Al-Anbar University
College of engineering
Electrical Engineering Department

fundamental of Electric Circuit 1
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Stage 1
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LECTURE 05

SOURCES IN SERIES AND PARALLEL
VOLTAGE DIVIDER RULE-CURRENT DIVIDER RULE

Topics

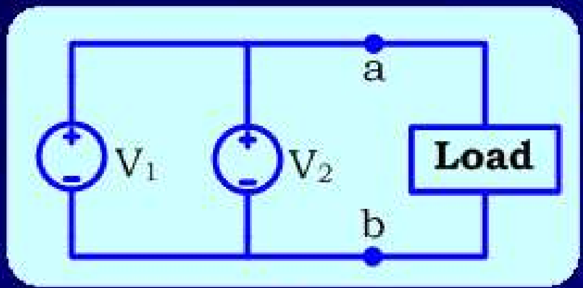
- Voltage Sources in Series and Parallel
- Current Sources in Series and Parallel
- Combining KVL and Ohm's Law
- Voltage Divider Rule
- Current Divider Rule

Objectives

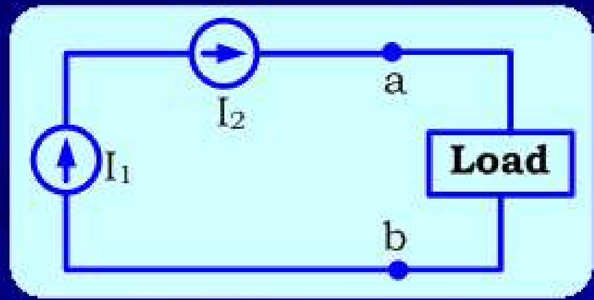
- Recognize invalid series and parallel source connections
- Combine voltage sources in series
- Combine current sources in parallel
- Directly incorporate Ohm's Law in KVL
- Use the Voltage Divider Rule to simplify circuit analysis
- Use the Current Divider Rule to simplify circuit analysis

Series and Parallel Connection of Sources

Both circuits are invalid, why?



(a)



(b)

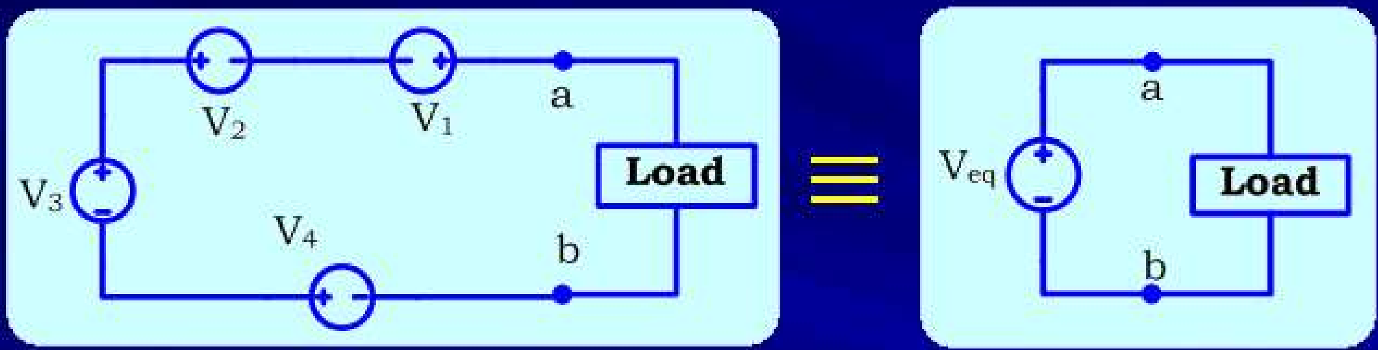
Circuit (a) violates KVL $\Longrightarrow$ Ideal voltage sources cannot be combined in parallel (unless they have the same voltage)

Circuit (b) violates KCL $\Longrightarrow$ Ideal current sources cannot be combined in series (unless they have the same current)

