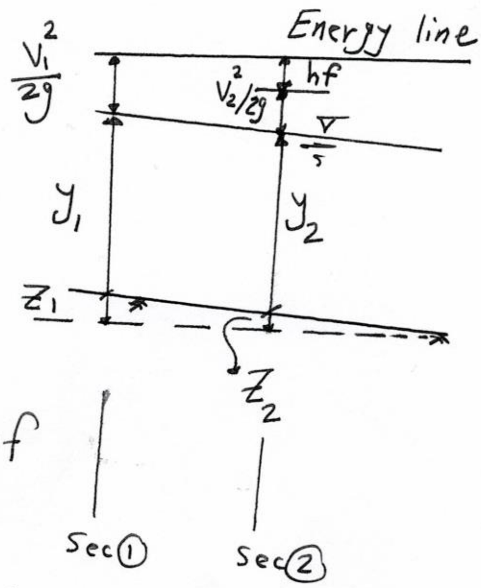
Energy Equation for steady Flow and Specific Energy (Critical flow)

Considering Sections 1 and 2 in the flow as shown in Fig. Bernoulli eq. is written as:



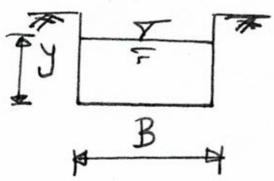
$$\frac{V_1^2}{2g} + y_1 + Z_1 = \frac{V_2^2}{2g} + y_2 + Z_2 + hf$$

- The term $\frac{V^2}{2g} + y$ is found to be an important Parameter is open channel flow, This quantity is defined as Specific Energy الطاقة النوعية OPEN CHANELS ONLY and, symbol used is E.

$\therefore$ Specific energy $= E = y + \frac{V^2}{2g}$ --- ①

- The case of rectangular section:

$$E = y + \frac{Q^2}{2g B^2 y^2} \quad \text{--- ②}$$



in this Process the value of minimum energy for a given flow is found as:

$$E_{min} = y_c + \frac{y_c^3}{2y_c^2} = \frac{3}{2} y_c \quad \text{---} \ (3)$$

Min. Energy with critical flow only

The value of y for minimum energy is called critical depth (y_c).

$$y_c = \sqrt[3]{q^2/g} \quad \text{—} \ (4)$$

where, $q = \frac{Q}{B}$ m^2/sec (flow rate for unit width)

The flow rate at this condition (Min. specific energy and critical depth) is:

$$q_{max} = \sqrt{g y_c^3} \quad \text{---} \ (5)$$

at yc :
the energy is Min.
the discharge is Max.

• المعادلات من (1 ← 5) خاصه بالمقاطع المستطيله ويمكن
هنا تمثيلها من خلال المخططين التالين وكالاتي :

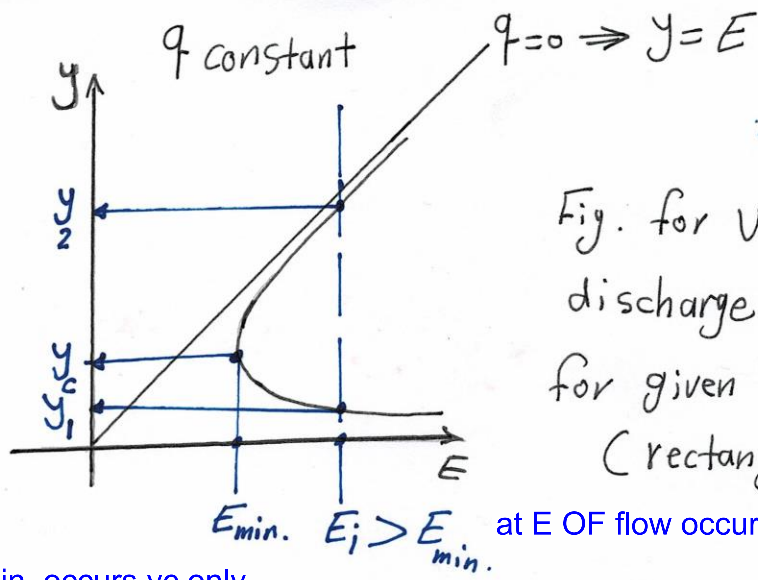


Fig. for Variation of discharge with depth for given Specific Energy. (rectangular section)

