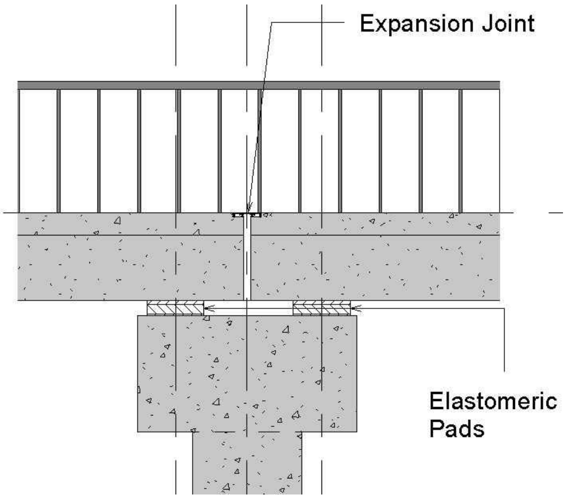
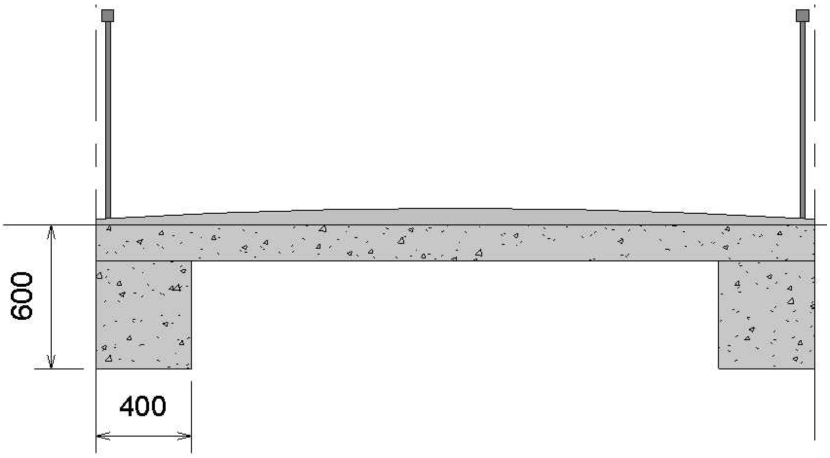


Callout View



Cross Sectional View

Assume that the designer intends to use:

1. Concrete of $f_c' = 28\ \text{MPa}$.
2. Steel of A615 Grade 60.
3. A width of 400mm and a height of 600mm (these dimensions have been determined based on deflection considerations).
4. Rebar of No. 25 for longitudinal reinforcement.
5. Rebar of No. 13 for stirrups.

Notes on the Problem

1. With using of the expansion joint and elastomeric pads shown in the callout view, the beam almost behaves as a simply supported one.
2. Surfacing and live loads should be simulated as load per unit area. Until Chapter 13 where student learns how to transfer loads from slabs to the supporting beams, loads will be given per unit length.
3. Beam is assumed to have a rectangular shape. This conservative assumption equivalent to neglecting slab flanging effect. More detailed modeling will be discussed in analysis and design of beams with "T" shapes.

Answers

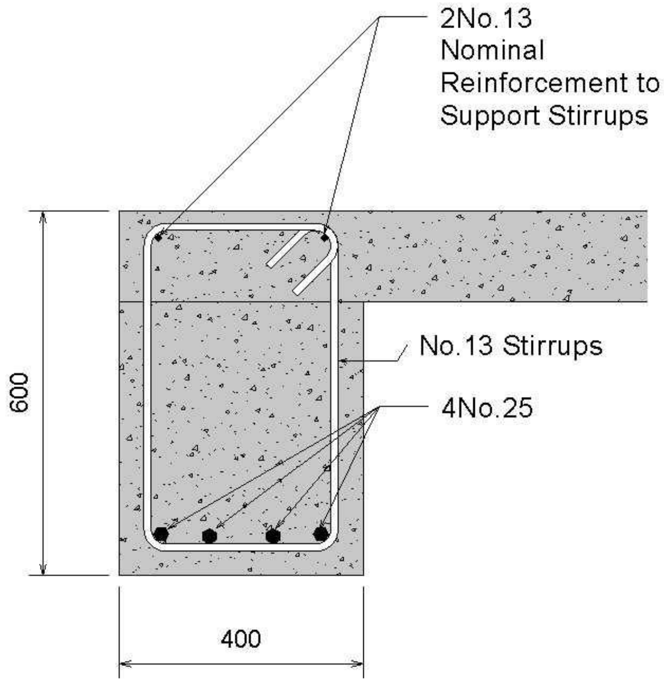
- Computed required factored applied moment (M_u):
  a. Moment due to Dead Loads:
  $$W_{\text{Selfweight}} = 5.76\frac{\text{kN}}{\text{m}} \Rightarrow W_{\text{Dead}} = 11.3\frac{\text{kN}}{\text{m}} \Rightarrow$$
  b. Moment due to Live Load:
  $$M_{\text{Live}} = 75\ \text{kN.m}$$
  c. Factored Moment M_u:
  $$M_u = \text{Maximum of } (1.4M_D \text{ or } 1.2M_D + 1.6M_L)$$
  $$= \text{Maximum of } [1.4 \times 141 \text{ or } (1.2 \times 141 + 1.6 \times 75)]$$
  $$= \text{Maximum of } [197 \text{ or } 289] = 289\ \text{kN.m} \blacksquare$$
- Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:

$M_n = \frac{M_u}{\emptyset}$

Strength reduction factored can be assumed 0.9, and checked later.

$M_n = 321\ kN.m$

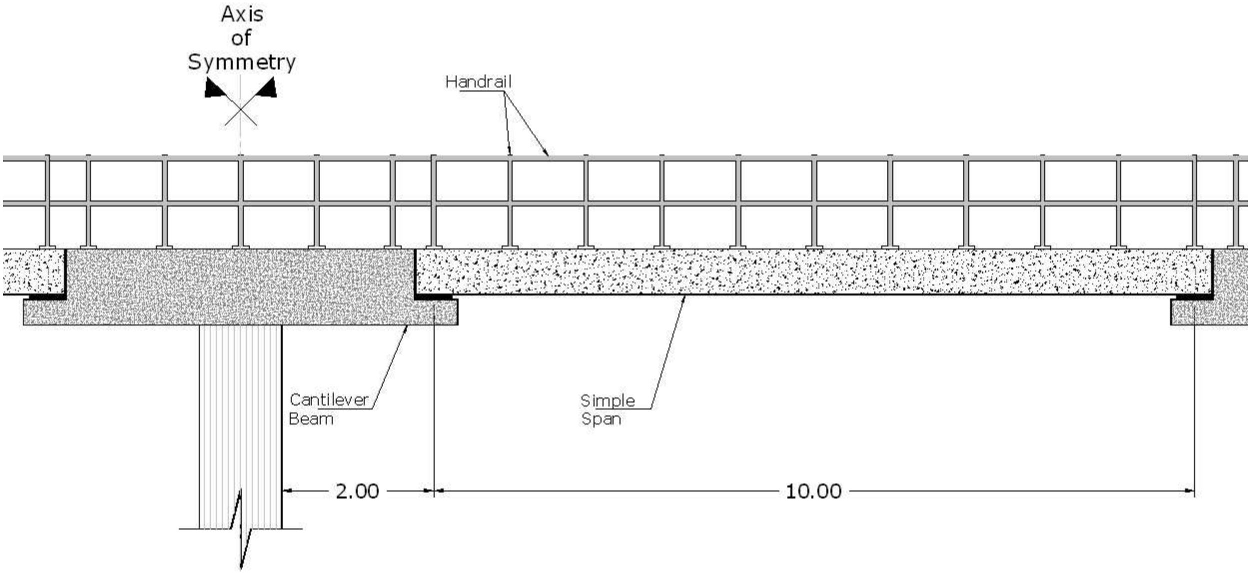
- Compute the effective beam depth "d":
  Assume that, reinforcement can be put in single reinforcement, then:
  $d_{for\ One\ Layer} = 600 - 40 - 13 - \frac{25}{2} = 534\ mm$
- Compute the Required Steel Ratio $\rho_{Required}$:
  $\rho_{Required} = 7.15 \times 10^{-3}$
- Check if the beam failure is secondary compression failure or compression failure:
  $\rho_{max} = 20.6 \times 10^{-3} > \rho_{Required}$ Ok.
- Compute the required steel area:
  $A_{S\ Required} = 1\,527\ mm^2$
- Compute required rebars number:
  $A_{Bar} = \frac{\pi \times 25^2}{4} = 490\ mm^2$
  $\text{No. of Rebars} = 3.12$
  Try $4\emptyset 25mm$.
  $A_{S\ Provided} = 4 \times 490\ mm^2 = 1\,960\ mm^2$
- Check if the available width "b" is adequate to put the rebars in a single layer:
  $b_{required} = 281mm < 400mm\ Ok.$
- Check for S_{max}:
  $s_{\frac{c}{c}} = 92mm < s_{max}\ Ok.$
- Check with $A_{s\ minimum}$ requirements:
  $A_{s\ minimum} = 712\ mm^2 < A_{S\ Provided} = 1\,960\ mm^2\ Ok.$
- Check the assumption of $\emptyset = 0.9$:
  a. Compute steel stain based on the following relations:
     $a = 86.5\ mm$
     $c = 102\ mm$
     $\epsilon_t = 12.7 \times 10^{-3}$
  b. As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.
- Final Reinforcement Details:



Problem 4.4-3

Design a cantilever beam of pedestrian's bridge shown below to carry the following loads:

1. Dead load of slab and pavement surfacing is $5.0\ kN/m$.
2. Handrail weight can be taken as $0.5\frac{\mathrm{kN}}{\mathrm{m}}$.
3. Service live load of $6.0\ \mathrm{kN/m}$.

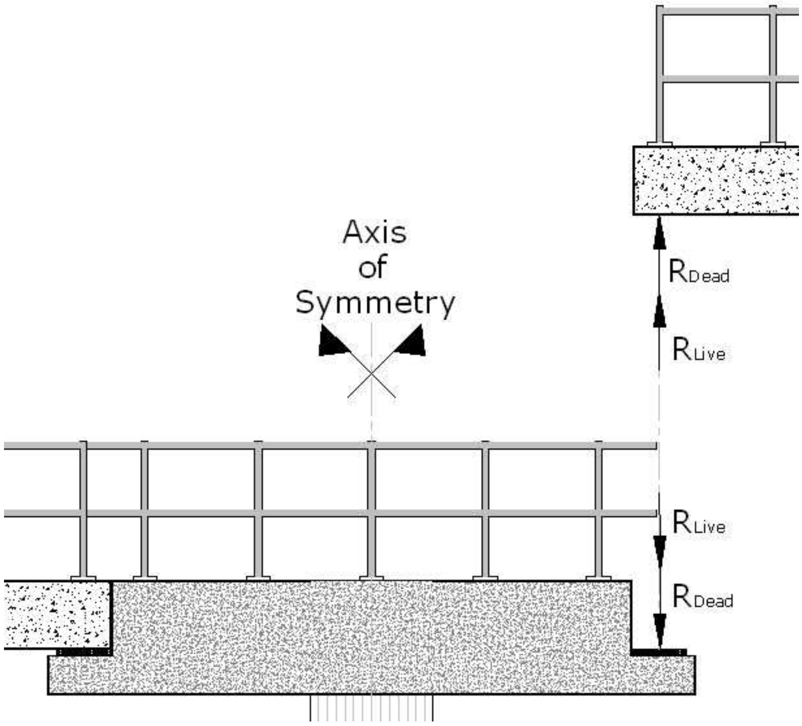


Assume that the designer intends to use:

1. Concrete of $f_c' = 28\ \mathrm{MPa}$.
2. Steel of A615 Grade 60.
3. A width of 400mm and a height of 800mm for cantilever span.
4. A width of 400mm and a height of 600mm for simple span.
5. Rebar of No. 25 for longitudinal reinforcement.
6. Rebar of No. 13 for stirrups.

Answers

- Computed required factored applied moment (M_u):
  a. Compute the reactions of simple span:



b. Reaction due to Dead Loads:

$$W_{Selfweight} = 0.6m \times 0.4m \times 24\frac{kN}{m^3} = 5.76\frac{kN}{m} \Rightarrow W_{Dead} = (5.76 + 5.0 + 0.5)\frac{kN}{m} = 11.3\frac{kN}{m}$$

$$R_{Dead} = \frac{11.3\frac{kN}{m} \times 10.0m}{2} = 56.5 \text{ kN}$$

c. Reaction due to Live Load:

$$R_{Live} = \frac{6.0\frac{kN}{m} \times 10.0m}{2} = 30 \text{ kN}$$

d. Compute the moments of cantilever span:
Generally, it is useful to note that a negative moment is computed at face of support and not at support centerline. This is due to the fact within support loads transferred in a bearing form instead of shear and bending form.

i. Moment due to Dead Loads:

$$W_{Selfweight} = 0.8m \times 0.4m \times 24\frac{kN}{m^3} = 7.68\frac{kN}{m}$$

$$W_{Dead} = (7.68 + 5.0 + 0.5)\frac{kN}{m} = 13.2\frac{kN}{m}$$

$$M_D = 56.5 \text{ kN} \times 2.0m + \frac{13.2\frac{kN}{m} \times 2.0^2 m^2}{2} = 139 \text{ kN.m}$$

ii. Moment due to Live Loads:

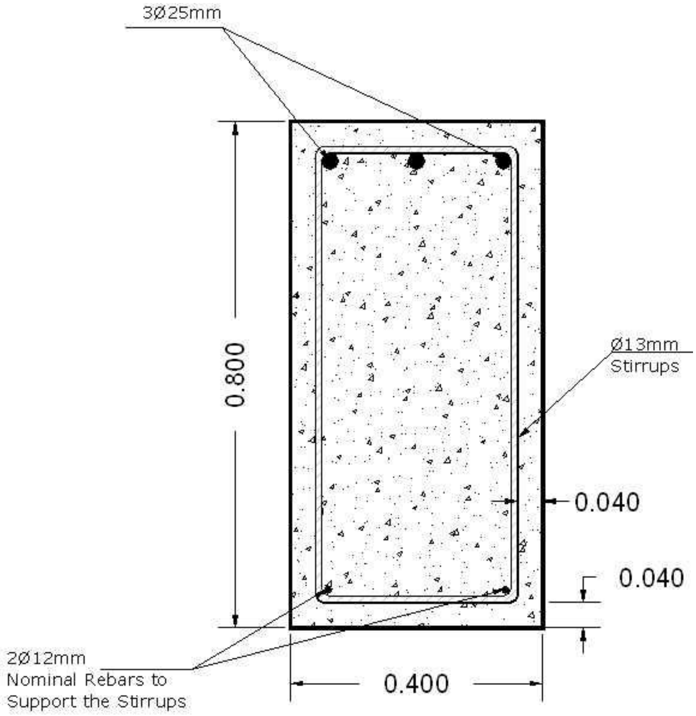
$$M_L = 30 \text{ kN} \times 2.0m + \frac{6.0\frac{kN}{m} \times 2.0^2 m^2}{2} = 72 \text{ kN.m}$$

iii. Factored Moment M_u:

$M_u = \text{Maximum of } (1.4M_D \text{ or } 1.2M_D + 1.6M_L)$
$M_u = \text{Maximum of } [1.4 \times 139 \text{ or } (1.2 \times 139 + 1.6 \times 72)]$
$M_u = \text{Maximum of } [195 \text{ or } 282] = 282 \text{ kN.m}$ ■

- Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:
  $$M_n = \frac{M_u}{\emptyset}$$
  Strength reduction factored can be assumed 0.9, and checked later.
  $M_n = 313 \text{ kN.m}$
- Compute the effective beam depth "d":
  Assume that, reinforcement can be put in single reinforcement, then:
  $d_{\text{for One Layer}} = 734 \text{ mm}$
- Compute the Required Steel Ratio $\rho_{Required}$:
  $\rho_{Required} = 3.57 \times 10^{-3}$
- Check if the beam failure is secondary compression failure or compression failure:
  $\rho_{max} = 20.6 \times 10^{-3} > \rho_{Required}$ Ok.
- Compute the required steel area:
  $A_{S\ Required} = 1\,048 \text{ mm}^2$
- Compute required rebars number:
  $$A_{Bar} = \frac{\pi \times 25^2}{4} = 490 \text{ mm}^2$$
  No. of Rebars $= 2.14$
  Try, 3Ø25mm.
  $A_{S\ Provided} = 3 \times 490 \text{ mm}^2 = 1\,470 \text{ mm}^2$
- Check if the available width "b" is adequate to put the rebars in a single layer:
  $b_{required} = 231\text{mm} < 400mm\ Ok.$
- Check for Smax:
  $s = 137\ mm < s_{max}$
- Check with $A_{s\ minimum}$ requirements:
  $A_{s\ minimum} = 979 \text{ mm}^2 < A_{S\ Provided} = 1\,470 \text{ mm}^2$ Ok.

- Check the assumption of $\emptyset = 0.9$:
  a. Compute steel stain based on the following relations:
     $a = 64.9\ \text{mm} \Rightarrow c = 76.3\ \text{mm} \Rightarrow \epsilon_t = 25.9 \times 10^{-3}$
  b. As $\epsilon_t\ \geq 0.005$, then $\emptyset = 0.9$ Ok.
- Final Reinforcement Details



Problem 4.4-4

Design a simply supported beam with a span of 6.1m to support following loads:

1. A Dead load of $8.22\frac{\text{kN}}{\text{m}}$.
2. Service live load of $24.1\ \text{kN/m}$.
3. Assume that the designer intends to use:
4. Concrete of $f_c' =\ 35\ \text{MPa}$.
5. Steel of $f_y =\ 420\text{MPa}$.
6. A width of 325mm and a height of 420mm.
7. Rebar of No. 20 for longitudinal reinforcement.
8. Rebar of No. 13 for stirrups.
9. Two layers of reinforcement.

Aim of the Problem

This problem aims to show how to design a beam within the transition zone, i.e. when the assumption of $\phi = 0.9$ is incorrect.

Answers

- Computed required factored applied moment (M_u):
  a. Moment due to Dead Loads:
     $W_{\text{Selfweight}} =\ 3.28\frac{\text{kN}}{\text{m}}\ W_{\text{Dead}} = 11.5\frac{\text{kN}}{\text{m}} \Rightarrow M_{\text{Dead}} = 53.5\ \ \text{kN.m}$
  b. Moment due to Live Load:
     $M_{\text{Live}} = 112\ \text{kN.m}$
  c. Factored Moment M_u:
     $M_u =\ \text{Maximum of}\ (1.4M_D\ \text{ or}\ 1.2M_D +\ 1.6M_L) =$
     $=\ \text{Maximum of}\ [1.4 \times 53.5\quad \text{or}\ (1.2 \times 53.5\ \ +\ 1.6 \times 112)]$
     $= \text{Maximum of}\ [74.9\ \text{ or}\ 243] = 243\ \text{kN.m}\ \blacksquare$
- Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:
  $M_n = \frac{M_u}{\emptyset}$
  Strength reduction factored can be assumed 0.9, and checked later.
  $M_n = 270\ \text{kN.m}$

- Compute the effective beam depth "d":

  $d_{\text{for Two Layer}} = 420 - 40 - 13 - 20 - \frac{25}{2} = 334 \text{ mm}$
- Compute the Required Steel Ratio ρ_{Required}:

  $\rho_{\text{Required}} = 20.8 \times 10^{-3}$

  Check if the beam failure is secondary compression failure or compression failure:

  $\beta_1 = 0.80$

  $\rho_{\max} = 24.3 \times 10^{-3} > \rho_{\text{Required}}$ Ok. ■
- Compute the required steel area:

  $A_{S\text{ Required}} = 2\,258 \text{ mm}^2$
- Compute required rebars number:

  $A_{\text{Bar}} = 314 \text{ mm}^2$

  $\text{No. of Rebars} = 7.19$

  Try $8\emptyset 20\text{mm}$.

