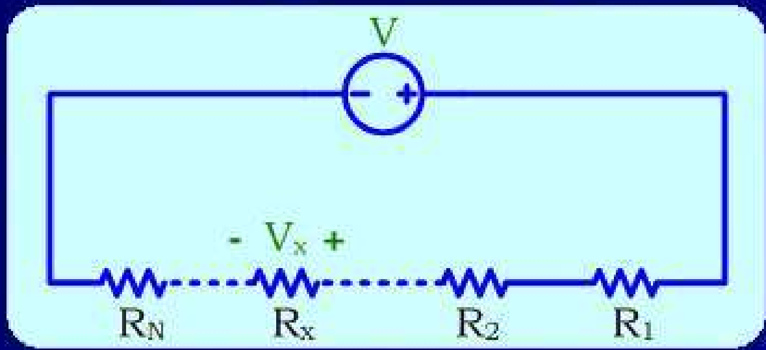
The Voltage Divider Rule

The total voltage across the *series* resistors $R_1, R_2, \ldots, R_N$ is V



$$i = \frac{V}{R_{eq}} = \frac{V}{\sum_{i=1}^{N} R_i}$$

$$v_x = iR_x = \frac{V}{\sum_{i=1}^{N} R_i} R_x = \left(\frac{R_x}{\sum_{i=1}^{N} R_i} \right) V$$

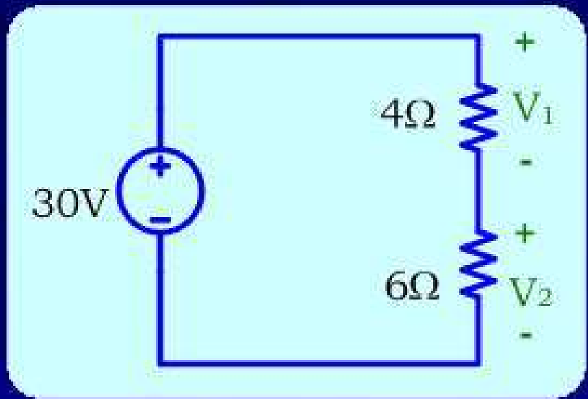
$$\text{VDR} \Rightarrow \; v_x = \left(\frac{R_x}{\sum_{i=1}^{N} R_i} \right) V$$

$$\therefore v_{resistor} = \frac{\text{resistor}}{\text{sum}} \times (\text{total voltage})$$

The VDR is valid for *any* number of resistors in *series*

Example 3

Calculate the unknown voltages



Solution

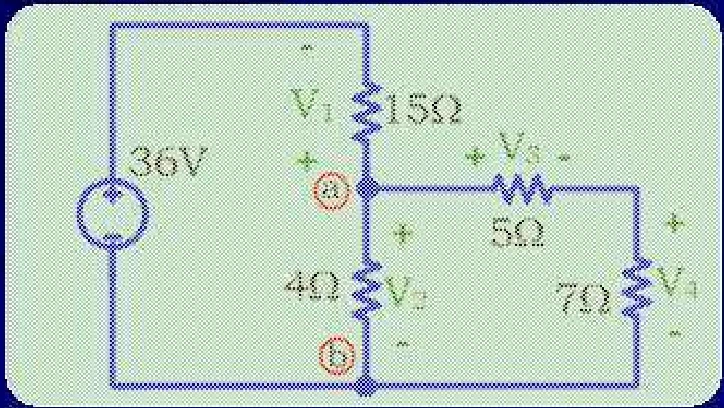
$$\text{VDR} \Rightarrow v_1 = \frac{4}{4+6} \times 30 = 12\text{V}$$