Voltage sources in series

We can connect ideal voltage sources in series

Voltage sources in series can be reduced to a single voltage source

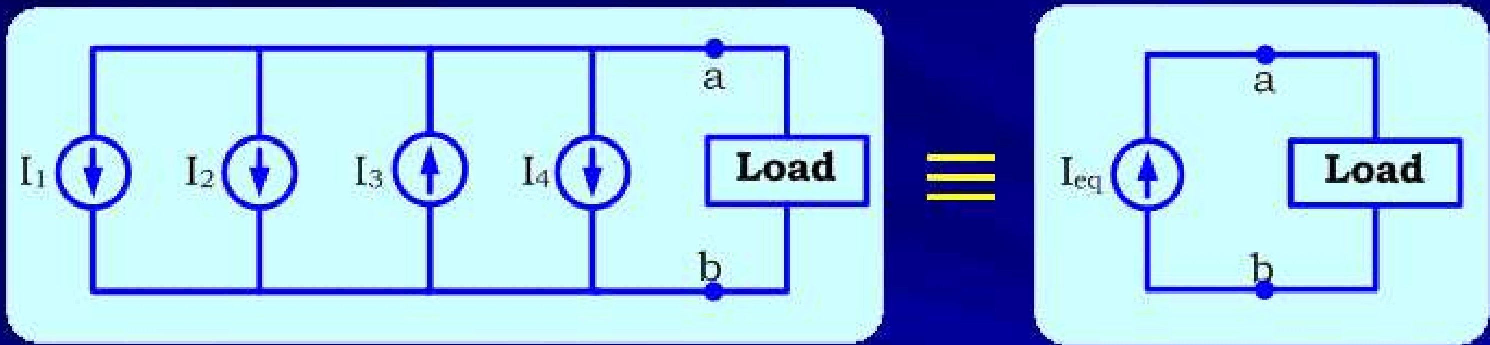


$$V_{eq} = V_1 - V_2 + V_3 + V_4$$

Current sources in parallel

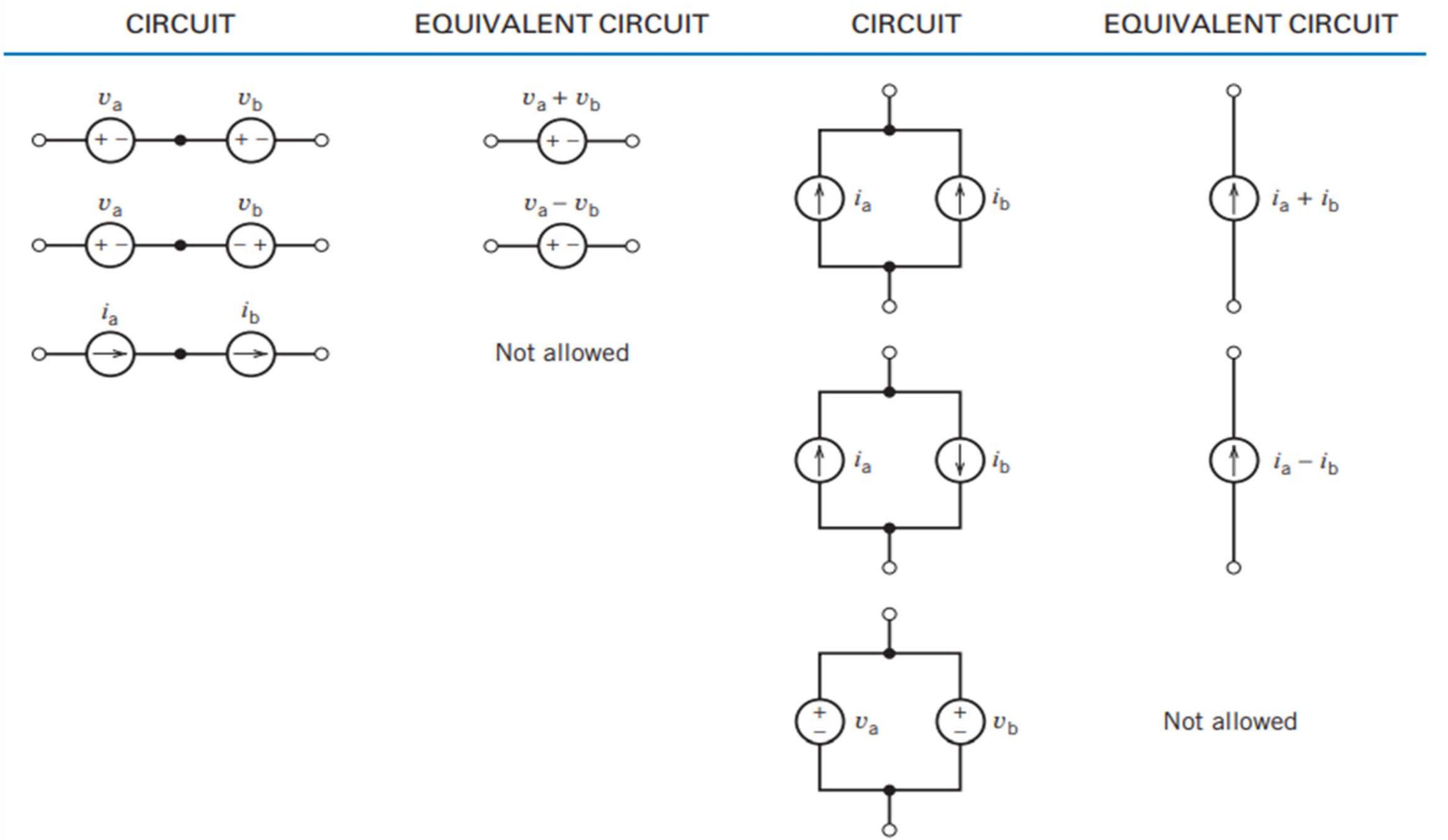
We can connect ideal current sources in parallel

