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fundamental of Electric
Circuit 1 Assist. Lect.
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Stage 1
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LECTURE 08
CIRCUIT ANALYSIS TECHNIQUE
NODAL ANALYSIS (INTRODUCTION)

Topics

- Definition of Nodal Voltages
- Nodal Analysis in the absence of Voltage Sources

Objectives

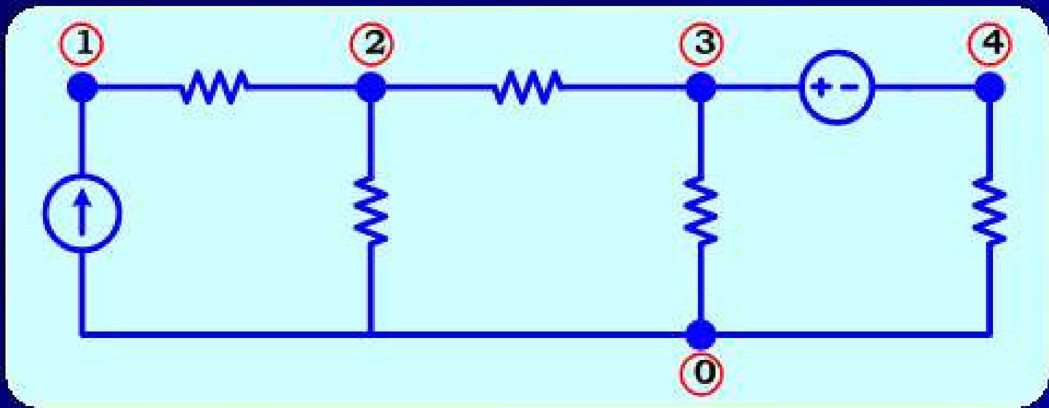
- Understand the meaning of a nodal voltage
- Understand the meaning of a reference node
- Differentiate between voltages across elements and nodal voltages
- Relate nodal voltages to voltages across elements
- Determine the number of unknown nodal voltages
- Apply the Nodal Analysis Procedure in the absence of voltage sources
- Apply the Nodal Analysis Procedure directly

Definition of Essential Nodes

The essential nodes of the circuit are labelled '0', '1', '2', '3', '4', etc.

All points that are connected by a short circuit belong to the same essential node.

All points in the lower part of the circuit are connected by a short circuit, they all belong to node 0. The same applies to nodes "1", "2", "3", "4".



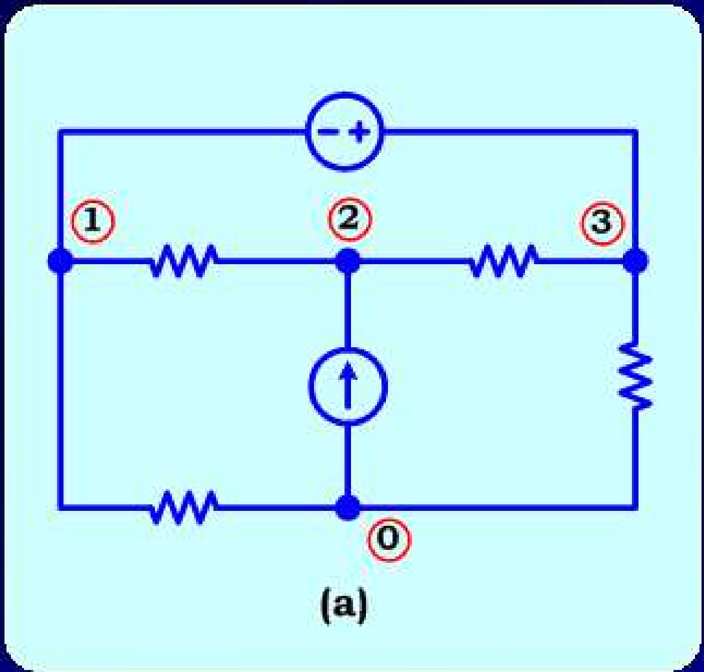
In this circuit, there are five essential nodes.

