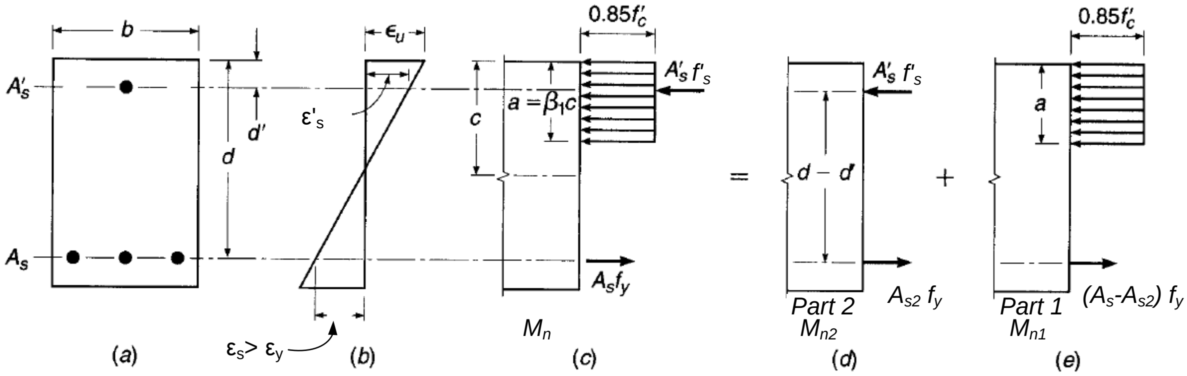
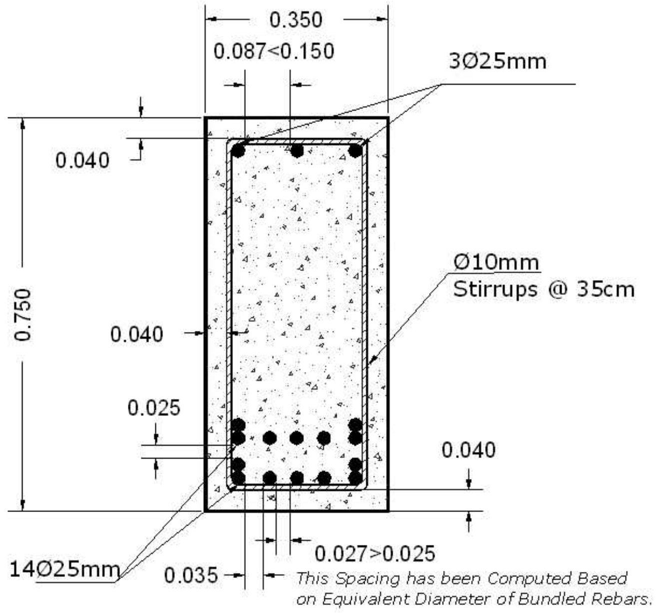
The nominal flexure strength M_n is considered to consist of the two parts shown below:



5. Compute of Tension Reinforcement As:
   a. Compute the nominal moment and tension reinforcement for part 1:
   $A_{s1} = A_{smax} = \rho_{max} bd = 5\,630 \text{ mm}^2$
   $$M_{n1} = \rho_{max} f_y bd^2 \left(1 - 0.59\,\frac{\rho_{max} f_y}{f_c'}\right) = 1\,278 \text{ kN.m}$$
   b. Compute the nominal moment and tension reinforcement for part 2:
   $M_{n2} = M_n - M_{n1} = 288 \text{ kN.m}$
   $d' = 62.5$
   $$A_{s2} = \frac{M_{n2}}{f_y(d - d')} = 1\,160 \text{ mm}^2$$
   c. Compute the Total Tension Reinforcement A_s:
   $A_s = A_{s1} + A_{s2} = 6\,790 \text{ mm}^2$
6. Compute of Compression Reinforcement A_s':
   a. Check if compression reinforcement is yielded or not:
   Compute of "a" based on force diagram of Part 1:
   $$a = 227 \text{ mm} \Rightarrow c = 284 \text{ mm} \Rightarrow f_s' = \epsilon_u E_s \frac{c - d'}{c} = 468 \text{ MPa} > f_y$$
   b. Then compression reinforcement is yielded and it's area will be:
   $f_s' = f_y = 414 \text{ MPa} \Rightarrow A_s' = A_{s2} = 1\,160 \text{ mm}^2$
7. Compute the Required Rebars Numbers.
   $\text{Number of Tension Rebars} = 13.8$
   Then use $14\emptyset25\text{mm}$ for tension reinforcement.
   Check if these rebars can be put in two layers:
   $b_{Required} = 40 \times 2 + 10 \times 2 + 7 \times 25 + 6 \times 25 = 425\text{mm} > b_{Provided}$
   Then, the rebars must be put in more than two layers. This problem can be solved through using of Bundled Bars (See Section Details)
   $\text{Number of Compression Rebars} = 2.36$
   Then use $3\emptyset25\text{mm}$ for compression reinforcement.
8. Design of Required Ties:
   a. Select bar diameter for ties:
   $\because \emptyset_{Bar} = 25\text{mm} \le 32\text{mm}$
   It is useful to note that ties design is depending on diameter of compression reinforcement and not on tension reinforcement. Therefore, the designer compare with diameter of compression reinforcement instead of comparison with equivalent diameter of Bundled Bars.
   Then
   $\emptyset_{Tie} = 10\text{mm Ok.}$
   b. Compute the required spacing of the ties:
   $S_{Required\ for\ Ties} = \min[16 d_{bar}, 48 d_{ties}, \text{Least dimension of column}]$
   $S_{Required\ for\ Ties} = \min[16 \times 25, 48 \times 10\ , 350] = 350 \text{ mm}$

Use $\emptyset 10\text{mm} @ 350\text{mm}$ for ties. This spacing must be checked with shear requirement as will be discussed in Chapter 5.

9. Draw the final section details:



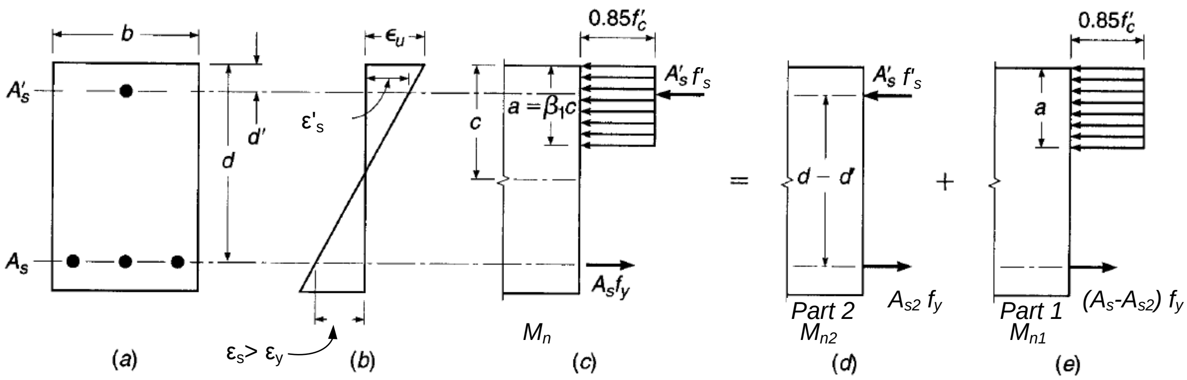
Problem 4.7-2

Resolve previous problem with using of:

1. Materials of $f_c' = 21\ \text{MPa}$ and $f_y = 420\ \text{MPa}$.
2. Two layers of longitudinal reinforcement.
3. Bar diameter of 32 mm for longitudinal reinforcement ($A_{bar} = 819\ \text{mm}^2$).
4. Bar diameter of 12 mm for stirrups.

Answers

1. Compute the required factored moment M_u:
   As for previous problem:
   $M_u = 1\,278\ kN.m$
2. Compute the required nominal moment based on following relation:
   $M_n = \frac{M_u}{\emptyset} = \frac{1\,278}{0.9} = 1\,420\ kN.m$
   where $\emptyset$ will be assumed 0.9 to be checked later.
3. Check if the section can be design as a singly reinforced section or not based on following reasoning:
   $d = 655\ mm$
   $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}} = \frac{1 - 0.257i}{1.18 \times \frac{420}{21}}$$
   As the quantity under square root of above relation has a negative value, then the section cannot be designed as Singly Reinforced Section.
4. Re-compute the required nominal for the section based on $\emptyset = 0.816$:
   $M_n = \frac{M_u}{\emptyset} = 1\,566\ kN.m$
   The nominal flexure strength M_n is considered to consist of the two parts shown below:



5. Compute of Tension Reinforcement As:
   a. Compute the nominal moment and tension reinforcement for part 1:

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 15.5 \times 10^{-3}$$

$$A_{s1} = A_{smax} = \rho_{max} bd = 3\,548\, mm^2$$

$$M_{n1} = \rho_{max} f_y bd^2 \left(1 - 0.59 \frac{\rho_{max} f_y}{f_c'}\right) = 796\, kN.m$$

   b. Compute the nominal moment and tension reinforcement for part 2:

$$M_{n2} = M_n - M_{n1} = 770\, kN.m$$

$$d' = 64.5$$

$$A_{s2} = \frac{M_{n2}}{f_y(d - d')} = 3\,110\, mm^2$$

   c. Compute the Total Tension Reinforcement $\boldsymbol{A_s}$:

$$A_s = A_{s1} + A_{s2} = 3\,548\, mm^2 + 3\,110\, mm^2 = 6\,658\, \text{mm}^2$$

6. Compute of Compression Reinforcement A_s':
   a. Check if compression reinforcement is yielded or not:
   Compute of "a" based on force diagram of Part 1:

$$a = \frac{A_{s1} f_y}{0.85 f_c' b} = 239\, mm \Rightarrow c = \frac{a}{\beta_1} = 281\, mm$$

   And compute of compressive stress in compression reinforcement:

$$f_s' = \epsilon_u E_s \frac{c - d'}{c} = 462\, MPa > f_y$$

   b. Then compression reinforcement is yielded and it's area will be:

$$A_s' = A_{s2} = 3\,110\, mm^2$$

7. Compute the Required Rebars Numbers.

$$Number\ of\ Tension\ Rebars = \frac{6\,658\ \text{mm}^2}{819} = 8.13$$

Then use $9\emptyset 32mm$ for tension reinforcement.

Check if these rebars can be put in two layers:

$$b_{Required} = 40 \times 2 + 12 \times 2 + 5 \times 32 + 4 \times 32 = 392mm > b_{Provided}\ \text{Not Ok.}$$

To solve a problem that related to distribution Bundled Rebars will be used.

$$Number\ of\ Compression\ Rebars = \frac{3\,110\, mm^2}{819\ \text{mm}^2} = 3.80$$

Then use $4\emptyset 32mm$ for compression reinforcement.

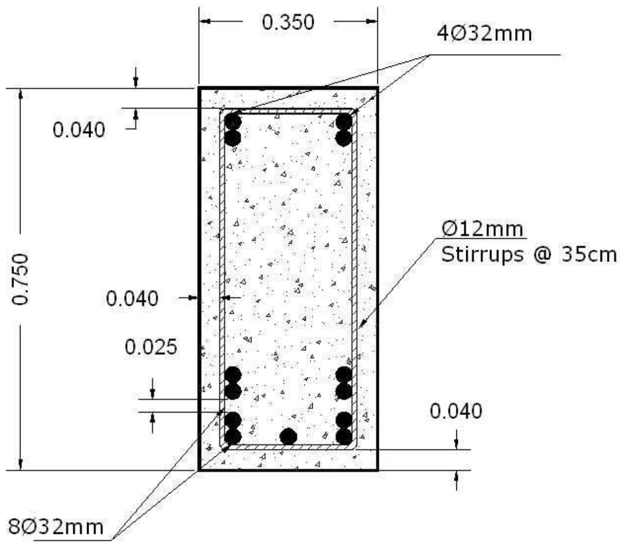


Figure 4.7-3: Detailed section for Problem 4.7-2.