  $A_{S\text{ Provided}} = 2\,512 \text{ mm}^2$
- Check if the available width "b" is adequate to put the rebars in a single layer:

  $b_{\text{required}} = 2 \times 40\text{mm} + 2 \times 13 + 8 \times 20\text{mm} + 7 \times 25\text{mm} = 441\text{mm} > 325mm$

  Then the reinforcement must be put in two layers as the designer is assumed.
- Check for smax:

  $s = 70mm < s_{max}\ Ok.$
- Check with $A_{s\text{ minimum}}$ requirements:

  $A_{s\text{ minimum}} = 382 \text{ mm}^2 < A_{S\text{ Provided}} = 2\,512 \text{ mm}^2$ Ok.
- Check the assumption of $\emptyset = 0.9$:
  a. Compute steel stain based on the following relations:

     $a = 109 \text{ mm} \Rightarrow c = 136 \text{ mm} \Rightarrow \epsilon_t = 4.37 \times 10^{-3} > 4.0 \times 10^{-3}$ Ok.
  b. As $\epsilon_t < 0.005$, then:

     $\emptyset = 0.483 + 83.3\epsilon_t = 0.847$

     Then the reinforcement will be re-designed based on new $\emptyset$.
- Reinforcement Re-design Based on New $\emptyset$:
  a. Re-computed the required nominal flexure strength (M_n):

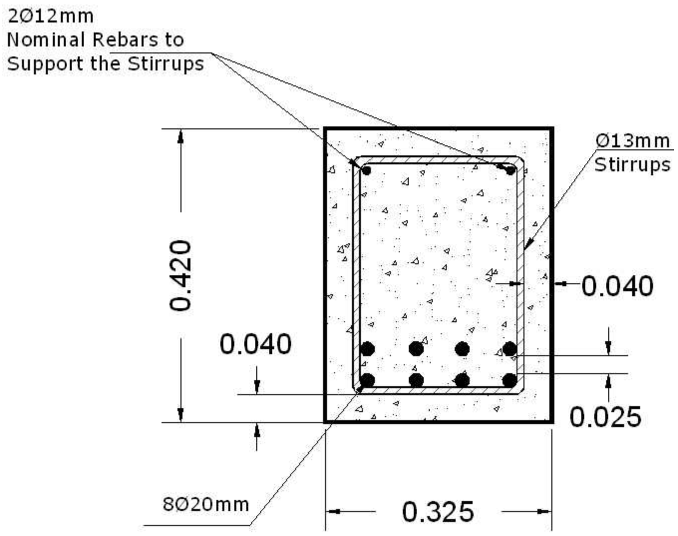
     $M_n = 287 \text{ kN.m}$
  a. Re-compute the Required Steel Ratio ρ_{Required}:

     $\rho_{\text{Required}} = 22.4 \times 10^{-3} < \rho_{\max} = 24.3 \times 10^{-3}$ Ok.
  b. Re-compute the required steel area:

     $A_{S\text{ Required}} = 2\,432 \text{ mm}^2$
  c. Compute required rebars number:

     $\text{No. of Rebars} = 7.74$

  Use $8\emptyset 20\text{mm}$.
- Final Reinforcement Details:



4.5 Practical Flexure Design of a Rectangular Beam with Tension Reinforcement Only and with No Pre-specified Dimensions

4.5.1 Essence of the Problem

- A second type of problems may occur when there are no previous functional or architectural limitations on beam dimensions.
- Then, the designer has ***three design parameters***, namely beam width "b", beam depth "h", and beam reinforcement.

4.5.2 Design Procedure

As we have only one relation, namely

$$M_n = \rho f_y b d^2 \left(1 - 0.59\ \frac{\rho f_y}{f'_c}\right)$$

and have three unknowns, then two assumptions to be adopted as summarized in design procedure presented below:

1. Computed the factored moment M_u based on given spans, dead, live, and other loads. Beam selfweight is assumed and checked later.
2. Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:
   $$M_n = \frac{M_u}{\emptyset}$$
   Strength reduction factored can be assumed 0.9 and checked later.
   It will be shown in Step 3 below that we are usually working in the range far from ρ_{max}, then the assumption of $\Phi = 0.9$ seems fair.
3. Select a Reinforcement Ratio (***First Assumption***):
   Theoretically, reinforcement ratio can be selected anywhere between maximum and minimum steel ratios (ρ_{max} or ρ_{min}). However, based on economical and deflection requirements, it is preferred to use a reinforcement ratio in the range of:
   a. **For Economical Purposes:**
      It can be shown, that an economical design will typically have reinforcement ratios between $0.5\rho_{max}$ to $0.75\rho_{max}$. This recommended range is based on American literature and different recommendations may be adopted for different locations or for the same location but at different periods.
   b. **For Deflection Control:**
      From mechanics of materials it is known there is an inverse proportionality between the beam deflection, Δ, and its moment of inertia, I:
      $$\Delta \propto \frac{1}{I}$$
      As the concrete dimensions are more effective in increasing the moment of inertia, I, than the reinforcement area, therefore a larger steel ratio is used a smaller moment of inertia resulted with larger potential deflection problem.
      Following criterion can be used to determine the required steel ratio to avoid the potential deflection problem.
      $$\rho_{\text{For Deflection Control}} \le \frac{0.18 f'_c}{f_y}$$
      This criterion is based on previous ACI Code (ACI Code 1963). This stipulation was deleted in more current ACI Code. Nevertheless, it remains a valid guide for selecting a preliminary value for reinforcement ratio.
4. Solve the following relation to compute the required (bd^2):
   $$M_n = \rho f_y (bd^2)_{\text{Required}} \left(1 - 0.59\ \frac{\rho f_y}{f'_c}\right)$$
5. Experience and judgment developed over the years have also established a range of acceptable and economical depth/width ratios for rectangular beams.
   Although there is no code requirement for the d/b ratio to be within a given range, rectangular beams commonly have d/b ratios of (***Second Assumption***):

$$1.0 \le \frac{d}{b} \le 3.0$$

Desirable d/b ratios lie between:

$$1.5 \le \frac{d}{b} \le 2.2$$

6. Compute the required steel area:
$A_{s\ \text{Required}} = \rho \times (bd)$
7. Compute the required rebars number:
$$\text{No. of Rebars } = \frac{A_s}{A_{Bar}}$$
8. Check if rebars can be put in one or two layers:
$$b_{required} = 2 \times \text{Side Cover} + 2 \times \text{Stirrups Diamter} + \text{No. of Rebars} \times \text{Bar Diameter} + (\text{No. of Rebars} - 1) \times \text{Spacing between Rebars}$$
If
$b_{required} < b_{available}$
Then reinforcement cannot be put in a single layer.
9. Check Spacing "s" with s_{max} limitations of the ACI Code:
If
$s < s_{max}$
Ok.
Else, you should used a larger number of smaller bars.
10. Compute the required beam depth "h". Depend on reinforcement layers, one of following two relations can be used:
$$h_{\text{for One Layer}} = d + \text{Cover} + \text{Stirrups} + \frac{\text{Bar Diameter}}{2}$$
$$h_{\text{for Two Layer}} = d + \text{Cover} + \text{Stirrups} + \text{Bar Diameter} + \frac{\text{Spacing between Layers}}{2}$$
Round the computed "h" to a practical number.
11. Check the Assumption of $\emptyset = 0.9$:
In the previous sections, the strain of tensile reinforcement has been determined directly based similar triangles of the strain distribution.
In below, another indirect method is proposed to classify the section based on computing the reinforcement ratio required to have a tensile reinforcement strain of 0.005:
    a. Compute the provided effective depth, d.
    b. Compute the provided steel ratio:
    $$\rho_{\text{Provided}} = \frac{A_{s\ \text{Provided}}}{b \times d_{\text{Provided}}}$$
    c. Compute the steel ratio required for steel strain of 0.005:
    $$\rho_{\text{for } \epsilon_t = 0.005} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.005}$$
    If
    $\rho_{\text{Provided}} \le \rho_{\text{for } \epsilon_t = 0.005}$
    Then the assumption of $\emptyset = 0.9$ is correct.
    Else, compute the more accurate value of $\emptyset$ and retain to step 2.

This indirect approach may be used by engineers who familiar with older code versions and they do not prefer to use strains in their design calculations.
12. Draw the final reinforcement details.

4.5.3 Skin Reinforcement, (ACI318M, 2014), Article *9.7.2.3*

4.5.3.1 Aim of Skin Reinforcement

- Where h of a beam or joist exceeds 900 mm, longitudinal skin reinforcement shall be uniformly distributed along both side faces of the member.
- These rebars are necessary for crack control. Without such reinforcement, cracks widths in the web wider than those at the level of the main bars have been observed (see ***Figure 4.5-1 below***).

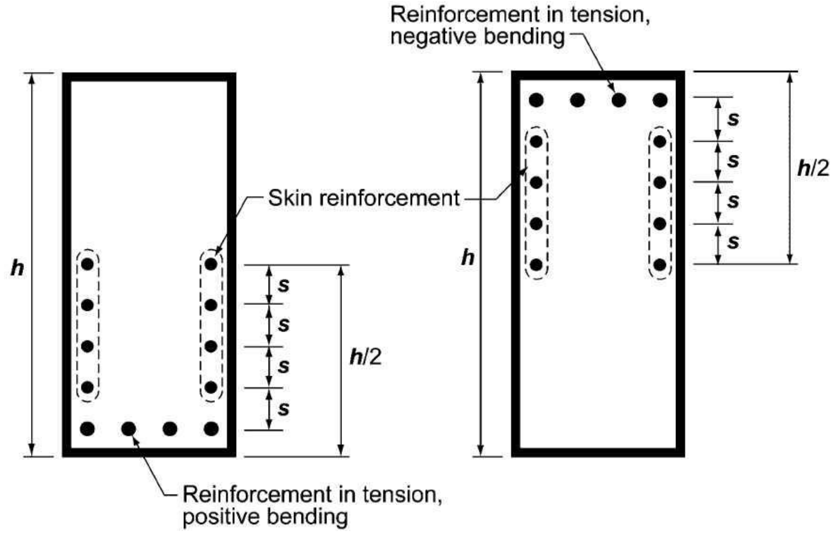


Figure 4.5-1: Skin reinforcement according to Article 9.7.2.3 of ACI code.

4.5.3.2 Extension of Skin Reinforcement

Skin reinforcement shall extend for a distance h/2 from the tension face (see Figure 4.5-1 above).

4.5.3.3 Spacing between Skin Reinforcement

The spacing, s, shall be as provided in Article **24.3.2** of ACI code (i.e. requirements for $s_{Maximum}$), where c_c is the least distance from the surface of the skin reinforcement to the side face.

4.5.3.4 Diameter for Skin Reinforcement

- The size of the skin reinforcement is not specified according to ACI Code. Research has indicated that the spacing rather than bar size is of primary importance.
- According to ACI Commentary (**R9.7.2.3**) bar sizes No. 10 to No. 16 (or welded wire reinforcement with a minimum area of 210 mm^2 per meter of depth) are typically provided.

4.5.3.5 Strength Usefulness of Skin Reinforcement

It shall be permitted to include such reinforcement in strength computations if a strain compatibility analysis is made to determine stress in the individual bars.

4.5.4 Examples

Example 4.5-1

Design a simply supported beam with a span of 7m. Service design loads can be taken as:

$$W_{Dead} = 20\frac{kN}{m}(\text{Including beam selfweight}) \quad W_{Live} = 29.0\frac{kN}{m}$$

Assume that the designer intends to use:

1. Concrete of $f_c' = 35\text{ MPa}$.
2. Steel of $f_y = 400\text{MPa}$.
3. A reinforcement ratio of $0.5\rho_{max}$.
4. Rebar of No. 25 for longitudinal reinforcement.
5. Rebar of No. 10 for stirrups.

Solution

1. Computed the factored moment M_u:

$$M_{Dead} = \frac{20\frac{kN}{m} \times 7.0^2 m^2}{8} = 123\text{ kN.m} \quad M_{Live} = \frac{29\frac{kN}{m} \times 7.0^2 m^2}{8} = 178\text{ kN.m}$$

$$M_u = \text{Maximum of }(1.4M_D \text{ or } 1.2M_D + 1.6M_L)$$

$$M_u = \text{Maximum of }[1.4 \times 123 \quad \text{or } (1.2 \times 123 + 1.6 \times 178)] = \text{Maximum of }[172 \text{ or } 432] = 432\text{ kN.m} \blacksquare$$

2. Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:

$$M_n = \frac{M_u}{\emptyset}$$

Strength reduction factored can be assumed 0.9 and checked later.

$$M_n = \frac{432\ \text{kN.m}}{0.9} = 480\ \text{kN.m}$$

3. Select a Reinforcement Ratio:
   Assume that:
   $$\rho = 0.5\rho_{max}$$
   $$\rho_{max} = 0.85\beta_1\ \frac{f'_c}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004}$$
   $$\beta_1 = 0.85 - \frac{35-28}{7} \times 0.05 = 0.80$$
   $$\rho_{max} = 0.85 \times 0.80\ \frac{35}{400}\ \frac{0.003}{0.003+0.004} = 25.5\ \times 10^{-3}$$
   $$\rho = 0.5 \times 25.5\ \times 10^{-3} = 12.8 \times 10^{-3}$$
4. Solve the following relation to compute the required (bd^2):
   $$M_n = \rho f_y (bd^2)_{Required}\left(1 - 0.59\ \frac{\rho f_y}{f'_c}\right)$$
   $$480 \times 10^6 \text{N.mm} = 12.8 \times 10^{-3} \times 400 \times (\text{bd}^2)_{\text{Required}}\left(1 - 0.59\ \frac{12.8 \times 10^{-3} \times 400}{35}\right)$$
   $$(\text{bd}^2)_{\text{Required}} = 103 \times 10^6\ \text{mm}^3$$
   Use
   $$\frac{d}{b} = 2.0$$
   and solve for "b":
   $$(\text{b} \times (2\text{b})^2)_{\text{Required}} = 103 \times 10^6\ \text{mm}^3$$
   $$\text{b} = 294\ \text{mm}$$
   Try
   $$\text{b} = 300\ \text{mm}$$
   Then "d" will be:
   $$\text{d} = \sqrt{\frac{103 \times 10^6\ \text{mm}^3}{300\ \text{mm}}} = 586\ \text{mm}$$
5. Compute the required steel area:
   $$A_{s\ \text{Required}} = 12.8 \times 10^{-3} \times (300\ \text{mm} \times 586\ \text{mm}) = 2\ 250\ \text{mm}^2$$
6. Compute the required rebars number:
   $$\text{No. of Rebars} = \frac{A_s}{A_{Bar}}, A_{Bar} = \frac{\pi \times 25^2}{4} = 490\text{mm}^2\ \text{No. of Rebars} = \frac{A_s}{A_{Bar}} = \frac{2\ 250\ \text{mm}^2}{490\ \text{mm}^2} = 4.59$$
   Try $5\emptyset25$.
7. Check if rebars can be put in one or two layers:
   $$b_{required} = 2 \times \text{Side Cover} + 2 \times \text{Stirrups Diamter} + \text{No. of Rebars} \times \text{Bar Diameter} + (\text{No. of Rebars} - 1) \times \text{Spacing between Rebars}$$
   $$b_{required} = 2 \times 40 + 2 \times 10 + 5 \times 25 + 4 \times 25 = 325 > 300$$
   Then reinforcement cannot be put in a single layer and should be put in two layers (2Φ25 + 3Φ25 see Figure below).
8. Check Spacing "s" with s_{max} limitations of the ACI Code:
   For beams with more than single layer of reinforcement (as in this example), ACI requires that maximum spacing limitations should be checked for the layer most closet to the tension face. Then, for this example, s_{max} will be checked for the layer that has 3Φ25. By inspection, this requirement is satisfied for the beam.
9. Compute the required beam depth "h". depend on reinforcement layers:
   $$h_{\text{for Two Layer}} = \text{d} + \text{Cover} + \text{Stirrups} + \text{Bar Diameter} + \frac{\text{Spacing between Layers}}{2}$$
   $$h_{\text{for Two Layer}} = 586\ \text{mm} + 40 + 10 + 25 + \frac{25}{2} = 673.5\ \text{mm}$$
   Use $300\text{mm} \times 675\text{mm}$ with $5\emptyset25$.
10. Check the Assumption of $\emptyset = 0.9$:

a. Compute the provided effective depth:

$$d_{Provided} = 675 - 40 - 10 - 25 - \frac{25}{2} = 587 \text{ mm}$$

b. Compute the provided steel ratio:

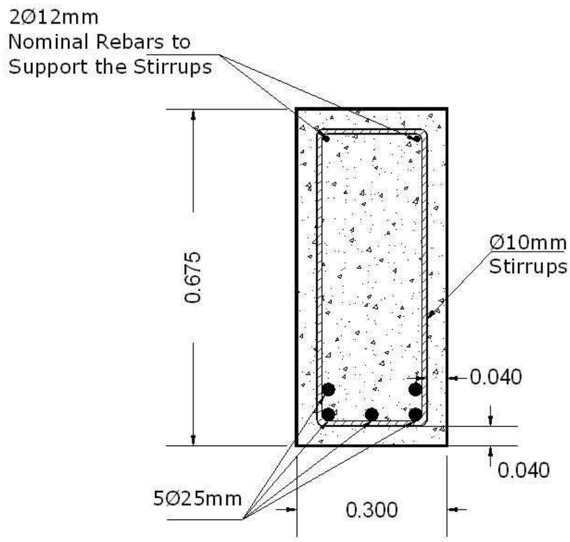
$$\rho_{Provided} = \frac{5 \times 490}{300 \times 587} = 13.9 \times 10^{-3}$$

c. Compute the steel ratio required for steel strain of 0.005:

$$\rho_{for\ \epsilon_t = 0.005} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.005} = 0.85 \times 0.80 \frac{35}{400} \frac{0.003}{0.008} = 22.3 \times 10^{-3}$$

$$> \rho_{Provided} \implies \emptyset = 0.9$$

11. Draw the final reinforcement details:



Notes:

Smaller section (i.e. a section with $\rho > 0.5\rho_{max}$) can be used if:

- Economic studies show that this section is better. As these studies depended on cost rates of concrete, steel, forms, and labor and as these rates differ from state to state, then a steel ratio that is most economical in a place may not be the best in another place. Usually these studies are out the scope of traditional courses in Reinforced Concrete Design. For more details about these issues, see for example ***Engineering Economy*** by ***Thuesen***.
- Deflection calculations show that this section is adequate based on serviceability requirements.

Example 4.5-2

Design beam shown in Figure 4.5-2 for flexure requirements according to ACI 318M-14.

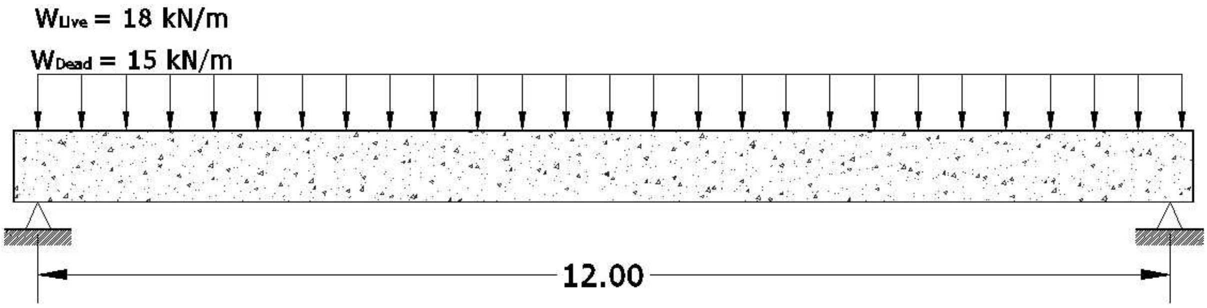


Figure 4.5-2: A Simply supported beam of Example 4.5-2.