Current sources in parallel can be combined as a single current source



$$I_{eq} = -I_1 - I_2 + I_3 - I_4$$

Parallel and serious voltage and current sources



Example

Figures 3.5-3*a* and *c* show two similar circuits. Both contain series voltage sources and parallel current sources. In each circuit, replace the series voltage sources with an equivalent voltage source and the parallel current sources with an equivalent current source.

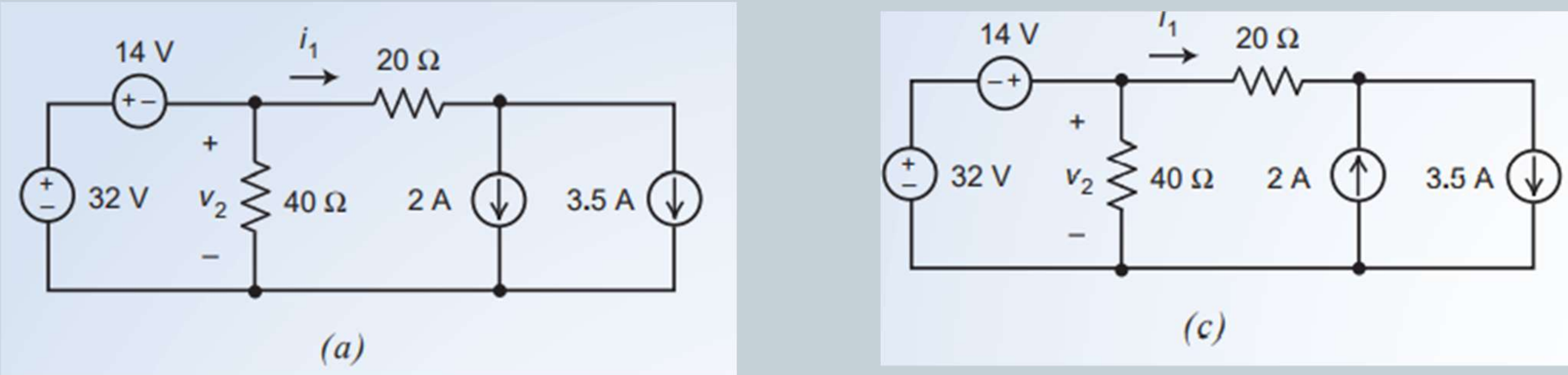


Figure 3.5-3

Solution

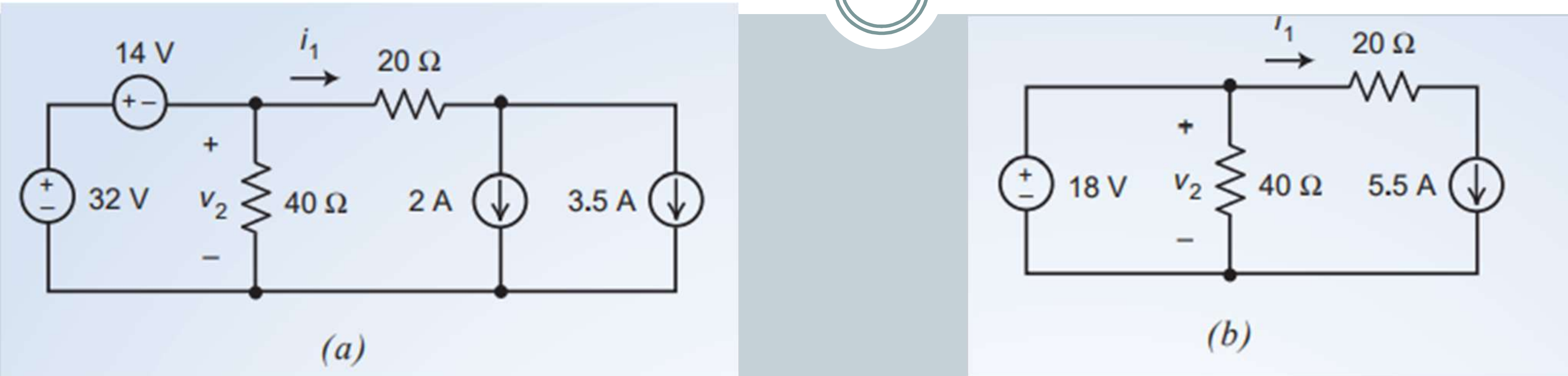


Figure 3.5-3

Consider first the circuit in Figure 3.5-3*a*. Apply KVL to the left mesh to get

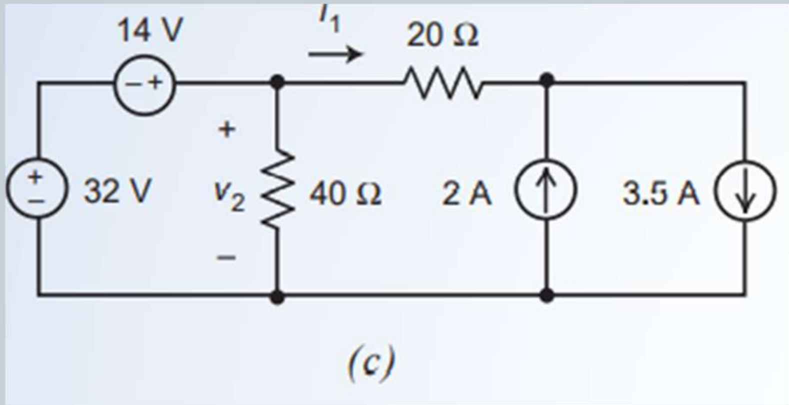
$$14 + v_2 - 32 = 0 \quad \Rightarrow \quad v_2 - 18 = 0$$

Next apply KCL at the right node of the 20 Ω to get

$$i_1 = 2 + 3.5 \quad \Rightarrow \quad i_1 = 5.5$$