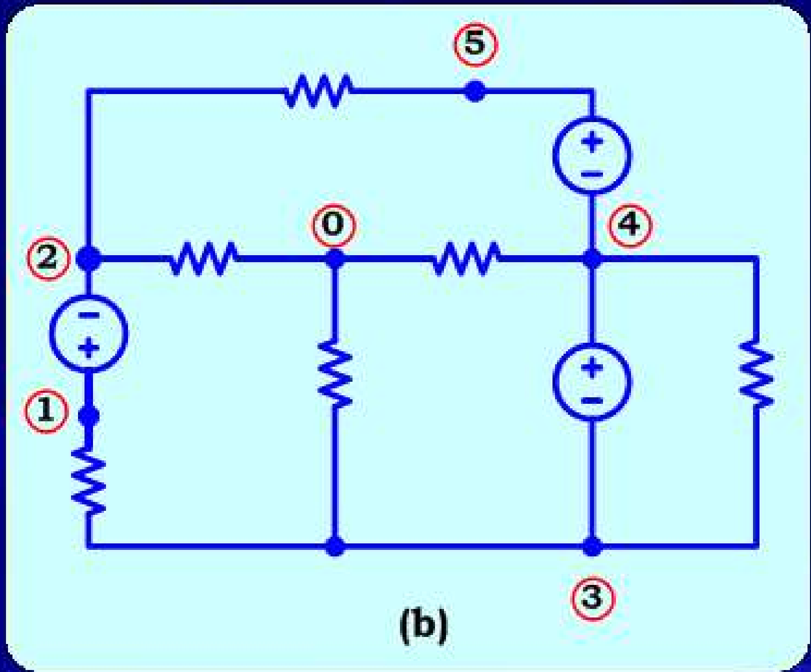
Example: Label the essential nodes starting from node 0.

Solution:

Circuit (a) has 4 essential nodes.

Circuit (b) has 6 essential nodes.





The essential nodes can be labelled as we like.

Reference Node

The node labelled '0' is called a *reference node*.

We will see later that the *reference* node always has a *zero Nodal Voltage*.

Possible labels for the reference node are shown below:

Labels for reference node

Nodal Voltages

We associate a voltage with every essential node. These voltages are called *Nodal Voltages.*

V_0, V_1, V_2, V_3 $\Leftrightarrow$ Nodal voltages of essential nodes 0, 1, 2, 3.

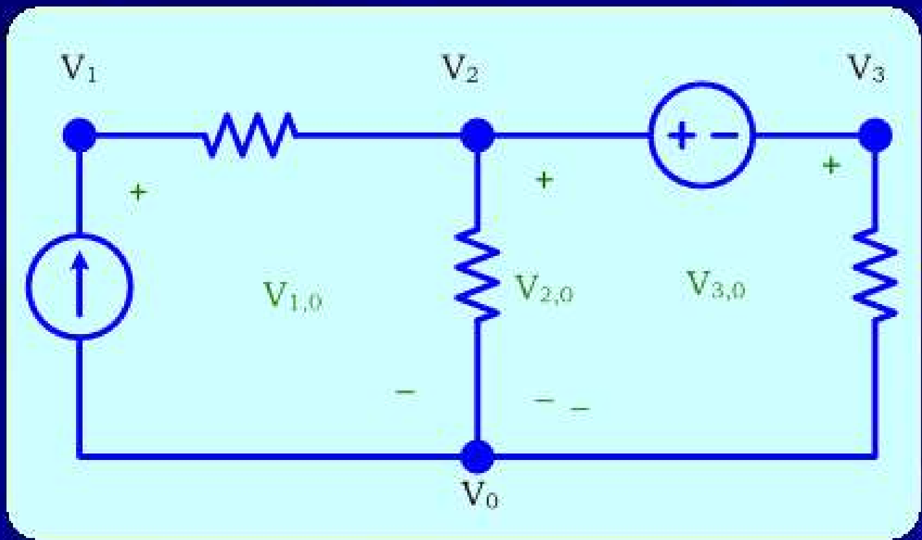
The nodal voltage V_i is the voltage drop *from* node "i" *to* the reference node "0".

