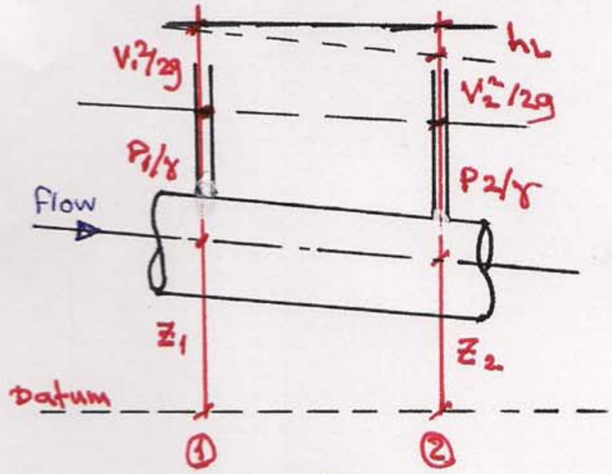
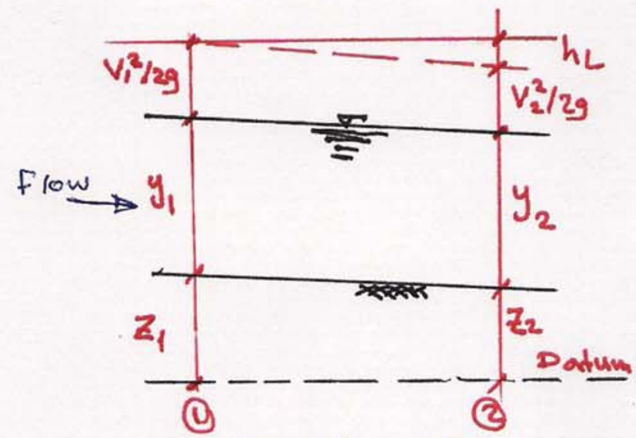
open channel flow is an important area of fluid mechanics for civil engineers. It describes the flow in rivers, man made channels and Partially full pipes as well as the behaviour of hydraulic structures such as weirs, spillways and sluices. Open channel flow must have a free surface subjected to atmospheric pressure. The flow is gravity driven, with the discharge and flow depth dependent on the balance between the downslope component of gravity and bed friction.

Pipe flow & open channel flow

See the illustration figures for demonstration the flow features of closed conduil and open channel flow :-



Pipe flow



Open Channel flow

Despite the similarity between the two kinds of flow, it must be pointing out the following nots :-

* The difficulties to solve problems of flow in open channels than in Pipe because the flow conditions in open channel are complicated due to:

- Variability of position of free surface which will change with time and space.
- The depth of flow, the discharge, and the slope of channel bottom and the free surface ar all inter dependent

* Physical conditions in open channel vary much more than in pipe where, the cross section of pipe is usually regular but for open channel may be irregular.
* The roughness in open channel is considered the main variable that may be differs in flow manner from section to another (e.g: the roughness may depend on the depth of flow).

Types of flow

The following classifications are made according to change in flow depth with respect to time and distance :-

* Steady & Unsteady : Time is the criterion

flow is said to be steady if the depth of flow at a section does not change for the time under consideration. Otherwise is said to be un-steady.

* Uniform flow : Distance is the Criterion

The flow is said to be uniform if the depth of flow and hence velocity are the same at every section of channel (e.g: The flow in prismatic channels)