These equations suggest that we replace the series voltage sources by a single 18-V source and replace the parallel current sources by a single 5.5-A source. Figure 3.5-3*b* shows the result.

Solution



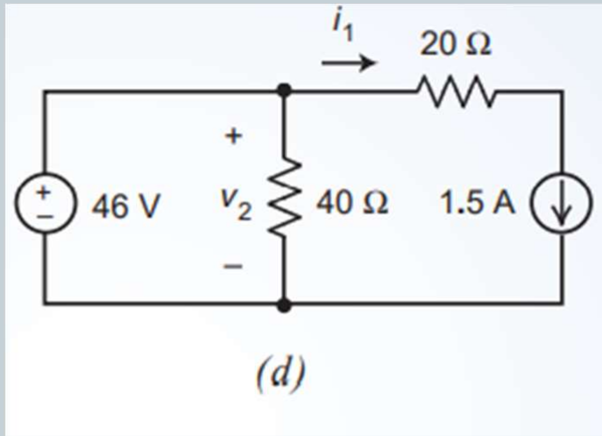


Figure 3.5-3

Next, consider first the circuit in Figure 3.5-3*c*. Apply KVL to the left mesh to get

$$-14 + v_2 - 32 = 0 \quad \Rightarrow \quad v_2 - 46 = 0$$

Next apply KCL at the right node of the 20 Ω to get

$$i_1 + 2 = 3.5 \quad \Rightarrow \quad i_1 = 1.5$$

Combining Ohm's Law and KVL

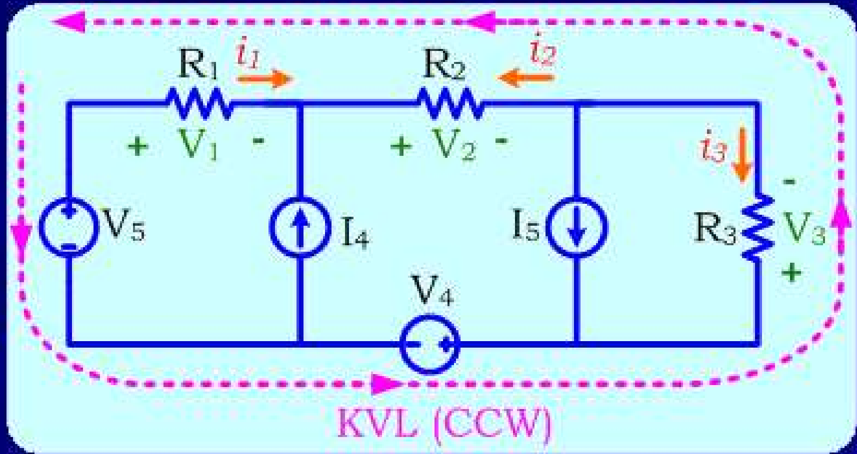
KVL around outer circuit (CW)

$$-v_5 + v_1 + v_2 - v_3 + v_4 = 0$$

Using Ohm's Law

$$-v_5 + (i_1 R_1) + (-i_2 R_2) - (-i_3 R_3) + v_4 = 0$$

$$\Rightarrow -v_5 + i_1 R_1 - i_2 R_2 + i_3 R_3 + v_4 = 0 \cdots\cdots (1)$$



KVL equation can be written directly in terms of the resistor currents i_1, i_2 and i_3

i (through R) same as KVL direction	$\Rightarrow$	$+iR$
i (through R) opposite to KVL direction	$\Rightarrow$	$-iR$