8. Design of Required Ties:
    a. Select bar diameter for ties:
    As designer intend to use bundled compression rebars, then
    $\emptyset_{Tie} = 12\ mm\ \ Ok.$
    b. Compute the required spacing of the ties:

$S_{Required\ for\ Ties} = \min[16d_{bar}, 48d_{ties}\ , Least\ dimension\ of\ column]$

$\mathrm{S_{Required\ for\ Ties}} = \min[400, 576, 350]$

$\mathrm{S_{Required\ for\ Ties}} = 350\ \mathrm{mm}$

Use $\emptyset 12mm\ @\ 350mm$ for ties. This spacing must be checked with shear requirement as will be discussed in Chapter 4.

9. Draw the final section details as indicated in Figure 4.7-3.

4.8 FLEXURE ANALYSIS OF A SECTION WITH T SHAPE

4.8.1 Construction Stages

- During construction, the concrete in columns is placed and allowed to harden before the concrete in the floor beam is placed. In next operation, concrete is placed in the slab and beams in a monolithic pour, article 26.5.7.2 of (ACI318M, 2014).
- As a result, the slab serves as the top flange of the beam as indicated by shading area in the Figure 4.8-1 below:

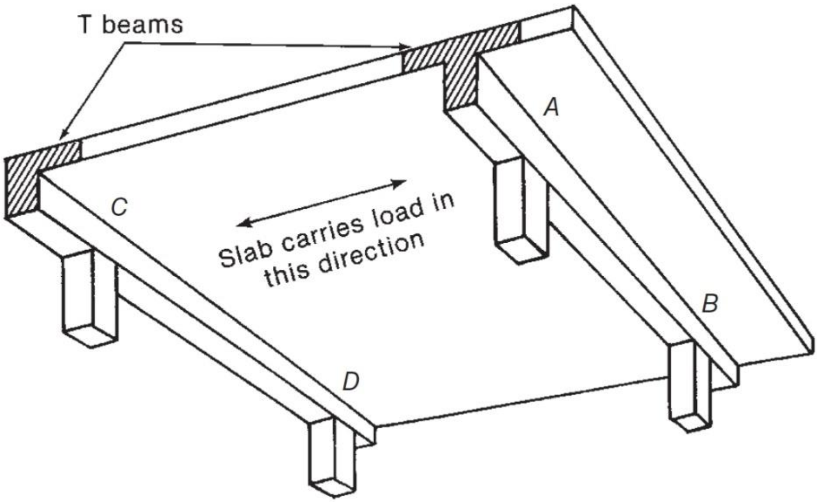


Figure 4.8-1: Slab beam interaction due to monolithic casting.

- Such a beam is referred to as a ***T Beam***. The interior beam, AB, of the Figure 4.8-1 above, has a flange on both sides. The spandrel beam, CD, with flange on one side only, is also referred to as a T Beam.

4.8.2 Behavior of Tee Beams

- An exaggerated deflected view of the interior beam “AB” is shown in Figure 4.8-2 below:

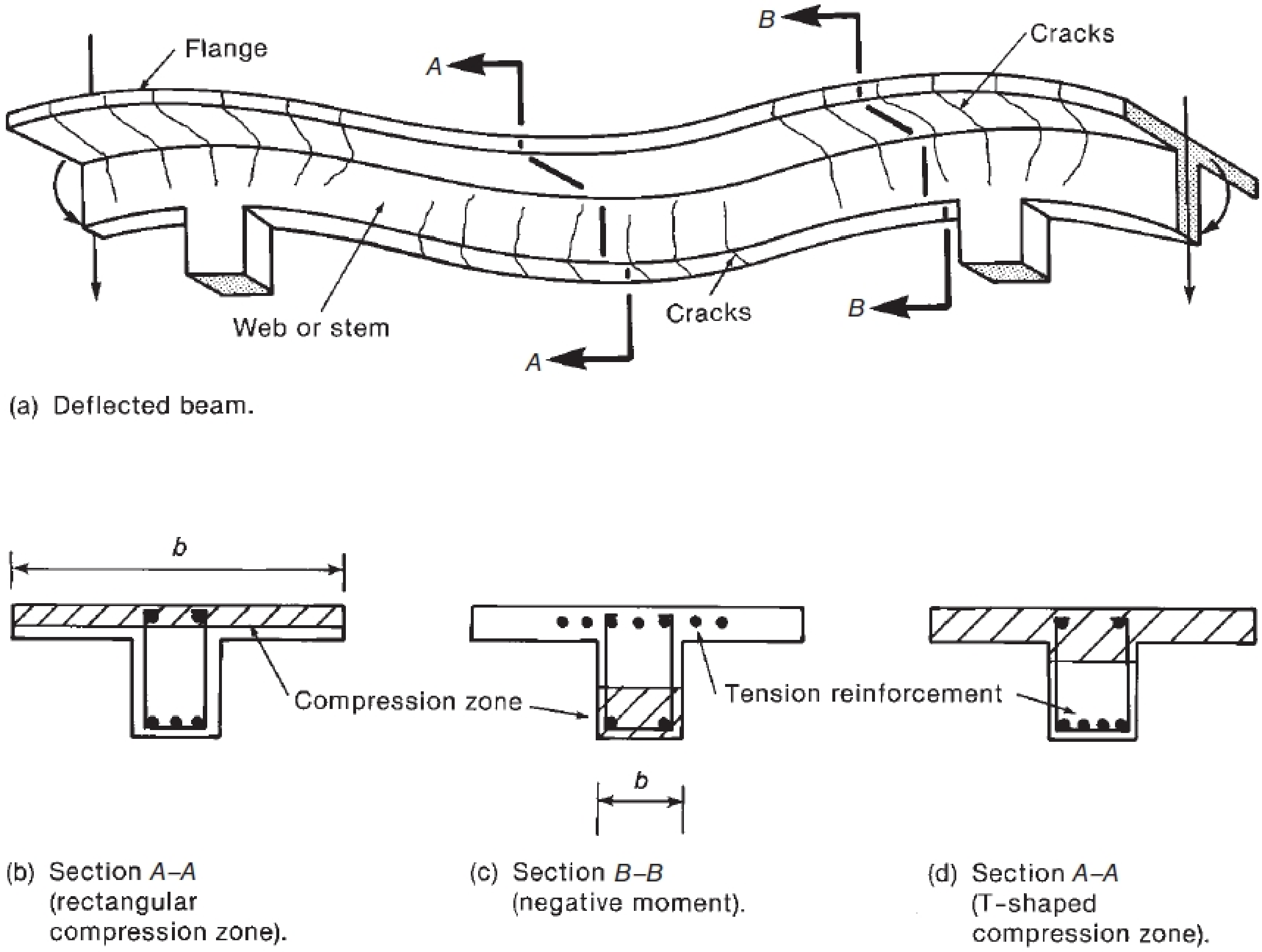


Figure 4.8-2: Exaggerated deflected view for a continuous beam with Tee shape.

- Form above deflected shape, following points can be concluded:
  - At mid-span, the compression zone is in flange as shown in Figure 4.8-2 "b" and "d" above.
  - Generally, it is rectangular as shown in Figure "b", although in a few cases, the neutral axis may shift down into the web, giving a T-shaped compression zone.
  - At the support, the compression zone is at the bottom of the beam and is rectangular, as shown in Figure "c"

4.8.3 Notations Adopted in Design of Tee Beams

Notations indicated in Figure 4.8-3 below are adopted in analysis and design of T Section.

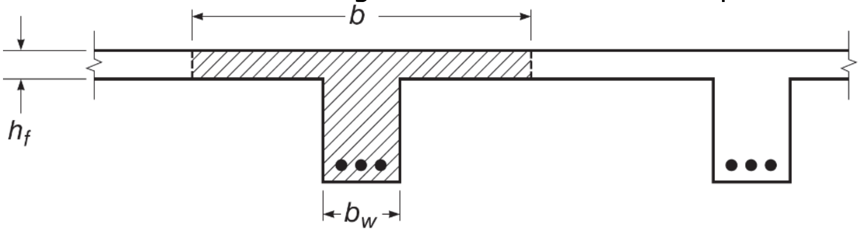


Figure 4.8-3: Notations adopted in analysis and design of Tee beams.

4.8.4 Procedure for Analysis of a Beam with T-Shape

Checking the adequacy of a T-Shape beam according to the requirements of ACI Code can be summarized as follows:

1. Definition of Section Dimensions:
   a. The first question that must be answered in the analysis of T section is "What is the part of the slab that will act as a compression flange for the T beam?"
   Due shearing deformation of the flange, which relieves the more remote elements of some compressive stress, ***shear-lag phenomenon***, actual compression stress in the beam flange varies as indicated in Figure 4.8-4 below.

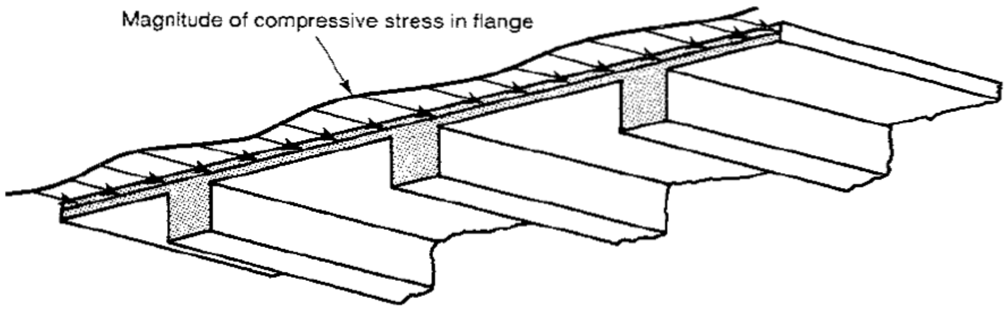


Figure 4.8-4: Actual distribution of compressive stresses in Tee flange.

According to ACI, the variable compressive stresses that acting on the overall width, b_o, in Figure 4.8-5 below can be replaced by an equivalent uniformly distributed compressive force that acting on an effective width, b.

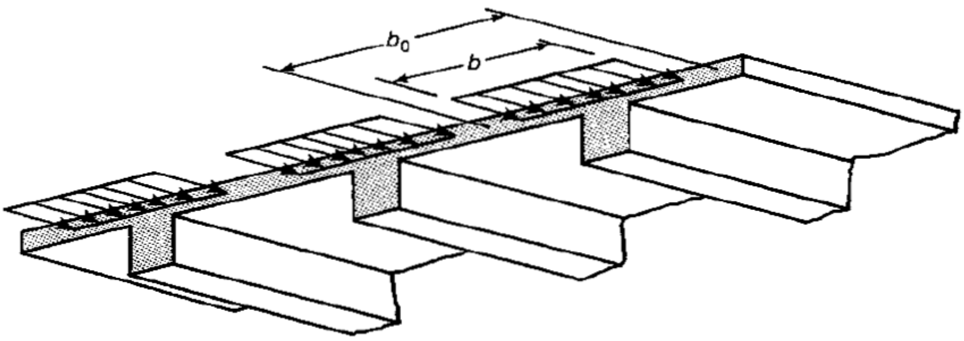


Figure 4.8-5: Equivalent uniform flange stresses adopted by the ACI code, 3D view.