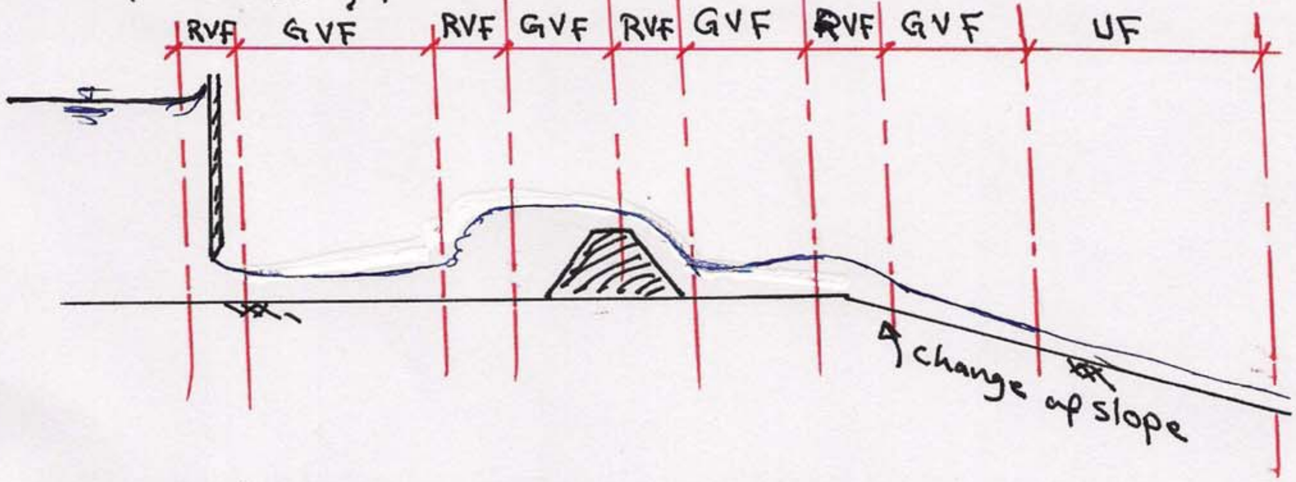
from the above definition "Stead-uniform" flow the depth & velocity will be constant with both Time & distance. This fundamental type of flow in an open channel occurs when gravity forces are in equilibrium with resistance forces

* Steady non uniform flow (Varied flow)

The depth varies with distance but not with time. This type of flow may be either Gradually Varied or Rapidly varied.

The gradually varied flow requires to applicate the energy and frictional resistance equations, while the rapidly varied flow rquires to applicate the energy & momentum Eq.

(see fig. below for demonstration the feature type of flow in practice).



Geometric properties necessary for analysis

The commonly needed geometric properties are shown as follows:-

- Depth (y): The vertical distance from lowest point of channel Section to the free surface.
- stage (Z): The vertical distance from the free surface to an arbitrary datum.
- Area (A): The cross-sectional area of flow, normal to the direction of flow.
- Wetted perimeter (P): The length of the wetted Surface measured normal to the direction of flow.
- Top width (T): width of the channel section at the free surface.
- Hydraulic Radius (R): The ratio of area to wetted perimeter where:- $R = \frac{A}{P}$
- Hydraulic depth (D): The ratio of area to top width where:-

Fundamental Equations

The equations which describe the flow of water are derived from three fundamental laws of Physics:

* Conservation of Mass.
* Conservation of energy.
* Conservation of momentum.

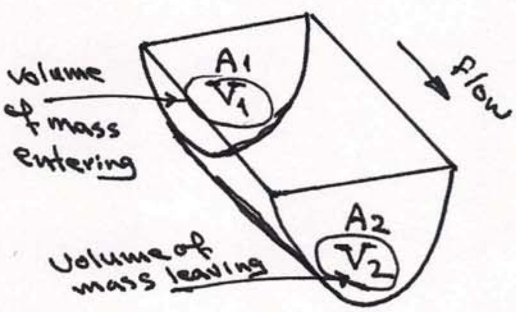
① The Continuity Equation (Conservation of mass)

for any control volume during the small time interval according to this law "The mass entering the control volume - the mass leaving the control volume = the change of mass within the control volume".

