

المعادلات الخاصة بالقفز الهيدروليكي في المقاطع المستطيلة :-

المعادلة الخاصة بحساب النسبة بين العمقين y_1 و y_2 هي كالتالي :

$$\frac{y_2}{y_1} = \frac{1}{2} \left[\sqrt{1 + 8 Fr_1^2} - 1 \right] \quad \text{--- (1)}$$

and, head loss can be determined as :-

$$\frac{h_L}{y_1} = \frac{y_1}{4 y_2} \left[\frac{y_2}{y_1} - 1 \right]^3 \quad \text{--- (2)}$$

EX16 in the flow through a sluic in a large reservoir the velocity is (5.33 m/sec) while the depth is (0.056 m). determine the downstream conditions if a hydraulic jump takes place downstream?

Soln.

$$Fr_1 = \frac{V}{\sqrt{g y_1}} = \frac{5.33}{\sqrt{9.81 \times 0.056}} = 7.172$$

$$\therefore \frac{y_2}{y_1} = \frac{1}{2} \left[\sqrt{1 + 8 Fr_1^2} - 1 \right]$$

$$\frac{y_2}{0.056} = \frac{1}{2} \left[\sqrt{1 + 8 \times (7.172)^2} - 1 \right] \Rightarrow y_2 = 0.544 \text{ m}$$

$$\frac{h_L}{y_1} = \frac{y_1}{4y_2} \left[\frac{y_2}{y_1} - 1 \right]^3$$

$$\frac{h_L}{0.0563} = \frac{0.0563}{4 \times 0.544} \left[\frac{0.544}{0.0563} - 1 \right]^3 \Rightarrow h_L = 0.945 \text{ m}$$

ولايجاد النسبة المئوية للخسارة بسبب حدوث القفز نعمل ما يلي:

$$E_1 = \frac{V_1^2}{2g} + y_1 = \frac{(5.33)^2}{2g} + 0.0563 = 1.504 \text{ m}$$

$$\% \text{ dissipation} = \frac{h_L}{E_1} \times 100 = \frac{0.945}{1.504} \times 100 = 63\%$$

about 63% of mechanical energy is dissipated by hyd. Jump.

Ex 17 A rectangular channel of 5m width discharge water at the rate of $(1.5 \text{ m}^3/\text{sec})$ into a (5m) wide with $(1/3000)$ slope at velocity of (5 m/sec) , Determine the height of hyd. Jump and energy loss?

soln. $Q = V_1 A_2$

$$\therefore 1.5 = 5 \times (5 \times y_1) \Rightarrow y_1 = 0.06 \text{ m}$$

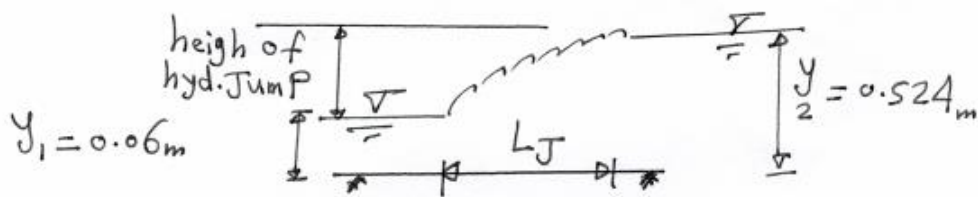
$$F_r = \frac{V_1}{\sqrt{g y_1}} = \frac{5}{\sqrt{9.81 \times 0.06}} = 6.52$$

- ↓
- ① Hence flow is supercritical
 - ② Hence hyd. Jump is Possible.

$$\frac{y_2}{y_1} = \frac{1}{2} \left[\sqrt{1 + 8F_r^2} - 1 \right]$$

$$\frac{y_2}{0.06} = \frac{1}{2} \left[\sqrt{1 + 8(6.52)^2} - 1 \right] \Rightarrow y_2 = 0.524 \text{ m}$$

$$\therefore \text{Height of hyd. Jump} = y_2 - y_1 = 0.524 - 0.06 = 0.464 \text{ m}$$



$$\frac{h_L}{y_1} = \frac{y_1}{4y_2} \left[\frac{y_2}{y_1} - 1 \right]^3$$

$$\frac{h_L}{0.06} = \frac{0.06}{4 \times 0.524} \left[\frac{0.524}{0.06} - 1 \right]^3 \Rightarrow h_L = 0.794 \text{ m}$$

$$\text{and, } L_J = 6.9(y_2 - y_1) = 6.9(0.524 - 0.06) = 3.2 \text{ m}$$

hydraulic Jump in non-rectangular channel ⁴²

لايجاد العمق الثاني للقضه وايجاد مقدار الخسارة سلا لفتون
سبه المغرزه نطبق المعطى الاي والمرنق :

EX18 Find the sequent depth corresponding to depth of (0.5m) in (3m) wide trapezoidal channel ($Z=1.5$) at discharge of $20 \text{ m}^3/\text{sec}$?

