Al-Anbar University
College of engineering
Electrical Engineering Department

fundamental of Electric Circuit 1
Assist. Lect. Yasameen Kamil
Stage 1
2021-2022

LECTURE 9
MESH ANALYSIS

Topics

- Mesh Analysis without Current Sources
- Mesh Analysis with Current Sources

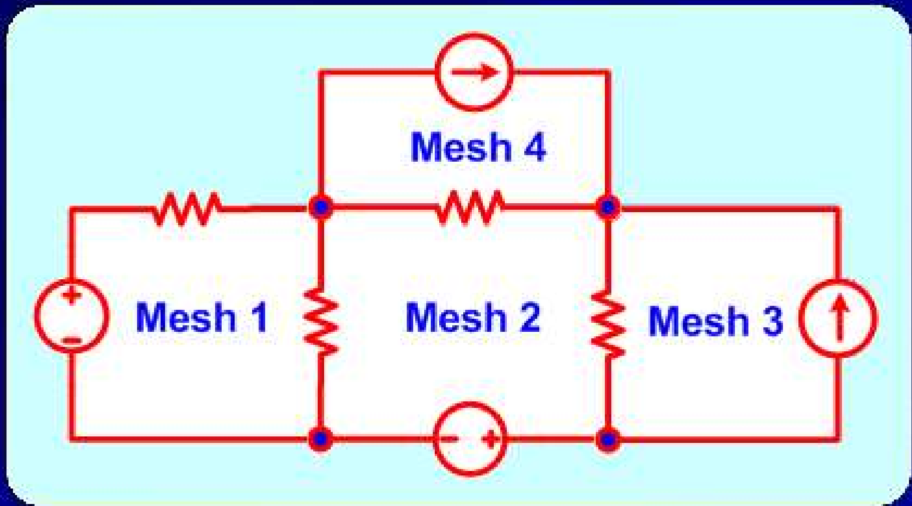
Objectives

- Understand mesh currents
- Relate currents through elements to mesh currents
- Apply Mesh Analysis in the absence of current sources
- Understand the concept of a super mesh
- Apply Mesh Analysis in the presence of current sources

Definition of a Mesh

A mesh is simply a window in an electric circuit.

This circuit contains four windows (meshes).



Currents through Elements and Mesh Currents

The currents i_a, i_b, and i_c are currents through elements.

KCL at node 1:

$$\Rightarrow i_a = i_b + i_c \quad \Rightarrow \quad i_b = i_a - i_c$$

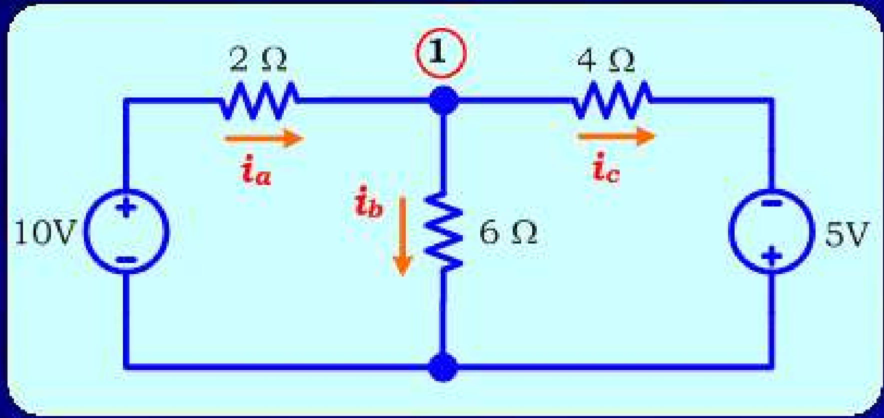
The imaginary currents i_1, and i_2 are mesh currents.