Using this rule,

KVL around outer circuit (CCW) $\Rightarrow +v_5 - v_4 - i_3 R_3 + i_2 R_2 - i_1 R_1 = 0$ [The same as (1)]

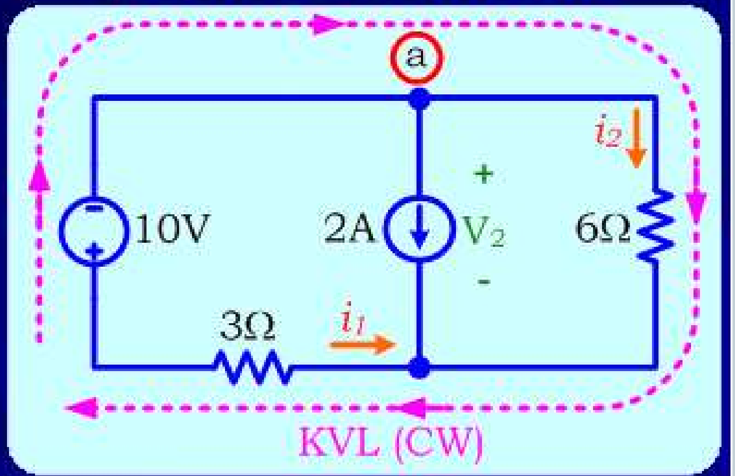
Ohm's Law can also be combined with KCL. This case will be covered in later lectures

Example 2

In the given circuit calculate

(a) i_1 and i_2

(b) the power absorbed by the current source



Solution

(a) KVL around outer circuit (CW)

$$10 + 6i_2 - 3i_1 = 0 \cdots\cdots (1)$$

KCL at node 'a'

$$i_1 + 2 + i_2 = 0 \cdots\cdots (2)$$

Solving (1) and (2) $\Rightarrow 10 + 6(-i_1 - 2) - 3i_1 = 0 \Rightarrow \boxed{i_1 = -\frac{2}{9}\text{A}}$

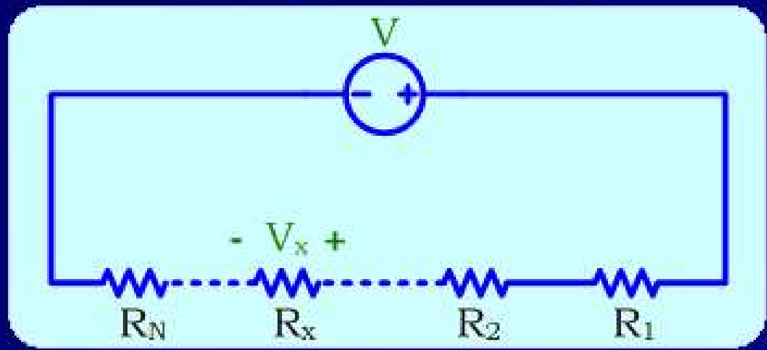
Substituting in (2) $\Rightarrow -\frac{2}{9} + 2 + i_2 = 0 \Rightarrow \boxed{i_2 = -2 + \frac{2}{9} = -\frac{16}{9}\text{A}}$

(b) Ohm's Law $\Rightarrow v_2 = 6i_2 \Rightarrow v_2 = 6(-\frac{16}{9}) = -\frac{32}{3}\text{V}$

$$\boxed{p_{2A} = +iv = +(2)(-\frac{32}{3}) = -21.33\text{W}}$$

The Voltage Divider Rule

The total voltage across the *series* resistors R_1, R_2, ..., R_N is V



$$i = \frac{V}{R_{eq}} = \frac{V}{\sum_{i=1}^{N} R_i}$$

$$v_x = iR_x = \frac{V}{\sum_{i=1}^{N} R_i} R_x = \left(\frac{R_x}{\sum_{i=1}^{N} R_i} \right) V$$