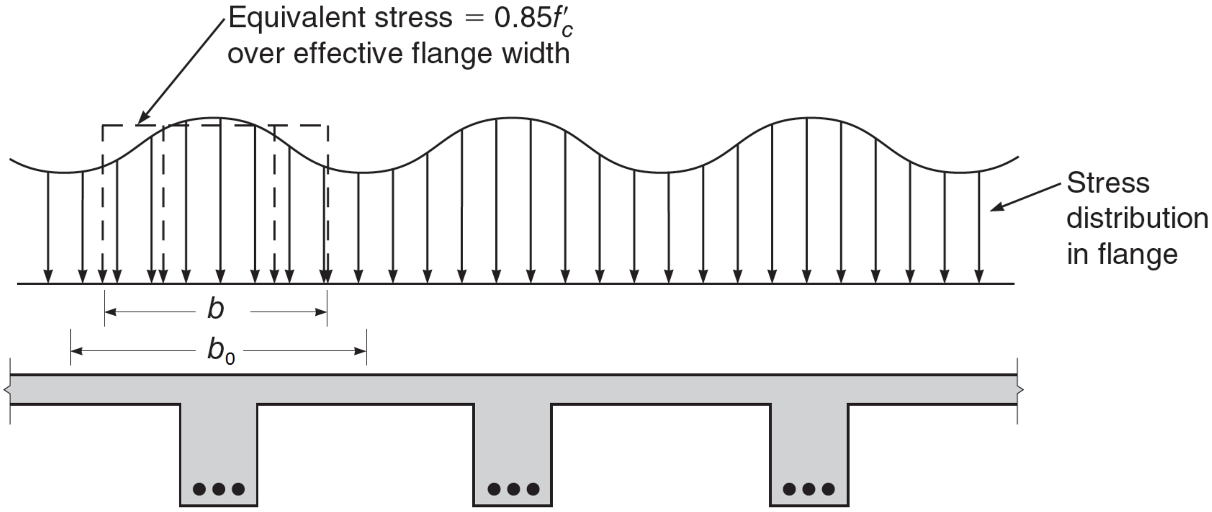


Figure 4.8-5: Equivalent uniform flange stresses adopted by the ACI code, a sectional view.

b. According to ACI Code (6.3.2.1), for nonprestressed T-beams supporting monolithic or composite slabs, the effective flange width, b, shall include the beam web width, b_w, plus an effective overhanging flange width in accordance with Table 4.8-1 below, where h is the slab thickness and s_w is the clear distance to the adjacent web:

Table 4.8-1: Dimensional limits for effective overhanging flange width for T-beams, Table 6.3.2.1 of the (ACI318M, 2014).

Flange location	Effective overhanging flange width, beyond face of web	
Each side of web	Least of:	$8h$
		$s_w/2$
		$\ell_n/8$
One side of web	Least of:	$6h$
		$s_w/2$
		$\ell_n/12$

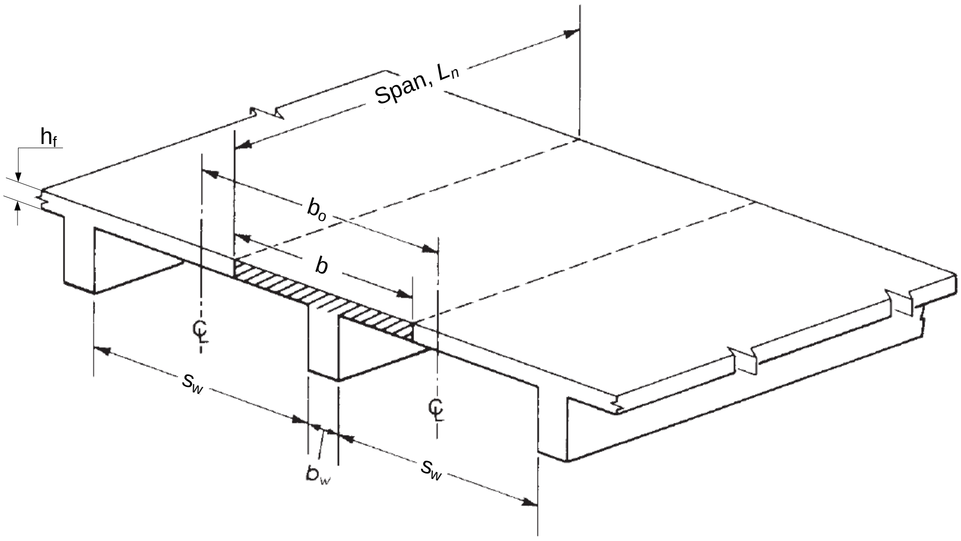


Figure 4.8-6: Notations of Table 4.8-1.

c. According to article 6.3.2.2 of the (ACI318M, 2014), isolated nonprestressed T-beams in which the flange is used to provide additional compression area shall have a flange thickness greater than or equal to $0.5b_w$ and an effective flange width less than or equal to $4b_w$.

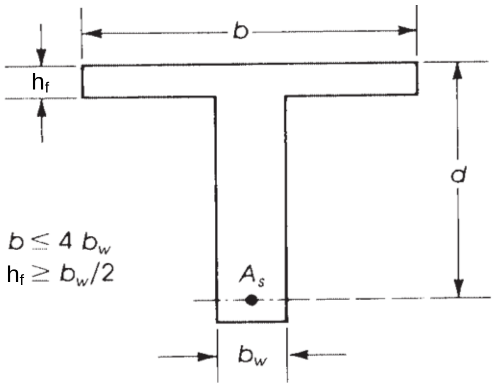


Figure 4.8-7: Isolated T-shaped sections.

2. Checking the Section Type:
   Check if the failure is secondary compression failure or compression failure through following comparison:
   $\rho_w \ ? \ \rho_{w\,max}$
   where
   $$\rho_w = \frac{A_s}{b_w d}$$
   To derive a relation for computing of $\rho_{w\,max}$ it is useful to imagine that the T section is consists of following two parts indicated in Figure 4.8-8 below
   Then, based on $\sum F_x = 0$, one can show that:
   $$\rho_{w\,max} = \frac{A_{s\,max}}{b_w d} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$
   or $\rho_{w\,max} = \rho_{max} + \rho_f$■
   where $\rho_{w\,max} = \frac{A_{s\,max}}{b_w d}$
   $$\rho_f = \frac{A_{sf}}{b_w d}, A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y}$$

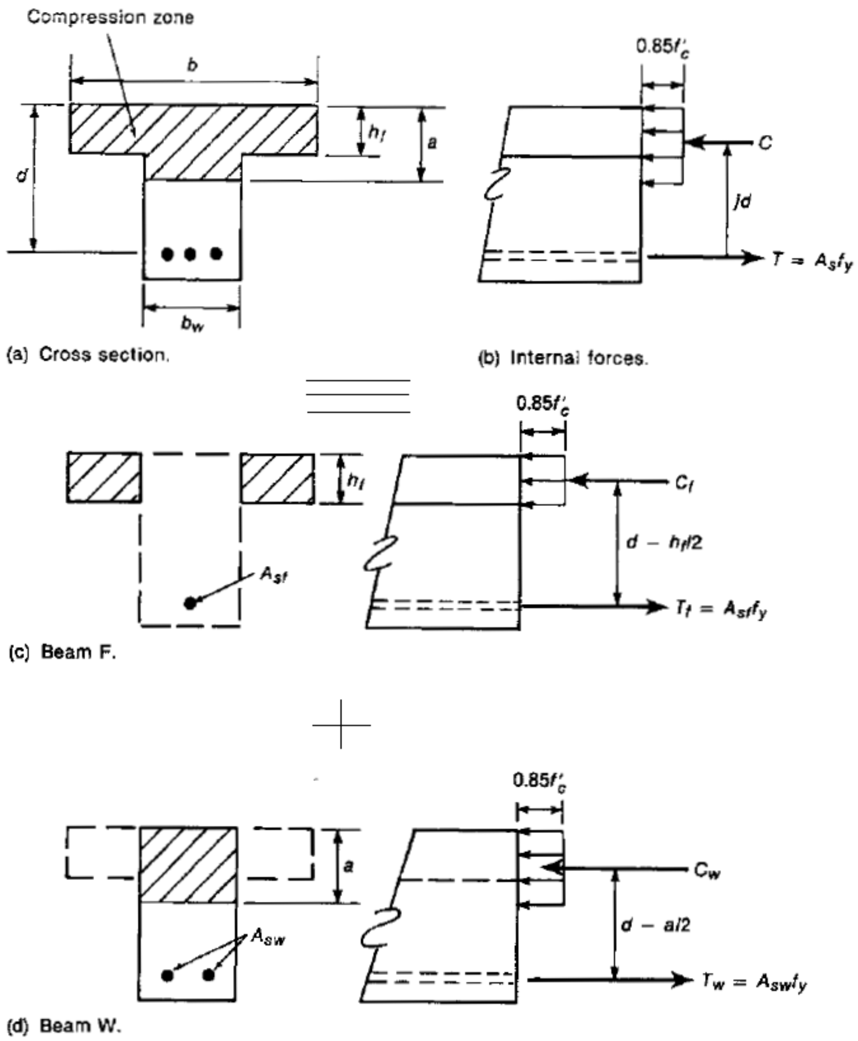


Figure 4.8-8: Imaginary two parts for Tee beam for analysis purpose.

3. Checking of $A_{s\,minimum}$ limitation
   As the flange is under compression stress, then the minimum steel area shall be compute based on ACI (9.6.1.2):
   $$A_{s\,minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$
4. Computing of Nominal Flexure Strength "M_n":
   As the relation for computing of M_n depends on location of compression block, if it is in the flange or extend to the web. Then the analyzer must first check to see if "a" is less than h_f or not (See Figure 4.8-9 below).

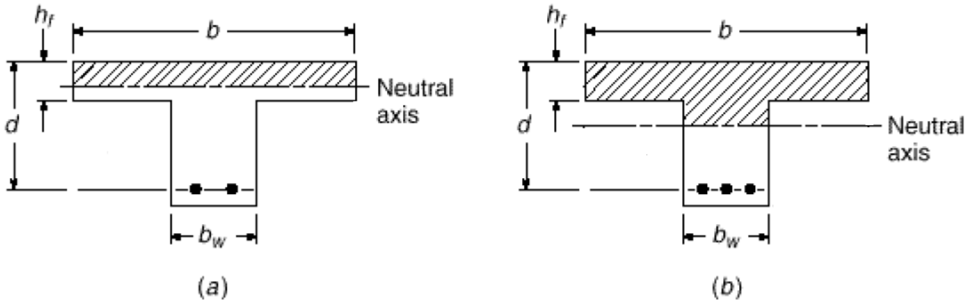


Figure 4.8-9: Possible location of compression block of Tee beams.

a. Assume that $a \leq h_f$ (based on experience, this can be considered as the general case):

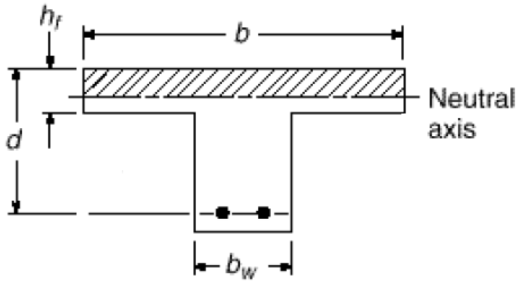


Figure 4.8-10: Compression block within section flange.

