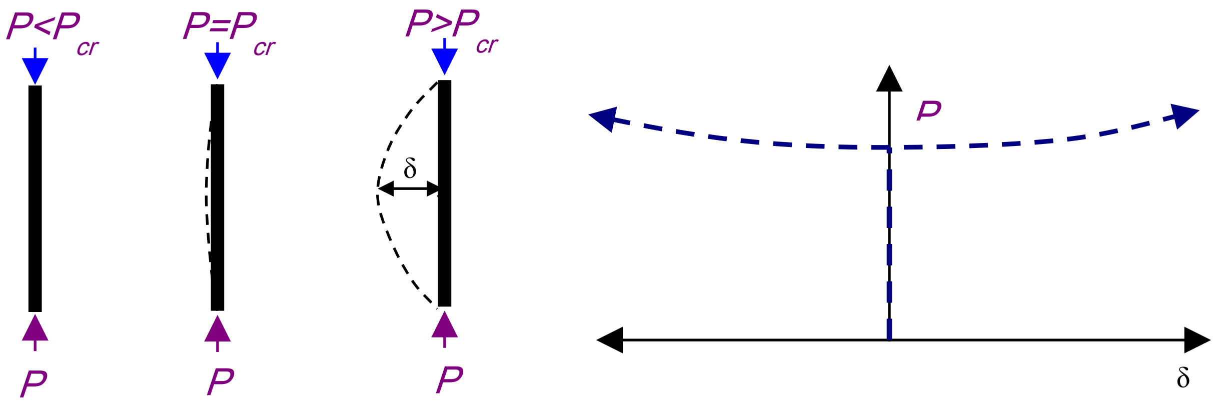

Figure 2: Illustration of buckling state of column.

TYPES OF INSTABILITY

Structure subjected to compressive forces can undergo:

1. Buckling – bifurcation of equilibrium from deformation state-1 to state-2. Bifurcation buckling occurs for columns, beams, and symmetric frames under gravity loads only. When member or

structure subjected to loads. As the load is increased, it reaches a *critical* value where:

- The deformation changes suddenly from state-1 to state-2.
- And, the equilibrium load-deformation path bifurcates.

At post-buckling load-deform, paths are *symmetric* about load axis so:

- If the load capacity increases after buckling then stable symmetric bifurcation.
- If the load capacity decreases after buckling then unstable symmetric bifurcation.

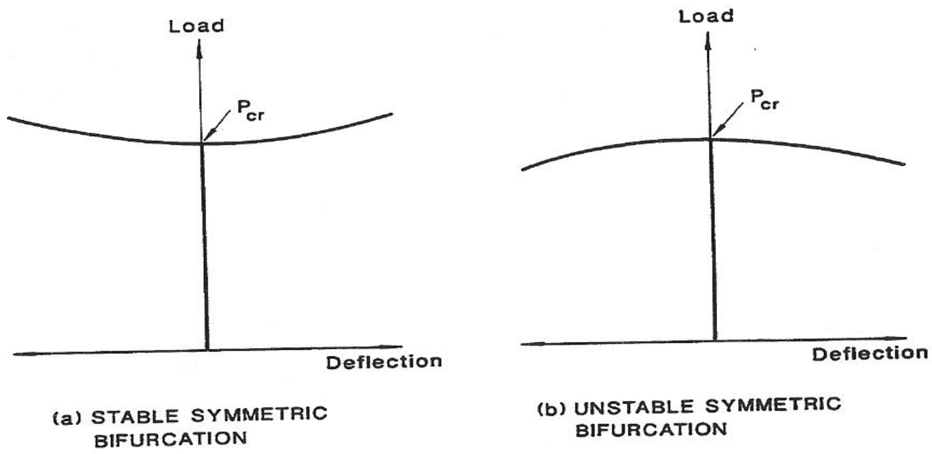

Figure 3: Illustration of buckling bifurcation

2. Failure due to instability of equilibrium state-1 due to large deformations or material inelasticity
   a. Elastic instability occurs for beam-columns, and frames subjected to gravity and lateral loads.
   b. Inelastic instability can occur for all members and the frame.

The structure stiffness decreases as the loads are increased. The change is stiffness is due to large deformations and / or material inelasticity.

- The structure stiffness decreases to zero and becomes negative.
- The load capacity is reached when the stiffness becomes zero.
- Neutral equilibrium when stiffness becomes zero and unstable equilibrium when stiffness is negative.
- Structural stability failure – when stiffness becomes negative.

DESECREPTION OF THE COURSE

The subject of this course is the stability of structures subjected to external loading that induces compressive stresses in the body of the structures. The structural elements examined are beams, columns, beam-columns and frames. Emphasis is on understanding the behavior of structures in terms of load-displacement characteristics; on formulation of the governing equations; and on calculation of the critical load.

Buckling is essentially flexural behavior. Therefore, it is imperative to examine the condition of equilibrium in a flexurally deformed configuration (adjacent equilibrium position). The governing stability equations are derived by both the equilibrium method and the energy method.

Stability analysis is a topic that fundamentally belongs to nonlinear analysis. The fact that the eigenvalue procedure in modern matrix and/or finite element analysis is a fortuitous by-product of incremental nonlinear analysis is a reaffirming testimony. The modern emphasis on fast-track education designed to limit the number of required credit hours for core courses in curriculums left many budding practicing structural analysts with gaping gaps in their understanding of the theory of elastic stability. Many advanced works on structural stability describe clearly the fundamental aspects of general nonlinear structural analysis. There is a need for understanding the fundamentals of structural stability analysis within the context of elementary nonlinear flexural analysis. It is believed that a firm grasp of these fundamentals and principles is essential to performing the important interpretation required of analysts when computer solutions are adopted.