6. Economical Cross-section (Most Efficient section) المقطع الامثل

The hydraulic engineer upon to determine the shape and, size of cross section of channel which transport the Maximum discharge when (P) is minimum.

This illustrated in the Problem by atrial Process assuming depth and width.

EX8 A smooth cement rectangular channel with a slope (1 : 2500), Manning's coefficient is (0.011), total area ($4 m^2$), assuming different ratios of depth and width, determin the flow rate?

Sol:

$$Q = \frac{1}{n} R_h^{2/3} S^{1/2} A$$

$$A = B \times y = 4 m^2 \quad \text{——— (1)}$$

$$P = B + 2y$$

$$R_h = \frac{A}{P} = \frac{4}{P} \quad \text{——— (2)}$$

هنا لايجاد التصريف يجب معرفة قيمة الارتفاع y والعرض B
و كالاتي :

S:

$y:B$	y	B	P	R_h	$Q = \frac{1}{n} R_h^{2/3} S^{1/2} (4)$
1:1.5	1.633	2.45	5.716	0.699	5.73
1:1.75	1.512	2.645	5.67	0.706	5.764
1:2	1.414	2.828	5.66 Min.	0.707 $y/2$	5.772 → Max.
1:2.25	1.33	3	5.67	0.706	5.766
1:2.5	1.265	3.16	5.69	0.703	5.748

$y/B = 1/1.5$ $B=1.5y$

$A=4 = B.y = (1.5y)y = 1.5y^2$

$y = 1.633$

$B=1.5y$ $B= 2.45$

نلاحظ ان عند نسبه (1:2) (الارتفاع : العرض) نحصل على الحاله المثلى
لأفضل مقطع مستطيل (اكبر تصريف لاقل محيط مبلل)

then, for rectangular section :. $\frac{y}{B} = \frac{1}{2}$

$$B = 2y$$

and, $R_h = \frac{y}{2}$

وبنفس الطريقه يمكن لنا ايجاد المعادلات الخاصه بالمقطع
الامثل للمقاطع شبه المنحرف والمثلث وكما مبين بالجدول
الاتي :

section	optimum geometry	normal depth y	R_h	A
rectangular	$B=2y$	$0.917\left[\dfrac{Qn}{S^{1/2}}\right]^{3/8}$	$\dfrac{y}{2}$	$1.68\left[\dfrac{Qn}{S^{1/2}}\right]^{3/4}$
Trapezoidal	$\theta=60^\circ$, $Z=\dfrac{1}{\sqrt{3}}$, $B=\dfrac{2y}{\sqrt{3}}$	$0.968\left[\dfrac{Qn}{S^{1/2}}\right]^{3/8}$	$\dfrac{y}{2}$	$1.622\left[\dfrac{Qn}{S^{1/2}}\right]^{3/4}$
Traingular	$Z=1$, $\theta=45^\circ$	$1.297\left[\dfrac{Qn}{S^{1/2}}\right]^{3/8}$	$\dfrac{y}{2\sqrt{2}}$	$Zy^2=y^2$

يتم تطبيق الجدول اعلاه ونختار الحالة المطلوبه حسب نوع المقطع اما حساب التصريف فيتم تطبيق معادلة ماننك بعد حساب المتطلبات في الجدول اعلاه.

Ex9 For given slope of 1 : 2500 and flow rate of ($4 m^3/sec$), determine the optimum cross section (Most Efficient Section) for ① rectangular ② trapezoidal ③ traingular, ($n = 0.011$) ?

Section	y	R_h	A
rectangular	$0.917\left[\frac{Qn}{S^{1/2}}\right]^{3/8}$ $y = 0.917\left[\frac{4(0.011)}{(\frac{1}{2500})^{1/2}}\right]^{3/8}$ $y = 1.235 m$ $B = 2y = 2.465$	$\frac{y}{2}$	$1.68\left[\frac{Qn}{S^{1/2}}\right]^{3/4}$ $A = 1.68\left[\frac{4(0.011)}{(\frac{1}{2500})^{1/2}}\right]^{3/4}$ $A = 3.04 m^2$
trapezoidal	$Z = \frac{1}{\sqrt{3}}$ $\theta = 60$ $B = \frac{2y}{\sqrt{3}}$ $y = 0.968\left[\frac{Qn}{S^{1/2}}\right]^{3/8}$ $y = 0.968\left[\frac{4(0.011)}{(\frac{1}{2500})^{1/2}}\right]^{3/8}$ $y = 1.302 m$ $B = \frac{2(1.502)}{\sqrt{3}}$ $B = 1.502 m$	$\frac{y}{2}$	$1.622\left[\frac{Qn}{S^{1/2}}\right]^{3/4}$ $A = 1.622\left[\frac{4(0.011)}{(\frac{1}{2500})^{1/2}}\right]^{3/4}$ $A = 2.93 m^2$

Ex 10 Find the depth of flow in the most efficient traingular section carring discharge of $(0.2\ m^3/sec.)$ on slope of $(1:2500)$, $n=0.014$?