at E OF flow occurs y1 and y2

at Emin. occurs yc only

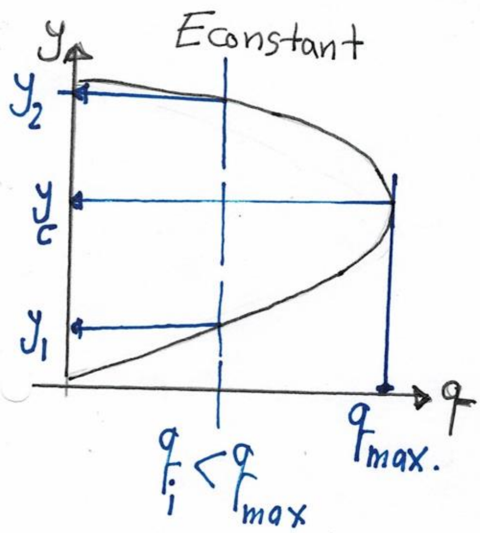


Fig. Variation of depth with Specific Energy for given flow rate (rectangular section)

●**ملاحظة مهمة** في الشكلين اعلاه نلاحظ تكون عمقين (y_1 و y_2) والثاني يمثل (alternate depth) او (Squent depth)

ويحسب من خلال المعادلة الاتية:

$$y^3 - E y^2 + q^2/2g = 0 \quad --- (6) \quad \text{(rectangular sec.)}$$

1 or 2

Ex 13 water flow in a rectangular channel at the rate of ($3 m^3/sec$) per m width, the depth being ($1.5 m$) y1. Determine whether the flow is subcritical or supercritical. Also determine the alternate depth and critical depth?

Sol^n

① $E = \frac{V^2}{2g} + y$

$V = \frac{Q}{A} = \frac{3}{1.5 \times 1} = 2 \ m/sec$

$\therefore E_i = \frac{4}{2g} + 1.5 = 1.704 m$ وهذه تبقى قيمة ثابتة حتى لو تغيرت السرعه والعمق فقط تتغير عند حالة الجريان الحرجه. $E_i = E_1 = E_2$

$Fr_1 = \frac{V}{\sqrt{gy}} = \frac{2}{\sqrt{9.81 \times 1.5}} = 0.52 < 1$ The flow is subcritical

② $y_c = \sqrt[3]{q^2/g}$ $\qquad q = \frac{Q}{B} = \frac{3}{1} = 3 \ m^2/sec$

$\therefore y_c = \sqrt[3]{3^2/9.81} = 0.972 m$

$E_{min.} = \frac{3}{2} y_c = \frac{3}{2}(0.972) = 1.458 m$

③ alternate depth y_2:

$y_2^3 - E y_2^2 + \frac{q^2}{2g} = 0 \Rightarrow y_2^3 - 1.703 y_2^2 + \frac{3^2}{2g} = 0$

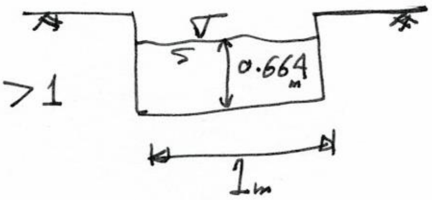
Solving by trail, $y_2 = 0.6643\,m$

وكما كان للعمق الاول (1.5m) حساب: حالة الجريان ممكنة الان

حساب حالة الجريان للعمق الثاني والذي مقداره (0.664m)

وكالاتي :

$$V = \frac{Q}{A} = \frac{3}{1 \times 0.664} = 4.52\ m/sec$$



Fr2

$$F_r = \frac{V}{\sqrt{g\,y_2}} = \frac{4.52}{\sqrt{9.81 \times 0.664}} = 1.8 > 1$$

The flow is super critical.