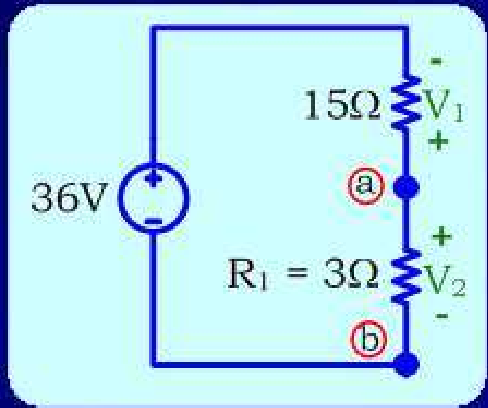
$$\text{VDR} \Rightarrow v_2 = \frac{6}{4+6} \times 30 = 18\text{V}$$

VDR ⇒ Higher voltage drop across the higher resistance

Example 4

Calculate the unknown voltages





Solution

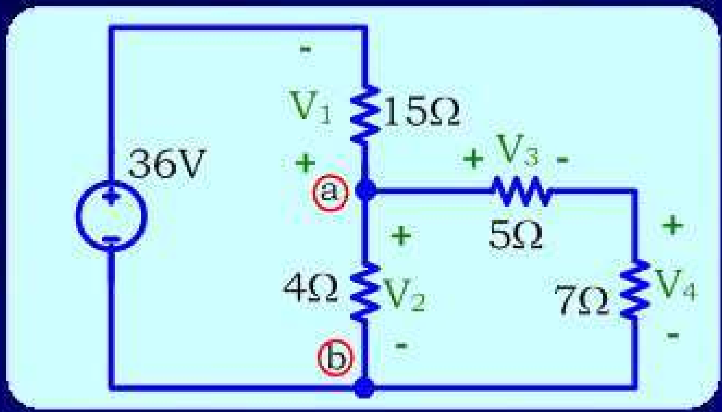
$$5+7=12\Omega \Rightarrow R_1=\frac{4\times 12}{4+12}=3\Omega$$

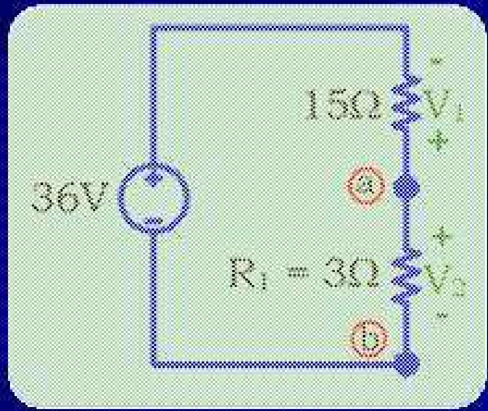
VDR $\Rightarrow v_1=-\frac{15}{15+3}\times 36$ (a minus sign is required here. Why?) $\Rightarrow \boxed{v_1=-30\text{V}}$

VDR $\Rightarrow v_2=\frac{3}{15+3}\times 36 \Rightarrow \boxed{v_2 = 6\text{ V}}$

Check: KVL $\Rightarrow -36-v_1+v_2=-36-(-30)+(6)=-36+30+6=0$

Example 4 (Contd...)





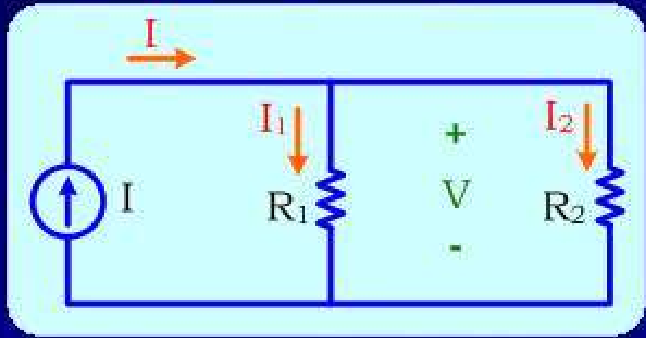
From previous slide $v_1 = -30\text{V}$ $v_2 = 6\text{ V}$