In your solution, assume that:

1. $\rho = 0.5\rho_{max}$ (for deflection requirements).
2. Beam selfweight is 10.0 kN/m.
3. h = 1000 mm.
4. Concrete of $f_c' = 28$ MPa.
5. Steel of $f_y = 420$MPa.
6. Rebar of No. 25 for longitudinal reinforcement.
7. Rebar of No. 10 for stirrups.
8. Two layers of reinforcement.

Solution

1. Computed the factored moment M_u:
   Beam selfweight is assumed:
   $$W_{Selfweight} = 10 \frac{kN}{m}$$
   Then, total dead load is:

$$W_{Dead} = 25\frac{kN}{m} \Rightarrow M_{Dead} = \frac{W_D l^2}{8} = \frac{25 \times 12^2}{8} = 450 \text{ kN.m}$$

$$M_{Live} = \frac{W_L l^2}{8} = \frac{18 \times 12^2}{8} = 324 \text{ kN.m}$$

$$M_u = \text{Maximum of } (1.4M_D \text{ or } 1.2M_D + 1.6M_L)$$

$$M_u = \text{Maximum of } [1.4 \times 450 \quad \text{or } (1.2 \times 450 + 1.6 \times 324)] = \text{Maximum of } [630 \text{ or } 1058] = 1058 \text{ kN.m} \blacksquare$$

2. Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:

$$M_n = \frac{M_u}{\emptyset}$$

Strength reduction factored can be assumed 0.9, and checked later.

$$M_n = \frac{1058}{0.9} = 1176\ kN.m$$

3. Select a Reinforcement Ratio:
For deflection control, the designer will start with reinforcement ratio of:

$$\rho = 0.5\rho_{max}$$

$$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$

$$\beta_1 = 0.85$$

$$\rho_{max} = 0.85^2 \frac{28}{420} \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3}$$

$$\rho = 0.5 \times 20.6 \times 10^{-3} = 10.3 \times 10^{-3}$$

4. Solve the following relation to compute the required (bd^2):

$$M_n = \rho f_y (bd^2)_{Required}\left(1 - 0.59\ \frac{\rho f_y}{f'_c}\right)$$

$$1176 \times 10^6 = 10.3 \times 10^{-3} \times 420 (bd^2)_{Required}\left(1 - 0.59\ \frac{10.3 \times 10^{-3} \times 420}{28}\right)$$

$$(bd^2)_{Required} = 299 \times 10^6 \text{ mm}^3$$

$$d = 1000 - 40 - 10 - 25 - \frac{25}{2} = 913\ mm$$

Solve for b:

$$b = 359\ mm$$

Say

$$b = 375\ mm$$

5. Compute the required steel area:

$$A_{s\ Required} = \rho bd = 10.3 \times 10^{-3} \times 913 \times 375 =$$

$$A_{s\ Required} = 3526\ mm^2$$

6. Compute the required rebars number:

$$No. of\ Rebars = \frac{A_s}{A_{Bar}}$$

$$No. of\ Rebars = \frac{3526}{490} = 7.19$$

Try 8Ø25.

$$A_{S\ Provided} = 490\ mm^2 \times 8 = 3920\ mm^2$$

7. Check if rebars can be put in one or two layers:

$$b_{required} = 40 \times 2 + 10 \times 2 + 4 \times 25 + 3 \times 25$$

$$b_{required} = 275\ mm < 375\ mm\ Ok.$$

8. Check for s_{max}:
By inspection, one can conclude that s_{max} requirement is satisfied.
9. Check the Assumption of $\emptyset = 0.9$:
   a. Compute the provided effective depth:
   $d = 913$ mm
   b. Compute the provided steel ratio:

   $$\rho_{Provided} = \frac{3920}{375 \times 913} = 11.5 \times 10^{-3}$$

   c. Compute the steel ratio required for steel strain of 0.005:

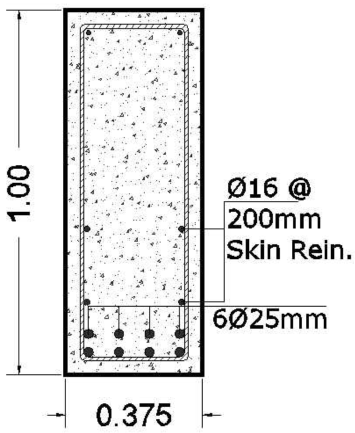
$$\rho_{\text{for } \epsilon_t = 0.005} = 0.85^2 \; \frac{28}{420} \; \frac{0.003}{0.003 + 0.005}$$

$$\rho_{\text{for } \epsilon_t = 0.005} = 18.1 \times 10^{-3}$$

$\because \rho_{\text{Provided}} < \rho_{\text{for } \epsilon_t = 0.005}$

$\therefore \; \emptyset = 0.9$ Ok.

As reinforcement ratio is in the range of $0.5\rho_{maximum}$, then the resulting strain at failure load will be greater than 0.005. From this one can conclude that this checking only has academic value.



10. Check the assumed selfweight:

$$W_{\text{Selfweight}} = 0.375 \times 1.0 \times 24 = 9\frac{\text{kN}}{\text{m}} < 10\frac{\text{kN}}{\text{m}} \; Ok$$

11. Draw the final reinforcement details. With skin reinforcement, beam section would be as indicated in below.

Example 4.5-3

Design beam shown in Figure 4.5-3 below for flexure requirements according ACI 318M-14.

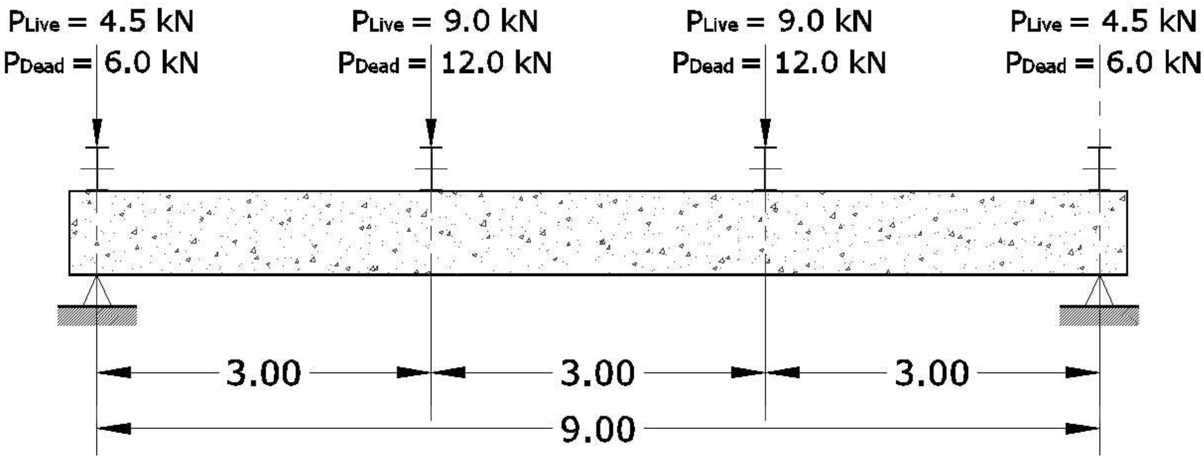


Figure 4.5-3: Simply supported beam for Example 4.5-3.

In your solution, assume that:

1. $\rho = 0.5\rho_{max}$ (for economical and serviceability requirements).
2. Beam selfweight is 3.0 kN/m.
3. b = 250 mm.
4. Concrete of $f_c' = 28$ MPa.
5. Steel of $f_y = 420$MPa.
6. Rebar of No. 25 for longitudinal reinforcement.
7. Rebar of No. 10 for stirrups.

Solution

1. Computed the factored moment M_u:

Beam selfweight is assumed:

$$W_{\text{Selfweight}} = 3\frac{\text{kN}}{\text{m}}$$

Then, total dead load is:

$$M_{\text{Dead}} = \frac{3 \times 9^2}{8} + 12.0 \times 3.0 = 66.4 \; kN.m$$

$$M_{\text{Live}} = 9 \times 3.0 = 27 \text{ kN.m}$$

$$M_u = \text{Maximum of } (1.4M_D \text{ or } 1.2M_D + 1.6M_L)$$

$$M_u = \text{Maximum of } [1.4 \times 66.4 \text{ or } (1.2 \times 66.4 + 1.6 \times 27)]$$

$$M_u = \text{Maximum of } [93.0 \text{ or } 123] = 123 \text{ kN.m} \; \blacksquare$$

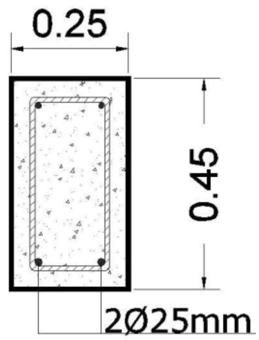
2. Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:
   $M_n = \frac{M_u}{\emptyset}$
   Strength reduction factored can be assumed 0.9, and checked later.
   $M_n = \frac{123}{0.9} = 137\ kN.m$
3. Select a Reinforcement Ratio:
   For deflection control, the designer starts with reinforcement ratio of:
   $\rho = 0.5\rho_{max}$
   $\rho_{max} = 0.85\beta_1\ \frac{f_c'}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004}$
   $\beta_1 = 0.85$
   $\rho_{max} = 0.85^2\ \frac{28}{420}\ \frac{0.003}{0.003 + 0.004} = 20.6\ \times 10^{-3}$
   $\rho = 0.5 \times 20.6 \times 10^{-3} = 10.3 \times 10^{-3}$
4. Solve the following relation to compute the required (bd^2):
   $M_n = \rho f_y (bd^2)_{Required}\left(1 - 0.59\ \frac{\rho f_y}{f_c'}\right)$
   $137 \times 10^6 = 10.3 \times 10^{-3} \times 420(bd^2)_{Required}$
   $\left(1 - 0.59\ \frac{10.3 \times 10^{-3} \times 420}{28}\right)$
   $(bd^2)_{Required} = 34.8 \times 10^6\ mm^3$
   $\because b = 250mm\ \therefore d\ 373\ mm$
5. Compute the required steel area:
   $A_{s\ Required} = \rho bd = 10.3 \times 10^{-3} \times 373 \times 250 =$
   $A_{s\ Required} = 960\ mm^2$
6. Compute the required rebars number:
   $No.of\ Rebars\ = \frac{A_s}{A_{Bar}}$
   $No.of\ Rebars\ = \frac{960}{490} = 1.96$
   Try 2∅25.
   $A_{S\ Provided} =\ 490\ mm^2 \times 2 = 980\ mm^2$
7. Check if rebars can be put in one or two layers:
   $b_{required} =\ 40 \times 2 + 10 \times 2 + 2 \times 25 + 25$
   $b_{required} =\ 175\ mm < 250\ mm\ Ok.$
8. Check for s_{max}:
   By inspection, one can conclude that S_{max} requirement is satisfied.
9. Compute Required "h":
   $h = 373 + \frac{25}{2} + 10 + 40 = 436\ mm$
   Say h = 450mm
10. Check the assumption of $\emptyset = 0.9$:
    a. Compute the provided effective depth:
       $d =\ 450 - 40 - 10 - \frac{25}{2} = 388\ mm$
    b. Compute the provided steel ratio:
       $\rho_{Provided} = \frac{980}{388 \times 250} = 10.1 \times 10^{-3}$
    c. Compute the steel ratio required for steel strain of 0.005:
       $\rho_{for\ \epsilon_t = 0.005} = 0.85^2\ \frac{28}{420}\ \frac{0.003}{0.003 + 0.005}$
       $\rho_{for\ \epsilon_t = 0.005} = 18.1 \times 10^{-3}$
       $\because \rho_{Provided} < \rho_{for\ \epsilon_t = 0.005}$
       $\therefore\ \emptyset = 0.9\ Ok.$

As reinforcement ratio is in the range of $0.5\rho_{maximum}$, then the resulting strain at failure load will be greater than 0.005. From this one can conclude that this checking only has academic value.

11. Check the assumed selfweight:

$$W_{Selfweight} = 0.45 \times 0.25 \times 24 = 2.7\frac{kN}{m} < 3\frac{kN}{m} \quad Ok$$

12. Draw the final reinforcement details:



4.5.5 Homework Problems

Problem 4.5-1

Design a simply supported rectangular reinforced concrete beam to carry a service dead load of $40\,kN/m$ and a service live load of $17.5\,kN/m$. The span is 12m. It is known that this beam is not exposed to weather and not in contact with ground. Select the beam preliminary steel ratio based on deflection requirements.

Assume that the designer intend to use:

1. Concrete of $f_c' = 21\,MPa$.
2. Steel of A615 Grade 60.
3. A width of 500mm.
4. Rebar of No. 25 for longitudinal reinforcement.
5. Rebar of No. 13 for stirrups.

Answers

- Computed the factored moment M_u:
  Beam selfweight is assumed:
  $$W_{Selfweight} = 8.0\frac{kN}{m}$$
  Then, total dead load is:
  $$W_{Dead} = 48\frac{kN}{m}$$
  $$M_{Dead} = 864\,kN.m$$
  $$M_{Live} = 315\,kN.m$$
  $$M_u = \text{Maximum of } (1.4M_D \text{ or } 1.2M_D + 1.6M_L)$$
  $$M_u = \text{Maximum of } [1.4 \times 864 \text{ or } (1.2 \times 864 + 1.6 \times 315)] =$$
  $$M_u = \text{Maximum of } [1\,210 \text{ or } 1\,541] = 1\,541\,kN.m \blacksquare$$
- Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:
  $$M_n = \frac{M_u}{\emptyset}$$
  Strength reduction factored can be assumed 0.9, and checked later.
  $$M_n = 1\,712\,kN.m$$
- Select a Reinforcement Ratio:
  For deflection control, the designer starts with reinforcement ratio of:
  $$\rho = \frac{0.18\,f_c'}{f_y} = \frac{0.18 \times 21\,MPa}{420\,MPa} = 9.0 \times 10^{-3}$$
  $$\rho_{max} = 15.5 \times 10^{-3} > \rho \quad Ok.$$
- Solve the following relation to compute the required (bd^2):
  $$(bd^2)_{Required} = 507 \times 10^6\,mm^3$$
  Use b = 500mm, then "d" will be:
  $$d = 1\,007\,mm$$
- Compute the required steel area:
  $$A_{s\,Required} = 4\,532\,mm^2$$
- Compute the required rebars number:
  $$\text{No. of Rebars} = \frac{A_s}{A_{Bar}}$$

$$\text{No. of Rebars } = \frac{A_s}{A_{Bar}} = 9.25$$

Try $10\emptyset 25$.

- Check if rebars can be put in one or two layers:
  $b_{required} = 581 > 500$
  Then reinforcement cannot be put in a single layer.
- Check for s_{max}:
  By inspection, one can conclude that s_{max} requirement is satisfied (see Figure below).
- Compute the required beam depth "h". depend on reinforcement layers:
  $h_{for\ Two\ Layer} = 1\,097.5\ \text{mm}$
  Try $500\text{mm} \times 1\,100\text{mm}$ with $10\emptyset 25$.
- Check the Assumption of $\emptyset = 0.9$:
  a. Compute the provided effective depth:
     $d_{Provided} = 1\,010\ \text{mm}$
  b. Compute the provided steel ratio:
     $\rho_{Provided} = 9.7 \times 10^{-3}$
  c. Compute the steel ratio required for steel strain of 0.005:
     $\rho_{for\ \epsilon_t = 0.005} = 13.5 \times 10^{-3}$
     $\because \rho_{Provided} < \rho_{for\ \epsilon_t = 0.005}$
     $\therefore \emptyset = 0.9$ Ok.
- Check the assumed selfweight:

$$W_{Selfweight} = 13.2\frac{kN}{m} > 8.0\frac{kN}{m}$$

$$W_{Dead} = 53.2\frac{kN}{m}$$

$$M_{Dead} = \frac{53.2\frac{kN}{m} \times 12^2 m^2}{8} = 958\ \text{kN.m}$$

$$M_u = \text{Maximum of } (1.4M_D \text{ or } 1.2M_D + 1.6M_L)$$

$$M_u = \text{Maximum of } [1.4 \times 958 \quad \text{or } (1.2 \times 958 + 1.6 \times 315)] =$$

$$M_u = \text{Maximum of } [1\,341 \text{ or } 1\,654] = 1\,654\ \text{kN.m} \blacksquare$$

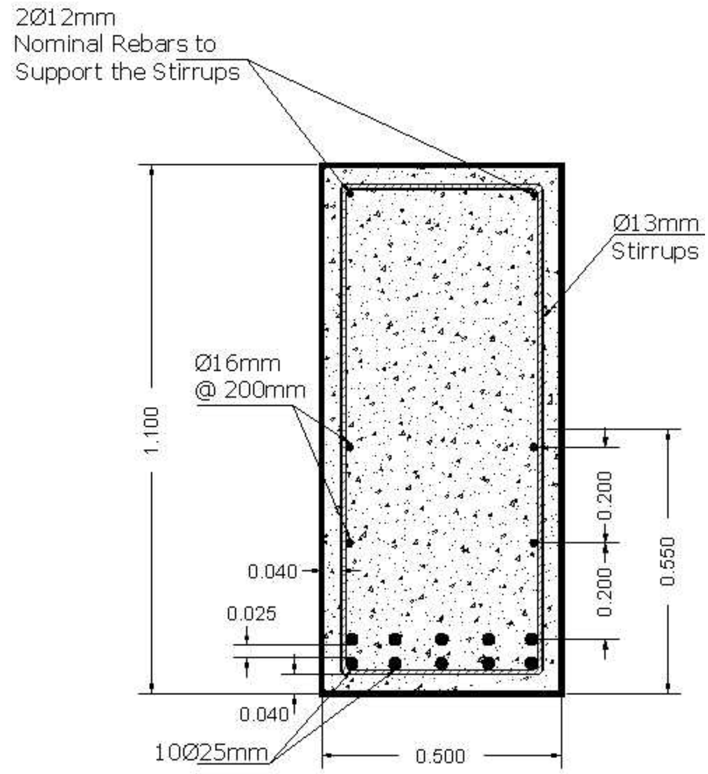
$$M_n = 1\,840\ \text{kN}$$

$$\emptyset M_n = 0.9 \times 1\,840\ \text{kN} = 1\,656\ \text{kN.m} > 1\,654\ kN.m\ Ok.$$

- Draw the final reinforcement details:
  With skin reinforcement, beam section would as indicated in below:

ADDITIONAL NOTES:

As was discussed previously, smaller section can be used if deflection calculations show that this section is adequate.