$$V_i \equiv V_{i,0}$$

$\Rightarrow$ $V_0 \equiv V_{0,0}$ $\quad V_1 \equiv V_{1,0}$ $\quad V_2 \equiv V_{2,0}$ $\quad V_3 \equiv V_{3,0}$

Reference node: $V_0 \equiv V_{0,0} = 0$

$\Rightarrow$ Reference node always has zero nodal voltage.

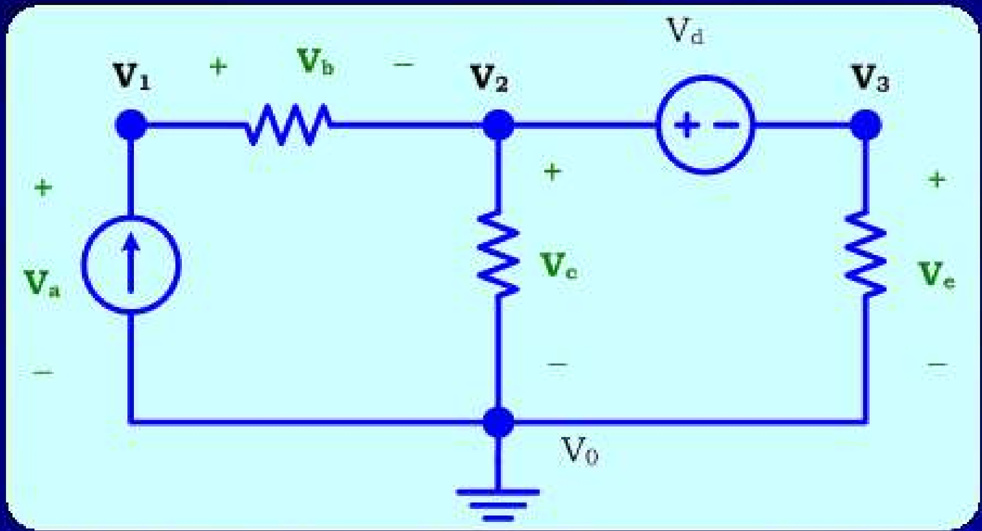


Relation between Nodal Voltages and Voltages across Elements

Nodal Voltages (NV) ⟹ V_1, V_2, V_3

The reference nodal voltage $V_0 = 0$.

Voltage across elements (VAE) ⟹ V_a, V_b, V_c, V_d, V_e



Another label is used to mark the reference node in this case.

Relation between Nodal Voltages and Voltage across Elements

Applying KVL:

$-V_a + V_{10} = 0 \Rightarrow -V_a + V_1 = 0 \Rightarrow V_a = V_1$

$V_a = V_1 \Rightarrow \boxed{V_a = V_1 - V_0}$

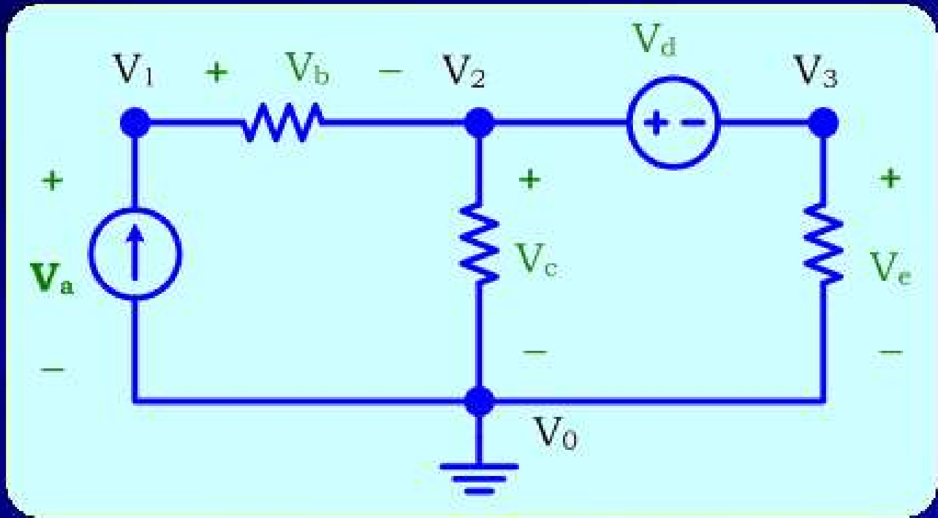
$-V_1 + V_b + V_2 = 0$

$\Rightarrow \boxed{V_b = V_1 - V_2}$

$V_c = V_2 \Rightarrow \boxed{V_c = V_2 - V_0}$

$-V_2 + V_d + V_3 = 0$

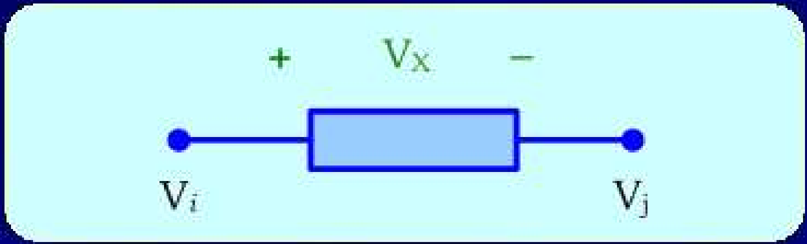
$\Rightarrow \boxed{V_d = V_2 - V_3}$



$\boxed{V_e = ?}$

Voltage across Elements: General Equation

$$\Rightarrow \quad V_x = V_i - V_j$$



where:

V_x = *Voltage Across Element*

V_i = *Nodal Voltage on the '+' side of* V_x

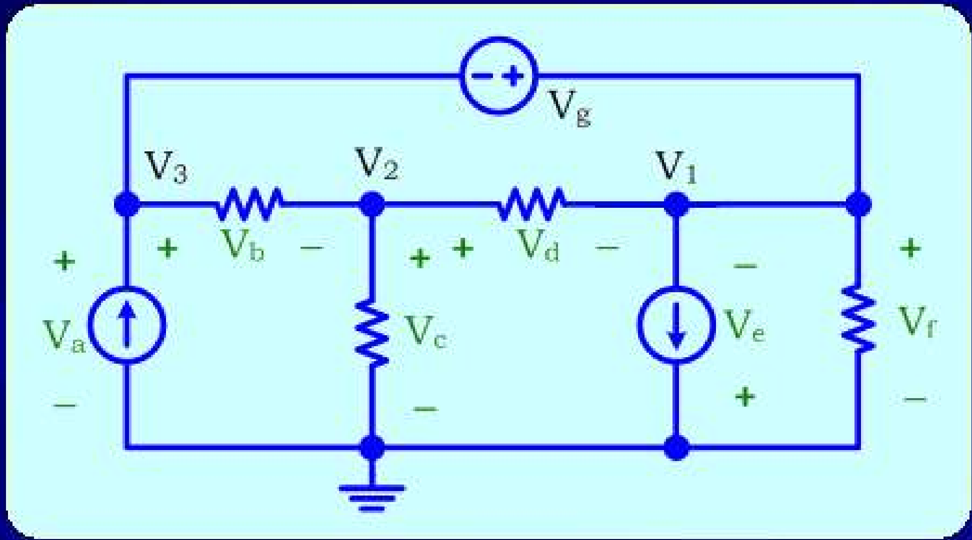
V_j = *Nodal Voltage on the '-' side of* V_x

Example

Express the VAE $V_a, V_b, V_c, V_d, V_e, V_f, V_g$ in terms of the NV V_1, V_2, V_3

$V_a = V_3 - V_0 = V_3 - 0 = V_3$
$V_b = V_3 - V_2$
$V_c = V_2 - 0 = V_2$
$V_d = V_2 - V_1$
$V_e = 0 - V_1 = - V_1$ *(why?)*
$V_f = V_1 - 0 = V_1$
$V_g = V_1 - V_3$

$V_x = V_i - V_j$



If we know all NV implies we know all VAE.