If the flow is steady & incompressible the mass of flow entering is equal to the mass leaving, where:-

$$\text{mass entering} = \text{mass leaving} \quad \text{——} ①$$

for control volume of flow in open channel show in fig. below



The eq.1 can be written as:-

$$\boxed{\rho Q_{ent.} = \rho Q_{leav.}} \quad \text{——} (1a)$$

also: the discharge known as: $Q = V.A$

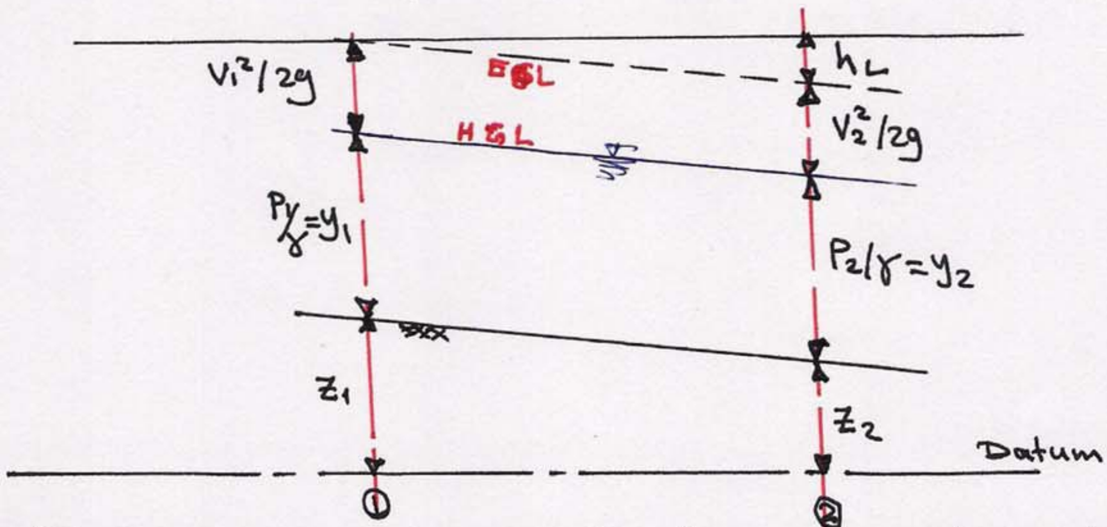
Therefore the continuity equation can be written as:-

$$\boxed{V_1 * A_1 = V_2 * A_2} \quad \text{——} ②$$

② The Energy Equation (Conservation of energy)

For any given system, the change in energy is equal to the difference between the heat transferred to the system and the work done by the system. The energy in open channel flow represent the total energy of the system, which is the sum of the potential energy, kinetic energy, and pressure energy.

In hydraulic application the "energy" converted to "head" in order to get a better feel for the resulting behavior of the system (see fig. below)



The energy at any point within the cross-section of flow is as can be seen in figure often expressed in three parts

- Elevation head, Z
- Pressure head (depth of flow), P/γ or y
- Velocity head (kinetic energy), $V^2/2g$

Because energy is conserved, it across any two points in the flow control volume must balance, where the energy between any two sections of control volume take the following form:-

$$\boxed{Z_1 + y_1 + V_1^2/2g = Z_2 + y_2 + V_2^2/2g + h_L} \quad (3)$$

③ The momentum Equation (conservation of momentum)

Again consider the control volume, during the time "t" :-

momentum Entering = $\rho Q_1 t V_1$

momentum leaving = $\rho Q_2 t V_2$

By the continuity $Q_1 = Q_2 = Q$

and by Newton's second law F = rate of change of momentum

where:- $$F = \frac{M_{leaving} - M_{entering}}{t}$$

then $$\boxed{F = \rho Q (V_2 - V_1)} \quad \text{—————} \quad (4)$$

Eq.4 is the momentum equation for steady flow for a region of uniform velocity.

Laminar & Turbulent flow

As in pipes and all flow, the flow in an open channel may be either laminar or turbulent. The criterion for determining the type of flow is the Reynold's number, Re.

In practice the limit for turbulent flow is not so well defined in channel as it is in pipes so $\boxed{Re = 2000}$ is often taken as the threshold for turbulent flow. Also in practice, flow in open channels is usually in the rough turbulent zone and consequently simpler friction formula may be applied to relate frictional losses to velocity and channel shape.