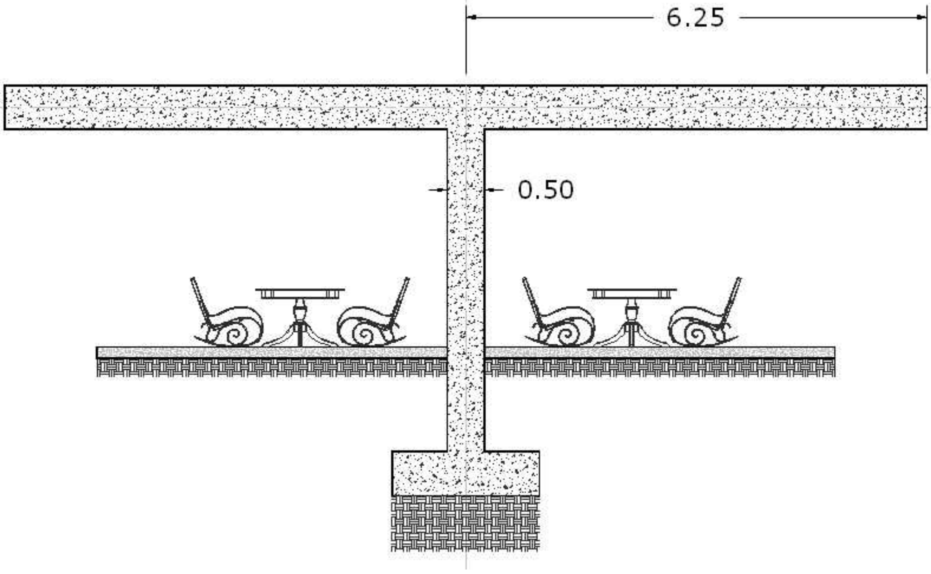


Problem 4.5-2

Design the cantilever beam of canopy structure shown in below to carry a service dead load of $50\,\mathrm{kN/m}$ and a service live load of $7.5\,\mathrm{kN/m}$. Select beam preliminary steel ratio based on deflection requirements.

Assume that the designer intends to use:

1. Concrete of $f_c' = 21\ \mathrm{MPa}$.
2. Steel of A615 Grade 60.
3. A width of 500mm.
4. Rebar of No. 25 for longitudinal reinforcement.
5. Rebar of No. 13 for stirrups.



Answers

- Computed the factored moment M_u:

  Beam selfweight is assumed:

  $W_{Selfweight} = 6.00\,\frac{\mathrm{kN}}{\mathrm{m}}$

  Then, total dead load is:

  $W_{Dead} = 56.0\,\frac{\mathrm{kN}}{\mathrm{m}}$

  $M_{Dead} = 1\,008\ \mathrm{kN.m}$

  $M_{Live} = 135\ \mathrm{kN.m}$

  $M_u = \text{Maximum of}\ (1.4M_D\ \text{ or } 1.2M_D + 1.6M_L)$

  $M_u = \text{Maximum of}\ [1.4 \times 1\,008\ \text{ or } (1.2 \times 1\,008 + 1.6 \times 135)] =$

  $M_u = \text{Maximum of}\ [1\,411\ \text{ or } 1\,426] = 1\,426\ \mathrm{kN.m}$ ■

- Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:

  $M_n = \frac{M_u}{\emptyset}$

  Strength reduction factored can be assumed 0.9, and checked later.

  $M_n = 1\,584\ \mathrm{kN.m}$

- Select a Reinforcement Ratio:

  For deflection control, the designer starts with reinforcement ratio of:

  $\rho = \frac{0.18\,f_c'}{f_y} = \frac{0.18 \times 21\ \mathrm{MPa}}{420\ \mathrm{MPa}} = 9.0 \times 10^{-3}$

  $\rho_{max} = 15.5 \times 10^{-3} > \rho\ \ Ok.$

- Solve the following relation to compute the required (bd^2):

  $(bd^2)_{Required} = 469 \times 10^6\ \mathrm{mm^3}$

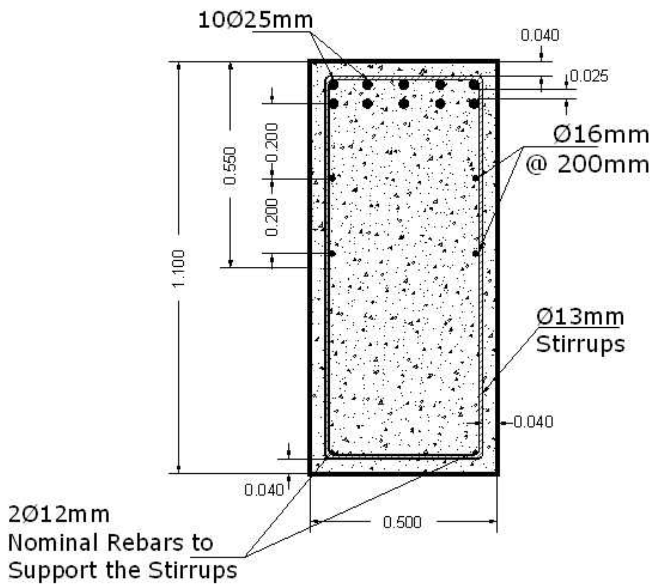
  Use b = 500mm, then "d" will be:

  $d = 969\ \mathrm{mm}$

- Compute the required steel area:

  $A_{s\,Required} = 4\,360\ \mathrm{mm^2}$

- Compute the required rebars number:
  $\text{No. of Rebars} = 9$
  Try $9\emptyset25$.
- Check if rebars can be put in one or two layers:
  $b_{required} = 531 > 500$
  Then reinforcement cannot be put in a single layer.
- Check for s_{max}:
  By inspection, one can conclude that s_{max} requirement is satisfied (see Figure below).
- Compute the required beam depth "h". depend on reinforcement layers:
  $h_{for\ Two\ Layer} = 1\,059.5\ \text{mm}$
  Try $500\text{mm} \times 1\,100\text{mm}$ with $9\emptyset25$.
- Check the Assumption of $\emptyset = 0.9$:
  a. Compute the provided effective depth:
     $d_{Provided} = 1\,010\ \text{mm}$
  b. Compute the provided steel ratio:
     $\rho_{Provided} = 8.73 \times 10^{-3}$
  c. Compute the steel ratio required for steel strain of 0.005:
     $\rho_{for\ \epsilon_t = 0.005} = 13.5 \times 10^{-3}$
     $\because \rho_{Provided} < \rho_{for\ \epsilon_t = 0.005}$
     $\therefore \emptyset = 0.9\ \text{Ok.}$
- Check the assumed selfweight:
  $W_{Selfweight} = 13.2\,\frac{\text{kN}}{\text{m}} > 6.0\,\frac{\text{kN}}{\text{m}}$
  $W_{Dead} = 63.2\,\frac{\text{kN}}{\text{m}}$
  $M_{Dead} = 1\,138\ \text{kN.m}$
  $M_u = \text{Maximum of } (1.4M_D \text{ or } 1.2M_D + 1.6M_L)$
  $M_u = \text{Maximum of } [1.4 \times 1\,138 \quad \text{or } (1.2 \times 1\,138 + 1.6 \times 135)] =$
  $M_u = \text{Maximum of } [1\,593 \text{ or } 1\,582] = 1\,593\ \text{kN.m}$ ■
  $M_n = 1\,677\ \text{kN}$
  $\emptyset M_n = 0.9 \times 1\,677\ \text{kN} = 1\,509\ \text{kN.m}$
  $< 1\,593\ kN.m\ \ Not\ Ok.$
  Try $500\text{mm} \times 1\,100\text{mm}$ with $10\emptyset25$:
  $\rho_{Provided} = 9.7 \times 10^{-3}$
  $M_n = 1\,840\ \text{kN}$
  $\emptyset M_n = 0.9 \times 1\,840\ \text{kN} = 1\,656\ \text{kN.m}$
  $> 1\,593\ kN.m\ \ Ok.$
  Use $500\text{mm} \times 1\,100\text{mm}$ with $10\emptyset25$ ■
- Draw the final reinforcement details: With skin reinforcement, beam section is presented in below.



4.6 Analysis of a Rectangular Beam with Tension and Compression Reinforcements (a Doubly Reinforced Beam)

4.6.1 Basic Concepts

- Occasionally, beams are built with both tension reinforcement and compression reinforcement. These beams are called as beams with tension and compression reinforcement or doubly reinforced beams.
- Area and ratio of compression reinforcement have notations of A_s' and ρ' respectively (See Figure 4.6-1 below):

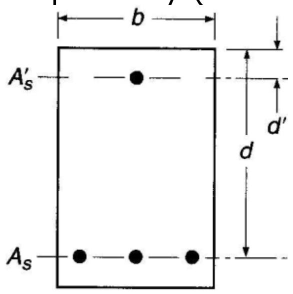


Figure 4.6-1: A doubly reinforced section.

- To be consistence with notations adopted in single reinforced beams, reinforcement ratio for tension reinforcement, ρ, is defined as:

$$\boldsymbol{\rho = \frac{A_s}{bd}}$$

- To simplify algebraic **operation through adopting same denominator, reinforcement notation for compression reinforcement, ρ', is defined as:**

$$\rho' = \frac{A_s'}{bd} \quad \textbf{Eq. 4.6-1}$$

- There are four reasons for using compression reinforcement in beams:
  - **Reduce Sustained-Load Deflection**
    First and most important, the addition of compression reinforcement reduces the long-term deflections of a beam subjected to sustained loads, see Figure 4.6-2 below.

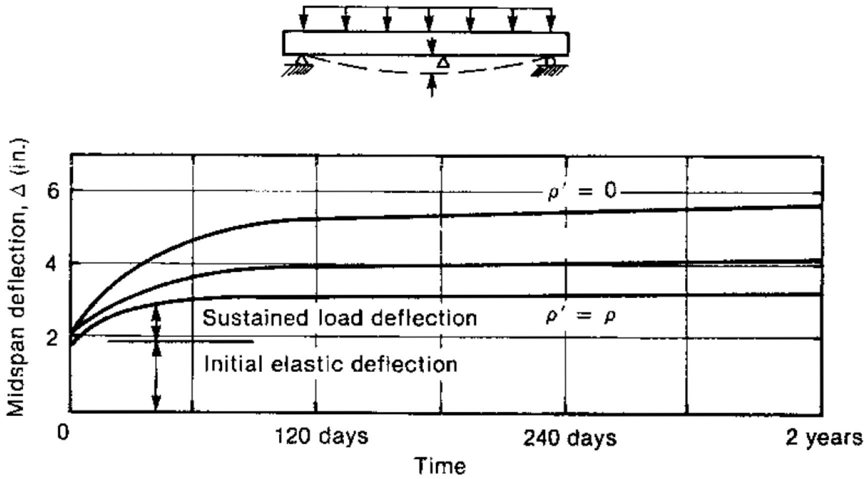


Figure 4.6-2: Compression reinforcement effects on deflection due to sustained loads.

  - **Fabrication Ease**
    When assembling the reinforcing cage for a beam, it is customary to provide bars in the corners of stirrups to hold stirrups in place in the form see Figure 4.6-3 below.

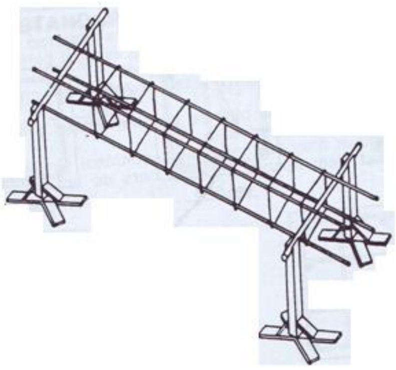


Figure 4.6-3: A reinforcement cage for fabrication ease.

- **Increase Ductility**
  It can be shown that the addition of compression reinforcement causes a reduction in the depth of the compression stress block "a". As "a" decreases the strain in the tension reinforcement at failure increases, resulting in more ductile behavior, see Figure 4.6-4 below.

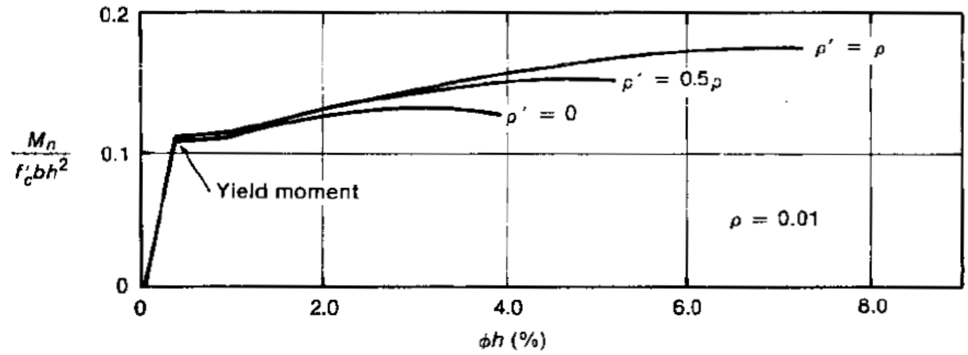


Figure 4.6-4: Ductility increase versus increasing in compression reinforcement.

- **Change the Mode of Failure from a Compression Failure to Secondary Compression Failure:**
  - When $\rho > \rho_b$, a beam fails in brittle manner through crushing of the compressive zone before the steel yields.
  - Adding of compression steel to such beam reduces the depth of the compression stress block "a".
  - As "a" decreases the strain in the tension reinforcement at failure increases, resulting in a more ductile behavior, see Figure 4.6-5 below.
  - The use of compression reinforcement for this reason has decreased markedly with use of strength design method.

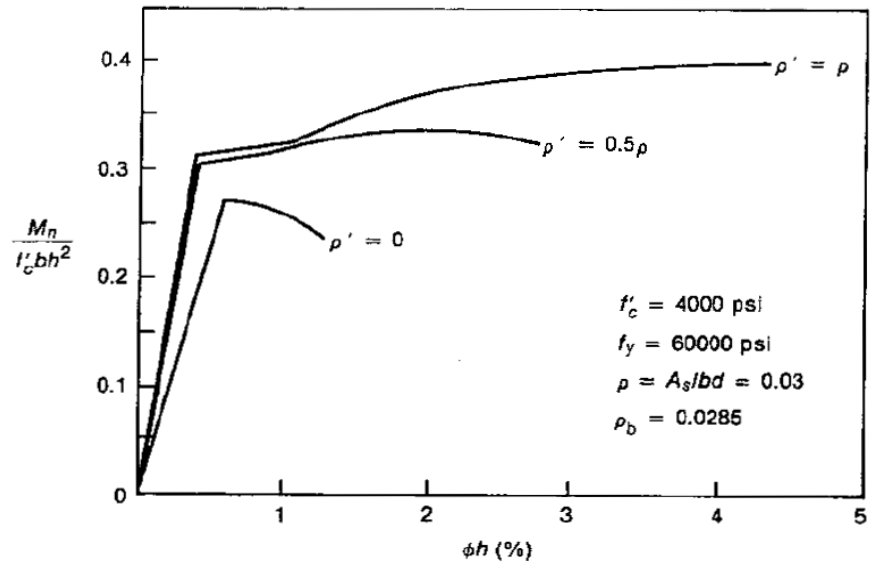


Figure 4.6-5: Changing in failure mode versus compression reinforcement.

- Analysis of a beam with tension and compression reinforcement starts with a checking to diagnose the cause for using the compression reinforcement based on the following argument. if
  $$\rho > \rho_{\max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$
  then the compression reinforcement has been used to ***Change the Mode of Failure from Compression Failure to Secondary Compression Failure***. Then this reinforcement must be included in the beam analysis. Else, if
  $$\rho < \rho_{\max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$
  then the compression reinforcement has been ***used either to reduce sustained-load deflection or to fabrication ease or to increase ductility*** and ***its effects can be neglected in the beam analysis***.

- Generally, compression reinforcement increases the value of maximum of steel ratio ρ_{max} and increases the value of nominal strength M_n. These effects will be discussed in paragraphs below.

4.6.2 Maximum Steel Ratio ($\bar{\rho}_{max}$) of a Rectangular Beam with Tension and Compression Reinforcement:

Based on basic tenets of ***Compatibility***, ***Stress-Strain Relation***, and ***Equilibrium***, one can prove that using of compression reinforcement increases the maximum permissible steel ratio from the value of:

$$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$

to a ratio of:

$$\bar{\rho}_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \rho' \frac{f'_s}{f_y}$$

or

$$\bar{\rho}_{max} = \rho_{max} + \rho' \frac{f'_s}{f_y} \blacksquare$$

where f'_s is stress in the compression reinforcement at strains of ρ_{max}. It can be computed from strain distribution and as shown in relation below:

$$f'_s = E_s \left[\epsilon_u - \frac{d'}{d}(\epsilon_u + 0.004)\right] \le f_y \blacksquare$$

4.6.3 Nominal Flexure Strength of a Rectangular Beam with Tension and Compression Reinforcement:

- The M_n relation of a doubly reinforced beam depends on the yielding of compression reinforcement.
- Then there are two relations for computing of M_n,
  - One for the doubly reinforced beam with compression steel at yield stress,
  - The other for the doubly reinforced beam with compression steel below yield stress.

4.6.3.1 M_n for a Beam with Compression Steel at Yield Stress

- Strains, stresses, and forces diagrams for a beam with tension and compression reinforcement at yield stress can be summarized in Figure 4.6-6 below.

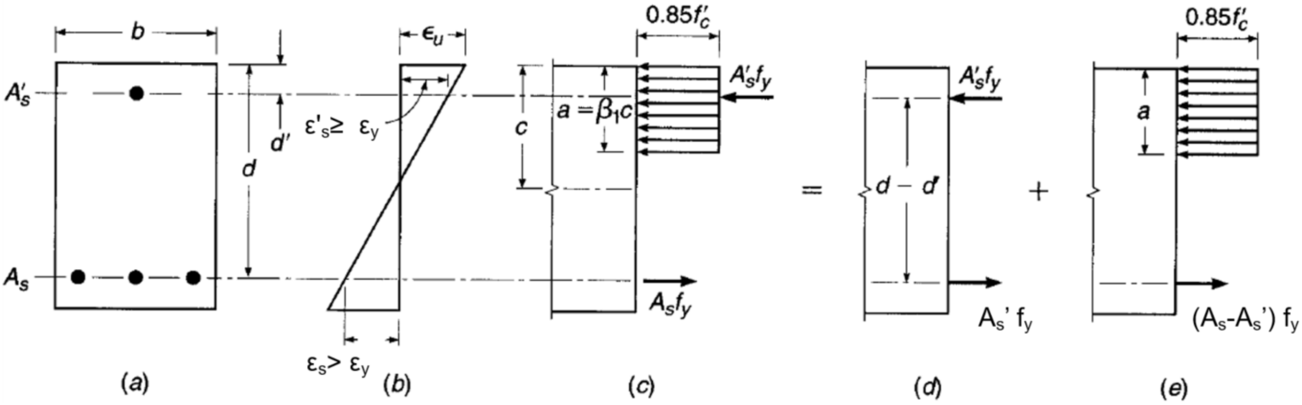


Figure 4.6-6: Strains and stresses for a doubly reinforced rectangular beam with yielded compression reinforcement.

- Then, based on superposition one can conclude that M_n for the section can be computed based on following relation:

$$\sum M_{about\ A'_s} = 0$$

$$M_n = M_{n1} + M_{n2} = A'_s f_y (d - d') + (A_s - A'_s) f_y \left(d - \frac{a}{2}\right) \blacksquare$$

where

$$a = \frac{(A_s - A_s') f_y}{0.85 f'_c b} \blacksquare$$

4.6.3.2 M_n for a Beam with Compression Steel below Yield Stress

- Strains, stresses, and forces diagrams for a beam with compression reinforcement below yield stress can be summarized in Figure 4.6-7 below.

