University of Anbar

College of Agriculture

Dept. of Agricultural Economics

Operations research

Third stage

Dept. of Agricultural Economics

Sixth Lecture

Simplex Method- Dual Model/2

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The relationship between the original optimal solution and the Dual Model

The elements of the optimal solution of the binary model (dual model) can be deduced from the elements of the optimal solution of the elementary model, as follows: -

- $opt\ Z = opt\ \acute{Z}$
- The coefficients of the initial solution variables S_i and A_i in the optimal solution table of the elementary model are equal to the value of the solution variables of the binary model and vice versa, i.e. the coefficients of the variables of the elementary solution in the optimal solution of the binary model are equal to the value of the optimal solution variables of the elementary model.

Example: Solve the following linear model using the simplex method (Big M)

$$\min Z = 6X_1 + 5X_2$$

Subject to

$$2X_1 + X_2 \geq 5$$

$$3X_1 + 4X_2 \geq 9$$

$$X_1, X_2 \geq 0$$

Solution:

$$\min Z = 6X_1 + 5X_2 + 0S_1 + 0S_2 + mA_1 + mA_2$$

Subject to

$$2X_1 + X_2 - S_1 + A_1 = 5$$

$$3X_1 + 4X_2 - S_2 + A_2 = 9$$

$$X_1, X_2, S_1, S_2, A_1, A_2 \geq 0$$

min	6	5	0	m	0	m	
B.V	X_1	X_2	S_1	A_1	S_2	A_2	RHS
$m \leftrightarrow A_1$	2	1	-1	1	0	0	5
$m \leftrightarrow A_2$	3	4	0	0	-1	1	9
Z	$5m$	$5m$	$-m$	m	$-m$	m	$14m$
$C - Z$	$6 - 5M$	$5 - 5m$	m	0	m	0	

Let the value of $m = 100$

$$6 - 5(100) = -494$$

$$5 - 5(100) = -495$$

The rule here is that the input variable is the one with the least negative value in class $C - Z$ and here the lowest negative value is -495

min	6	5	0	m	0	m		Ratio
B.V	X_1	X_2	S_1	A_1	S_2	A_2	RHS	
$m \leftrightarrow A_1$	2	1	-1	1	0	0	5	$5 \div 1 = 5$
$m \leftrightarrow A_2$	3	4	0	0	-1	1	9	$9 \div 4 = 2.25$
Z	$5m$	$5m$	$-m$	m	$-m$	m	$14m$	
$C - Z$	$6 - 5M$	$5 - 5m$	m	0	m	0		

The entering variable is X_2 because it corresponds to the lowest positive value in the ratio column. The outside variable is A_2 and the fulcrum is 4.

We extract the row numbers opposite in front of $5 - X_2$ (the second row in the following table by dividing the row numbers by the fulcrum, as follows:

min	6	5	0	m	0	m		Ratio
B.V	X_1	X_2	S_1	A_1	S_2	A_2	RHS	
$m \leftrightarrow A_1$	1.25	0	-1	1	0.25	-0.25	2.75	
$5 - X_2$	0.75	1	0	0	-0.25	0.25	2.25	
Z	$1.25m + 3.75$	5	$-m$	m	$0.25m + (-1.25)$	$-0.25m + 1.25$		
$C - Z$	$2.25 - 1.25m$	0	m	0	$1.25 - 0.25m$	$1.25m - 1.25$		

The row numbers in front of $m - A_1$ are extracted using the previous rule, but the intersection number in the old row is 1.

min	6	5	0	m	0	m		Ratio
B.V	X_1	X_2	S_1	A_1	S_2	A_2	RHS	
$m \leftrightarrow A_1$	1.25	0	-1	1	0.25	-0.25	2.75	$2.75 \div 1.25 = 2.5$
$5 - X_2$	0.75	1	0	0	-0.25	0.25	2.25	$2.25 \div 0.75 = 3$
Z	$1.25m + 3.75$	5	$-m$	m	$0.25m + (-1.25)$	$-0.25m + 1.25$		
$C - Z$	$2.25 - 1.25m$	0	m	0	$1.25 - 0.25m$	$1.25m - 1.25$		

The previous table is not the best solution because the values in front of the row $C - Z$ are still negative and the new variable will be entered which is $X_1$and we divide the RHS numbers by the numbers X_1