Solⁿ. $\frac{Z y_1}{B} = \frac{1.5 \times 0.5}{3} = 0.25$

$$\frac{Z \Phi}{\sqrt{g} B^{5/2}} = \frac{(1.5)^{3/2} (20)}{\sqrt{9.81} (3)^{5/2}} = 0.75$$

∴ from diagram $\Rightarrow \frac{Z h_L}{B} = 1.8 \Rightarrow h_L = 3.6 \text{ m}$

and, from $\frac{Z h_L}{B} = 1.8$ and $\frac{Z \Phi}{\sqrt{g} B^{5/2}} = 0.75$ find:-

$$\frac{Z y_2}{B} = 1.34 \Rightarrow y_2 = 2.68 \text{ m}$$

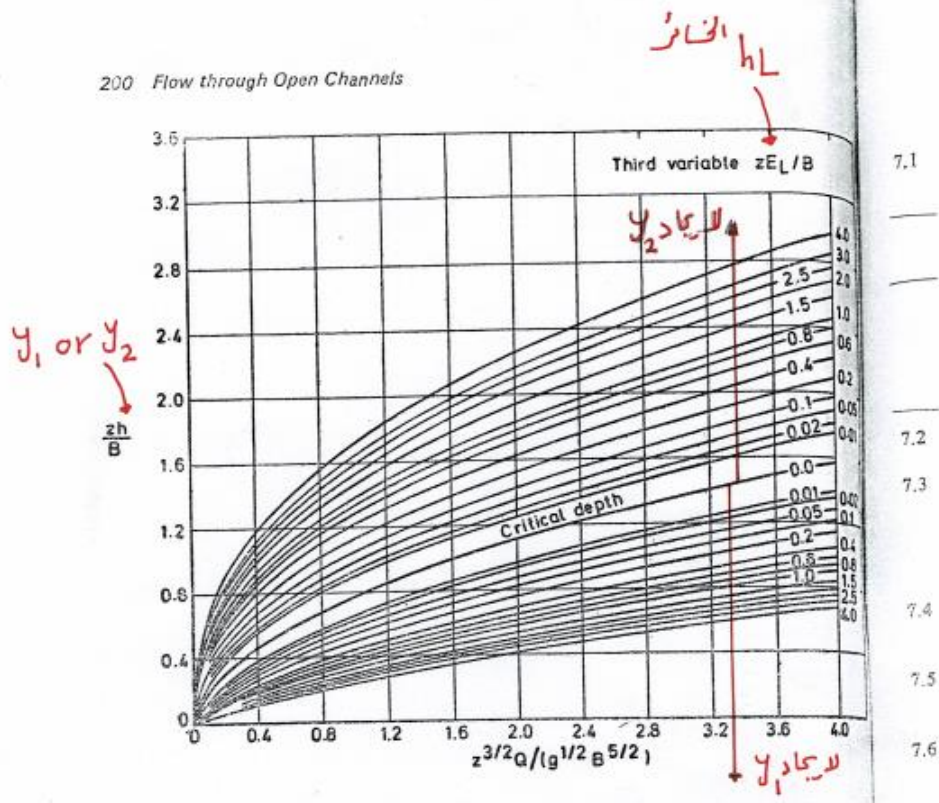


Fig. 7.29 Pre-jump and post-jump depths in trapezoidal channels.

Solution

$$zh_1/B = \frac{1.5 \times 0.5}{3.0} = 0.25$$

$$z^{3/2}Q/g^{1/2}B^{5/2} = \frac{1.5^{3/2} \times 20}{(9.8)^{1/2} 3^{5/2}} = 0.75$$

The corresponding value of $zE_L/B = 1.8$

$$\therefore E_L = \frac{1.8 \times 3}{1.5} = 3.6 \text{ m}$$

Corresponding to $zE_L/B = 1.8$ and $z^{3/2}Q/g^{1/2}B^{5/2} = 0.75$

$$zh_2/B = 1.34$$

$$\therefore h_2 = \frac{3 \times 1.34}{1.5} = 2.68 \text{ m}$$

Solving Eq. (15.35) for c after discarding terms with $(\Delta y)^2$, assuming an infinitesimally small wave, yields the wave velocity c as follows:

$$c = \sqrt{gy} \quad (15.36)$$

It has thus been shown that the speed of a small solitary wave is equal to the square root of the product of the depth and g .