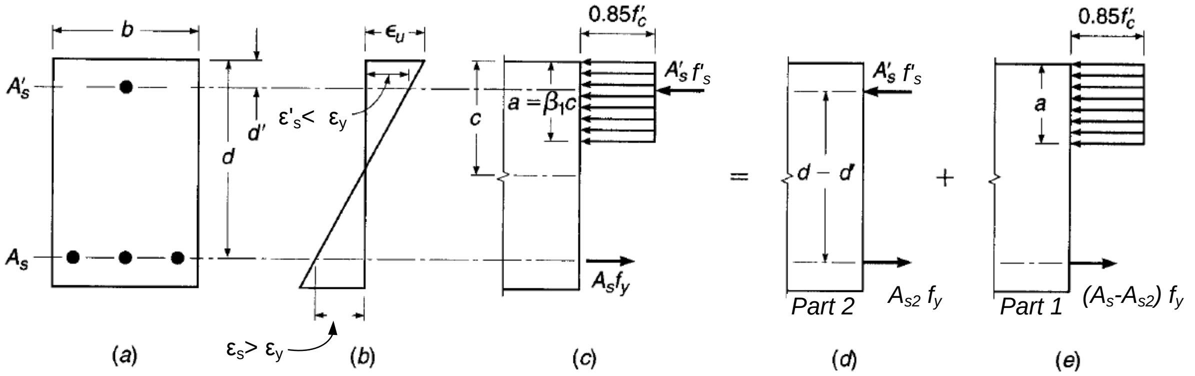


Figure 4.6-7: Strains and stresses for a doubly reinforced rectangular beam with compression reinforcement below the yield.

- Using the superposition, M_n for section can be computed based on following relation:

$$\sum M_{about\ A_s} = 0$$

$$M_n = M_{n1} + M_{n2} = 0.85f'_c ab(d - \frac{a}{2}) + A'_s f'_s (d - d') \blacksquare$$

where "a" and f_s' can be computed as follows:

  - From strain and stress diagram:

$$f'_s = \epsilon_u E_s \frac{(c - d')}{c} \blacksquare \qquad (1)$$

  - From equilibrium:

$$\sum F_x = 0.0$$

$$A_s f_y = 0.85\beta_1 f'_c bc + A'_s f'_s \qquad (2)$$

Substitute of (1) into (2):

$$A_s f_y = 0.85\beta_1 f'_c bc + A'_s \epsilon_u E_s \frac{(c - d')}{c} \blacksquare \qquad (3)$$

  - Solve this quadratic equation for "c" value:

$$c = \sqrt{Q + R^2} - R \blacksquare$$

where:

$$Q = \frac{600 d' A'_s}{0.85\beta_1 f'_c b} \blacksquare$$

and

$$R = \frac{600A'_s - f_y A_s}{1.7\beta_1 f'_c b} \blacksquare$$

  - Then substitute "c" into equation (1) to obtain f'_s. Finally "a" value can be computed from:

$$a = \beta_1 c$$

4.6.3.3 Criterion to Check if the Compression Steel is at Yield Stress or Not

- It is clear from above discussion; that the form of relation for computing of M_n is depended on checking of yielding of compression steel.
- Based on basic principles (Compatibility, Stress-Strain Relation, and Equilibrium), following criterion can be derived to check the yielding of compression reinforcement.

  If $\rho \geq \bar{\rho}_{cy}$, then $f'_s = f_y$ and the compression reinforcement is at yield stress.

  Else $f'_s < f_y$ and the compression reinforcement is below the yield stress.
- The minimum tensile reinforcement ratio $\bar{\rho}_{cy}$ that will ensure yielding of the compression reinforcement at failure can be computed as follows:

$$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{d'}{d} \frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho' \blacksquare$$

4.6.4 Ties for Compression Reinforcement

- If compression bars are used in a flexural member, precautions must be taken to ensure that these bars will not buckle outward under load spelling off the outer concrete, see Figure 4.6-8 below.

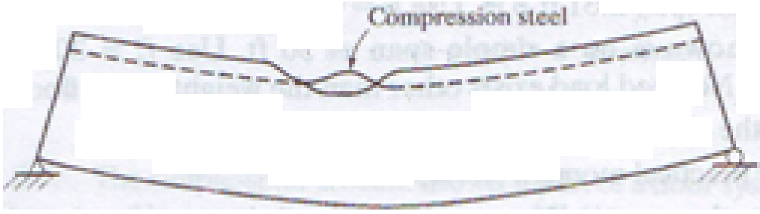


Figure 4.6-8: Buckling of beam compression reinforcement.

- ACI Code (***25.7.2.1***) imposes the requirement that such bars be anchored in the same way that compression bars in columns are anchored by lateral ties. Such ties are designed based on the following procedures:
  - Select bar diameter for ties (**25.7.2.2**):
    All bars of tied columns shall be enclosed by lateral ties at least No 10 in size for longitudinal bars up to No. 32 and at least No. 13 in size for Nos. 36, 43, and 57 and bundled longitudinal bars.
  - The spacing of the ties shall not exceed (**25.7.2.1**):
    $S_{Maximum} = \min[16d_{bar}, 48d_{ties}, \text{Least dimension of column}]$
  - Ties Arrangement (**25.7.2.3**):
    According to ACI (**25.7.2.3**), *rectilinear ties shall be arranged to satisfy (a) and (b):*
    (a) Every corner and alternate longitudinal bar shall have lateral support provided by the corner of a tie with an included angle of not more than 135 degrees.
    (b) No unsupported bar shall be farther than 150 mm clear on each side along the tie from a laterally supported bar.

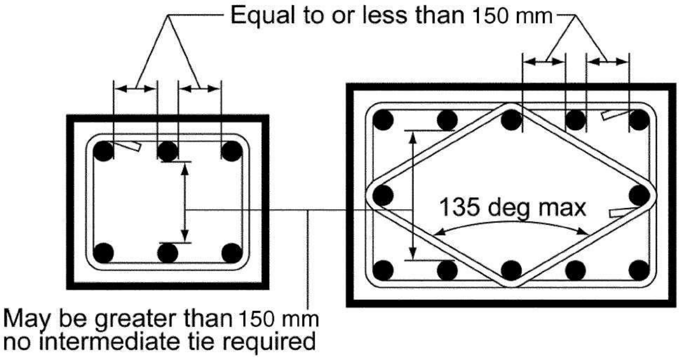


Figure 4.6-9: Ties arrangement according to requirements of ACI Code.

4.6.5 Examples

Example 4.6-1

Check the adequacy of beam shown in Figure 4.6-10 below and compute its design strength according to ACI Code. Assume that: $f'_c = 20\ \text{MPa}$ and $f_y = 300\ \text{MPa}$.

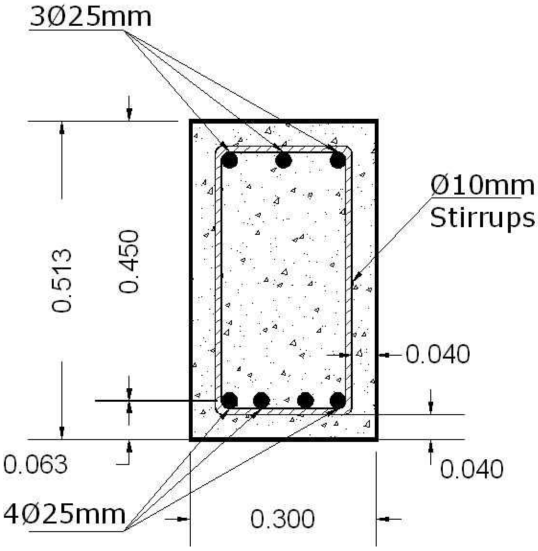


Figure 4.6-10: Cross section for beam of Example 4.6-1.

Solution

- Check the reason for using of compression reinforcement:

$$A_{s\ \text{Provided}} = 4 \times 490 = 1\,960\ \text{mm}^2 \Longrightarrow \rho_{\text{Provided}} = \frac{1\,960\ \text{mm}^2}{300 \times 450} = 14.5 \times 10^{-3}$$

$$\rho_{max} = 0.85\beta_1\ \frac{f'_c}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004} = \ 0.85 \times 0.85\ \frac{20}{300}\ \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} > \rho_{\text{Provided}}$$

Then, compression reinforcement has been added for a reason other than changing the failure mode from compression failure to secondary compression failure and its effects on section strength can be neglected. Therefore, the section can be analyzed as a singly reinforced section.

$$A_{s\ \text{minimum}} = \frac{0.25\sqrt{f'_c}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

$$\because\ f'_c < 31\ MPa$$

$$\therefore A_{s\ \text{minimum}} = \frac{1.4}{f_y} b_w d = \frac{1.4}{300} \times 300 \times 450 = 630\ \text{mm}^2 < A_{s\ \text{Provided}}\ \text{Ok.} \blacksquare$$

- Compute section nominal strength M_n :

$$M_n = \rho f_y b d^2 \left(1 - 0.59\ \frac{\rho f_y}{f'_c}\right)$$

$$M_n = 14.5 \times 10^{-3} \times 300 \times 300 \times 450^2 \left(1 - 0.59\ \frac{14.5 \times 10^{-3} \times 300}{20}\right) = 230\ \text{kN.m}$$

- Compute strength reduction factor Ø:
    - Compute steel stain based on the following relations:

    $$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{1\,960\ \text{mm}^2 \times 300\ \text{MPa}}{0.85 \times 20\ \text{MPa}\ \times 300\text{mm}} = 115\ \text{mm} \Longrightarrow c = \frac{a}{\beta_1}\frac{115\ \text{mm}}{0.85} = 135\ \text{mm}$$

    $$\epsilon_t = \frac{d - c}{c}\epsilon_u = \frac{450 - 135}{135} \times 0.003 = 7.0 \times 10^{-3}$$

    - $\epsilon_t > 0.005$, then $\emptyset = 0.9$.
- Compute section design strength $\emptyset M_n$:

$$\emptyset M_n = \ \emptyset \times M_n = \ 0.9 \times 230\ \ \text{kN.m} = 207\ \text{kN.m}\blacksquare$$

Example 4.6-2

Check the adequacy of beam shown in Figure 4.6-11 below and compute its design strength according to ACI Code. Assume that: $f'_c = 20\ \text{MPa}$ and $f_y = 300\ \text{MPa}$.

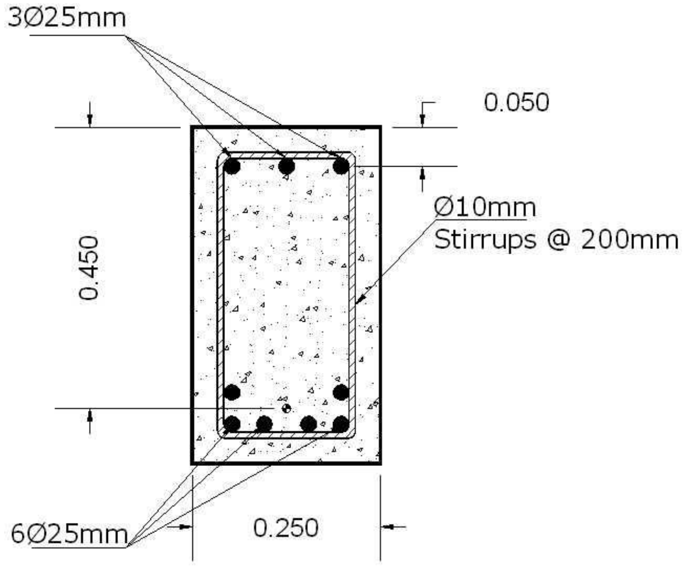


Figure 4.6-11: Cross section for beam of Example 4.6-2.

Solution

- Check the reason for using of compression reinforcement:

$$A_{s\ \text{Provided}} = 6 \times 490 = 2\,940\ \text{mm}^2 \Longrightarrow \rho_{\text{Provided}} = \frac{2\,940\ \text{mm}^2}{250 \times 450} = 26.1 \times 10^{-3}$$

$$\rho_{\max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} \Longrightarrow 0.85 \times 0.85\ \frac{20}{300} \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} < \rho_{\text{Provided}}$$

Then, compression reinforcement has been added for changing the failure mode from compression failure to secondary compression failure and its effects on section strength must be included.

- Checking the Section Type (i.e., check the effect of compression reinforcement on maximum permissible steel ratio):

$$\bar{\rho}_{\max} = \rho_{\max} + \rho' \frac{f'_s}{f_y} \blacksquare$$

where f'_s is stress in the compression reinforcement at strains of $\rho_{\max}$. It can be computed from strain distribution and as shown in relation below:

$$f'_s = E_s \left[\epsilon_u - \frac{d'}{d}(\epsilon_u + 0.004)\right] \leq f_y \blacksquare$$

$$f'_s = 200\,000\text{MPa}\left[0.003 - \frac{50}{450}(0.003 + 0.004)\right] = 444 > f_y$$

$$f'_s = f_y = 300\text{MPa}$$

$$\therefore\ \bar{\rho}_{\max} = \rho_{\max} + \rho'$$

$$A_s' = 3 \times 490 = 1\,470\text{mm}^2$$

$$\rho' = \frac{1\,470\text{mm}^2}{250 \times 450} = 13.1 \times 10^{-3}$$

$$\therefore\ \bar{\rho}_{\max} = 20.6 \times 10^{-3} + 13.1 \times 10^{-3} = 33.7 \times 10^{-3} > \rho_{\text{Provided}}\ \text{Ok.}$$

- Compute Section Nominal Strength M_n:

First of all, check if the compression reinforcement is yielded on not.

$$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f'_c}{f_y}\frac{d'}{d}\frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho' = 0.85 \times 0.85 \frac{20}{300}\frac{50}{450}\frac{0.003}{0.003 - \dfrac{300}{200\ 000}} + 13.1 \times 10^{-3}$$

$$\bar{\rho}_{cy} = 10.7 \times 10^{-3} + 13.1 \times 10^{-3} = 23.8 \times 10^{-3} < \rho_{\text{Provided}}$$

$$\therefore f'_s = f_y = 300\text{MPa}$$

Then use the relation that derived for yielded compression reinforcement:

$$M_n = M_{n1} + M_{n2} = A'_s f_y (d - d') + (A_s - A'_s) f_y \left(d - \frac{a}{2}\right)$$

where,

$$a = \frac{(A_s - A_s')f_y}{0.85 f'_c b} = \frac{(2\,940 - 1\,470) \times 300}{0.85 \times 20 \times 250} = 104\text{mm}$$

$$\mathrm{M_n = M_{n1} + M_{n2}} = 1\,470 \times 300 \times (450 - 50) + (2\,940 - 1\,470) \times 300 \times \left(450 - \frac{104}{2}\right)$$
$$M_n = M_{n1} + M_{n2} = 176.4 \times 10^6 N.mm + 175.5 \times 10^6 N.mm = 352\ kN.m$$

- Compute strength reduction factor $\emptyset$:
  - Compute steel stain based on the following relations:
  $$\mathrm{a = 104mm \Rightarrow c = \frac{a}{\beta_1} = \frac{104\ mm}{0.85} = 122\ mm}$$
  $$\epsilon_t = \frac{d-c}{c}\epsilon_u = \frac{450-122}{122} \times 0.003 = 8.06 \times 10^{-3}$$
  - $\epsilon_t > 0.005$, then $\emptyset = 0.9$.
- Compute section design strength $\emptyset M_n$:
  $\emptyset M_n = \emptyset \times M_n = 0.9 \times 352\ kN.m = 317\ kN.m$■
- Check Adequacy of Stirrups as Ties:
  $\because \emptyset_{for\ Longitudinal} = 25mm < No.32$
  $\therefore \emptyset_{for\ Ties} = 10mm\ Ok.$
  $S_{Maximum} = min[16d_{bar}, 48d_{ties}, Least\ dimension\ of\ column]$
  $S_{Maximum} = min[16 \times 25mm, 48 \times 10mm, 250mm] = min[400mm, 480mm, 250mm]$
  $S_{Maximum} = 250mm > S_{Provided} = 200mm\ \ Ok.$
  Checking if alternative rebar is supported or not
  $$S_{Clear} = (250 - 40 \times 2 - 10 \times 2 - 3 \times 25) \times \frac{1}{2} = 37.5\ mm < 150mm\ Ok.$$

Example 4.6-3

Recheck the adequacy of the beam of Example 4.6-2 above but with d'=65 mm.

Solution

- Check the reason for using of compression reinforcement:
  $$A_{s\ Provided} = 6 \times 490 = 2\,940\ mm^2 \Rightarrow \rho_{Provided} = \frac{2\,940\ mm^2}{250 \times 450} = 26.1 \times 10^{-3}$$
  $$\rho_{max} = 0.85\beta_1\ \frac{f_c'}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85 \times 0.85\ \frac{20}{300}\ \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} < \rho_{Provided}$$
  Then, compression reinforcement has been added for changing the failure mode from compression failure to secondary compression failure and its effects on section strength must be included.
- Checking the Section Type (i.e., check the effect of compression reinforcement on maximum permissible steel ratio):
  $$\bar{\rho}_{max} = \rho_{max} + \rho' \frac{f_s'}{f_y}\ ■$$
  where f_s' is stress in the compression reinforcement at strains of ρ_{max}. It can be computed from strain distribution and as shown in relation below:
  $$f_s' = E_s\left[\epsilon_u - \frac{d'}{d}(\epsilon_u + 0.004)\right] \le f_y\ ■$$
  $$f_s' = 200\,000MPa\left[0.003 - \frac{65}{450}(0.003 + 0.004)\right] = 398 > f_y \Rightarrow f_s' = f_y = 300MPa$$
  $\therefore\ \bar{\rho}_{max} = \rho_{max} + \rho'$
  $$A_s' = 3 \times 490 = 1\,470mm^2 \Rightarrow \rho' = \frac{1\,470mm^2}{250 \times 450} = 13.1 \times 10^{-3}$$
  $\therefore\ \bar{\rho}_{max} = 20.6 \times 10^{-3} + 13.1 \times 10^{-3} = 33.7 \times 10^{-3} > \rho_{Provided}\ \ Ok.$
- Compute of Section Nominal Strength M_n:
  First of all, check if the compression reinforcement is yielded on not.
  $$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y}\frac{d'}{d}\frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho' = 0.85 \times 0.85 \frac{20}{300}\frac{65}{450}\frac{0.003}{0.003 - \frac{300}{200\ 000}} + 13.1 \times 10^{-3}$$
  $\bar{\rho}_{cy} = 13.9 \times 10^{-3} + 13.1 \times 10^{-3} = 27.0 \times 10^{-3} > \rho_{Provided}$
  $\therefore f_s' < f_y = 300MPa$
  Compute of f_s' can be done based on following relations:
  - Compute "c" based on Quadratic Formula:
  $$c = \sqrt{Q + R^2} - R$$
  where:

$$Q = \frac{600d'A_s'}{0.85\beta_1 f_c' b} = \frac{600 \times 65 \text{ mm } \times 1\,470\text{mm}^2}{0.85 \times 0.85 \times 20 \frac{\text{N}}{\text{mm}^2} \times 250\text{mm}} = 15\,870$$

and

$$R = \frac{600A_s' - f_y A_s}{1.7\beta_1 f_c' b} = \frac{600 \times 1\,470\text{mm}^2 - 300 \times 2\,940}{1.7 \times 0.85 \times 20 \times 250} = 0$$

$$c = \sqrt{15\,870 + 0.0^2} - 0.0 = 126\text{mm}$$

- Compute f_s' can be computed based on following relation:

$$f_s' = \epsilon_u E_s \frac{(c-d')}{c} = 0.003 \times 200\,000 \times \frac{126-65}{126} = 290 \text{ MPa} < f_y \text{ Ok.}$$

Then use the relation that derived for yielded compression reinforcement:

$$M_n = M_{n1} + M_{n2} = 0.85 f_c' ab \left(d - \frac{a}{2}\right) + A_s' f_s' (d - d') ■$$

where

$$a = \beta_1 c = 0.85 \times 126\text{mm} = 107\text{mm}$$

$$M_n = M_{n1} + M_{n2} = 0.85 \times 20 \times 107 \times 250 \left(450 - \frac{107}{2}\right) + 1\,470 \times 290 \times (450 - 65)$$

$$M_n = M_{n1} + M_{n2} = 180.3 \times 10^6 \text{N.mm} + 164.1 \times 10^6 \text{N.mm} = 344 \text{ kN.m}$$

- Compute strength reduction factor $\emptyset$:
  - Compute steel stain based on the following relations:

$$c = 126\text{mm} \Rightarrow \epsilon_t = \frac{d-c}{c}\epsilon_u = \frac{450-126}{126} \times 0.003 = 7.71 \times 10^{-3}$$

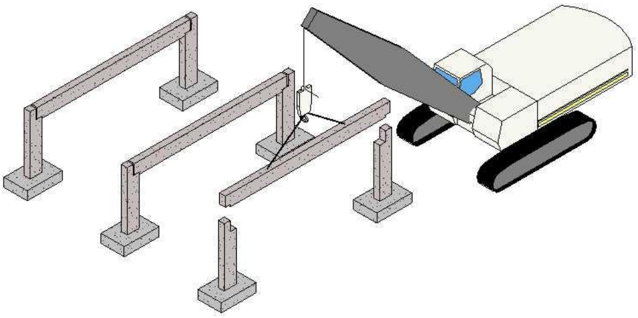
  - $\epsilon_t > 0.005$, then $\emptyset = 0.9$
- Compute section design strength $\emptyset M_n$:

$$\emptyset M_n = \emptyset \times M_n = 0.9 \times 344\, kN.m = 310\, kN.m ■$$

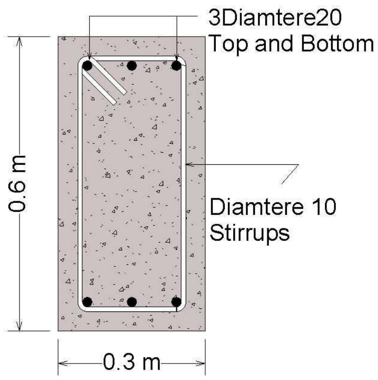
- Check Adequacy of Stirrups as Ties:
  See previous example for stirrups checking when used as ties.