$$\because \sum F_x = 0$$
$$\therefore 0.85 f_c' \, b \, a = A_s f_y$$
$$a = \frac{A_s f_y}{0.85 f_c' b}$$

b. If $a_{Computed} \leq h_f$, then above assumption is correct and nominal flexure strength M_n can be computed based on:
$$M_n = A_s f_y \left(d - \frac{a}{2}\right) \blacksquare$$

c. Else (i.e. $a_{Computed} > h_f$) then the nominal flexure strength M_n will be considered to be consist from two parts shown in Figure 4.8-11 below:
Compute A_{sf} based on:
$$\sum\nolimits_{\text{for Part Beam F}} F_x = 0$$
$$A_{sf} f_y = 0.85 f_c' h_f (b - b_w)$$
$$A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y} \blacksquare$$
Compute the correct value of "a" based on Part "Beam W":
$$\sum\nolimits_{\text{for Part Beam W}} F_x = 0$$
$$(A_s - A_{sf}) f_y = 0.85 f_c' a (b - b_w)$$
$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' (b_w)}$$
Compute M_n based on following relation:
$$M_n = [0.85 f_c' h_f (b - b_w)] \left(d - \frac{h_f}{2}\right)_{M_n for\ Part\ Beam\ F} + [0.85 f_c' a b_w] \left(d - \frac{a}{2}\right)_{M_n for\ Part\ Beam\ W} \blacksquare$$

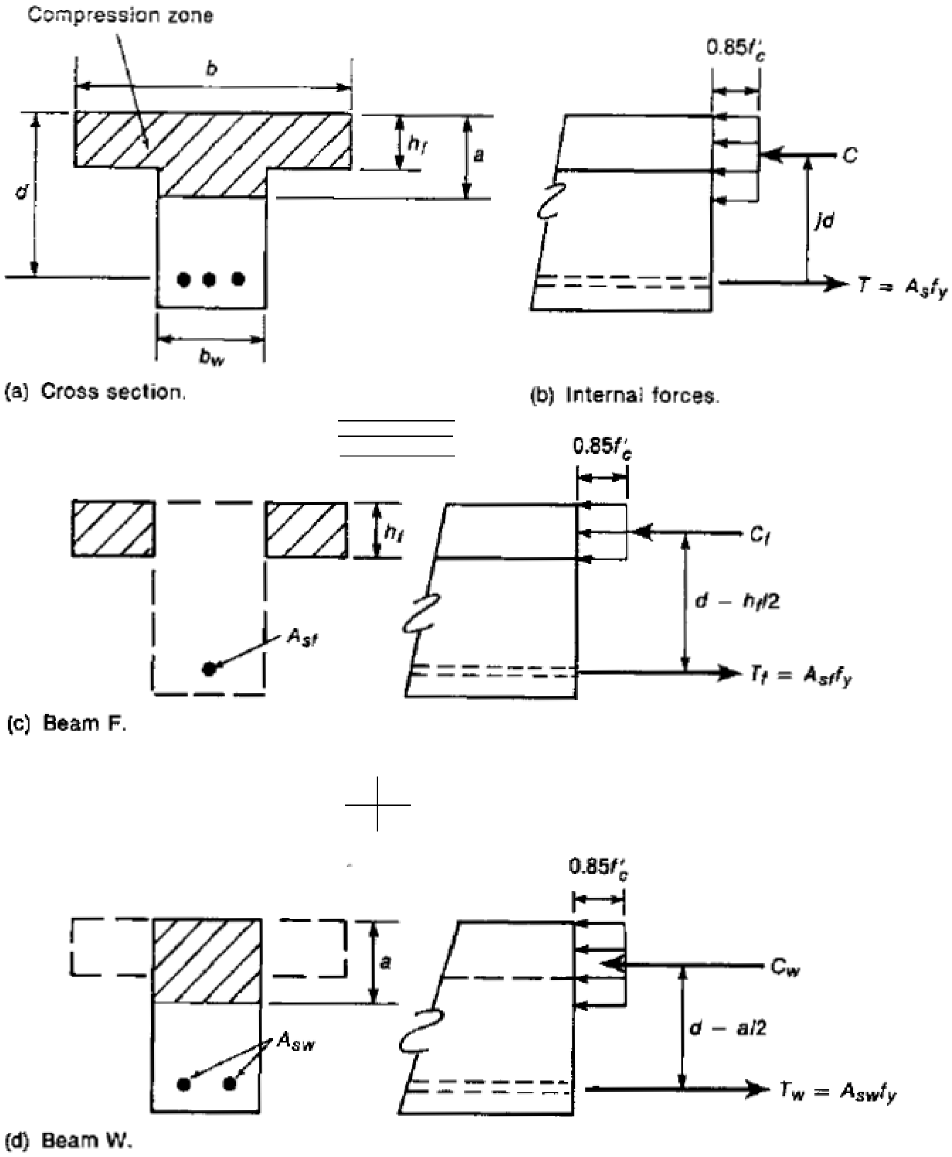


Figure 4.8-11: Analysis parts when compression block within flange.

5. Compute the Strength Reduction Factor Ø Based on Following Relation:
   a. Compute steel stain based on the following relations:
      $$c = \frac{a}{\beta_1}$$
      $$\epsilon_t = \frac{d - c}{c}\epsilon_u$$
   b. If $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.
   c. If $\epsilon_t < 0.005$, then compute more accurate Ø:
      $$\emptyset = 0.483 + 83.3\epsilon_t$$
6. Finally Compute the Design Flexure Strength of Section $\emptyset M_n$:
   $$\emptyset M_n = \emptyset \times M_n$$

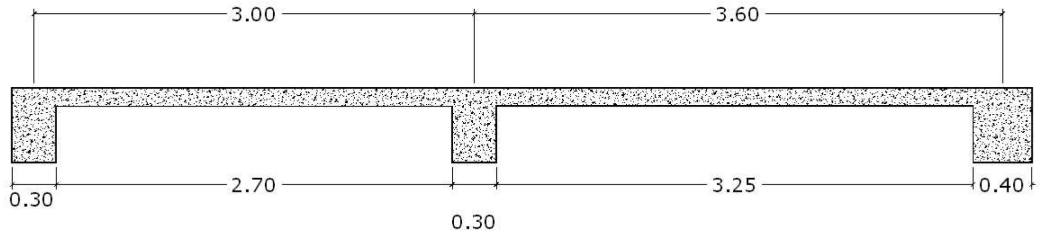
4.8.5 Examples

Example 4.8-1

Check the adequacy of the interior T-beam shown below for ACI Code requirements and determine its design strength.

Assume that:

- $f_y = 300$ Mpa
- $f_c' = 20$ Mpa.
- Beam Span 5.5m.
- $A_{Bar\ for\ \emptyset\ 19mm} = 284mm^2$.



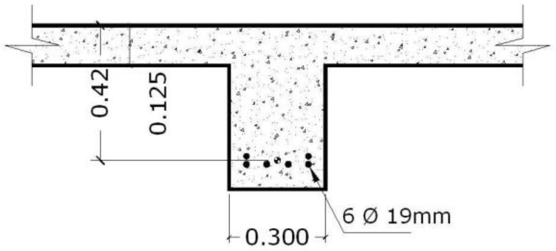
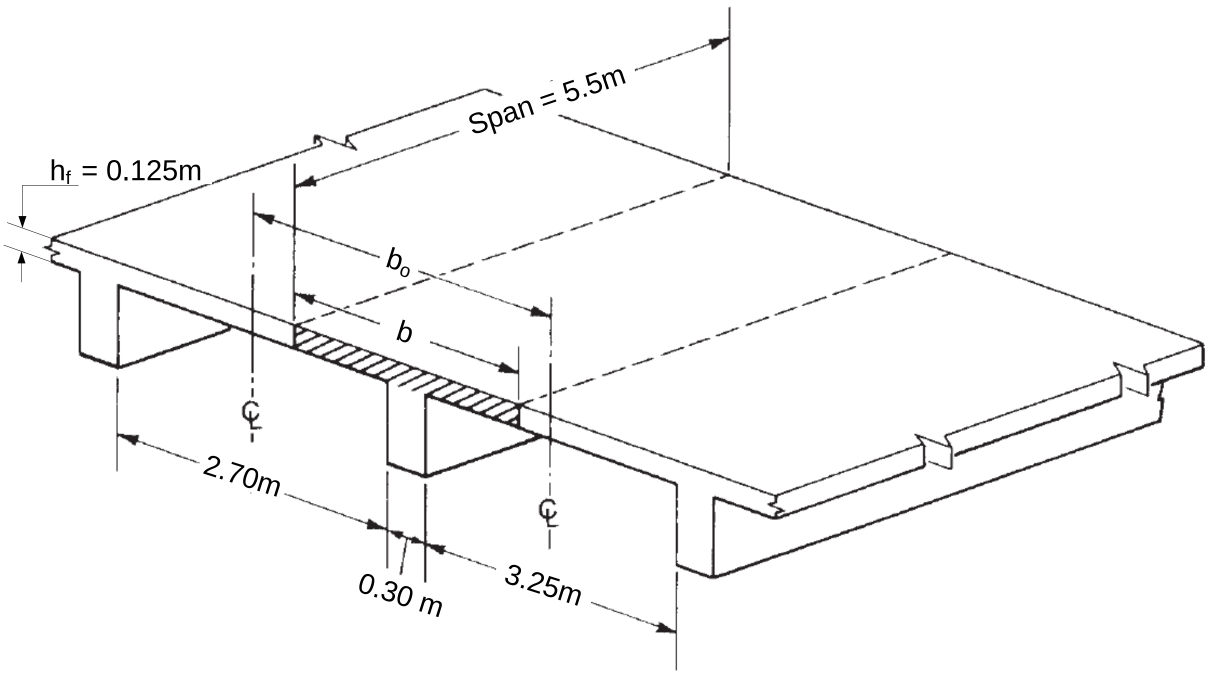


Figure 4.8-12: Monolithic Tee beam for Example 4.8-1.

Solution

1. Definition of Section Dimensions:

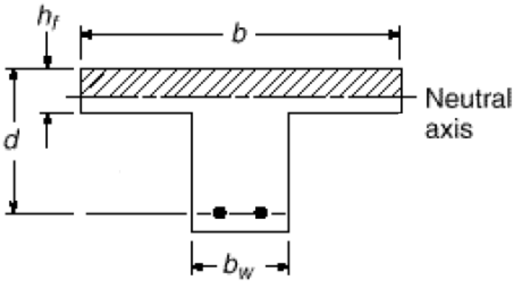


$$b = \mathrm{b_w} + minimum\left[\frac{s_{w\,left}}{2}\ or\ 8h\ or\ \frac{l_n}{8}\right] + minimum\left[\frac{s_{w\,right}}{2}\ or\ 8h\ or\ \frac{l_n}{8}\right]$$
$$b = 0.3 + minimum\left[\frac{2.7}{2}\ or\ 8\times 0.125\ or\ \frac{5.5}{8}\right] + minimum\left[\frac{3.25}{2}\ or\ 8\times 0.125\ or\ \frac{5.5}{8}\right]$$
$$b = 0.3 + minimum\,[1.35\ or\ 1.0\ or\ 0.688] + minimum[1.63\ or\ 1.0\ or\ 0.688]$$
$$b = 0.3 + 0.688 + 0.688 = 1.68\ m$$

2. Checking the Section Type:
   Check if the failure is secondary compression failure or compression failure through following comparison:

$$\rho_w\ ?\ \rho_{w\,max}$$
$$\rho_{w\,max} = \frac{A_{s\,max}}{b_w d} = 0.85\beta_1\ \frac{f'_c}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$
$$A_{sf} = \frac{0.85 f'_c h_f (b - b_w)}{f_y} = \frac{0.85\times 20\times 125\times(1680 - 300)}{300} = 9775\ \mathrm{mm^2}$$
$$\rho_{w\,max} = 0.85^2\times\frac{20}{300}\times\frac{0.003}{0.003+0.004} + \frac{9775}{300\times 420} = 20.6\times 10^{-3} + 77.6\times 10^{-3}$$
$$= 98.2\times 10^{-3}$$
$$\rho_w = \frac{A_s}{b_w d}$$
$$A_s = 6\times 284\mathrm{mm^2} = 1\,704\mathrm{mm^2}$$
$$\rho_w = \frac{1704\mathrm{mm^2}}{300\times 420} = 13.5\times 10^{-3} << \rho_{w\,max}\ \ \mathrm{Ok.}$$