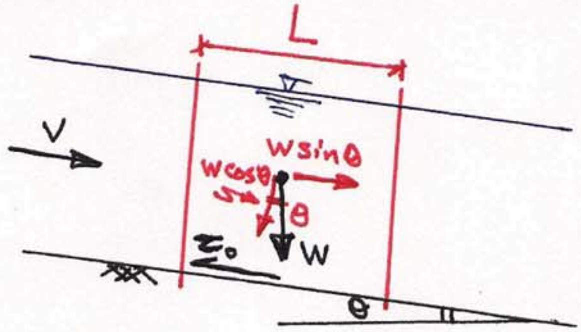
Uniform flow and the development of friction formula

When uniform flow occurs gravitational forces exactly balance with the frictional resistance forces, which apply as a shear force along the boundary (channel bed and walls).

Considering the below diagram :-

- The gravity force in the direction of flow is;

$$GF = \gamma A L \sin\theta \quad \text{———————} \quad (a)$$



- The boundary shear force ;

$$S.F = \tau_o P L \quad \text{------} \quad (b)$$

Then in uniform flow GF = S.F , that is ;

$$\gamma A L \sin\theta = \tau_o P L \quad \text{---} \quad (c)$$

for Small Slope channel that is refere to $\sin\theta \cong \tan\theta \simeq S_o$

where ,So, the bed slope , then

$$\tau_o = \frac{\gamma A S_o}{P} = \gamma R S_o \quad \text{---} \quad (d)$$

The Chezy Equation

In hydraulics and for "rough turbulent" flow by definition of the skin friction coefficient C_f , the bed shear stress can be related to average velocity :-

$$\tau_o = C_f \left(\tfrac{1}{2} \rho V^2\right)$$

$$\text{i.e} \quad \tau_o = K V^2 \quad \text{---------} \quad (e)$$

when substituting into Eq. d gives ;

$$V = \sqrt{\frac{\gamma}{K} R S_o} \quad \text{---------} \quad (f)$$

If grouping the constants together as denoted " C " Eq.f will be;

$$\boxed{V = C\sqrt{R S_o}} \quad \text{---------} \quad (5)$$

This is the " Chezy " equation and the " Chezy Coefficent C " as in ~~Eq.~~f depending on fluid properties and shear resistance.

The Manning Equation

many studies have been made of the evaluation of "C" for different open channels. Robert Manning (1891, 1895) derived the following empirical relation for "C" based upon experiments:

$$C = \frac{R^{1/6}}{n} \quad \text{------------} \quad (6)$$

Where "n" is the Manning's roughness coefficient. This coefficient is relats the effects of State, kind, shape, configuration of roughness of bed & sides of channel (Tables 802A – 802C) Illustrate a wide range of this coefficient.

when substiuting Eq. 6 into Eq. 5 gives the well known Manning's Equation :-

$$\boxed{V = \frac{1}{n} R^{2/3} S_o^{1/2}} \quad \text{---------} \quad (7)$$

It should be noted that the slope referred in Eq. 7 considered energy grade line slope, in uniform flow the lines EGL, HGL & bed slope line are all in parallel, So that $S_f = S_o$, where S_f is the slope of energy line.

Conveyance

If re-arranged eq. 7 as the following form

$$Q = V * A = \frac{1}{n} A \left(\frac{A}{P}\right)^{2/3} S^{1/2}$$

then $Q = \frac{1}{n} \cdot \frac{A^{5/3}}{P^{2/3}} \cdot S^{1/2}$

For the same flow condition, shape and roughness the discharge in open channel can be expressed as:-

$$\boxed{Q = K S^{1/2}} \quad \text{---------} \quad (8)$$

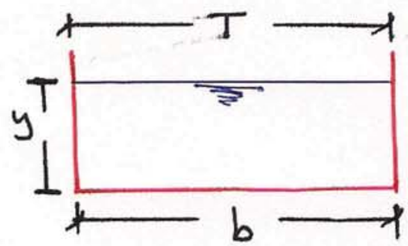
"K" is called Conveyance Coefficient it is a measure of the carrying capacity of a channel. For a specified "K" the only influencing parameter on discharg is the Slope "S"

$$\boxed{K = \frac{1}{n} \cdot \frac{A^{5/3}}{P^{2/3}}} \quad \text{------------} \quad (9)$$