15.6 Hydraulic Jump

Occurrence of the Hydraulic Jump

An interesting and important case of rapidly varied flow is the hydraulic jump. A hydraulic jump occurs when the flow is supercritical in an upstream section of a channel and is then forced to become subcritical in a downstream section (the change in depth can be forced by a sill in the downstream part of the channel or just by the prevailing depth in the stream further downstream), resulting in an abrupt increase in depth and considerable energy loss. Hydraulic jumps (Fig. 15.23) are often considered in the design of open channels and spillways of dams. If a channel is designed to carry water at supercritical velocities, the designer must be certain that the flow will not become subcritical prematurely. If it did, overtopping of the channel walls would undoubtedly occur, with consequent failure of the structure. Because the energy loss in the hydraulic jump is initially not known, the energy equation is not a suitable tool for analysis of the velocity-depth relationships. Because there is a significant difference in hydrostatic head on both sides of the equation causing opposing pressure forces, the momentum equation can be applied to the problem, as developed in the following sections.

Derivation of Depth Relationships in Hydraulic Jumps

Consider flow as shown in Fig. 15.23. Here, it is assumed that uniform flow occurs both upstream and downstream of the jump and that the resistance of the channel bottom over the relatively short distance L is negligible. The derivation is for a horizontal channel, but experiments show that the results of the derivation will apply to all channels of moderate slope ($S_0 < 0.02$). The derivation is started by applying the momentum equation in the x direction to the control volume shown in Fig. 15.24:

$$\sum P_x = m_2 V_2 - m_1 V_1$$

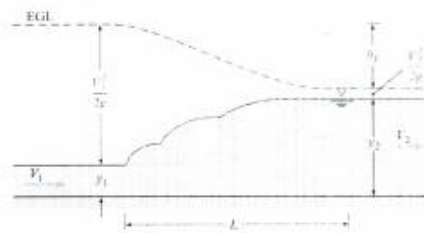


FIGURE 15.23
Derivation sketch for the hydraulic jump.

or y_2/y_1 (both terms are in common use) to each other, in contrast to the alternate depths obtained from the energy equation. Numerous experiments show that the relation represented by Eqs. (15.41) and (15.42) is valid over a wide range of Froude numbers.

Although no theory has been developed to predict the length of a hydraulic jump, experiments [see Chow (5)] show that the relative length of the jump, L/y_2 , is approximately 6 for Fr_1 ranging from 4 to 18.

Head Loss in a Hydraulic Jump

In addition to determining the geometric characteristics of the hydraulic jump, it is often desirable to determine the head loss produced by it. This is obtained by comparing the specific energy before the jump to that after the jump, the head loss being the difference between the two specific energies. It can be shown that the head loss for a jump in a rectangular channel is

$$h_L = \frac{(y_2 - y_1)^3}{4y_1 y_2} \quad (15.43)$$

For more information on the hydraulic jump, see Chow (5). The following example shows that Eq. (15.43) yields a magnitude that equals the difference between the specific energies at the two ends of the hydraulic jump.

EXAMPLE 15.11

Calculating Head Loss in a Hydraulic Jump

Problem Statement

Water flows in a rectangular channel at a depth of 30 cm with a velocity of 16 m/s, as shown in the following sketch. If a downstream sill (not shown) forces a hydraulic jump, what will be the depth and velocity downstream of the jump? What head loss is produced by the jump?



Define the Situation

A hydraulic jump is occurring in a rectangular channel.

State the Goal

- Calculate downstream depth and velocity.
- Calculate head loss produced by the jump.

Generate Ideas and Make a Plan

1. To calculate h_L using Eq. (15.43), calculate y_2 from the depth ratio equation (Eq. 15.42). This requires Fr_1 .
2. Check validity of head loss by comparing to $E_1 - E_2$.

Take Action (Execute the Plan)

1. Calculate Fr_1 , y_2 , V_2 , and h_L from Eqs. (Eq. 15.42) and (15.43):

$$Fr_1 = \frac{V}{\sqrt{gy_1}} = \frac{16}{\sqrt{9.81(0.30)}} = 9.33$$

$$y_2 = \frac{0.30}{2} \left[\sqrt{1 + 8(9.33)^2} - 1 \right] = 3.81 \text{ m}$$

$$V_2 = \frac{q}{y_2} = \frac{(16 \text{ m/s})(0.30 \text{ m})}{3.81 \text{ m}} = 1.26 \text{ m/s}$$

$$h_L = \frac{(3.81 - 0.30)^3}{4(0.30)(3.81)} = 9.46 \text{ m}$$

2. Compare the head loss to $E_1 - E_2$:

$$h_L = \left(0.30 + \frac{16^2}{2 \times 9.81} \right) - \left(3.81 + \frac{1.26^2}{2 \times 9.81} \right) = 9.46 \text{ m}$$

The value is the same, so validity of h_L equation is verified.

Use of Hydraulic Jump on Downstream End of Dam Spillway

Previously it was shown that the transition from supercritical to subcritical flow produces a hydraulic jump and that the relative height of the jump (y_2/y_1) is a function of Fr_1 . Because