Example 4.6-4

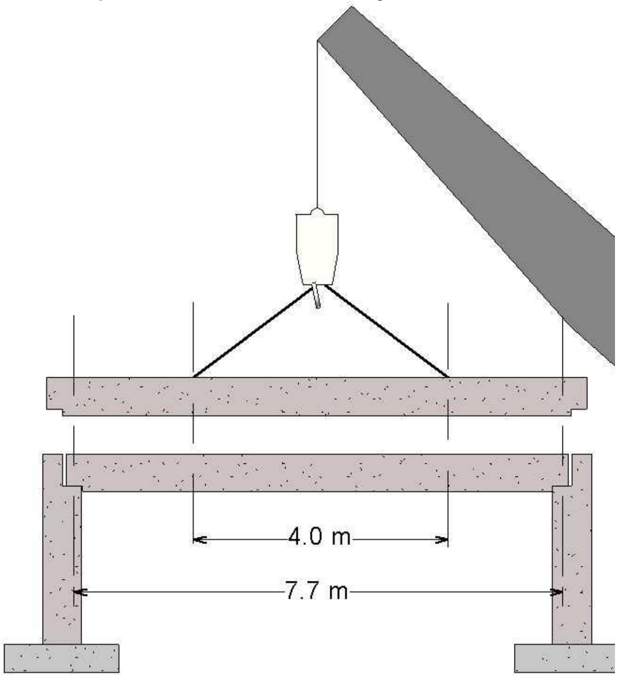
To counteract stresses during lifting process, a simply supported precast concrete beam shown in Figure 4.6-12 below has been symmetrically reinforced with $\mathbf{3\phi20}$ rebars.



3D View.



Beam Cross Section



Elevation View.

Figure 4.6-12: Precast beam of Example 4.6-4.

For this precast beam:

- With including effects of compressive reinforcement in your solution, compute section nominal flexural strength ϕM_n.

- What is the maximum uniformly distributed load "Wu" that could be applied on the beam during its work?
- Are the proposed reinforcement adequate during lifting process?

In your solution, assume that, $f_c' = 28\,MPa$ and $f_y = 420\,MPa$.

Solution

- **<u>Section Flexural Strength:</u>**

$$A_s = A_s' = 3 \times \frac{\pi \times 20^2}{4} = 942\,mm^2$$

$$d = 600 - 40 - 10 - \frac{20}{2} = 540\,mm, d' = 40 + 10 + \frac{20}{2} = 60\,mm$$

$$\rho = \rho' = \frac{942}{300 \times 540} = 5.81 \times 10^{-3}$$

$$\rho_{max} = 0.85\beta_1\ \frac{f_c'}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85^2 \times \frac{28}{420} \times \frac{3}{7} = \ 20.6 \times 10^{-3} > \rho$$

In spite of the compression, reinforcement has been used for a reason other than change failure mode; according to problem statement, the compression reinforcement should be included within solution.

$$\because \rho = \rho'$$

$$\therefore \bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y}\frac{d'}{d}\frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho' > \rho$$

$$f_s' < f_y$$

$$c = \sqrt{Q + R^2} - R$$

$$Q = \frac{600d'A_s'}{0.85\beta_1 f_c' b} = \frac{600 \times 60\ \times 942}{0.85 \times 0.85 \times 28 \times 300} = 5588$$

$$R = \frac{600A_s' - f_y A_s}{1.7\beta_1 f_c' b} = \frac{600 \times 942 - 420 \times 942}{1.7 \times 0.85 \times 28 \times 300} = 13.9$$

$$c = \sqrt{Q + R^2} - R = \sqrt{5588 + 13.9^2} - 13.9 = 62.1\text{ mm}$$

$$f_s' = \epsilon_u E_s \frac{(c - d')}{c} = 0.003 \times 200\,000 \times \frac{62.1 - 60}{62.1} = 20\text{ MPa} < f_y\ \text{Ok.}$$

$$M_n = M_{n1} + M_{n2} = 0.85 f_c' ab\left(d - \frac{a}{2}\right) + A_s' f_s'(d - d')$$

$$a = \beta_1 c = 0.85 \times 62.1 = 52.8\,mm$$

$$M_n = 0.85 \times 28 \times 52.8 \times 300\left(540 - \frac{52.8}{2}\right) + 942 \times 20 \times (540 - 60) = 203\text{ kN.m}$$

$$\epsilon_t = \frac{d - c}{c}\epsilon_u = \frac{540 - 62.1}{62.1} \times 0.003 = 23.1 \times 10^{-3} > 0.005 \Longrightarrow \phi = 0.9$$

$$\phi M_n = 0.9 \times 203 = 183\,kN.m\blacksquare$$

- **<u>Maximum Permissible Factored Load W_u:</u>**

$$M_u = \frac{W_u l^2}{8} = \phi M_n \Longrightarrow M_u = \frac{W_u \times 7.7^2}{8} = 183 \Longrightarrow W_u = 24.7\frac{kN}{m}\blacksquare$$

- **<u>Section Adequacy during Lifting Process:</u>**

During lifting process, factored load is equal to factored dead load:

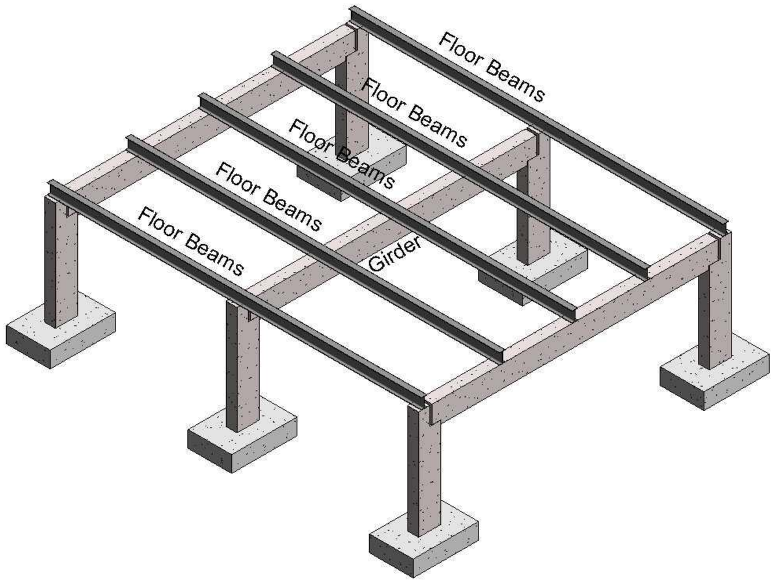
$$W_u = 1.4W_d = 1.4 \times (24 \times 0.6 \times 0.3) = 6.05\frac{kN}{m}$$

$$M_{u\,for\,cantilever\,part} = \frac{6.05 \times \left(0.5 \times (7.7 - 4)\right)^2}{2} = 10.4\,kN.m < \phi M_n \therefore Ok.$$

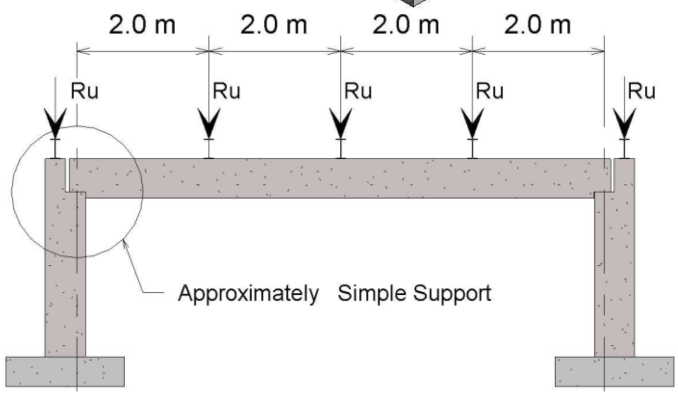
$$M_{u\,mid-span} = \frac{6.05 \times 4^2}{8} - 10.4 = 1.70 < \phi M_n \therefore Ok.$$

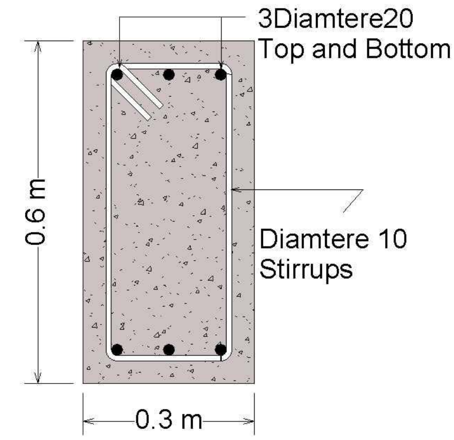
Example 4.6-5

For a frame shown in Figure 4.6-13 below, with neglecting selfweight and with including the effects of compression rebars and based on flexural strength only; what is the maximum factored floor beam reaction "Ru" that could be supported by the girder? In your solution, assume that, $f_c' = 28\,MPa$ and $f_y = 420\,MPa$.



3D View.





Elevation View. **Beam Cross Section.**

Figure 4.6-13: Frame system of Example 4.6-5.

Solution

- **Section Flexural Strength:**

$$A_s = A_s' = 3 \times \frac{\pi \times 20^2}{4} = 942\ mm^2$$

$$d = 600 - 40 - 10 - \frac{20}{2} = 540\ mm, d' = 40 + 10 + \frac{20}{2} = 60\ mm$$

$$\rho = \rho' = \frac{942}{300 \times 540} = 5.81 \times 10^{-3}$$

$$\rho_{max} = 0.85\beta_1\ \frac{f_c'}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85^2 \times \frac{28}{420} \times \frac{3}{7} = \ 20.6 \times 10^{-3} > \rho$$

In spite of the compression reinforcement has been used for a reason other than change failure mode, according to problem statement the compression reinforcement should be included within solution.

$$\because \rho = \rho'$$

$$\therefore \bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y}\frac{d'}{d}\frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho' > \rho \Longrightarrow f_s' < f_y$$

$$\mathrm{c} = \ \sqrt{\mathrm{Q} + \mathrm{R}^2} - \mathrm{R}$$

$$\mathrm{Q} = \frac{600\mathrm{d}'\mathrm{A}_\mathrm{s}'}{0.85\beta_1\mathrm{f}_\mathrm{c}'\mathrm{b}} = \frac{600 \times 60\ \times 942}{0.85 \times 0.85 \times 28 \times 300} = 5588, \mathrm{R} = \frac{600\mathrm{A}_\mathrm{s}' - \mathrm{f}_\mathrm{y}\mathrm{A}_\mathrm{s}}{1.7\beta_1\mathrm{f}_\mathrm{c}'\mathrm{b}} = \frac{600 \times 942 - 420 \times 942}{1.7 \times 0.85 \times 28 \times 300}$$
$$= 13.9$$

$$\mathrm{c} = \ \sqrt{\mathrm{Q} + \mathrm{R}^2} - \mathrm{R} = \sqrt{5588 + 13.9^2} - 13.9 = 62.1\ \mathrm{mm}$$

$$\mathrm{f}_\mathrm{s}^{'} = \epsilon_\mathrm{u}\mathrm{E}_\mathrm{s}\frac{(\mathrm{c} - \mathrm{d}')}{\mathrm{c}} = 0.003 \times 200\ 000 \times \frac{62.1 - 60}{62.1} = 20\ \mathrm{MPa} < \mathrm{f}_\mathrm{y}\ \ \mathrm{Ok.}$$

$$\mathrm{M}_\mathrm{n} = \mathrm{M}_{\mathrm{n}1} + \mathrm{M}_{\mathrm{n}2} = 0.85\mathrm{f}_\mathrm{c}'\mathrm{ab}\left(\mathrm{d} - \frac{\mathrm{a}}{2}\right) + \mathrm{A}_\mathrm{s}'\mathrm{f}_\mathrm{s}'(\mathrm{d} - \mathrm{d}')$$

$$a = \beta_1 c = 0.85 \times 62.1 = 52.8\ mm$$

$$\mathrm{M_n} = 0.85 \times 28 \times 52.8 \times 300 \left(540 - \frac{52.8}{2}\right) + 942 \times 20 \times (540 - 60) = 203\ \mathrm{kN.m}$$

$$\epsilon_t = \frac{d - c}{c}\epsilon_u = \frac{540 - 62.1}{62.1} \times 0.003 = 23.1 \times 10^{-3} > 0.005 \Rightarrow \phi = 0.9$$

$$\phi M_n = 0.9 \times 203 = 183\ kN.m \blacksquare$$

- **Maximum Floor Beam Reaction:**

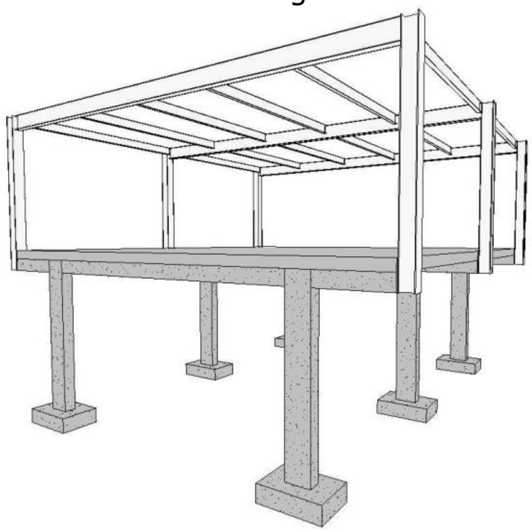
  Let $M_u = \phi M_n$

  With use of superposition principle, factored moment M_u is:

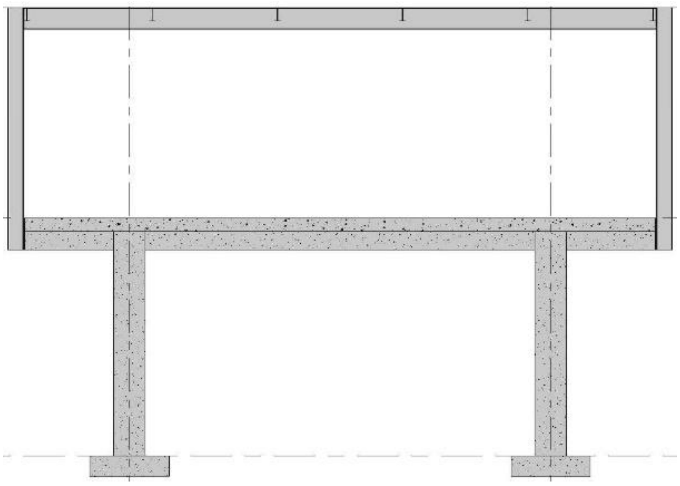
$$M_u = R_u a + \frac{R_u l}{4} = 2R_u + \frac{8}{4}R_u = 4R_u \Rightarrow M_u = 4R_u = \phi M_n = 183 \Rightarrow R_u = 45.8\ kN \blacksquare$$

Example 4.6-6

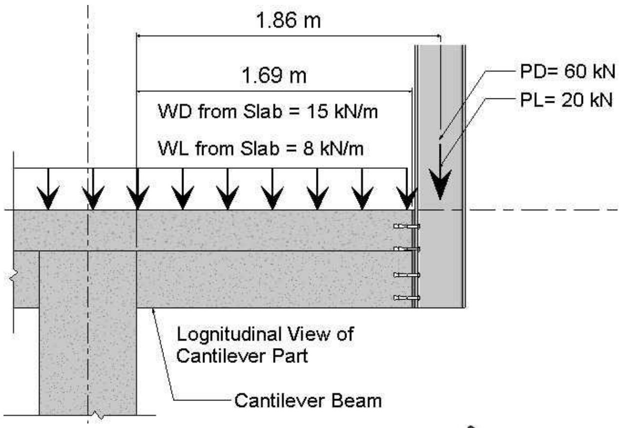
In an attempt to add a new floor for an existing reinforced concrete building, a steel frame shown in Figure 4.6-14 below has been proposed. The steel columns have been supported on cantilever concrete beams of the existing concrete floor. If the cantilever part of the beam is reinforced as shown; can it withstand the applied loads shown based on its flexural strength?

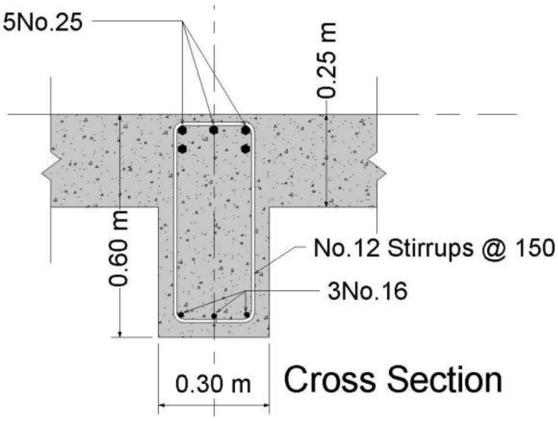


3D View



Elevation





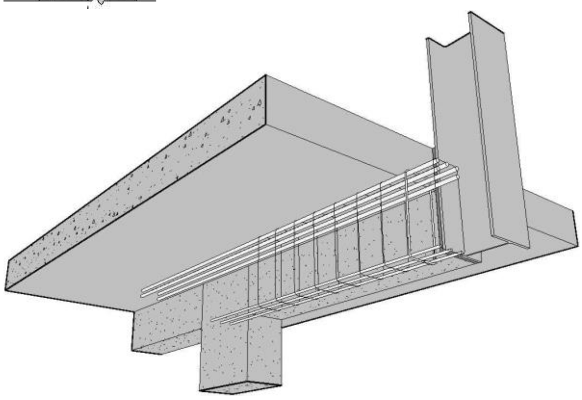


Figure 4.6-14: Building with additional new floor for Example 4.6-6.