* The application of conveyance may be a solution for calculating the discharge and stage of compound channels.

Particular Channel Shapes

In each case "y" is the depth of flow.



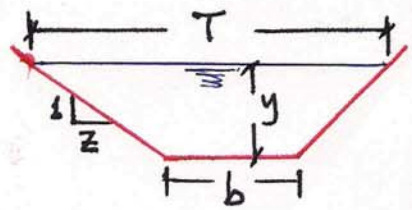
* **Rectangular channel**

$$R = \frac{A}{P} = \frac{by}{b+2y}$$

* **Wide channel**

The channel is ~~considered~~ as a wide if the limit of $y/b << 1$

then $R \simeq y$



* **Trapezoidal channel**

$$R = \frac{y(b+zy)}{b+2y\sqrt{1+z^2}}$$

The geometric elements of different channel section can be shown in Table 801.

Normal Depth

For any given discharge, there will ba a particular normal depth "y_n". To analyze open channel flow, it is usually necessary to know the normal depth, referring to conveyance Eq. 9, it can be re-writing (In SI-unints)

$$K = \frac{1}{n} \cdot R^{2/3} \cdot A$$

Note that , K, is a function of normal depth , properties of channel section and Manning's coefficient . Eq. 7 will be after multiplying by area of cross section ;

$$\boxed{AR^{2/3} = \frac{nQ}{\sqrt{S}} = nK} \quad \text{------ (10)}$$

Eq. 10 , illustrate that for a specified values of n, Q , and S it can be solve this equation to determine the normal depth

Computation of Normal depth

① Using Chow (1959) design curve (see fig. 6-1) .

② Trial & Error procedure

③ For a specified cross-section of channel , use the following Equations :-

* For wide rectangular channel ;

- $q = \frac{Q}{b}$ (discharge per unit width)
- $q = y_n V = \frac{1}{n} y_o R^{2/3} S^{1/2}$ (Manning's Eq.)
- $R \simeq y_n$ (for wide rectangular channel)

then :- $q = \frac{y_n^{5/3} S^{1/2}}{n}$ or $y_n = \left(\frac{nq}{\sqrt{S}}\right)^{3/5}$

than is refere Eq. 10 will become :-

$$\boxed{y_n = \left(\frac{nq}{\sqrt{S}}\right)^{3/5}} \quad \text{------ (11)}$$

Note that Eq. 11 used to calculate the normal depth for wide rectangular channel .

* For Rectangular channel :

referring to Table 801 the hydraulic radius of rectangular channel were calculated using the following expression :-

$$R = \frac{b\,y}{b+2y} = \frac{y_n}{1+\frac{2y_n}{b}}$$

then ; $$q = \frac{y_n^{5/3}\sqrt{S}}{n\left(1+\frac{2y_n}{b}\right)^{2/3}}$$

If need to calculate the normal depth re arrange the above formula to become as ;

$$\boxed{y_n = \left(\frac{n\,q}{\sqrt{S}}\right)^{3/5}\left(1+\frac{2y_n}{b}\right)^{2/5}} \quad \text{------ } (12)$$