3. Checking of $A_{s\ minimum}$ limitation:
   $\because f_c' < 31\ MPa$
   $\therefore A_{s\ minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{300} \times 300 \times 420 = 588\ \text{mm}^2 \ll A_{s\ Provided}$ Ok.
4. Computing of Nominal Flexure Strength "M_n":
   Assume that $a \leq h_f$ (based on experience, this can be considered as the general case):



   $$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1704 \times 300}{0.85 \times 20 \times 1680} = 17.9\ \text{mm} < 125 mm\ Ok.$$
   As $a_{Computed} \leq h_f$, then above assumption is correct and nominal flexural strength, M_n, can be computed based on:
   $$M_n = A_s f_y (d - \frac{a}{2}) = 1704 \times 300 \times \left(420 - \frac{17.9}{2}\right) = 210\ \text{kN.m}$$
5. Compute the strength reduction factor, ∅, based on following relation:
   Compute steel stain based on the following relations:
   $$c = \frac{a}{\beta_1} = \frac{17.9}{0.85} = 21.1 \Longrightarrow \epsilon_t = \frac{d-c}{c} \epsilon_u = \frac{420 - 21.1}{21.1} \times 0.003 = 56.7 \times 10^{-3}$$
   As $\epsilon_t \gg 0.005$, then $\emptyset = 0.9$
6. Finally Compute the Design Flexure Strength of Section $\emptyset M_n$:
   $\emptyset M_n = \emptyset \times M_n = 0.9 \times 210 = 189\ \text{kN.m}$ ■

Example 4.8-2

Check the adequacy of isolated T-beam shown below for ACI Code requirements and determine its design strength.

Assume that:

- $f_y = 420\ \text{Mpa}$
- $f_c' = 20\ \text{Mpa.}$
- Reinforcement is $6\emptyset 25$mm with $A_{Bar} = 500\text{mm}^2$.

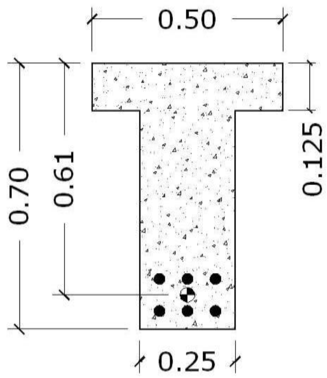


Figure 4.8-13: Isolated Tee beam for Example 4.8-2

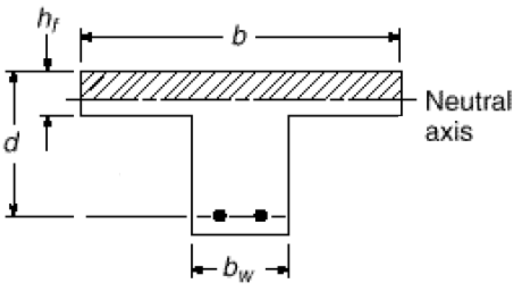
Solution

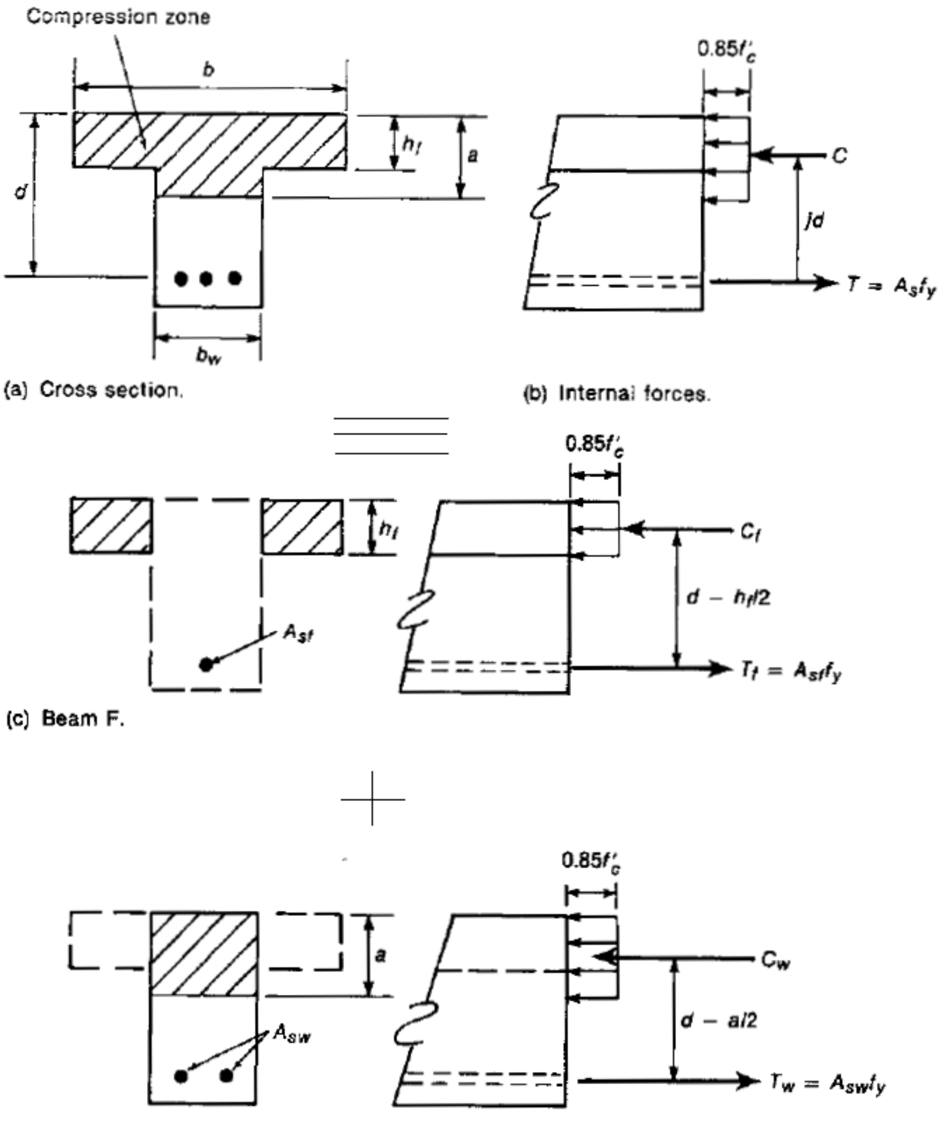
1. Check the Beam Dimensions:
   As the beam is an isolated T-beam, then its flange width and flange thickness must satisfy the following limitations:
   $$h_f \nless \frac{b_w}{2} \Longrightarrow h_f = 125\text{mm} = \frac{b_w}{2} = \frac{250}{2}\ \text{Ok.}$$
   $$b \ngtr 4b_w \Longrightarrow b = 500\text{mm} < 4 \times 250 mm\ Ok.$$
2. Checking Section Type:
   $\rho_w\ ?\ \rho_{w\ max}$
   $$\rho_{w\ max} = \frac{A_{s\ max}}{b_w d} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$
   $$A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y} = \frac{0.85 \times 20 \times 125 \times (500 - 250)}{420} = 1\ 265\ \text{mm}^2$$
   $$\rho_{w\ max} = 0.85^2 \frac{20}{420} \frac{0.003}{0.003 + 0.004} + \frac{1\ 265\ \text{mm}^2}{250 \times 610} = 14.7 \times 10^{-3} + 8.30 \times 10^{-3} = 23 \times 10^{-3}$$
   $$A_s = 6 \times 500\text{mm}^2 = 3\ 000\text{mm}^2$$
   $$\rho_w = \frac{A_s}{b_w d} = \frac{3\ 000\text{mm}^2}{250 \times 610} = 19.7 \times 10^{-3} < \rho_{w\ max}\ \text{Ok.}$$
3. Checking of $A_{s\ minimum}$ limitation:
   $\because f_c' < 31\ MPa$
   $\therefore A_{s\ minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 250 \times 610 = 508\ \text{mm}^2 < A_{s\ Provided}$ Ok.
4. Computing of Nominal Flexure Strength "M_n":

Assume that $a \leq h_f$ (based on experience, this can be considered as the general case):

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3\,000\text{mm}^2 \times 420 \text{ MPa}}{0.85 \times 20 \text{ MPa } \times 500\text{mm}}$$
$$= 148 \text{ mm} > 125 mm \ Not \ Ok.$$

As $a_{Computed} > h_f$ then the nominal flexure strength M_n will be considered to be consist from two parts shown below:





(a) Cross section. (b) Internal forces. (c) Beam F. (d) Beam W.

Compute the correct value of "a" based on Part "Beam W":

$$a = \frac{(A_s - A_{sf})f_y}{0.85 f_c'(b_w)} = \frac{(3\,000 - 1\,265) \times 420}{0.85 \times 20(250)} = 171 \text{ mm}$$

Compute M_n based on following relation:

$$M_n = \left[0.85 f_c' h_f (b - b_w)\right]\left(d - \frac{h_f}{2}\right)_{M_n for\ Part\ Beam\ F} + [0.85 f_c' a b_w]\left(d - \frac{a}{2}\right)_{M_n for\ Part\ Beam\ W}$$

$$M_n = [0.85 \times 20 \times 125 \times (500 - 250)]\left(610 - \frac{125}{2}\right)_{M_n for\ Part\ Beam\ F}$$
$$+ [0.85 \times 20 \ \times 171 \times 250]\left(610 - \frac{171}{2}\right)_{M_n for\ Part\ Beam\ W}$$

$M_n = 291 \text{ kN.m} + 381 \text{ kN.m} = 672 \text{ kN.m}$■

5. Compute the Strength Reduction Factor ∅ Based on Following Relation:
   Compute steel stain based on the following relations:

$$c = \frac{a}{\beta_1} = \frac{171}{0.85} = 201 \Rightarrow \epsilon_t = \frac{d-c}{c}\epsilon_u = \frac{610-201}{201} \times 0.003 = 0.006$$

As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.

6. Finally Compute the Design Flexure Strength of Section $\emptyset M_n$:

$\emptyset M_n = \emptyset \times M_n = 0.9 \times 672 = 605\ \text{kN.m}$■

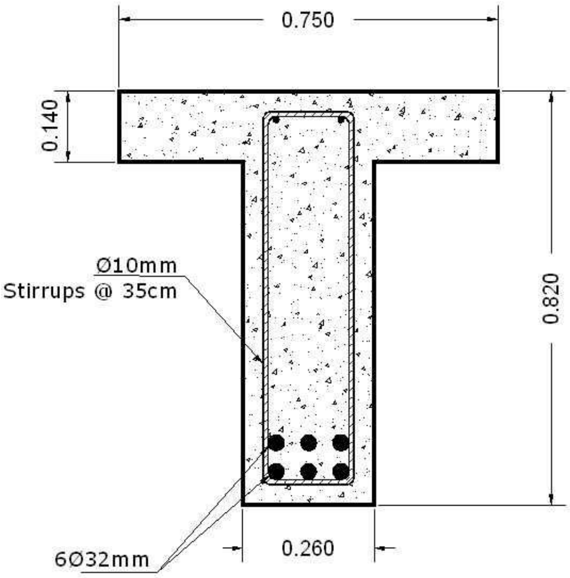
4.8.6 Problems for Solution

Problem 4.8-1

Check the adequacy of isolated T-beam shown below for ACI Code requirements and determine it's design strength.

Assume that:

- $f_y = 400$ Mpa
- $f_c' = 20$ Mpa.
- Reinforcement is 6∅32mm.