Then we extract the new numbers in the new table, as follows:

min	6	5	0	m	0	m		Ratio
B.V	X_1	X_2	S_1	A_1	S_2	A_2	RHS	
$6-X_1$	1	0	-0.8	0.8	0.2	-0.2	2.2	
$5-X_2$	0	1	0.6	-0.6	-0.4	0.4	0.6	
Z	6	5	-1.8	1.8	-0.8	0.8	16.2	
$C-Z$	0	0	1.8	$m - 1.8$	0.8	$m - 0.8$		

From the table above, we find that the numbers in front of the row $C-Z$ are positive, just as the numbers m In row Z are known numbers. So we came to the perfect solution

optimization test

$$X_1 = 2.2$$

$$X_2 = 0.6$$

$$Z = 16.2$$

We note that the values of the coefficients of the variables of the primary solution in the optimal solution table for this elementary model, i.e. the coefficients A_1,A_2 in this table (the optimal solution table) equal, respectively, 1.8 and 0.8, excluding the negative sign. After solving the binary model for this elementary model, we will see that the optimal values of its variables Y_1,Y_2 are exactly equal to 1.8 and 0.8, meaning the values of the optimal solution variables of the binary model (dual model) are equal to the coefficients of the variables of the primary solution in the optimal solution of the elementary model.

Now we are trying to solve the binary form of the previous elementary model to make sure that the elements of the optimal solution of the binary are the same that we found in the table of the optimal solution of the elementary model and thus can be extracted directly from it without resorting to the solution of the binary model and vice versa also.

So the binary form(dual model) of the previous elementary form is:

$$Max\ Z = 5Y_1 + 9Y_2$$

Subject to

$$2Y_1 + 3Y_2 \leq 6$$

$$Y_1 + 4Y_2 \leq 5$$

$$Y_1, Y_2 \geq 0$$

Solution:

$$Max\ Z = 5Y_1 + 9Y_2 + 0S_1 + 0S_2$$

Subject to

$$2Y_1 + 3Y_2 + S_1 = 6$$

$$Y_1 + 4Y_2 + S_2 = 5$$

By following the same method as the previous solution when maximizing the function, as follows:

Initial Plan		Price	5	9	0	0
Price		resources	Y_1	Y_2	S_1	S_2
0	S_1	6	2	3	1	0
0	S_2	5	1	4	0	1
	Z	0	0	0	0	0
	$Z-P$	0	-5	-9	0	0

By adopting the previous rules, the inner activity is Y_2 because it represents the highest value in the objective function and is equal to 9

Initial Plan		Price	5	9	0	0	Ratio
Price		resources	Y_1	Y_2	S_1	S_2	
0	S_1	6	2	3	1	0	$6 \div 3 = 2$
0	S_2	5	1	4	0	1	$5 \div 4 = 1.25$
	Z	0	0	0	0	0	
	$Z-P$	0	-5	-9	0	0	

The entering activity is Y_2 and the leaving activity is S_2 because it corresponds to the lowest value in the ratio column, which is 1.25

The fulcrum is 4 .

And we switch to the new table and extract its new numbers by adopting the previous rules

Initial Plan		Price	5	9	0	0	Ratio
Price		resources	Y_1	Y_2	S_1	S_2	
0	S_1	2.25	1.25	0	1	-0.75	$2.25 \div 1.25 = 1.8$
9	Y_2	1.25	0.25	1	0	0.25	$1.25 \div 0.25 = 5$
	Z	11.25	2.25	9	0	2.25	
	$Z - P$	11.25	-2.25	0	0	2.25	

Through the numbers of the row $Z - P$ we take the largest negative value which is here -2.25 under the column Y_1 and by dividing the numbers of the resource column by the numbers of the column Y_1 we get the Leaving activity and here it is S_1

The entering activity is Y_1 , the leaving activity is S_1 and the fulcrum is 1.25

We move to the new table and by following the same rules as above, we get the following:

Initial Plan		Price	5	9	0	0
Price		resources	Y_1	Y_2	S_1	S_2
5	Y_1	1.8	1	0	0.8	-0.6
9	Y_2	0.8	0	1	-0.2	0.44
	Z	16.2	5	9	2.2	0.96
	$Z - P$	16.2	0	0	2.2	0.96

The last results were obtained in the above table, which represents the optimal solution table, as the digits of the $Z - P$ row became all zeros..

$$Y_1 = 1.8 \ , \qquad Y_2 = 0.8 \qquad Z = 16.2$$

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