Solution

$P_u = maximum(1.4\ P_D\ or\ 1.2P_D + 1.6P_L) = maximum(1.4 \times 60\ or\ 1.2 \times 60 + 1.6 \times 20)$

$P_u = maximum(84\ or\ 104) = 104\ kN$

$$W_{self} = 0.3 \times (0.6 - 0.25) \times 24 = 2.52 \frac{kN}{m} \Longrightarrow W_D = 2.52 + 15 = 17.5 \frac{kN}{m}, W_L = 8 \frac{kN}{m}$$

$$W_u = maximum(1.4 \times 17.5 \; or \; 1.2 \times 17.5 + 1.6 \times 8) = maximum(24.5 \; or \; 33.8) = 33.8 \frac{kN}{m}$$

$$M_u = \frac{W_u l^2}{2} + P_u l = \frac{33.8 \times 1.69^2}{2} + 104 \times 1.86 = 242 \; kN.m$$

Check the reason for using of compression reinforcement:

$$A_{Bar} = \frac{\pi \times 25^2}{4} \approx 490 \; mm^2 \Longrightarrow \mathrm{A_{s\,Provided}} = 5 \times 490 = 2450 \; \mathrm{mm^2}$$

$$d = 600 - 40 - 12 - 25 - \frac{25}{2} = 510 \; mm \Longrightarrow \rho_{\mathrm{Provided}} = \frac{2450}{300 \times 510} = 16.0 \times 10^{-3}$$

$$\rho_{\max} = 0.85\beta_1 \; \frac{f_c'}{f_y} \; \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85 \times 0.85 \; \frac{28}{420} \; \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} > \rho_{\mathrm{Provided}}$$

Then, compression reinforcement has been added for a reason other than to chang the failure mode from compression failure to secondary compression failure and its effects on section strength can be neglected. Therefore, the section can be analyzed as a singly reinforced section.

As the flange is under tension, the span is a statically indeterminate one, and noting is mentioned about flange width, hence second term of second relation for $A_{s\,minimum}$ is adopted:

$$\mathrm{A_{s\,minimum}} = \frac{0.5\sqrt{f_c'}}{f_y} b_w d = \left(\frac{0.5 \times \sqrt{28}}{420}\right) \times (300 \times 510) = 9640 \; \mathrm{mm^2} < \mathrm{A_{s\,Provided}} \; \mathrm{Ok.} \blacksquare$$

Compute section nominal strength $\mathrm{M_n}$:

$$\mathrm{M_n} = \rho f_y b d^2 \left(1 - 0.59 \; \frac{\rho f_y}{f_c'}\right) = 16.0 \times 10^{-3} \times 420 \times 300 \times 510^2 \times \left(1 - 0.59 \; \frac{16.0 \times 10^{-3} \times 420}{28}\right)$$
$$= 450 \; \mathrm{kN.m}$$

Compute strength reduction factor $\emptyset$:

Compute steel stain based on the following relations:

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2450 \times 420}{0.85 \times 28 \; \times 300} = 144 \; \mathrm{mm} \Longrightarrow c = \frac{a}{\beta_1} = \frac{144}{0.85} = 169 \; \mathrm{mm}$$

$$\epsilon_t = \frac{d - c}{c} \epsilon_u = \frac{510 - 169}{169} \times 0.003 = 6.05 \times 10^{-3} \Longrightarrow \emptyset = 0.9$$

Compute section design strength $\emptyset \mathrm{M_n}$:

$$\emptyset \mathrm{M_n} = \emptyset \times \mathrm{M_n} = 0.9 \times 450 = 405 \; \mathrm{kN.m} > \mathrm{M_u} \; \therefore \mathrm{Ok.}$$

Therefore, based on its flexural strength, cantilever part is adequate to support intended steel frame.

Example 4.6-7

Based on flexure strength of section A-A, computed the maximum value of P_u that could be supported by the beam presented in Figure 4.6-15 below. In Your solution, assume that:

- $f_c' = 21MPa$ and $f_y = 420MPa$.
- Selfweight could be neglected.
- $A_{Bar} = 500 \; mm^2 \; for \; \phi \; 25mm.$

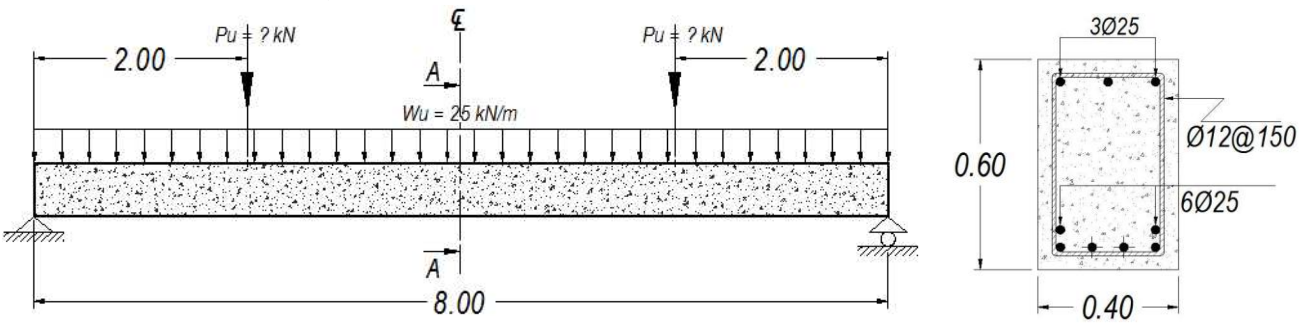


Figure 4.6-15: Simply supported beam for Example 4.6-7. **Section A-A**

Solution

- Check the reason for using of compression reinforcement:

$$A_{s\,Provided} = 6 \times 500 = 3000\ mm^2, d = 600 - 40 - 12 - 25 - \frac{25}{2} = 510$$

$$\rho_{Provided} = \frac{3000}{400 \times 510} = 14.7 \times 10^{-3}$$

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85 \times 0.85 \frac{21}{420} \frac{0.003}{0.003 + 0.004} = 15.5 \times 10^{-3} > \rho_{Provided}$$

Then, compression reinforcement has been added for a reason other than changing the failure mode from compression failure to secondary compression failure and its effects on section strength can be neglected.
Then the section can be analyzed as a singly reinforced section.

$$A_{s\,minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

$$\because f_c' < 31\ MPa$$

$$\therefore A_{s\,minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 400 \times 510 = 680\ mm^2 < A_{s\,Provided}\ \ Ok. \blacksquare$$

- Compute section nominal strength M_n :

$$M_n = \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right)$$

$$M_n = 14.7 \times 10^{-3} \times 420 \times 400 \times 510^2 \left(1 - 0.59 \frac{14.7 \times 10^{-3} \times 420}{21}\right) = 531\ kN.m$$

- Compute strength reduction factor $\emptyset$:
Compute steel stain based on the following relations:

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3000\ mm^2 \times 420\ MPa}{0.85 \times 21\ MPa \times 400mm} = 176\ mm$$

$$c = \frac{a}{\beta_1} = \frac{176\ mm}{0.85} = 207\ mm$$

$$\epsilon_t = \frac{d - c}{c}\epsilon_u = \frac{510 - 207}{207} \times 0.003 = 4.39 \times 10^{-3}$$

Then:

$$\emptyset = 0.483 + 83.3\epsilon_t = 0.483 + 83.3 \times 4.39 \times 10^{-3} = 0.849$$

- Compute section design strength $\emptyset M_n$:

$$\emptyset M_n = \emptyset \times M_n = 0.849 \times 531\ kN.m = 451\ kN.m \blacksquare$$

- Compute Pu:

$$M_u = P_u \times 2.0 + \frac{25 \times 8^2}{8} = 451\ kN.m \implies P_u = 125\ kN \blacksquare$$

Example 4.6-8

Compute the maximum factored load P_u that can be supported by a beam shown in Figure 4.6-16 below. In your solution:

- Neglect the selfweight.
- $f_c' = 21\ MPa$
- $f_y = 420\ MPa$.

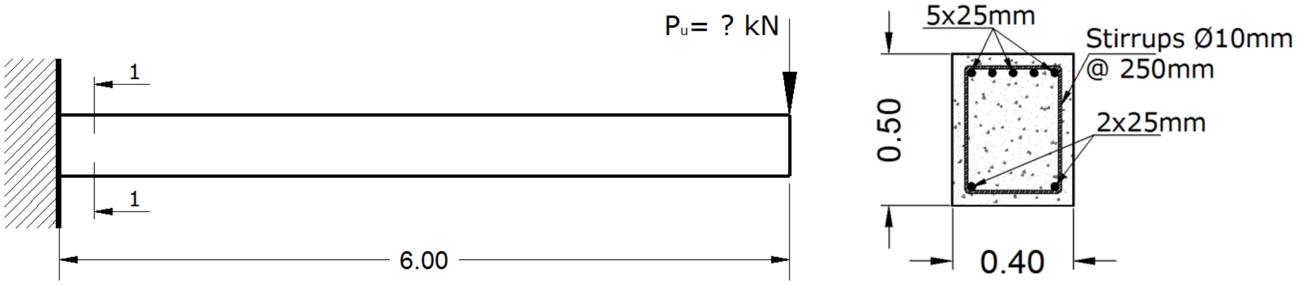


Figure 4.6-16: Cantilever beam for Example 4.6-8. **Section 1-1**

Solution

- Check the cause for using of compression reinforcement:

$$A_{Bar}=\frac{\pi\times 25^2}{4}=490\,mm^2, d=500-40-10-12.5=437.5\,mm$$

$$\rho_{Provided}=\frac{490\times 5}{437.5\times 400}=14\times 10^{-3}$$

$$\rho_{maximum}=0.85\beta_1\frac{f_c'}{f_y}\frac{0.003}{0.003+0.004}=0.85^2\ \frac{21}{420}\ \frac{0.003}{0.003+0.004}=15.5\times 10^{-3}$$

As $\rho_{Provided}<\rho_{max}$, then the compression reinforcement has been used to a cause other than the flexure strength. Then the section can be analyzed as singly reinforced section.

- Compute the section flexure nominal strength and design strength:

$$\sum F_x=0$$

$$0.85\times 21\times a\times 400=(490\times 5)\times 420 \Rightarrow a=144\,mm$$

$$M_n=(490\times 5)\times 420\times\left(437.5-\frac{144}{2}\right)=376\,kN.m$$

- Strength Reduction Factor ϕ:

$$c=\frac{144\,mm}{0.85}=169\,mm\Rightarrow \epsilon_t=\frac{d-c}{c}\times\epsilon_u=\frac{437\,mm-169\,mm}{169\,mm}\times 0.003=4.76\times 10^{-3}$$

$$\emptyset=0.483+83.3\epsilon_t=0.483+83.3\times 4.76\times 10^{-3}=0.879$$

- Compute ϕM_n:

$$\emptyset M_n=0.879\times 376\,kN.m=331\,kN.m$$

- Compute the maximum permissible force P_u:

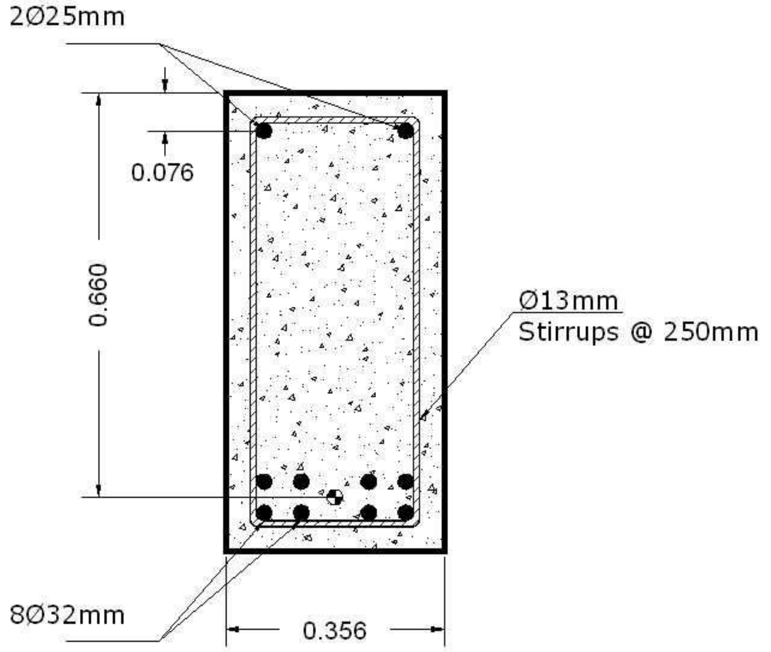
$$\emptyset M_n=331\,kN.m=P_u\times l=P_u\times 6m\Rightarrow P_u=55.2\,kN.m\quad\blacksquare$$

4.6.6 Homework Problems

Problem 4.6-1

Check the adequacy of the beam shown below and compute its design strength according to ACI Code. Assume that:

1. $f_c'=34.5\text{ MPa}$.
2. $f_y=414\text{MPa}$.
3. $A_{\text{of Bar No.25mm}}=510\text{mm}^2$.
4. $A_{\text{of Bar No.32mm}}=819\text{mm}^2$.



Answers

- Check the reason for using of compression reinforcement:

$A_{s\text{ Provided}}=6\,552\text{ mm}^2\Rightarrow\rho_{\text{Provided}}=27.9\times 10^{-3}$

$\beta_1=0.804\Rightarrow\ \rho_{\max}=24.4\times 10^{-3}<\rho_{\text{Provided}}$

Then, compression reinforcement has been added for changing the failure mode from compression failure to secondary compression failure and its effects on section strength must be included.

- Checking the Section Type (i.e., check the effect of compression reinforcement on maximum permissible steel ratio):

$\bar{\rho}_{max} = \rho_{max} + \rho' \frac{f_s'}{f_y}$ ■

$f_s' = f_y = 414\text{MPa} \Longrightarrow \bar{\rho}_{max} = \rho_{max} + \rho'$

$A_s{}' = 1\,020\text{mm}^2 \Longrightarrow \rho' = 4.34 \times 10^{-3} \Longrightarrow \therefore \bar{\rho}_{max} = 28.7 \times 10^{-3} > \rho_{Provided}$ Ok.

- Compute of Section Nominal Strength M_n:

First of all, check if the compression reinforcement is yielded on not.

$\bar{\rho}_{cy} = 25.5 \times 10^{-3} < \rho_{Provided} \Longrightarrow \therefore f_s' = f_y = 414\text{MPa}$

Then use the relation that derived for yielded compression reinforcement:

$M_n = M_{n1} + M_{n2} = A_s' f_y (d - d') + (A_s - A_s') f_y \left(d - \frac{a}{2}\right)$

where

$a = 219\text{mm}$

$M_n = M_{n1} + M_{n2} = 247 \times 10^6 \text{N.mm} + 1\,261 \times 10^6 \text{N.mm} = 1\,508\ \text{kN.m}$

- Compute strength reduction factor $\emptyset$:

Compute steel stain based on the following relations:

$a = 219\text{mm} \Longrightarrow c = 272\ \text{mm} \Longrightarrow \epsilon_t = 4.28 \times 10^{-3}$

$\epsilon_t < 0.005$, then:

$\emptyset = 0.84$

- Compute section design strength $\emptyset M_n$:

$\emptyset M_n = 1\,267\ \text{kN.m}$ ■

- Check Adequacy of Stirrups as Ties:

$\because \emptyset_{\text{for Longitudinal}} = 25\text{mm} < No.32$

$\therefore \emptyset_{\text{for Ties}} = 13\text{mm}$ Ok.

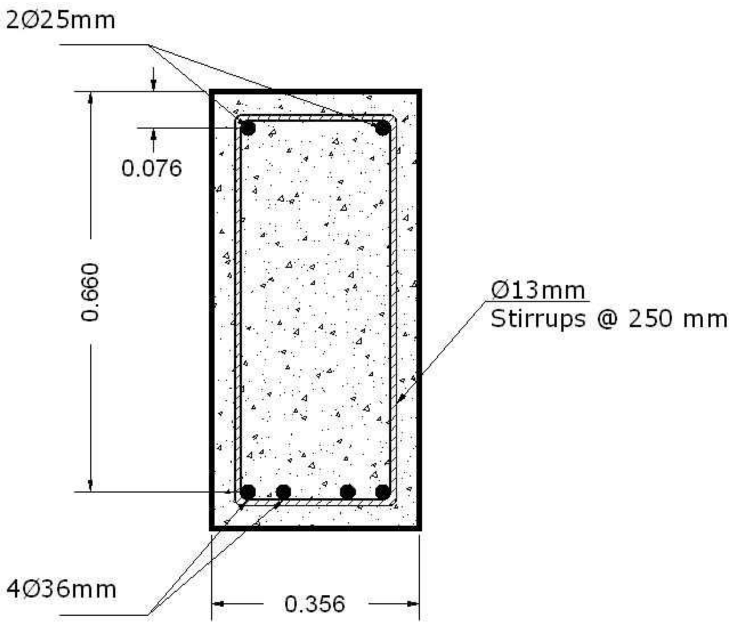
$S_{Maximum} = \min[16d_{bar}, 48d_{ties}, \text{Least dimension of column}]$

$S_{Maximum} = 356\text{mm} > S_{Provided} = 250\text{mm}$ Ok.

Problem 4.6-2

Check the adequacy of the beam shown below and compute its design strength according to ACI Code. Assume that:

1. $f_c' = 34.5$ MPa.
2. $f_y = 414$MPa.
3. $A_{\text{of Bar No.25mm}} = 510\text{mm}^2$.
4. $A_{\text{of Bar No.36mm}} = 1\,008\text{mm}^2$.



Answers

- Check the reason for using of compression reinforcement:

$A_{s\ Provided} = 4032\text{mm}^2$

$\rho_{Provided} = 17.2 \times 10^{-3}$

$\beta_1 = 0.804$

$\rho_{max} = 24.4 \times 10^{-3} > \rho_{Provided}$

Then, compression reinforcement has been added for a reason other than changing the failure mode from compression failure to secondary compression failure and its effects on section strength can be neglected.

Then the section can be analyzed as a singly reinforced section.

$\because f_c' > 31\ MPa \ \therefore A_{s\ minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d = 833\ \text{mm}^2 < A_{s\ Provided}$ Ok. ■

- Compute section nominal strength M_n:

$M_n = 970\ \text{kN.m}$

- Compute strength reduction factor $\emptyset$:
  Compute steel stain based on the following relations:
  $a = 160 \text{ mm} \Rightarrow c = 199 \text{ mm} \Rightarrow \epsilon_t = 6.95 \times 10^{-3}$
  $\epsilon_t > 0.005$, then:
  $\emptyset = 0.9$
- Compute section design strength $\emptyset M_n$:
  $\emptyset M_n = \emptyset \times M_n = 0.9 \times 970 \text{ kN.m} = 873 \text{ kN.m} \blacksquare$

Problem 4.6-3

Re-compute design strength of beam above according to ACI Code with including the effect of compression reinforcement even it has been used for a purpose other than strength requirement.