- Nodal analysis procedure

Steps to Determine Node Voltages:

1. Select a node as the reference node. Assign voltages $v_1, v_2, \ldots, v_{n-1}$ to the remaining $n-1$ nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the $n-1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Example 1

Derive the nodal equations. (do not simplify and do not solve).

Solution:

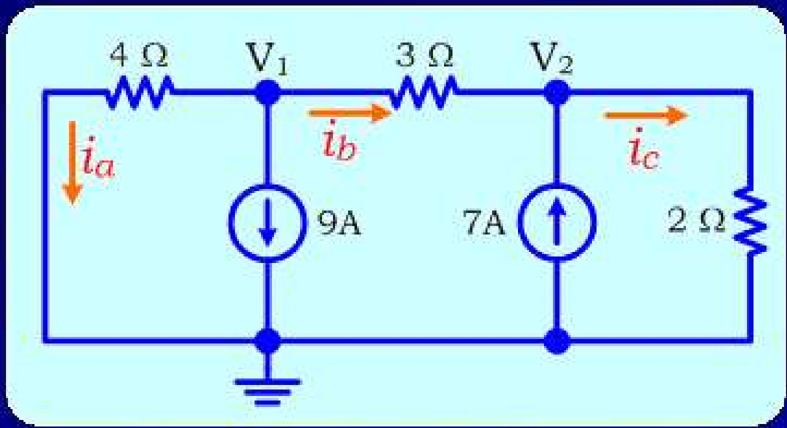
This time, we will combine steps 2 & 3 (Ohm's law and KVL) into a single step.
The voltage across resistances will not be shown explicitly.

Node 1:

KCL $\Rightarrow$ $i_a + 9 + i_b = 0$

Ohms' Law then KVL

$$\Rightarrow \frac{V_1 - 0}{4} + 9 + \frac{V_1 - V_2}{3} = 0 \qquad (1)$$



Node 2:

KCL $\Rightarrow$ $-i_b - 7 + i_c = 0$

Ohms' Law then KVL

$$\Rightarrow -\frac{V_1 - V_2}{3} - 7 + \frac{V_2 - 0}{2} = 0 \qquad (2)$$

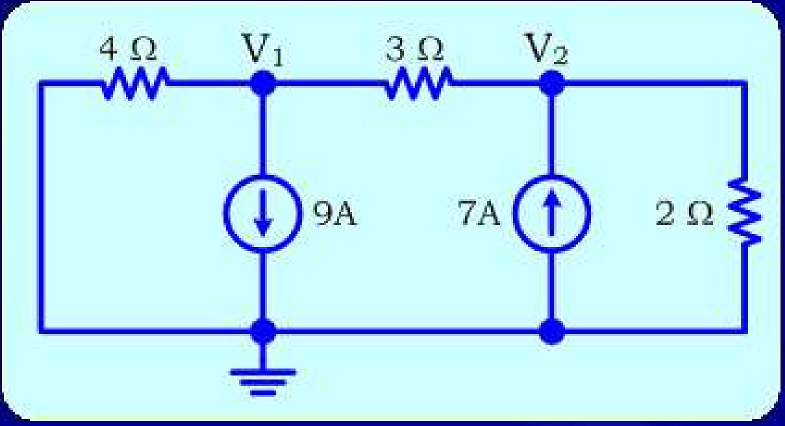
Example 2

Repeat the previous example by combining steps 1, 2, and 3 (KCL, Ohm's law, and KVL) into a single step.

Solution:

This time we will not show current through resistances or voltages across resistances.

Important: We will imagine the currents through resistors to be leaving the node under consideration.