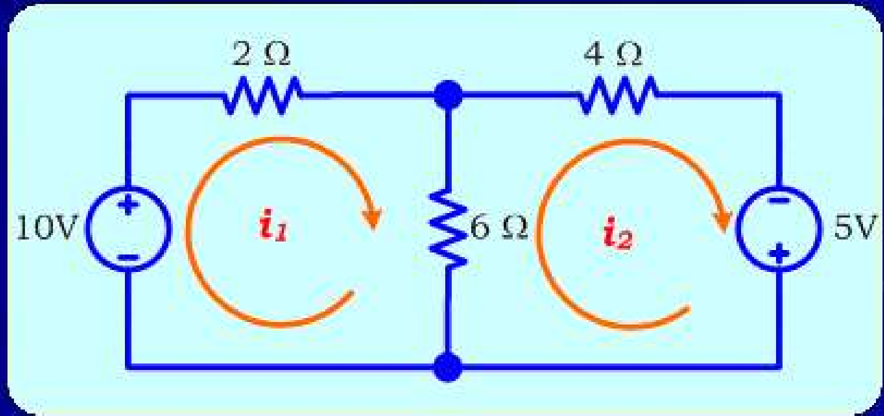
We imagine i_1 to circulate around mesh 1 (Clockwise).

We imagine i_2 to circulate around mesh 2 (also Clockwise).

$$i_a = i_1 \qquad i_c = i_2$$

$$i_b = i_a - i_b = i_1 - i_2$$





Example:

Express the currents through elements (CTE) i_w , i_x , i_y , and i_z in terms of mesh currents (MC) currents i_1 , i_2 , and i_3

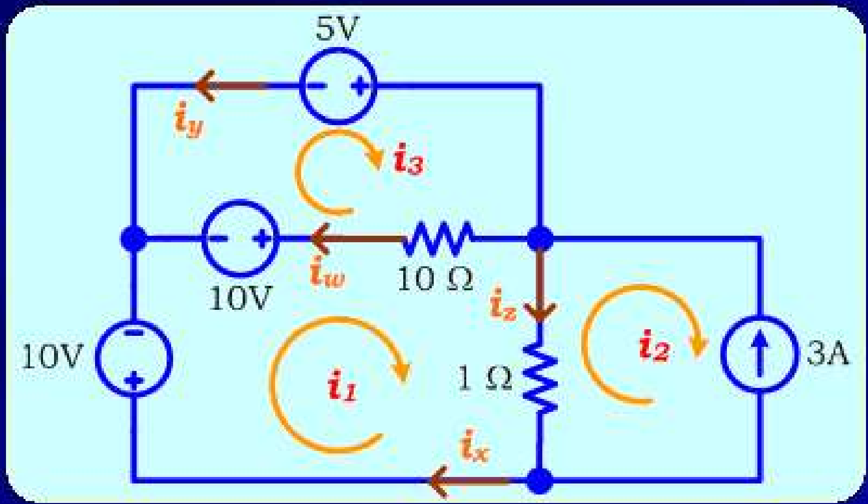
Solution:

$$i_x = i_1$$

$$i_y = -i_3$$

$$i_z = i_1 - i_2$$

$$i_w = i_3 - i_1$$



Number of MC $\leq$ Number of CTE

We know all MC $\Rightarrow$ We know all CTE

Mesh Analysis without Current Sources

The Mesh Analysis procedure for circuits without current sources will be considered first. This procedure is illustrated below:

Mesh Analysis

Step 1: KVL

↓

Step 2: Ohm's Law

↓

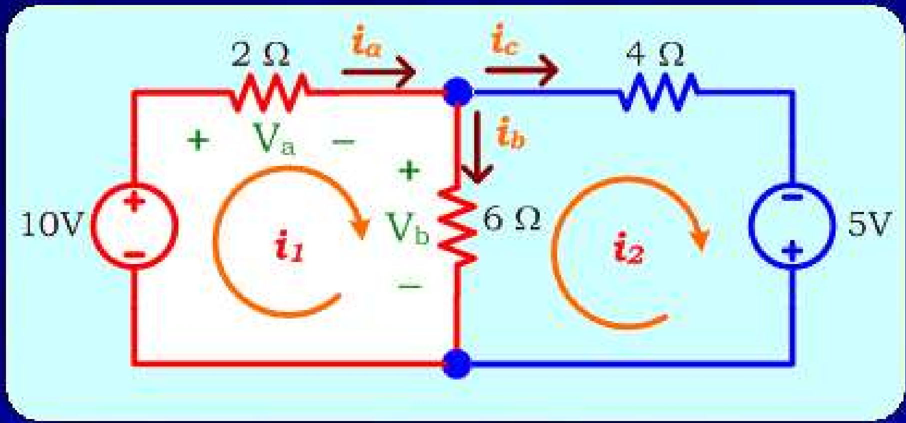
Step 3: KCL

Mesh Analysis ⇒ VOC

Example 1: Calculate the mesh currents i_1 and i_2

Solution:

Procedure:
First we will deal with Mesh 1



1. KVL: $\Rightarrow$ $-10+V_a+V_b=0$

2. Ohm's Law:

$\Rightarrow$ $-10+2i_a+6i_b=0$

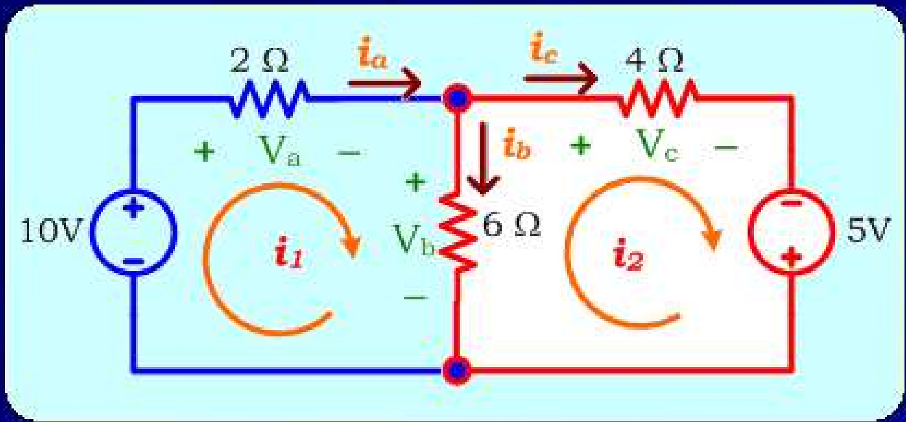
3. KCL:

$\Rightarrow$ $-10+2i_1+6(i_1-i_2)=0$ (CTE are expressed in terms of MC)

Simplify: $\Rightarrow$ $8i_1-6i_2=10$ (1)

Solution (contd):

We will repeat the procedure for Mesh 2



1. KVL: $\Rightarrow$ $-V_b + V_c - 5 = 0$

2. Ohm's Law:

$\Rightarrow$ $-6i_b + 4i_c - 5 = 0$

3. KCL: $\Rightarrow$ $-6(i_1 - i_2) + 4i_2 - 5 = 0$ (CTE are expressed in terms of MC)

Simplify: $\Rightarrow$ $-6i_1 + 10i_2 = 5$ *(2)*

From the previous slide

$8i_1 - 6i_2 = 10$ *(1)*

Equations (1) and (2) contain only mesh unknowns i_1 and i_2

Solving (1) and (2), we get: $\Rightarrow$ $i_1 = 2.955\text{A}$ $i_2 = 2.273\text{A}$

Example 2:

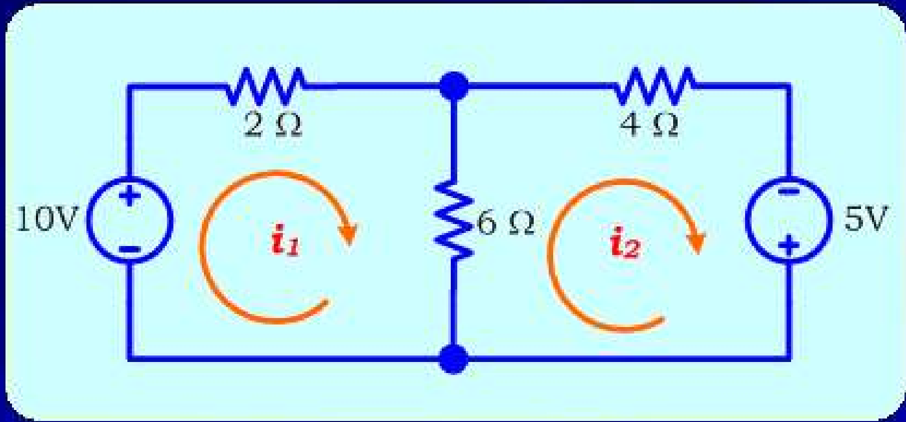
Calculate the mesh currents i_1 and i_2. Repeat the previous example by combining steps 1, 2, and 3.

Solution:

Mesh 1: KVL, & Ohm's Law, & KCL

$$\Rightarrow -10 + 2i_1 + 6(i_1 - i_2) = 0$$

$$8i_1 - 6i_2 = 10 \qquad (1)$$



Mesh 2: KVL, & Ohm's Law, & KCL

$$\Rightarrow 6(i_2 - i_1) + 4i_2 - 5 = 0$$

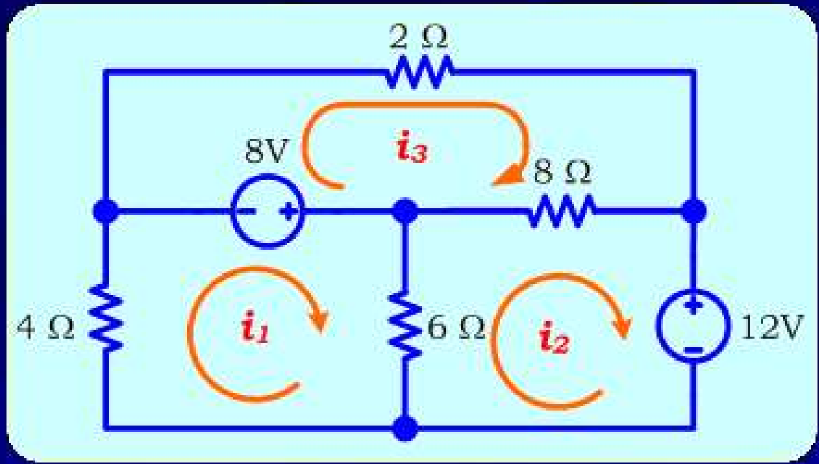
$$-6i_1 + 10i_2 = 5 \qquad (2)$$

Current through resistors = CTR

Always imagine CTR to be in the same direction as KVL.

Express the imagined CTR in terms of MC (Mesh Currents).

Example 3: Calculate the mesh currents i_1 , i_2 and i_3 .



Solution:

Mesh 1: $\Rightarrow 4i_1 - 8 + 6(i_1 - i_2) = 0 \quad \Rightarrow 10i_1 - 6i_2 = 8 \quad (1)$

Mesh 2: $\Rightarrow 6(i_2 - i_1) + 8(i_2 - i_3) + 12 = 0 \quad \Rightarrow -6i_1 + 14i_2 - 8i_3 = -12 \quad (2)$

Mesh 3: $\Rightarrow 2i_3 + 8(i_3 - i_2) + 8 = 0 \quad \Rightarrow -8i_2 + 10i_3 = -8 \quad (3)$