Answers

- Compute of Section Nominal Strength M_n:
  First of all, check if the compression reinforcement is yielded on not.
  $\bar{\rho}_{cy} = 21.2 \times 10^{-3} + 4.31 \times 10^{-3} = 25.5 \times 10^{-3} > \rho_{Provided}$
  $\therefore f'_s < f_y$
  Compute of f'_s can be done based on following relations:
  a. Compute "c" based on Quadratic Formula:
     $c = \sqrt{Q + R^2} - R$
     where:
     $$Q = \frac{600 d' A'_s}{0.85 \beta_1 f'_c b} = 5\,541$$
     and
     $$R = \frac{600 A'_s - f_y A_s}{1.7 \beta_1 f'_c b} = -63.0$$
     $c = 160 \text{mm}$
  b. Compute f'_s can be computed based on following relation:
     $$f'_s = \epsilon_u E_s \frac{(c - d')}{c} = 315 \text{ MPa} < f_y \text{ Ok.}$$
     Then use the relation that derived for not yielded compression reinforcement:
     $$M_n = M_{n1} + M_{n2} = 0.85 f'_c ab \left(d - \frac{a}{2}\right) + A'_s f'_s (d - d') \blacksquare$$
     where
     $a = \beta_1 c = 129 \text{mm}$
     $M_n = M_{n1} + M_{n2} = 802 \times 10^6 \text{N.mm} + 188 \times 10^6 \text{N.mm} = 990 \text{ kN.m}$
- Compute strength reduction factor $\emptyset$:
  a. Compute steel stain based on the following relations:
     $c = 160 \text{mm} \Rightarrow \epsilon_t = 9.38 \times 10^{-3}$
     It is useful to note, that using of compression reinforcement has increased strain of tensile reinforcement for $\epsilon_t = 6.95 \times 10^{-3}$ to a strain of $\epsilon_t = 9.38 \times 10^{-3}$. Then using of compression reinforcement has increased section ductility (as was discussed in reasons for using of compression reinforcement).
  b. $\epsilon_t > 0.005$, then $\emptyset = 0.9$
- Compute section design strength $\emptyset M_n$:
  $\emptyset M_n = \emptyset \times M_n = 891 \text{ kN.m} \blacksquare$

4.7 DESIGN OF A DOUBLY REINFORCED RECTANGULAR SECTION

4.7.1 Essence of the Problem

- This article discusses the design of a doubly reinforced concrete beam to solve a problem related to the fourth one of the four reasons discussed in previous article, i.e. this article discusses the computing of compression reinforcement A_s' when the designer needs a reinforcement ratio greater than ρ_{max} to resist the applied factored moment M_u.
- Therefore, the knowns of the design problem are:
  - Applied factored moment that must be resisted "M_u".
  - Materials strength f_c' and f_y.
  - Pre-specified beam dimensions b and h determined based on architectural or other limitations. These dimensions have been selected relatively small such that the section cannot resist the required moment with tension reinforcement only.
- While, the main unknowns of the design problem are the tension and compression reinforcements and their details. Selection of adequate stirrups that can act as ties for compression reinforcement is a part of the design process.

4.7.2 Design Procedure

This procedure has been written assuming the designer has no previous indication that the proposed dimensions are inadequate and that the section should be designed as a doubly reinforced section.

1. Compute the required factored moment M_u based on the given spans and loads. As the dimensions have been pre-specified, then beam selfweight can be computed and added to applied loads.
2. Compute the required nominal moment based on following relation:
   $$M_n = \frac{M_u}{\emptyset}$$
   where $\emptyset$ will be assumed 0.9 to be checked later.
3. Check if the section can be designed as a singly reinforced section or not based on following reasoning:
   a. If the square roof of following relation has an imaginary value, then the section cannot be designed as singly reinforced section.
   $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}}$$
   b. If the required steel ratio
   $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}}$$
   is greater than the maximum steel ratio
   $$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$
   then the section cannot be designed as singly reinforced section.
4. Re-compute the required nominal moment for the section that must be designed as a doubly reinforced section based on:
   $$M_n = \frac{M_u}{\emptyset}$$
   As the section is at tensile strain range of ρ_{max}, i.e. at tensile strain "ϵ_t" of 0.004, then the strength reduction factor would be as indicated in Figure 4.7-1 below.
   $\emptyset = 0.816$

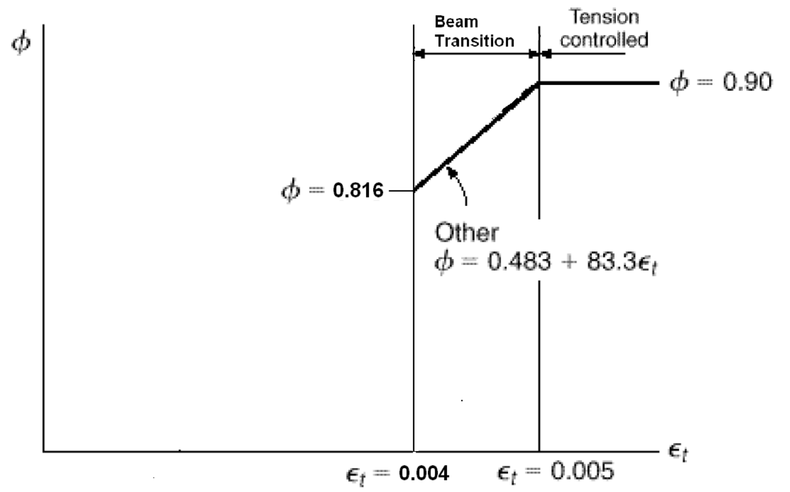


Figure 4.7-1: Strain versus strength reduction factor for beams according to ACI code, reproduced for convenience.

In design process of a doubly reinforced section, it is useful to imagine that the nominal flexure strength M_n is consisting of two parts shown below:

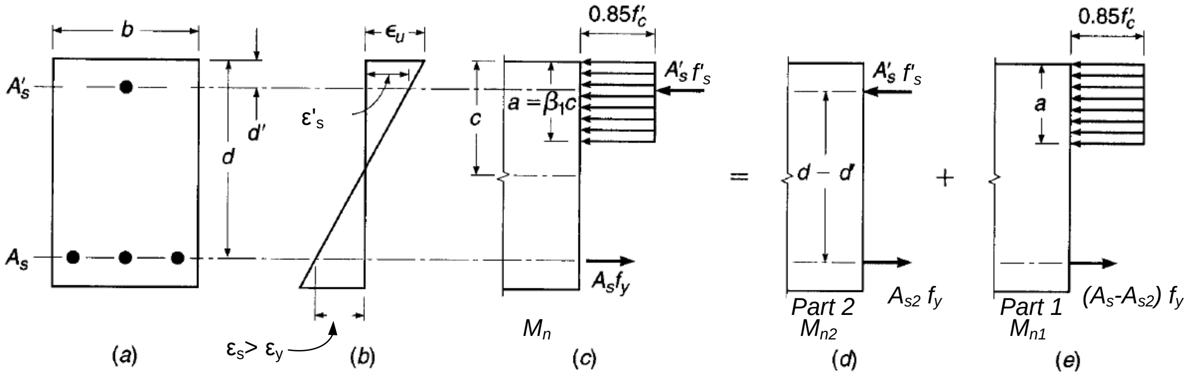


Figure 4.7-2: Strain, stress, and force distribution adopted in design of a doubly reinforced rectangular beam.

5. Compute of Tension Reinforcement A_s:
   a. Compute the nominal moment and tension reinforcement for part 1:
   $A_{s1} = A_{smax} = \rho_{max} bd$
   $M_{n1} = \rho_{max} f_y bd^2 \left(1 - 0.59\ \frac{\rho_{max} f_y}{f_c'}\right)$
   b. Compute the nominal moment and tension reinforcement for part 2:
   $M_{n2} = M_n - M_{n1}$
   $A_{s2} = \frac{M_{n2}}{f_y(d - d')}$
   c. Compute the ***Total Tension Reinforcement A_s***:
   $A_s = A_{s1} + A_{s2}$ ■
6. Compute of Compression Reinforcement A_s':
   a. Check if compression reinforcement is yielded or not:
   Compute of "a" based on force diagram of Part 1:
   $a = \frac{A_{s1} f_y}{0.85 f_c' b}$
   then compute "c":
   $c = \frac{a}{\beta_1}$
   and compute of compressive stress in compression reinforcement:
   $f_s' = \epsilon_u E_s \frac{c - d'}{c}$
   b. If $f_s' \geq f_y$, then the compression reinforcement has yielded:
   $f_s' = f_y$
   $A_s' = A_{s2}$ ■
   c. Else, the compression reinforcement is not yielded:
   $A_s' = A_{s2} \frac{f_y}{f_s'}$ ■

7. Compute the Required Rebars Numbers.
8. Ties Design:
    a. Select bar diameter for ties:
    If single compression rebars with diameter of:
    $\emptyset_{\text{Bar}} \leq 32\text{mm}$
    then
    $\emptyset_{\text{Tie}} = 10\text{mm}$
    else, use:
    $\emptyset_{\text{Tie}} = 13\text{mm}$
    b. Compute the required spacing of the ties:
    $S_{\text{Required for Ties}} = \min[16d_{\text{bar}}, 48d_{\text{ties}}, \text{Least dimension of column}]$
    This spacing must be checked with the shear requirement also. Actual design practice is to select "S" based on shear requirement (As will be discussed in Chapter 5) and then check its adequacy for ties requirements.
    c. Use a suitable ties arrangement as discussed previously.
9. Draw the final section details.

4.7.3 Example

Example 4.7-1

A rectangular beam, that must carry a service live load of $36.0\ \text{kN/m}$ and a dead load of $15.3\ \text{kN/m}$ (including its selfweight) on a simple span of $5.49\ \text{m}$, is limited in cross section for architectural reasons to 250mm width and 500mm depth. Design this beam for flexure. In your design, assume the following:

- $f_y = 414\ \text{Mpa},\ f_c{}' = 27.5\ \text{Mpa}$
- No. 29 for longitudinal tension reinforcement.
- No. 19 for compression reinforcement if required.
- No. 10 for stirrups (it's adequacy must be checked when used as a tie).
- Two layers of tension reinforcement.

Solution

- Compute the required factored moment M_u:

$$M_{\text{Dead}} = \frac{15.3\frac{\text{kN}}{\text{m}} \times 5.49^2\text{m}^2}{8} = 57.6\ \text{kN.m}\quad M_{\text{Live}} = \frac{36.0\frac{\text{kN}}{\text{m}} \times 5.49^2\text{m}^2}{8} = 136\ \text{kN.m}$$

$$M_u = \text{maximum of } [1.4M_{\text{Dead}} \text{or } 1.2M_{\text{Dead}} + 1.6M_{\text{Live}}]$$

$$M_u = maximum\ of\ [1.4 \times 57.6\ kN.m\ or\ 1.2 \times 57.6\ kN.m + 1.6 \times 136\ kN.m]$$

$$M_u = \text{maximum of } [80.6\ \text{kN.m or } 287\ \text{kN.m}] = 287\ \text{kN.m}$$

- Compute the required nominal moment based on following relation:

$$M_n = \frac{M_u}{\emptyset} = \frac{287}{0.9} = 319\ \text{kN.m}$$

where $\emptyset$ will be assumed 0.9 to be checked later.

- Check if the section can be design as a singly reinforced section or not based on following reasoning:

$$d = 500 - 40 - 10 - 29 - \frac{25}{2} = 409\text{mm}$$

$$\rho_{\text{Required}} = \frac{1 - \sqrt{1 - 2.36\frac{M_n}{f_c'bd^2}}}{1.18 \times \frac{f_y}{f_c'}} = \frac{1 - \sqrt{1 - 2.36\frac{319 \times 10^6}{27.5 \times 250 \times 409^2}}}{1.18 \times \frac{414}{27.5}} = 23.2 \times 10^{-3}$$

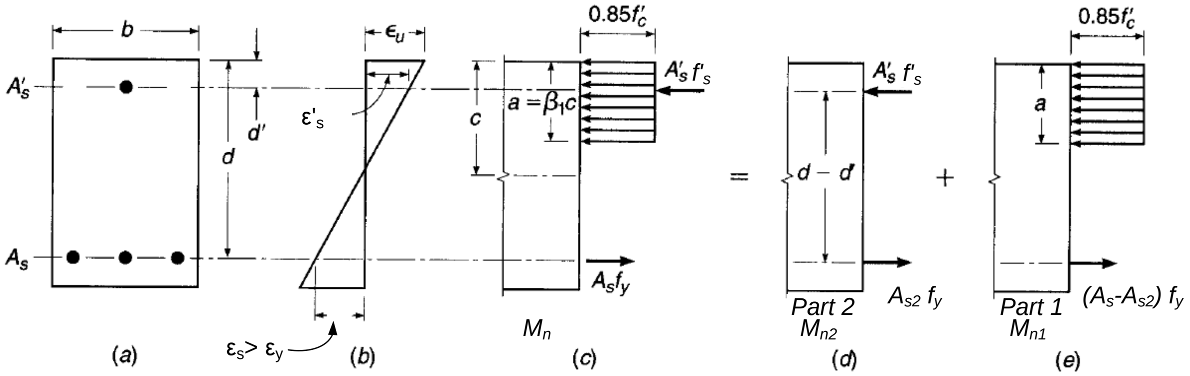
$$\rho_{\max} = 0.85\beta_1\ \frac{f_c'}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85 \times 0.85\ \frac{27.5}{414}\ \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} < \rho_{\text{Required}}$$

then the section must be design as a doubly reinforced section.

- Re-compute the required nominal for the section based on $\emptyset = 0.816$:

$$M_n = \frac{M_u}{\emptyset} = \frac{287}{0.816} = 352\ \text{kN.m}$$

It is useful to imagine that the nominal flexure strength M_n is consisting of two parts shown below:



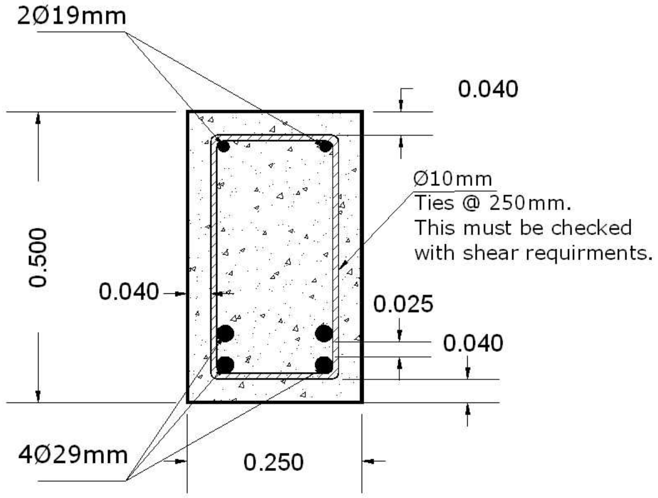
- Compute of Tension Reinforcement As:
  - Compute the nominal moment and tension reinforcement for part 1:
    $A_{s1} = A_{smax} = \rho_{max}bd == 20.6 \times 10^{-3} \times 250 \times 409 = 2\,106\ mm^2$
    $$M_{n1} = \rho_{max}f_ybd^2\left(1 - 0.59\ \frac{\rho_{max}f_y}{f_c'}\right)$$
    $$M_{n1} = 20.6 \times 10^{-3} \times 414 \times 250 \times 409^2\left(1 - 0.59\ \frac{20.6 \times 10^{-3} \times 414}{27.5}\right) = 291\ kN.m$$
  - Compute the nominal moment and tension reinforcement for part 2:
    $M_{n2} = M_n - M_{n1} = 352\ kN.m - 291\ kN.m = 61\ kN.m$
    $$d' = 40 + 10 + \frac{19}{2} = 59.2$$
    $$A_{s2} = \frac{M_{n2}}{f_y(d - d')} = \frac{61 \times 10^6}{414 \times (409 - 59.2)} = 421\ mm^2$$
  - Compute the Total Tension Reinforcement A_s:
    $A_s = A_{s1} + A_{s2} = 2\,106\ mm^2 + 421\ mm^2 = 2\,527\ mm^2$
- Compute of Compression Reinforcement A_s':
  - Check if compression reinforcement is yielded or not:
    Compute of "a" based on force diagram of Part 1:
    $$a = \frac{A_{s1}f_y}{0.85f_c'b} = \frac{2\,106\ mm^2 \times 414\ MPa}{0.85 \times 27.5MPa \times 250mm} = 149\ mm$$
    then compute "c":
    $$c = \frac{a}{\beta_1} = \frac{149\ mm}{0.85} = 175\ mm$$
    and compute of compressive stress in compression reinforcement:
    $$f_s' = \epsilon_u E_s\frac{c - d'}{c} = 0.003 \times 200\,000 \times \frac{175\ mm - 59.5\ mm}{175\ mm} = 396\ MPa < f_y$$
  - Then compression reinforcement is not yielded and compression reinforcement will be:
    $$A_s' = A_{s2}\frac{f_y}{f_s'} = 421\ mm^2 \times \frac{414\ MPa}{396\ MPa} = 440\ mm^2$$
- Compute the Required Rebars Numbers.
  $$\text{Number of Tension Rebars} = \frac{(2\,527\ mm^2)}{\frac{\pi \times 29^2}{4}} = \frac{(2\,527\ mm^2)}{660\ mm^2} = 3.83$$
  Then use $4\emptyset 29mm$ for tension reinforcement.
  Check if these rebars can be put in one layer:
  $b_{Required} = 40 \times 2 + 10 \times 2 + 29 \times 4 + 29 \times 3 = 303mm > b_{Provided}$
  Then, the rebars must be put in two layers as the designer has assumed.
  $$\text{Number of Compression Rebars} = \frac{(440\ mm^2)}{\frac{\pi \times 19^2}{4}} = \frac{(440\ mm^2)}{283\ mm^2} = 1.55$$
  Then use $2\emptyset 19mm$ for compression reinforcement.
- Design of Required Ties:
  - Select bar diameter for ties:
    $\because \emptyset_{Bar} = 19mm < 32mm$ and single rebar.
    then $\emptyset_{Tie} = 10mm\ \ Ok.$

Compute the required spacing of the ties:

$$S_{\text{Required for Ties}} = \min[16d_{bar}, 48d_{ties}, \text{Least dimension of column}]$$
$$= \min[16 \times 19, 48 \times 10, 250]$$
$$S_{\text{Required for Ties}} = \min[394, 480, 250] = 250 \text{ mm}$$

Use $\emptyset 10\text{mm} @ 250\text{mm}$ for ties. This spacing must be checked with shear requirement as will be discussed in Chapter 4.

- Draw the final section details:



4.7.4 Homework Problems

Problem 4.7-1

Design a rectangular beam to carry a service live load moment of 561 kN.m and a service dead load of 317 kN.m (including moment due to beam selfweight). In your design assume the following:

1. A width of 350mm and a depth of 750mm (these dimensions have been determined based on architectural limitations).
2. Materials of $f_c' = 34.5 \text{ MPa}$ and $f_y = 414 \text{ MPa}$.
3. Two layers of longitudinal reinforcement.
4. Bar diameter of 25mm for longitudinal reinforcement.
5. Bar diameter of 10 mm for stirrups.

Answers

1. Compute the required factored moment M_u:
$$M_u = \text{maximum of } [1.4M_{Dead} \text{ or } 1.2M_{Dead} + 1.6M_{Live}]$$
$$M_u = maximum\ of\ [1.4 \times 317\ kN.m\ or\ 1.2 \times 317\ kN.m + 1.6 \times 561\ kN.m]$$
$$M_u = \text{maximum of } [444 \text{ kN.m or } 1\,278 \text{kN.m}] = 1\,278 \text{ kN.m}$$
2. Compute the required nominal moment based on following relation:
$$M_n = \frac{M_u}{\emptyset} = 1\,420 \text{ kN.m}$$
where $\emptyset$ will be assumed 0.9 to be checked later.
3. Check if the section can be design as a singly reinforced section or not based on following reasoning:
$$d = 662 \text{ mm}$$
$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}} = 27.9 \times 10^{-3}$$
$$\beta_1 = 0.8$$
$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 24.3 \times 10^{-3} < \rho_{Required}$$
then the section must be design as doubly reinforced section
4. Re-compute the required nominal for the section based on $\emptyset = 0.816$:
$$M_n = \frac{M_u}{\emptyset} = 1\,566 \text{ kN.m}$$