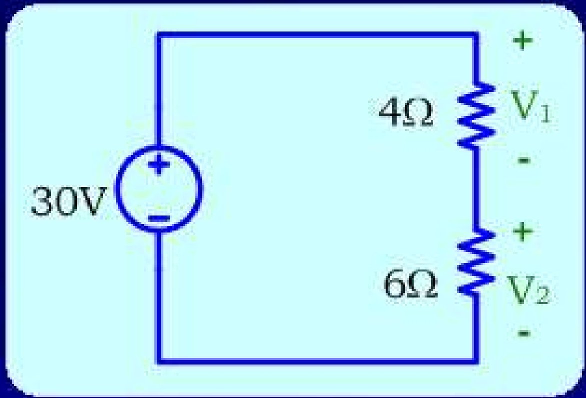
$$\text{VDR} \Rightarrow v_x = \left(\frac{R_x}{\sum_{i=1}^{N} R_i} \right) V$$

$$\therefore v_{resistor} = \frac{\text{resistor}}{\text{sum}} \times (\text{total voltage})$$

The VDR is valid for *any* number of resistors in *series*

Example 3

Calculate the unknown voltages



Solution

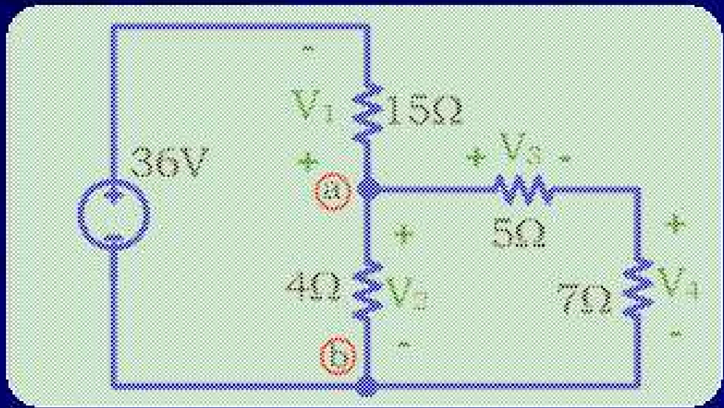
$$\text{VDR} \Rightarrow v_1 = \frac{4}{4+6} \times 30 = 12\text{V}$$

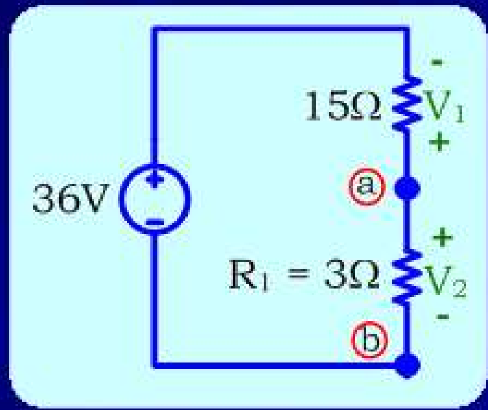
$$\text{VDR} \Rightarrow v_2 = \frac{6}{4+6} \times 30 = 18\text{V}$$

VDR $\Rightarrow$ Higher voltage drop across the higher resistance

Example 4

Calculate the unknown voltages





Solution

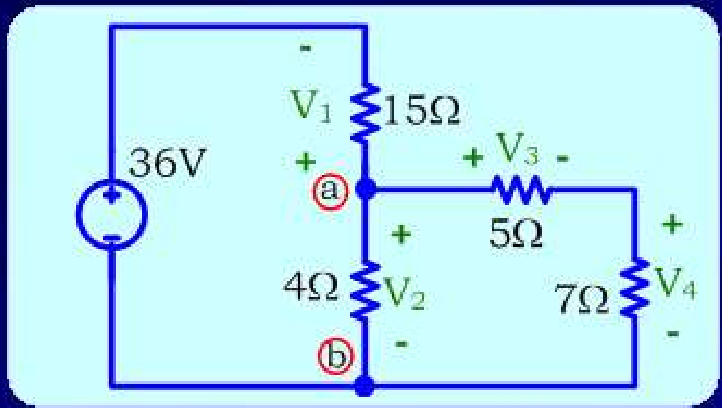
$$5+7=12\Omega \Rightarrow R_1=\frac{4\times 12}{4+12}=3\Omega$$

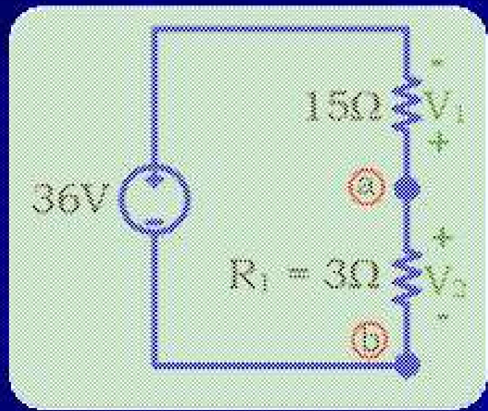
VDR $\Rightarrow v_1=-\frac{15}{15+3}\times 36$ (a minus sign is required here. Why?) $\Rightarrow \boxed{v_1=-30\text{V}}$

VDR $\Rightarrow v_2=\frac{3}{15+3}\times 36 \Rightarrow \boxed{v_2 = 6\text{ V}}$

Check: KVL $\Rightarrow -36-v_1+v_2=-36-(-30)+(6)=-36+30+6=0$

Example 4 (Contd...)