Node 1:

$$\Rightarrow \frac{V_1 - 0}{4} + 9 + \frac{V_1 - V_2}{3} = 0 \qquad (1)$$

Node 2:

$$\Rightarrow \frac{V_2 - V_1}{3} - 7 + \frac{V_2 - 0}{2} = 0 \qquad (2)$$

which are the same equations obtained in the previous example.

Nodal Analysis: Some Conclusions

From the examples shown in this lecture, it is easy to conclude that:

1. $N_u = N_{ess} - 1$

   N_u = number of unknown nodal voltages

   N_{ess} = number of essential nodes

2. $N_u \leq N_{ele}$

   N_{ele} = number of unknown voltages across elements

Thus, nodal analysis is efficient because the number of unknown voltages is reduced.

Voltage Sources connected to the Reference node

The case of voltage sources connected to the reference node is taken up first and it is illustrated with the help of an example.

Example3 Calculate the nodal voltages V_1, V_2, V_3.

Solution:

Nodes 1 & 2

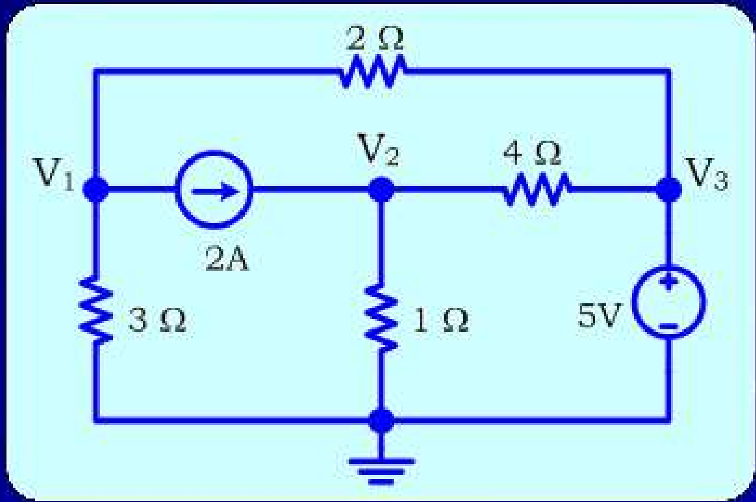
⇒ No voltage sources connected

⇒ No special treatment required

Node 3

⇒ Voltage source connected

⇒ Needs special treatment



Solution: Applying KCL

KCL at node 1:

$$\Rightarrow \frac{V_1-0}{3}+2+\frac{V_1-V_3}{2}=0$$

$$\Rightarrow 5V_1-3V_3=-6 \qquad (1)$$

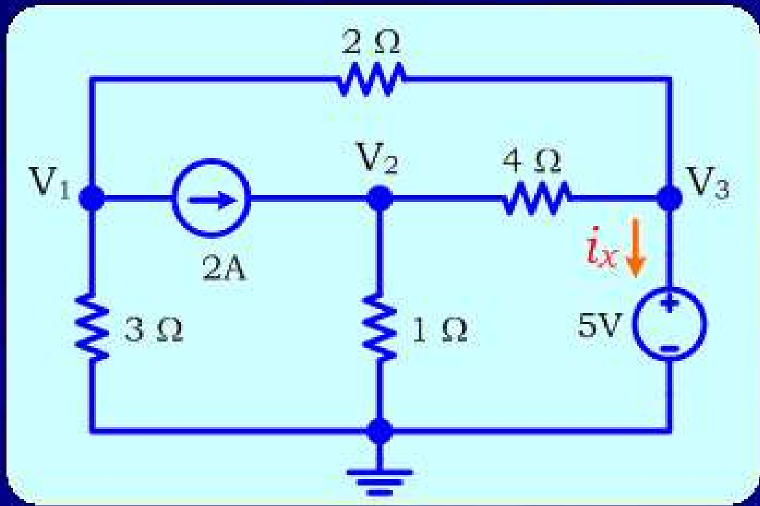
KCL at node 2:

$$\Rightarrow -2+\frac{V_2-0}{1}+\frac{V_2-V_3}{4}=0$$

$$\Rightarrow 5V_2-V_3=8 \qquad (2)$$

KCL at node 3:

$$\Rightarrow \frac{V_3-V_2}{4}+\frac{V_3-V_1}{2}+i_x=0$$ (problem!)