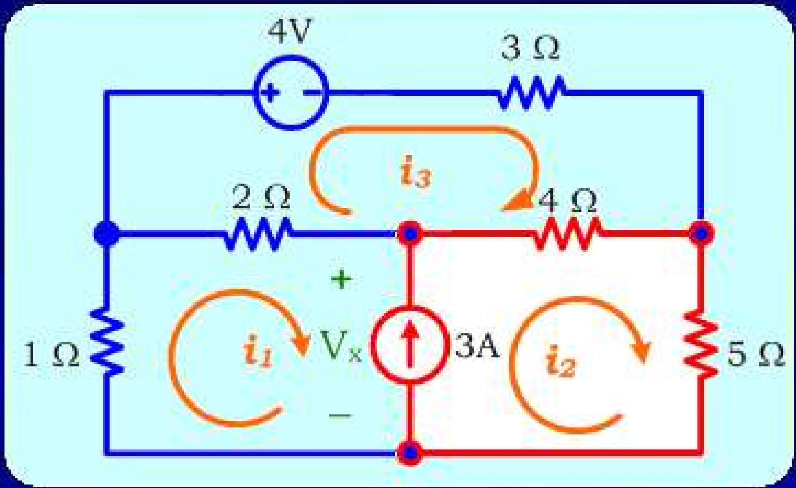
Solving (1), (2), and (3), we get: $\Rightarrow i_1 = -1.24\text{A} \quad i_2 = -3.40\text{A} \quad i_3 = -3.52\text{A}$

Mesh Analysis with Current Sources

When the circuit contains current sources, the previous procedure is modified.

Example 1: Calculate the mesh currents i_1 , i_2 and i_3

Solution:



KVL around Mesh 1:

$$1i_1 + 2(i_1 - i_3) + V_x = 0$$ (problem!)

We cannot directly replace V_x by mesh currents, because Ohm's law does not apply to current sources.

KVL around Mesh 2:

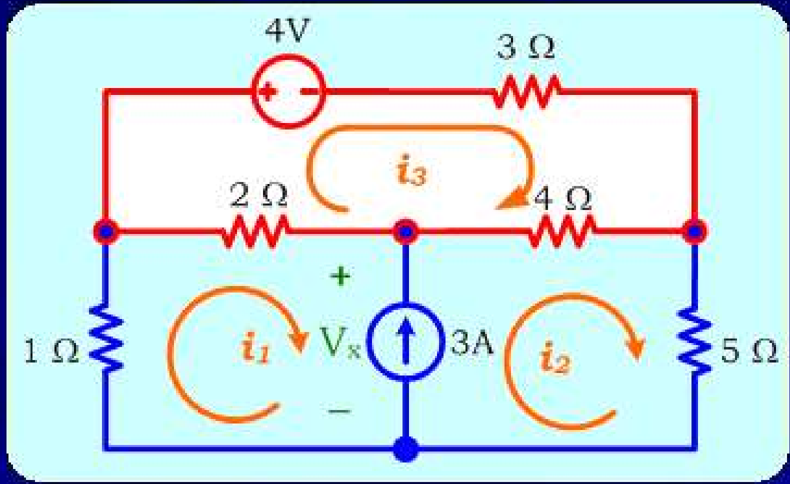
$$-V_x + 4(i_2 - i_3) + 5i_2 = 0$$ (similar problem!)

Solution (contd):

Mesh 1 & 2 contain a current source (they share the *3A* source)

What to do in this case?

Combine Mesh 1 & Mesh 2 into a Super Mesh (SM).



To avoid V_x, apply KVL around SM

$$\Rightarrow 1i_1 + 2(i_1 - i_3) + 4(i_2 - i_3) + 5i_2 = 0$$

$$3i_1 + 9i_2 - 6i_3 = 0 \qquad (1)$$

We need one more equation.

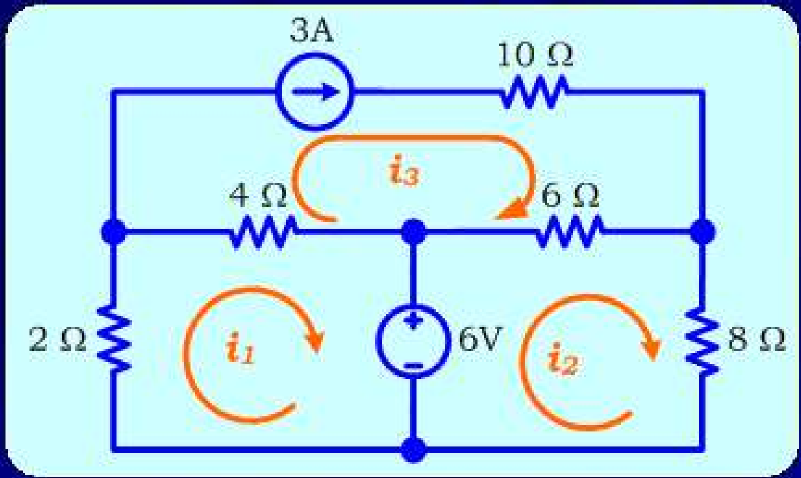
Apply KCL $\Rightarrow i_2 - i_1 = 3 \qquad (2)$