Answers

1. Check the Beam Dimensions:

As the beam is an isolated T-beam, then its flange width and flange thickness must satisfy the following limitations:

$$h_f \nless \frac{b_w}{2}$$

$$h_f = 140\text{mm} > \frac{b_w}{2} = \frac{260}{2} \quad \text{Ok.}$$

$$b \ngtr 4b_w \Longrightarrow b = 750\text{mm} < 4 \times 260mm\ Ok.$$

2. Checking Section Type:

$\rho_w\ ?\ \rho_{w\,max}$

$$\rho_{w\,max} = \frac{A_{s\,max}}{b_w d} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$

$$A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y} = 2\,916\ \text{mm}^2$$

$d = 725$ mm

$\rho_{w\,max} = 15.5 \times 10^{-3} + 15.5 \times 10^{-3} = 31.0 \times 10^{-3}$

$$\rho_w = \frac{A_s}{b_w d}$$

$A_s = 4\,824\text{mm}^2$

$\rho_w = 25.6 \times 10^{-3} < \rho_{w\,max}$ Ok.

3. Checking of $A_{s\,minimum}$ limitation:

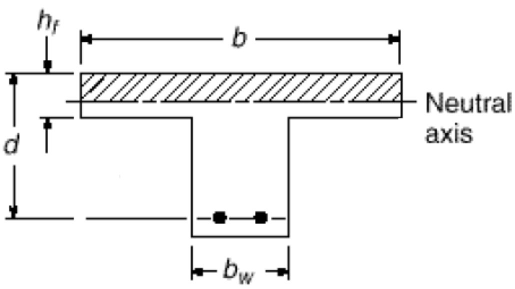
$\because f_c' < 31\ MPa$

$$\therefore A_{s\,minimum} = \frac{1.4}{f_y} b_w d = 660\ \text{mm}^2 < A_{s\,Provided}\ \text{Ok.}$$

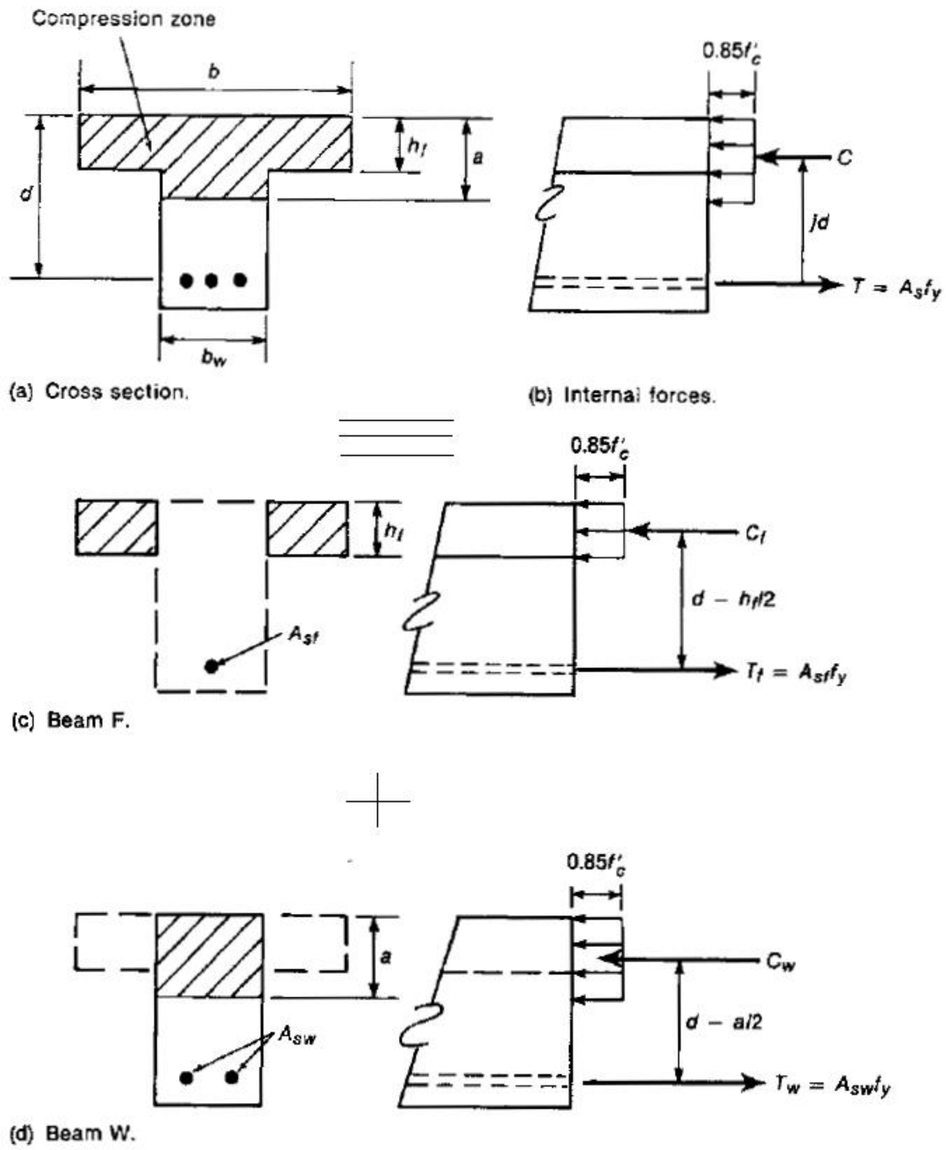
4. Computing of Nominal Flexure Strength "M_n":

Assume that $a \leq h_f$ (based on experience, this can be considered as the general case):

$$a = \frac{A_s f_y}{0.85 f_c' b} = 151\ \text{mm} > 140mm\ Not\ Ok.$$



As $a_{Computed} > h_f$ then the nominal flexure strength M_n will be considered to be consist from two parts shown below:



Compute the correct value of "a" based on Part "Beam W":

$$a = \frac{(A_s - A_{sf})f_y}{0.85f'_c b_w} = 173 \text{ mm}$$

Compute M_n based on following relation:

$$M_n = \left[0.85f'_c h_f (b - b_w)\right]\left(d - \frac{h_f}{2}\right)_{M_n for\ Part\ Beam\ F} + \left[0.85f'_c a b_w\right]\left(d - \frac{a}{2}\right)_{M_n for\ Part\ Beam\ W}$$

$$M_n = 764 \text{ kN.m} + 488 \text{ kN.m} = 1\,252 \text{ kN.m}\blacksquare$$

5. Compute the Strength Reduction Factor ∅ Based on Following Relation:
   Compute steel stain based on the following relations:
   $$c = \frac{a}{\beta_1} = 204 \Longrightarrow \epsilon_t = \frac{d - c}{c}\epsilon_u = 7.66 \times 10^{-3}$$
   As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.
6. Finally Compute the Design Flexure Strength of Section $\emptyset M_n$:
   $$\emptyset M_n = \emptyset \times M_n = 1\,127 \text{ kN.m}\blacksquare$$

Problem 4.8-2

Resolve previous problem but with $h_f = 180mm$.

Answers

1. Check the Beam Dimensions:
   As the beam is an isolated T-beam, then its flange width and flange thickness must satisfy the following limitations:
   $$h_f \nless \frac{b_w}{2} \Longrightarrow h_f = 180\text{mm} > \frac{b_w}{2} = \frac{260}{2} \text{ Ok.}$$

$b \ngtr 4b_w \Longrightarrow b = 750\text{mm} < 4 \times 260mm\ Ok.$

2. Checking Section Type:

$\rho_w\ ?\ \rho_{w\,max}$

$$\rho_{w\,max} = \frac{A_{s\,max}}{b_w d} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$

$$A_{sf} = \frac{0.85 f'_c h_f (b - b_w)}{f_y} = 3\,749\ \text{mm}^2$$

$d = 725\ \text{mm}$

$\rho_{w\,max} = 15.5 \times 10^{-3} + 19.9 \times 10^{-3} = 35.4 \times 10^{-3}$

$$\rho_w = \frac{A_s}{b_w d}$$

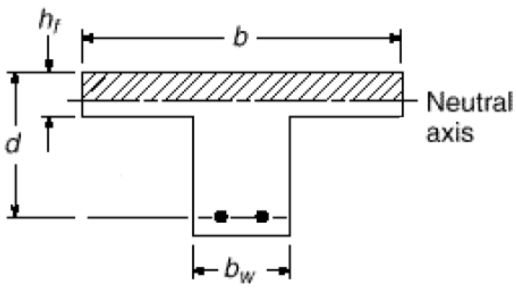
$A_s = 4\,824\text{mm}^2$

$\rho_w = 25.6 \times 10^{-3} < \rho_{w\,max}$ Ok.

3. Checking of $A_{s\,minimum}$ limitation:

$\because f'_c < 31\ MPa \Longrightarrow \therefore A_{s\,minimum} = \frac{1.4}{f_y} b_w d = 660\ \text{mm}^2 < A_{s\,Provided}$ Ok.

4. Computing of Nominal Flexure Strength "M_n":



Assume that $a \le h_f$ (based on experience, this can be considered as the general case):

$$a = \frac{A_s f_y}{0.85 f'_c b} = 151\ \text{mm} < 180mm\ Ok.$$

As $a_{Computed} \le h_f$, then above assumption is correct and nominal flexure strength M_n can be computed based on:

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 1\,253\ \text{kN.m}$$

5. Compute the Strength Reduction Factor Ø Based on Following Relation:

Compute steel stain based on the following relations:

$$c = \frac{a}{\beta_1} = 178\text{mm} \Longrightarrow \epsilon_t = \frac{d - c}{c}\epsilon_u = 9.22 \times 10^{-3}$$

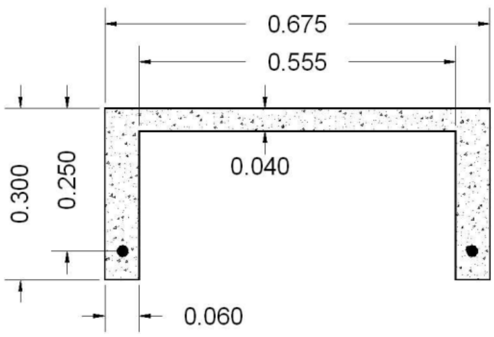
As $\epsilon_t \ge 0.005$, then $\emptyset = 0.9$ Ok.

6. Finally Compute the Design Flexure Strength of Section $\emptyset M_n$:

$\emptyset M_n = \emptyset \times M_n = 1\,128\ \text{kN.m}$ ■

Problem 4.8-3

Check the adequacy of the precast beam shown below according to ACI Code requirements and compute its flexure design strength. Assume that:

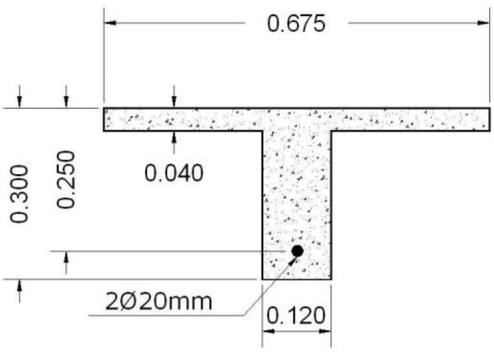


- $f_y = 400$ Mpa
- $f_c' = 20$ Mpa.
- Each leg has been reinforced with one of 20 mm rebar.

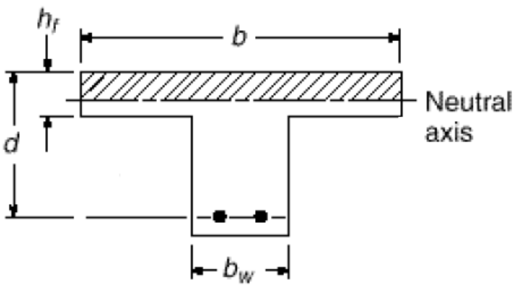
Answers

Note: It is easily to show that the horizontal movements of an area in a reinforced concrete beam has no effects on strain or stress distribution if the section remains to have a vertical axis of symmetry.