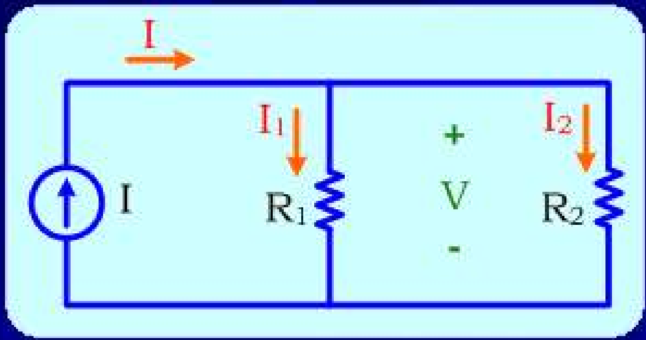
From previous slide $v_1 = -30\text{V}$ $v_2 = 6\text{ V}$

VDR $\Rightarrow v_3 = \dfrac{5}{5+7} \times v_2 = \dfrac{5}{12} \times 6 \Rightarrow v_3 = 2.5\text{V}$

VDR $\Rightarrow v_4 = \dfrac{7}{5+7} \times v_2 = \dfrac{7}{12} \times 6 \Rightarrow v_4 = 3.5\text{V}$

The Current Divider Rule

The total current entering into the *parallel* combination of resistors R_1 & R_2 is I



$$V = IR_{eq} = I\frac{R_1R_2}{R_1+R_2}$$

$$I_1 = \frac{V}{R_1} \quad \& \quad I_2 = \frac{V}{R_2}$$

$$I_1 = \frac{1}{R_1}\times\frac{R_1R_2}{R_1+R_2}I \Rightarrow I_1 = \frac{R_2}{R_1+R_2}I \cdots\cdots (1)$$

Similarly $$I_2 = \frac{R_1}{R_1+R_2}I \cdots\cdots (2)$$

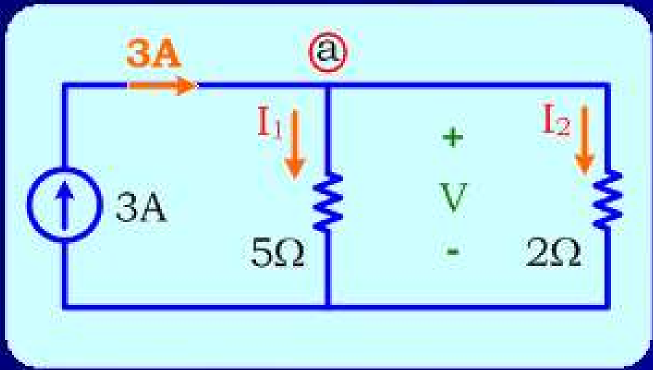
CDR $\Rightarrow$ $$I = \frac{\text{other resistor}}{\text{sum}}\times\text{total current}$$

CDR applies to *only two* resistors in *parallel*

Example 5

(a) Use CDR to calculate I_1 and I_2

(b) Verify your resluts by checking KCL



Solution

(a) CDR $\Rightarrow I_1 = \frac{2}{2+5} \times 3 \Rightarrow I_1 = \frac{6}{7}\text{A}$