TYPICAL ROUGHNESS COEFFICIENTS FOR OPEN CHANNELS

TYPE OF CHANNEL AND DESCRIPTION	MINIMUM	NORMAL	MAXIMUM
EXCAVATED OR DREDGED			
a. Earth, straight and uniform			
1. Clean, recently completed	0.016	0.018	0.020
2. Clean, after weathering	0.018	0.022	0.025
3. Gravel, uniform section, clean	0.022	0.025	0.030
4. With short grass, few weeds	0.022	0.027	0.033
b. Earth, winding and sluggish			
1. No vegetation	0.023	0.025	0.030
2. Grass, some weeds	0.025	0.030	0.033
3. Dense weeds or aquatic plans in deep channels	0.030	0.035	0.040
4. Earth bottom and rubble sides	0.028	0.030	0.035
5. Stony bottom and weedy banks	0.025	0.035	0.040
6. Cobble bottom and clean sides	0.030	0.040	0.050
c. Dragline-excavated or dredged			
1. No vegetation	0.025	0.028	0.033
2. Light brush on banks	0.035	0.050	0.060
d. Rock cuts			
1. Smooth and uniform	0.025	0.035	0.040
2. Jagged and irregular	0.035	0.040	0.050
e. Channels not maintained, weeds and brush			
1. Dense weeds, high as flow depth	0.050	0.080	0.120
2. Clean bottom, brush on sides	0.040	0.050	0.080
3. Same as above, but highest state of flow	0.045	0.070	0.110
4. Dense brush, high state	0.080	0.100	0.140

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TABLE 802A

TYPICAL ROUGHNESS COEFFECIENTS FOR OPEN CHANNELS

TYPE OF CHANNEL & DESCRIPTION	MINIMUM	NORMAL	MAXIMUM
Brass, smooth	0.009	0.010	0.013
Steel:			
Lockbar and welded	0.010	0.012	0.014
Riveted and spiral	0.013	0.016	0.017
Cast Iron:			
Coated	0.010	0.013	0.014
Uncoated	0.011	0.014	0.016
Wrought Iron:			
Black	0.012	0.014	0.015
Galvanized	0.013	0.016	0.017
Corrugated Metal:			
Sub-drain	0.017	0.019	0.021
Storm Drain	0.021	0.024	0.030
Lucite	0.008	0.009	0.010
Glass	0.009	0.010	0.013
Cement:			
Neat, surface	0.010	0.011	0.013
Mortar	0.011	0.013	0.015
Concrete:			
Culvert, straight and free of debris	0.010	0.011	0.013
Culvert with bends, connections, and some debris	0.011	0.013	0.014
Finished	0.011	0.012	0.014
Sewer with manholes, inlet, etc., straight	0.013	0.015	0.017
Unfinished, steel form	0.012	0.013	0.014
Unfinished, smooth wood form	0.012	0.014	0.016
Unfinished, rough wood form	0.015	0.017	0.020
Wood:			
Stave	0.010	0.012	0.014
Laminated, treated	0.015	0.017	0.020
Clay:			
Common drainage tile	0.011	0.013	0.017
Vitrified sewer	0.011	0.014	0.017
Vitrified sewer with manholes, inlet, etc.	0.013	0.015	0.017
Vitrified subdrain with open joint	0.014	0.016	0.018
Brickwork:			
Glazed	0.011	0.013	0.015
Lined with cement mortar	0.012	0.015	0.017
Sanitary sewers coated with sewage slime with bends and connections	0.012	0.013	0.016
Paved invert, sewer, smooth bottom	0.016	0.019	0.020
Rubble masonry, cemented	0.018	0.025	0.030