Then the section will be transformed for the shape below and analyzed as a T shape with web width of 120 mm:



1. Checking Section Type:
   $\rho_w \ ? \ \rho_{w\,max}$
   $\rho_{w\,max} = \frac{A_{s\,max}}{b_w d} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$
   $A_{sf} = \frac{0.85 f'_c h_f (b - b_w)}{f_y} = 945\ \text{mm}^2$
   $d = 250\ \text{mm}$
   $\rho_{w\,max} = 15.5 \times 10^{-3} + 31.5 \times 10^{-3} = 47.0 \times 10^{-3}$
   $\rho_w = \frac{A_s}{b_w d}$
   $A_s = 628\ \text{mm}^2$
   $\rho_w = 20.9 \times 10^{-3} < \rho_{w\,max}$ Ok.
   Checking of $A_{s\,minimum}$ limitation:
   $\because f'_c < 31\ MPa \Longrightarrow \therefore A_{s\,minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{400} \times (60 \times 2) \times 250 = 105\ \text{mm}^2$
   $< A_{s\,Provided}$ Ok.
2. Computing of Nominal Flexure Strength "M_n":
   Assume that $a \le h_f$ (based on experience, this can be considered as the general case):



   $a = \frac{A_s f_y}{0.85 f'_c b} = 21.9\ \text{mm} < 40 mm\ Ok.$
   As $a_{Computed} \le h_f$, then above assumption is correct and nominal flexure strength M_n can be computed based on:
   $M_n = A_s f_y (d - \frac{a}{2}) = 60.0\ \text{kN.m}$
3. Compute the Strength Reduction Factor $\emptyset$ Based on Following Relations:
   Compute steel stain based on the following relations:
   $c = \frac{a}{\beta_1} = 25.8\ \text{mm} \Longrightarrow \epsilon_t = \frac{d - c}{c} \epsilon_u = 26.1 \times 10^{-3}$
   As $\epsilon_t \ge 0.005$, then $\emptyset = 0.9$ Ok.
4. Finally Compute the Design Flexure Strength of Section $\emptyset M_n$:
   $\emptyset M_n = \emptyset \times M_n = 54.0\ \text{kN.m}$■

4.9 DESIGN OF A BEAM WITH T-SHAPE

4.9.1 Essence of Problem

- Generally, all design problems for T-section can be classified as a design of a section with pre-specified dimensions (h_f, b, b_w, and h). Usually these dimensions have been determined as follows:
  - h_f, b are both determined from slab design that logically be executed before the design of supporting beams.
  - b_w, and h are determined based on one of following criteria:
    - Based on architectural requirements.
    - Based on strength or deflection requirements in supports region (i.e., region of negative moment), in a continuous T beam.
    - Based on shear requirements (as will be discussed in Chapter 4).
- Therefore, the main unknown of design problem is to determine the required reinforcement and its details.

4.9.2 Design Procedures

Based on above known and unknown, the design procedure can be summarized as follows:

- Computed of M_u:
  Based on given loads and spans the applied factored moment M_u can be computed. As slab weight has been already included in the applied dead load, therefore only selfweight of beam stem should be added.

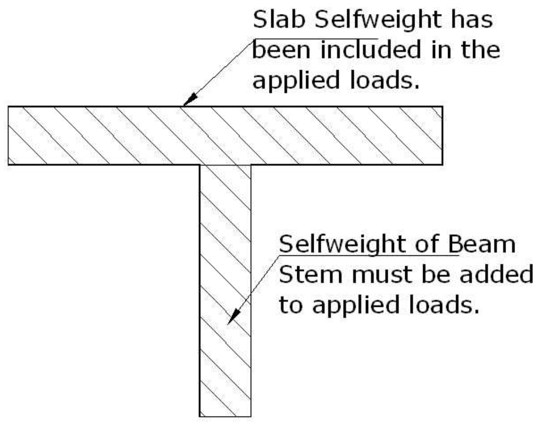


Figure 4.9-1: Selfweight of Tee beams.

- Based on slab and beam data, determine the effective flange width "b" and as was discussed in previous article. For isolated T beam, beam dimensions must be checked based on ACI requirements.
- Compute M_n based on following relation:
  $$M_n = \frac{M_u}{\emptyset}$$
  where $\emptyset$ will be assumed 0.9 to be checked later. This assumption is generally satisfied in the design of T section.
- Check if this section can be design with a compression block in section flange or extend to section web based on following comparison:
  If
  $$M_n \leq 0.85 f_c' \, h_f b \left(d - \frac{h_f}{2}\right)$$
  then $a \leq h_f$. Else $a > h_f$

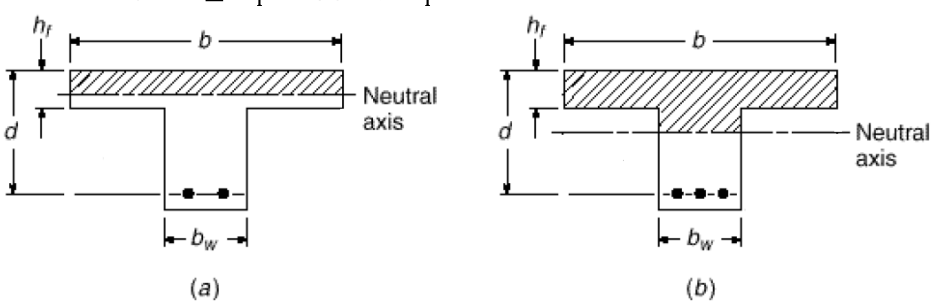


Figure 4.9-2: Possible location of compression block of Tee beams, reproduction of Figure 4.8-9 for quick reference.

- Design of a section with $a \leq h_f$:
  This section can be designed as a rectangular section with dimensions of b and d.
  $$\rho_{Required} = \frac{1-\sqrt{1-2.36\frac{M_n}{f_c' bd^2}}}{1.18 \times \frac{f_y}{f_c'}}$$
  $A_{S\ Required} = \rho_{Required} bd$ ■
- Design of a section with $a > h_f$:
  - Compute the nominal moment that can be supported by flange overhangs:
    $$M_{n1} = 0.85 f_c' h_f (b - b_w)(d - \frac{h_f}{2})$$
    Steel reinforcement for this part will be:
    $$A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y}$$ ■

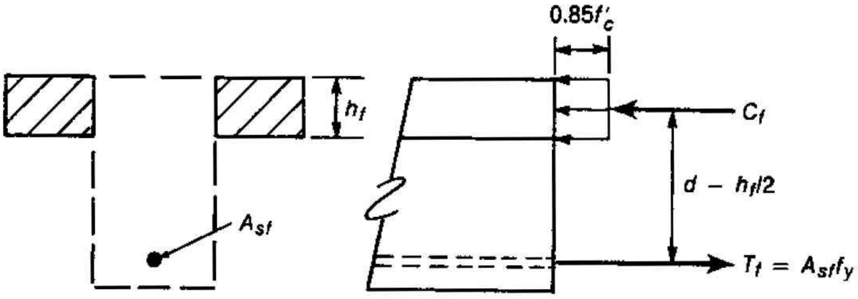


Figure 4.9-3: Forces acting on overhang parts and corresponding streel area.

  - Compute the remaining nominal strength that must be supported by section web:
    $M_{n2} = M_n - M_{n1}$

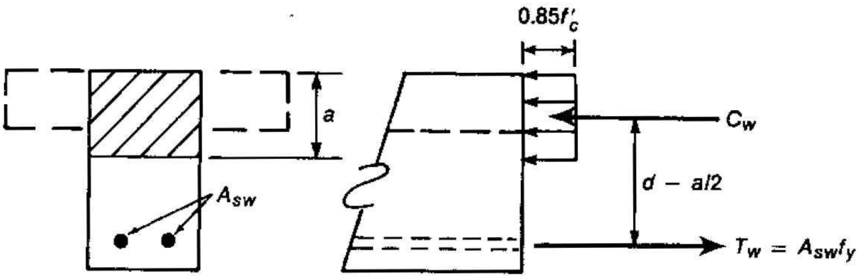


Figure 4.9-4: Forces acting on web part and corresponding steel area.

For this moment "M_{n2}", the section can be designed as a rectangular section with dimensions of b_w and d:
$$\rho_{Required} = \frac{1-\sqrt{1-2.36\frac{M_{n2}}{f_c' b_w d^2}}}{1.18 \times \frac{f_y}{f_c'}}$$
$A_{s2} = \rho_{Required} b_w d$
then:
$A_{s\ Required} = A_{sf} + A_{s2}$ ■

- Check $A_{s\ Required}$ with minimum steel area permitted by the ACI Code:
  $$A_{s\ minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$
  If $A_{s\ Required} > A_{s\ minimum}$ Ok. Else, use:
  $A_{s\ Required} = A_{s\ minimum}$
- Check the $A_{s\ Required}$ with the maximum steel area permitted by ACI Code:
  $$\rho_w = \frac{A_{s\ Required}}{b_w d} \ ?\ \rho_{w\ max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$
  If
  $\rho_w \leq \rho_{w\ max}$ Ok.
  Else, the designer must increase one or more of beam dimensions, i.e., in practice, compression reinforcement is not used in T sections.

- Check the assumption of $\emptyset = 0.9$:
  - Compute "a":
    If $a \le h_f$
    $$a = \frac{A_s f_y}{0.85 f_c' b}$$
    If $a > h_f$
    $$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' (b_w)}$$
  - Compute steel stain based on the following relations:
    $$c = \frac{a}{\beta_1}$$
    $$\epsilon_t = \frac{d - c}{c} \epsilon_u$$
  - If $\epsilon_t \ge 0.005$, then $\emptyset = 0.9$ Ok.
  - If $\epsilon_t < 0.005$, then re-compute a more accurate $\emptyset$:
    $\emptyset = 0.483 + 83.3\epsilon_t$
    and return to step of computing M_n.
- Finally, compute the required number of rebars and reinforcement layers and draw section details.

4.9.3 Examples

Example 4.9-1

Design the T-beam for the floor system shown in Figure 4.9-5 below. The floor slab supported by 6.71 m simple span beams. Service loads are: $W_{Live} = 14.6 \frac{kN}{m}$ and $W_{Dead} = 29.2 \frac{kN}{m}$.

Assume that the designer intends to use:

- $f_y = 414$ Mpa $f_c' = 21\ Mpa$.
- $\emptyset 25mm$ for longitudinal reinforcement ($A_{Bar} = 510mm^2$) and $\emptyset 10mm$ for stirrups.
- One layer of reinforcement.

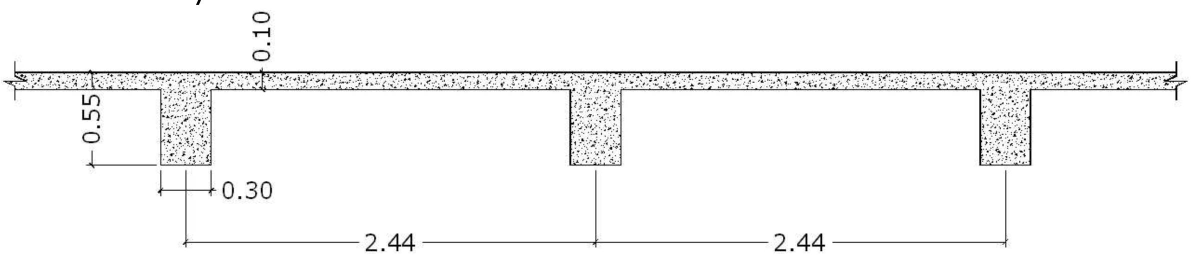


Figure 4.9-5: Floor system for Example 4.9-1.