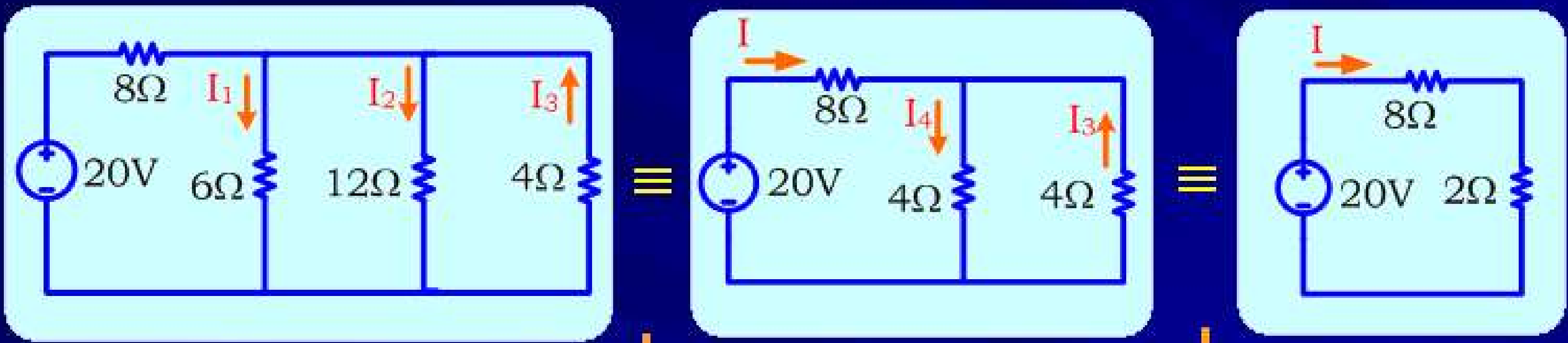
CDR $\Rightarrow I_2 = \frac{5}{2+5} \times 3 \Rightarrow I_2 = \frac{15}{7}\text{A}$

(b) KCL at node a $\Rightarrow I_s - I_1 - I_2 = 3 - \frac{6}{7} - \frac{15}{7} = 3 - \frac{21}{7} = 0$ (KCL verified)

CDR $\Rightarrow$ Higher current passes through the lower resistance

Example 6

Use CDR to calculate I_1, I_2 and I_3



6Ω & 12Ω are in parallel $\Rightarrow \frac{6\times 12}{6+12} = \frac{72}{18} = 4\Omega$

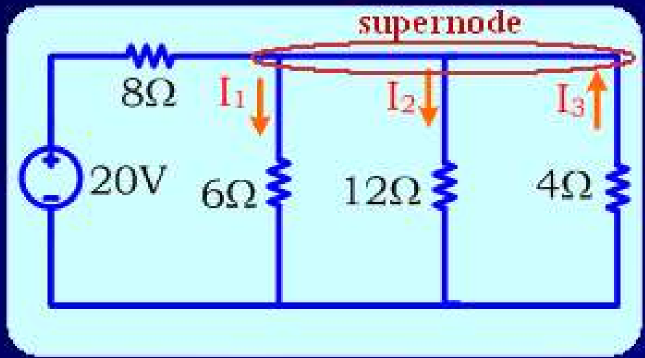
4Ω & 4Ω are in parallel $\Rightarrow \frac{4\times 4}{4+4} = \frac{16}{8} = 2\Omega$

$\therefore R_{eq} = 8 + 2 = 10\Omega$

$\Rightarrow I = \frac{20}{R_{eq}} = \frac{20}{10} = 2A$

Example 6

Use CDR to calculate I_1, I_2 and I_3



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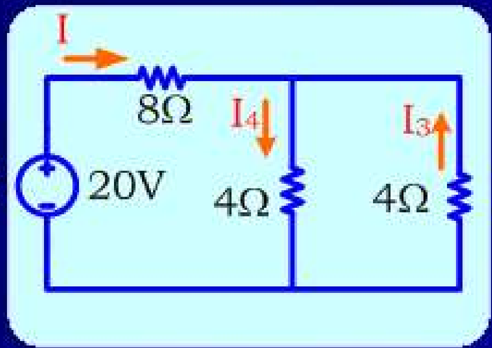

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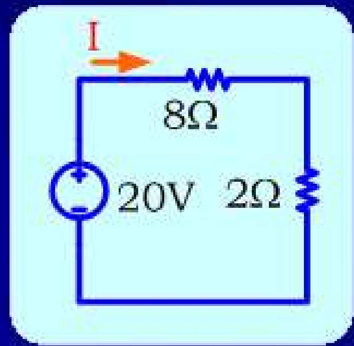

$$I = \frac{20}{R_{eq}} = \frac{20}{10} = 2\text{A}$$

CDR ⇒ $I_4 = \frac{4}{4+4} \times 2 = 1\text{A}$, $I_3 = -\frac{4}{4+4} \times 2 = -1\text{A}$ (the minus sign is necessary for I_3, why?)

CDR ⇒ $I_1 = \frac{12}{6+12} \times I_4 = \frac{2}{3}\text{A}$, $I_2 = \frac{6}{6+12} \times I_4 = \frac{1}{3}\text{A}$ (I_4 is the total current through 6Ω and 12Ω)

Check KCL at supernode ⇒ $I - I_1 - I_2 + I_3 = 2 - \frac{2}{3} - \frac{1}{3} + (-1) = 1 - 1 = 0$ (KCL is verified)