VDR $\Rightarrow v_3 = \frac{5}{5+7} \times v_2 = \frac{5}{12} \times 6 \Rightarrow v_3 = 2.5\text{V}$

VDR $\Rightarrow v_4 = \frac{7}{5+7} \times v_2 = \frac{7}{12} \times 6 \Rightarrow v_4 = 3.5\text{V}$

The Current Divider Rule

The total current entering into the *parallel* combination of resistors R_1 & R_2 is I



$$V = IR_{eq} = I\frac{R_1R_2}{R_1+R_2}$$

$$I_1 = \frac{V}{R_1} \quad \& \quad I_2 = \frac{V}{R_2}$$

$$I_1 = \frac{1}{R_1}\times\frac{R_1R_2}{R_1+R_2}I \Rightarrow I_1 = \frac{R_2}{R_1+R_2}I \cdots\cdots (1)$$

Similarly $$I_2 = \frac{R_1}{R_1+R_2}I \cdots\cdots (2)$$

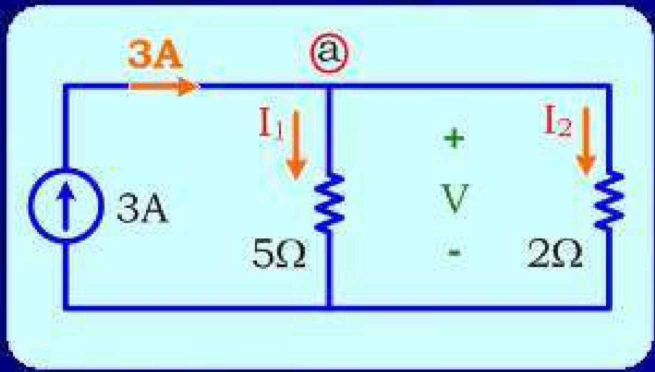
CDR $\Rightarrow$ $$I = \frac{\text{other resistor}}{\text{sum}}\times\text{total current}$$

CDR applies to *only two* resistors in *parallel*

Example 5

(a) Use CDR to calculate I_1 and I_2

(b) Verify your resluts by checking KCL



Solution

(a) CDR $\Rightarrow I_1 = \frac{2}{2+5} \times 3 \Rightarrow I_1 = \frac{6}{7}\text{A}$

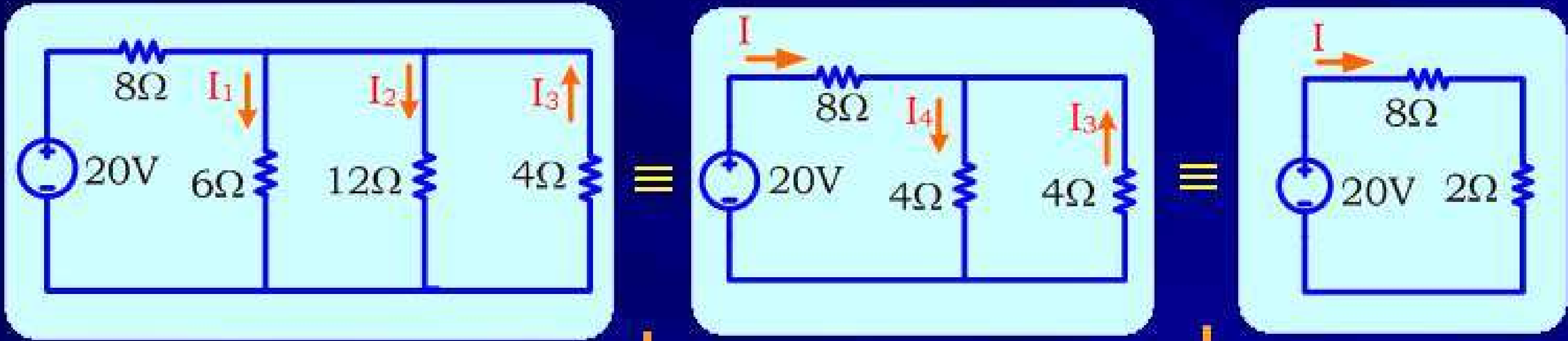
CDR $\Rightarrow I_2 = \frac{5}{2+5} \times 3 \Rightarrow I_2 = \frac{15}{7}\text{A}$