Soln. for traingular section :-

$Z=1 \qquad \Theta = 45^\circ$

$$y = 1.297\left[\frac{Qn}{S^{1/2}}\right]^{3/8} = 1.297\left[\frac{0.2(0.014)}{\left(\frac{1}{2500}\right)^{1/2}}\right]^{3/8}$$

$y = 0.62\ m$

Ex 11 Arectangular section is to be built of rough un sized timber. if given a drop of (2m in 1Km) what will be width and depth for best section to carry $(1.1\ m^3/sec)$, $n=0.011$?

Sol. $S = \frac{2m}{1Km} = \frac{2}{1000} = \frac{1}{500}$

$$y = 0.917\left[\frac{Qn}{S^{1/2}}\right]^{3/8} = 0.917\left[\frac{1.1(0.011)}{\left(\frac{1}{500}\right)^{1/2}}\right]^{3/8} = 0.56\ m$$

$B = 2y = 2(0.56) = 1.12\ m$

Ex12 A trapezoidal channel of Best section has a discharge of ($25\,m^3/sec$) with slope ($1:1500$) Design the section if $n = 0.0135$?

Soln. for trapezoidal section :-

$$Z = \frac{1}{\sqrt{3}}$$

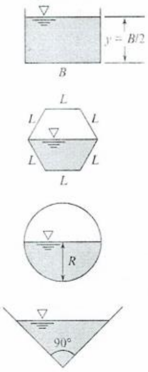
$$y = 0.968\left[\frac{Qn}{S^{1/2}}\right]^{3/8} = 0.968\left[\frac{25(0.0135)}{\left(\frac{1}{1500}\right)^{1/2}}\right]^{3/8}$$

$$\therefore y = 2.54\,m$$

$$B = \frac{2y}{\sqrt{3}} = \frac{2(2.54)}{\sqrt{3}} = 2.93\,m$$

FIGURE 15.5

Best hydraulic sections for different geometries.



Best Hydraulic Section for Uniform Flow

The best hydraulic section is the channel geometry that gives the maximum discharge for a given cross-sectional area. Maximum discharge occurs when a geometry has the minimum wetted perimeter. Therefore, it yields the least viscous energy loss for a given area. Consider the quantity $AR_h^{2/3}$ in Manning's equation given in Eqs. (15.15 and 15.16), which is referred to as the section factor. Because $R_h = A/P$, the section factor relating to uniform flow is given by $A(A/P)^{2/3}$. Thus, for a channel of given resistance and slope, the discharge will increase with increasing cross-sectional area but decrease with increasing wetted perimeter P. For a given area A and a given shape of channel—for example, rectangular cross section—there will be a certain ratio of depth to width (y/B) for which the section factor will be maximum. This ratio is the best hydraulic section.

Example 15.6 shows that the best hydraulic section for a rectangular channel occurs when $y = \frac{1}{2}B$. It can be shown that the best hydraulic section for a trapezoidal channel is half a hexagon as shown; for the circular section, it is the half circle with depth equal to radius; and for the triangular section, it is a triangle with a vertex of 90° (Fig. 15.5). Of all the various shapes, the half circle has the best hydraulic section because it has the smallest perimeter for a given area.

The best hydraulic section can be relevant to the cost of the channel. For example, if a trapezoidal channel were to be excavated and if the water surface were to be at adjacent ground level, the minimum amount of excavation (and excavation cost) would result if the channel of best hydraulic section were used.

EXAMPLE 15.6

Finding the Best Hydraulic Section for a Rectangular Channel

Problem Statement

Determine the best hydraulic section for a rectangular channel with depth y and width B.

Define the Situation

Water flows in a rectangular channel. Depth $= y$. Width $= B$.

State the Goal

Find the best hydraulic section (relate B and y).

Generate Ideas and Make a Plan

1. Set $A = By$ and $P = B + 2y$ so that both are a function of y.
2. Let A be constant, and minimize P:
   - Differentiate P with respect to y and set the derivative equal to zero.
   - Express the result of minimizing P as a relation between y and B.

Take Action (Execute the Plan)

1. Relate A and P in terms of y:

$$P = \frac{A}{y} + 2y$$

2a. Minimize P:

$$\frac{dP}{dy} = \frac{-A}{y^2} + 2 = 0$$

$$\frac{A}{y^2} = 2$$

2b. Express result in terms of y and B:

$$A = By, \text{ so}$$

$$\frac{By}{y^2} = 2 \quad \text{or} \quad \boxed{y = \frac{1}{2}B}$$

Review the Solution and the Process

Knowledge. The best hydraulic section for a rectangular channel occurs when the depth is one-half the width of the channel (see Fig. 15.5).

Uniform Flow in Culverts and Sewers

Sewers are conduits that carry sewage (liquid domestic, commercial, or industrial waste) from households, businesses, and factories to sewage disposal sites. These conduits are often circular in cross section, but elliptical and rectangular conduits are also used. The volume rate of sewage varies throughout the day and season, but of course sewers are designed to carry the