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TYPICAL ROUGHNESS COEFFICIENTS FOR OPEN CHANNELS

TYPE OF CHANNEL AND DESCRIPTION	MINIMUM	NORMAL	MAXIMUM
LINED OR BUILT-UP CHANNELS			
a. CONCRETE			
1. TROWEL FINISH	0.011	0.013	0.015
2. FLOAT FINISH	0.013	0.015	0.016
3. GUNITE, GOOD SECTION	0.016	0.019	0.023
4. GUNITE, WAVY SECTION	0.018	0.022	0.023
b. CONCRETE BOTTOM FLOAT FINISHED WITH SIDE OF			
1. DRESSED STONE IN MORTAR	0.015	0.017	0.020
2. RANSOM STONE IN MORTAR	0.017	0.020	0.024
3. DRY RUBBLE OR RIPRAP	0.020	0.030	0.035
c. GRAVEL BOTTOM WITH SIDES OF			
1. FORMED CONCRETE	0.017	0.020	0.025
2. RANDOM STONE IN MORTAR	0.020	0.023	0.026
3. DRY RUBBLE OR RIPRAP	0.023	0.033	0.036
d. ASPHALT			
1. SMOOTH	0.013	0.013	--
2. ROUGH	0.016	0.016	--
e. GRASSED	0.030	0.040	0.050

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GEOMETRIC ELEMENTS OF CHANNEL SECTIONS

GEOMETRIC ELEMENTS OF CHANNEL SECTIONS

SECTION	AREA, A	WETTED PERIMETER, P	HYDRAULIC RADIUS, R	TOP WIDTH, T	HYDRAULIC DEPTH, D	SECTION FACTOR, Z
Rectangle	by	$b+2y$	$\frac{by}{b+2y}$	b	y	$by^{1.5}$
Trapezoid	$(b+zy)y$	$b+2y\sqrt{1+z^2}$	$\frac{(b+zy)y}{b+2y\sqrt{1+z^2}}$	$b+2zy$	$\frac{(b+zy)y}{b+2zy}$	$\frac{[(b+zy)y]^{1.5}}{\sqrt{b+2zy}}$
Triangle	zy^2	$2y\sqrt{1+z^2}$	$\frac{zy}{2\sqrt{1+z^2}}$	$2zy$	$\frac{y}{2}$	$\frac{\sqrt{2}}{2}zy^{2.5}$
Circle	$\frac{1}{8}(\theta-\sin\theta)d_0^2$	$\frac{1}{2}\theta d_0$	$\frac{1}{4}\left(1-\frac{\sin\theta}{\theta}\right)d_0$	$(\sin\frac{1}{2}\theta)d_0$ or $2\sqrt{y(d_0-y)}$	$\frac{1}{8}\left(\frac{\theta-\sin\theta}{\sin\frac{1}{2}\theta}\right)d_0$	$\frac{\sqrt{2}}{32}\frac{(\theta-\sin\theta)^{1.5}}{(\sin\frac{1}{2}\theta)^{0.5}}d_0^{2.5}$
Parabola	$\frac{2}{3}Ty$	$T+\frac{8}{3}\frac{y^2}{T}$ *	$\frac{2T^2y}{3T^2+8y^2}$ *	$\frac{3}{2}\frac{A}{y}$	$\frac{2y}{3}$	$\frac{2}{9}\sqrt{6}\,Ty^{1.5}$
Round-cornered rectangle ($y>r$)	$\left(\frac{\pi}{2}-2\right)r^2+(b+2r)y$	$(\pi-2)r+b+2y$	$\frac{(\pi/2-2)r^2+(b+2r)y}{(\pi-2)r+b+2y}$	$b+2r$	$\frac{(\pi/2-2)r^2}{b+2r}+y$	$\frac{[(\pi/2-2)r^2+(b+2r)y]^{1.5}}{\sqrt{b+2r}}$
Round-bottom triangle	$\frac{T^2}{4z}-\frac{r^2}{z}(1-z\cot^{-1}z)$	$\frac{T}{z}\sqrt{1+z^2}-\frac{2r}{z}(1-z\cot^{-1}z)$	$\frac{A}{P}$	$2[z(y-r)+r\sqrt{1+z^2}]$	$\frac{A}{T}$	$A\sqrt{\frac{A}{T}}$

*Satisfactory approximation for the interval $0<x\leq1$, where $x=4y/T$. When $x>1$, use the exact expression $P=(T/2)[\sqrt{1+x^2}+1/x\ \ln(x+\sqrt{1+x^2})]$