(b) KCL at node a $\Rightarrow I_s - I_1 - I_2 = 3 - \frac{6}{7} - \frac{15}{7} = 3 - \frac{21}{7} = 0$ (KCL verified)

CDR $\Rightarrow$ Higher current passes through the lower resistance

Example 6

Use CDR to calculate I_1, I_2 and I_3



6Ω & 12Ω are in parallel $\Rightarrow \frac{6\times 12}{6+12}=\frac{72}{18}=4\Omega$

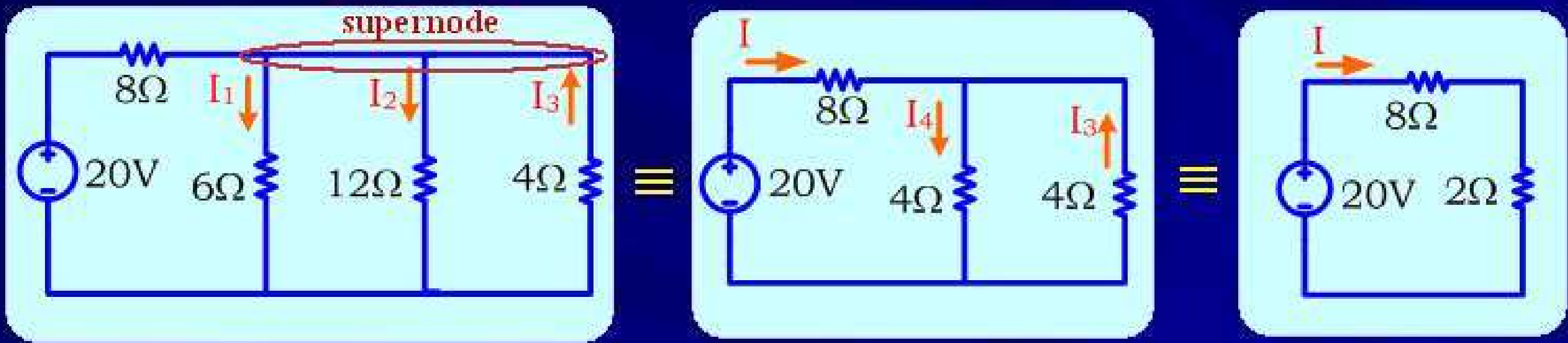
4Ω & 4Ω are in parallel $\Rightarrow \frac{4\times 4}{4+4}=\frac{16}{8}=2\Omega$

$\therefore R_{eq}=8+2=10\Omega$

$\Rightarrow I=\frac{20}{R_{eq}}=\frac{20}{10}=2A$

Example 6

Use CDR to calculate I_1, I_2 and I_3



$$I = \frac{20}{R_{eq}} = \frac{20}{10} = 2\text{A}$$

CDR $\Rightarrow$ $I_4 = \frac{4}{4+4} \times 2 = 1\text{A}$, $I_3 = -\frac{4}{4+4} \times 2 = -1\text{A}$ (the minus sign is necessary for I_3, why?)

CDR $\Rightarrow$ $I_1 = \frac{12}{6+12} \times I_4 = \frac{2}{3}\text{A}$, $I_2 = \frac{6}{6+12} \times I_4 = \frac{1}{3}\text{A}$ (I_4 is the total current through 6Ω and 12Ω)

Check KCL at supernode $\Rightarrow I - I_1 - I_2 + I_3 = 2 - \frac{2}{3} - \frac{1}{3} + (-1) = 1 - 1 = 0$ (KCL is verified)