Mesh 3 does not contain a current source $\Rightarrow$ No special treatment

KVL around Mesh 3 $\Rightarrow 4 + 3i_3 + 4(i_3 - i_2) + 2(i_3 - i_1) = 0 \Rightarrow -2i_1 - 4i_2 + 9i_3 = -4 \quad (3)$

Solving (1), (2) and (3), we get: $\Rightarrow i_1 = -2.708A \quad i_2 = 0.292A \quad i_3 = -0.917A$

Example 2: Calculate the mesh currents i_1 , i_2 and i_3 .



Solution: Mesh 1 & 2 does not contain current sources.

$\Rightarrow$ Just apply KVL around Mesh 1 & 2

KVL around Mesh 1:

$$\Rightarrow \quad 2i_1 + 4(i_1 - i_3) + 6 = 0$$

$$\Rightarrow \quad 6i_1 - 4i_3 = -6 \qquad (1)$$

KVL around Mesh 2:

$$\Rightarrow \quad -6 + 6(i_2 - i_3) + 8i_2 = 0$$

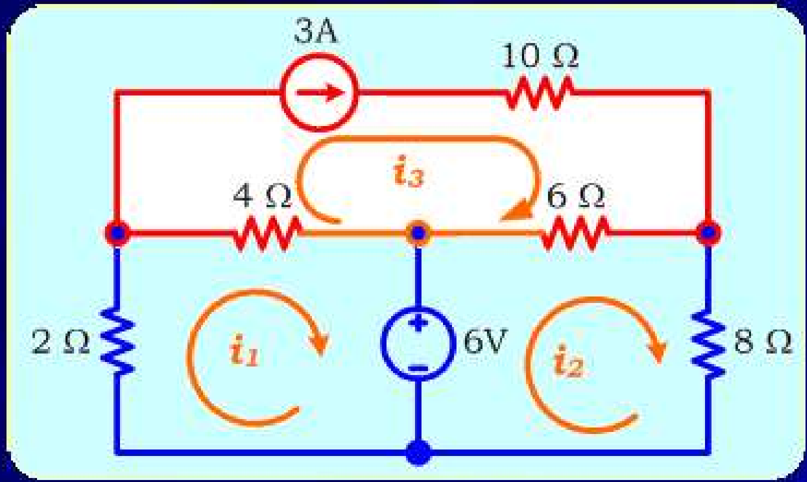
$$\Rightarrow \quad 14i_2 - 6i_3 = 6 \qquad (2)$$

Solution (contd):

From the previous slide,

$$6i_1 - 4i_3 = -6 \quad (1)$$

$$14i_2 - 6i_3 = 6 \quad (2)$$



Mesh 3 contains a 3A current source (not shared by another mesh)

Do not apply KVL (because KVL involves voltage across the current source).

Apply only KCL $\Rightarrow$ $i_3 = 3 \quad (3)$

[Note: Since we need only one equation from mesh 3, KCL provides it]

Solving (1), (2), and (3), we get: $\Rightarrow$ $i_1 = 1.000A \quad i_2 = 1.714A \quad i_3 = 3.000A$

Mesh Analysis with Current Sources : Summary

If a current source is shared by two meshes, then follow the procedure described below:

1. Combine the two meshes into a Super Mesh
2. Apply KVL around the Super Mesh
3. Apply KCL

If a current source is in one mesh only (not shared), then:

$\Longrightarrow$ Apply KCL only (do NOT apply KVL)