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TABLE 801

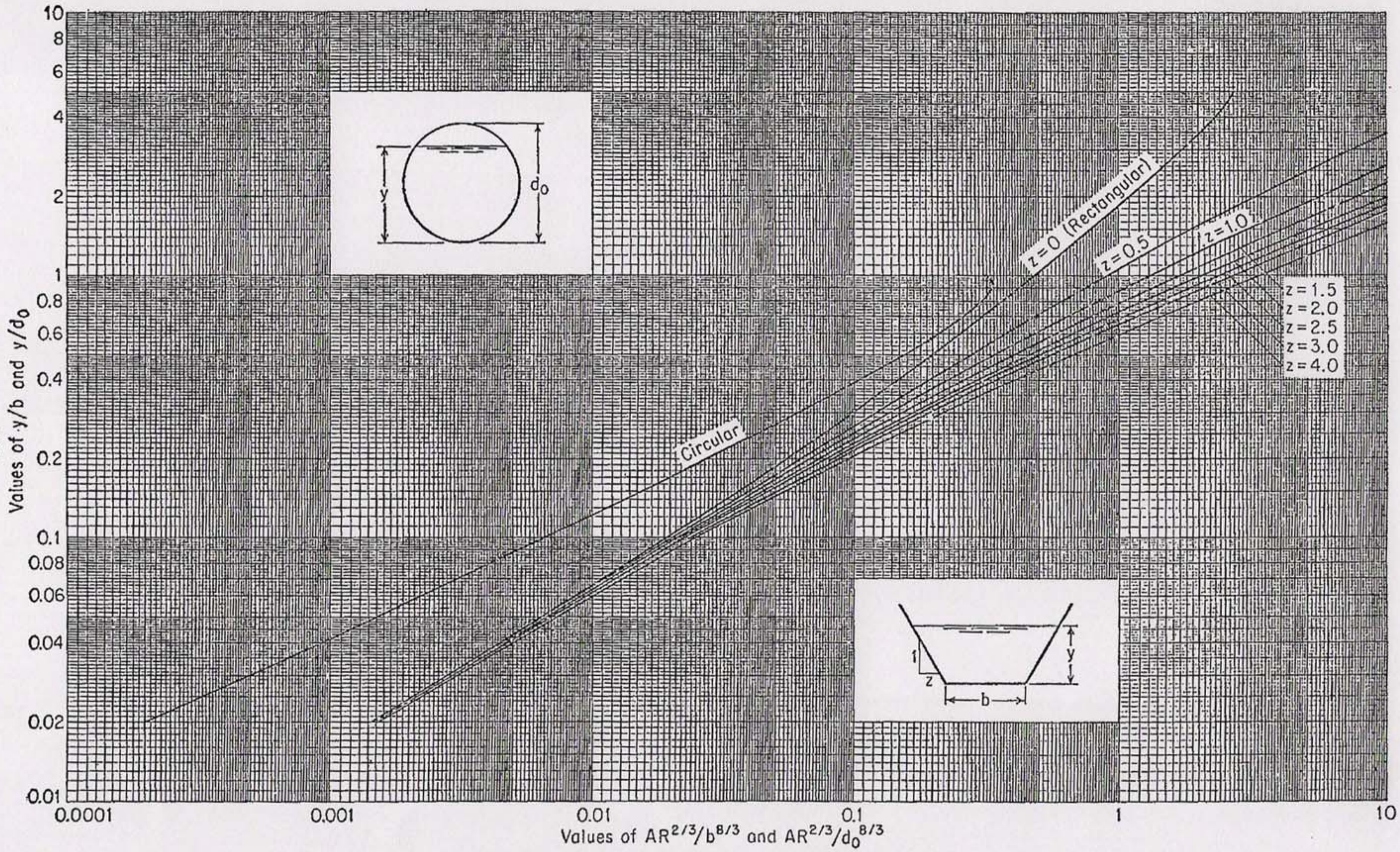

FIG. 6-1. Curves for determining the normal depth.