i_x cannot directly be replaced with nodal voltages, because Ohm's law does not apply to voltages sources

Solution: Solve equations

We have 3 unknowns $\Longrightarrow$ We need 3 equations $\Longrightarrow$ one equation is missing

For node 3, the basic Nodal Analysis procedure must be revised.

The 5V source is connected to the reference node.

Apply KVL: Node 3 and reference node

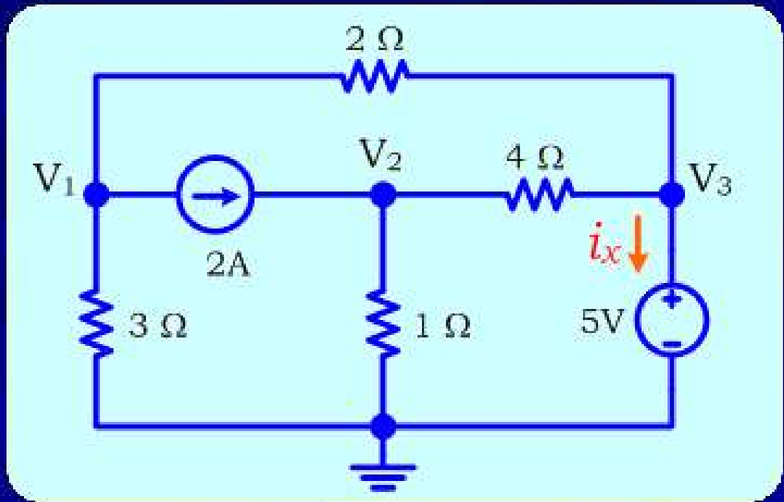
$\Rightarrow V_3 - 0 = 5$

$\Rightarrow V_3 = 5$ (3)

From the previous slide,

$5V_1 - 3V_3 = -6$ (1)

$5V_2 - V_3 = 8$ (2)



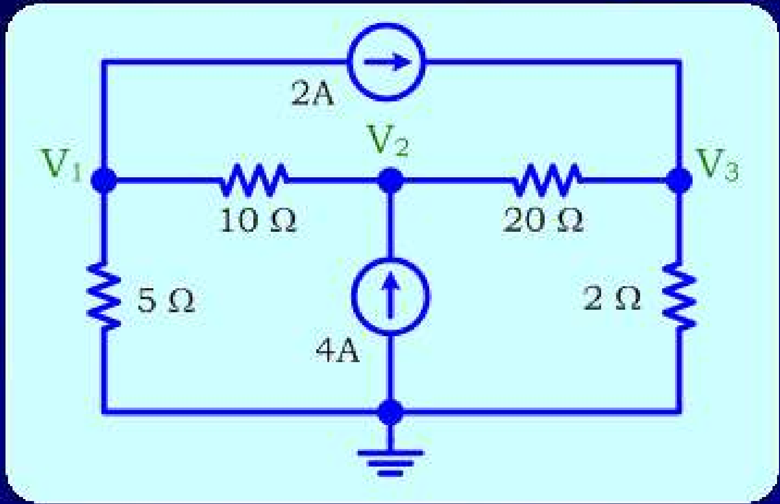
Solving the above set of equations, we get:

$V_1 = 1.8V$ & $V_2 = 2.6V$ & $V_3 = 5V$

Voltage source connected to reference $\Longrightarrow$ Use KVL only (do not use KCL)

Practice Problem

In the circuit shown, calculate the nodal voltages V_1, V_2 & V_3



Answer:

$V_1 = 6.67V$ $V_2 = 40V$ $V_3 = 26.67V$