Solution

- Computed of M_u
  $$W_{Self} = (0.55 - 0.1)m \times 0.3m \times 24 \frac{kN}{m^3} = 3.24 \frac{kN}{m} \Rightarrow W_{Dead} = 29.2 \frac{kN}{m} + 3.24 \frac{kN}{m} = 32.4 \frac{kN}{m}$$
  $$M_{Dead} = \frac{32.4 \frac{kN}{m} \times 6.71^2 m^2}{8} = 182\ kN.m, M_{Live} = \frac{14.6 \frac{kN}{m} \times 6.71^2 m^2}{8} = 82.2\ kN.m$$
  $$M_u = \text{maximum of } [1.4 M_{Dead} \text{or } 1.2 M_{Dead} + 1.6 M_{Live}]$$
  $$M_u = \text{maximum of } [1.4 \times 182\ kN.m \text{ or } 1.2 \times 182\ kN.m + 1.6 \times 82.2\ kN.m]$$
  $$M_u = \text{maximum of } [255\ kN.m \text{ or } 350 kN.m] = 350\ kN.m$$
- Compute of Required Nominal Flexure Strength M_n:
  $$M_n = \frac{M_u}{\emptyset} = \frac{350\ kN.m}{0.9} = 389\ kN.m$$
  where $\emptyset$ will be assumed 0.9 to be checked later.
- Compute the effective flange width "b"
  $$b = b_w + minimum \left[\frac{s_w}{2} \ or\ 8h\ or \frac{l_n}{8}\right] \times 2$$
  $$b = 300 + minimum \left[\frac{(2440 - 300)}{2} \ or\ 8 \times 100\ or \frac{6710}{8}\right] \times 2$$
  $$b = 300 + minimum\ [1070\ or\ 800\ or\ 839] \times 2 = 300 + 800 \times 2 = 1900$$

- Check if this section can be design with a compression block in section flange or extend to section web based on following comparison:

$$M_n ?\ 0.85f_c'\ h_f b\left(d-\frac{h_f}{2}\right)$$

$$d = 550mm - 40mm - 10mm - \frac{25}{2}mm = 487mm$$

$$M_n = 389\ kN.m\ ?\ 0.85f_c'\ h_f b\left(d-\frac{h_f}{2}\right) = 0.85 \times 21 \times 100 \times 1900\left(487-\frac{100}{2}\right) = 1482\ kN.m$$

$$M_n = 389\ kN.m < \ 0.85f_c'\ h_f b\left(d-\frac{h_f}{2}\right) = 1482\ kN.m$$

Then $a < \ h_f$

- Design of a section with $a\ \leq\ h_f$:
  This section can be designed as a rectangular section with dimensions of b and d.

$$\rho_{Required} = \frac{1-\sqrt{1-2.36\frac{M_n}{f_c'bd^2}}}{1.18 \times \frac{f_y}{f_c'}} = \frac{1-\sqrt{1-2.36\frac{389 \times 10^6}{21 \times 1900 \times 487^2}}}{1.18 \times \frac{414}{21}} = 2.13 \times 10^{-3}$$

$$A_{S\ Required} = \rho_{Required} bd = 2.13 \times 10^{-3} \times 1900 \times 487 = 1971\ mm^2$$

$$\text{No of Rebars} = \frac{1971}{510} = 3.86$$

Try $4\emptyset 25mm$

$$A_{s\ provided} = \ 4 \times 510mm^2 = 2040\ mm^2$$

$$b_{Required} = 40 \times 2 + 10 \times 2 + 4 \times 25 + 3 \times 25 = 275\ mm < 300 mm\ Ok.$$

- Check $A_{s\ Provided}$ with minimum steel area permitted by the ACI Code:

$$A_{s\ minimum} = \frac{0.25\sqrt{f_c'}}{f_y}b_w d \geq \frac{1.4}{f_y}b_w d \Longrightarrow A_{s\ minimum} = \frac{1.4}{f_y}b_w d = \frac{1.4}{414} \times 300 \times 487 = 494\ mm^2$$

As $A_{s\ Provided} > \ A_{s\ minimum}$ Ok.

- Check the $A_{s\ Provided}$ with the maximum steel area permitted by ACI Code:

$$\rho_w = \frac{A_{s\ Provided}}{b_w d}\ ?\ \rho_{w\ max} = \ 0.85\beta_1\ \frac{f_c'}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$

$$A_{sf} = \frac{0.85f_c'h_f(b-b_w)}{f_y} = \frac{0.85 \times 21 \times 100 \times (1900-300)}{414} = 6899\ mm^2$$

$$\rho_w = \frac{2\ 040\ mm^2}{300 \times 487}\ ?\ \rho_{w\ max} = \ 0.85 \times 0.85 \times \ \frac{21}{414}\ \frac{0.003}{0.003+0.004} + \frac{6899}{300 \times 487}$$

$$\rho_w = 13.9 \times 10^{-3}\ \ll\ \rho_{w\ max} = \ 15.7 \times 10^{-3} + 47.2 \times 10^{-3} 62.9 \times 10^{-3}\ Ok.$$

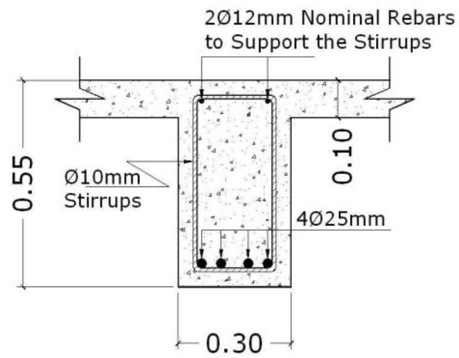
- Check the assumption of $\emptyset = 0.9$:
  - Compute "a":

$$\sum F_x = 0 \Longrightarrow 0.85 \times 21 \times a \times 1900$$
$$= \ 2040\ \ \times 414 \Longrightarrow a = \ 24.9\ mm$$

  - Compute steel stain based on the following relations:

$$c = \frac{a}{\beta_1} = \frac{24.9}{0.85} = 29.3mm \Longrightarrow \epsilon_t = \frac{487-29.3}{29.3} \times 0.003 = 46.9 \times 10^{-3}$$

  - As $\epsilon_t > 0.005$, then $\emptyset = 0.9$ Ok.
- Draw the Section Details:



Example 4.9-2

Design a T-beam having a cross section shown in Figure 4.9-6 below to support a total factored moment M_u of 461 kN.m. Assume that the effective flange width has been computed and as shown in Figure below.

Assume that the designer intends to use:

- $f_y = \ 414\ Mpa\ f_c' \ = \ 21\ Mpa$.
- $\emptyset 35mm$ for longitudinal reinforcement ($A_{Bar} = \ 1000mm^2$) and $\emptyset 10mm$ for stirrups.
- One layer of reinforcement.

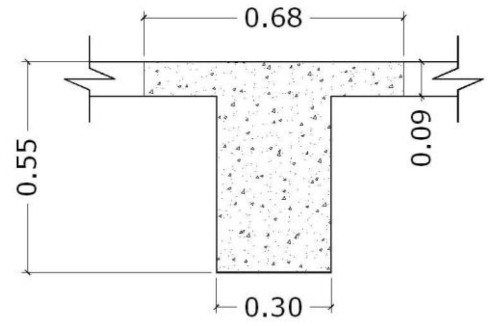


Figure 4.9-6: T section for Example 4.9-2.

Solution

- Compute of Required Nominal Flexure Strength M_n:

  $$\mathrm{M_n} = \frac{\mathrm{M_u}}{\emptyset} = \frac{461\ \mathrm{kN.m}}{0.9} = 512\ \mathrm{kN.m}$$

  where Ø will be assumed 0.9 to be checked later.
- Check if this section can be design with a compression block in section flange or extend to section web based on following comparison:

  $$\mathrm{M_n}?\ 0.85f_c'\ h_f b \left(d - \frac{h_f}{2}\right)$$

  $$d = 550\mathrm{mm} - 40\mathrm{mm} - 10\mathrm{mm} - \frac{35}{2}\mathrm{mm} = 482\mathrm{mm}$$

  $$\mathrm{M_n} = 512\ \mathrm{kN.m} > \ 0.85f_c'\ h_f b \left(d - \frac{h_f}{2}\right) = 0.85 \times 21 \times 90 \times 680 \left(482 - \frac{90}{2}\right) = 477\ \mathrm{kN.m}$$

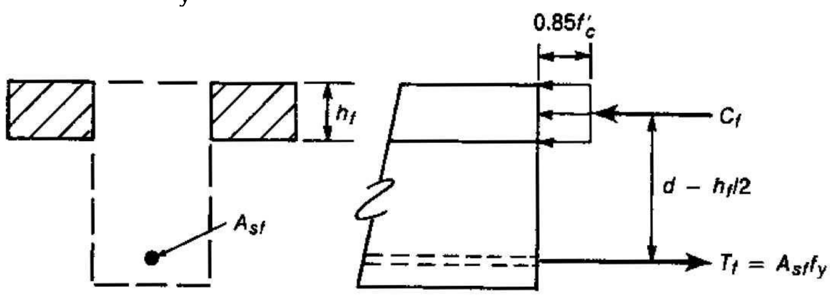
- Design of a section with $a > h_f$:
  - Compute the nominal moment that can be supported by flange overhangs:

    $$\mathrm{M_{n1}} = 0.85f_c'h_f(b - b_w)\left(d - \frac{h_f}{2}\right) = 0.85 \times 21 \times 90 \times (680 - 300)(482 - \frac{90}{2})$$

    $$\mathrm{M_{n1}} = 267\ \mathrm{kN.m}$$

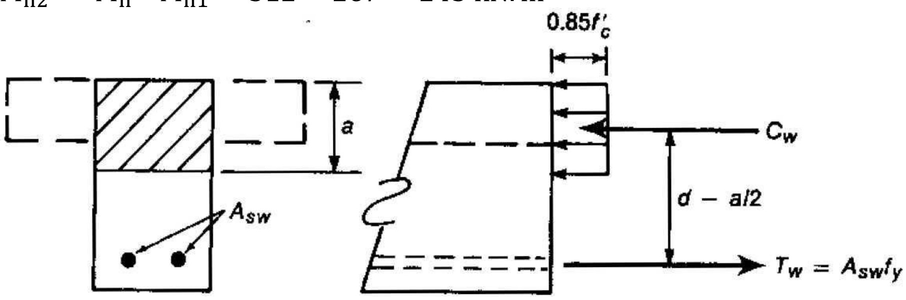
    Steel reinforcement for this part will be:

    $$A_{sf} = \frac{0.85f_c'h_f(b - b_w)}{f_y} = \frac{0.85 \times 21 \times 90 \times (680 - 300)}{414} = 1\,474\ \mathrm{mm^2}$$

  - Compute the remaining nominal strength that must be supported by section web:

    $$\mathrm{M_{n2}} = \ \mathrm{M_n} - \ \mathrm{M_{n1}} = 512 - 267 = 245\ \mathrm{kN.m}$$

    For this moment "$\mathrm{M_{n2}}$", the section can be designed as a rectangular section with dimensions of b_w and d:

    $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_{n2}}{f_c' b_w d^2}}}{1.18 \times \frac{f_y}{f_c'}} = \frac{1 - \sqrt{1 - 2.36\frac{245 \times 10^6}{21 \times 300 \times 482^2}}}{1.18 \times \frac{414}{21}} = 9.55 \times 10^{-3}$$

    $$A_{s2} = \ \rho_{Required} b_w d = \ 9.55 \times 10^{-3} \times 300 \times 482 = 1\,381\ \mathrm{mm^2}$$

Then:

$$A_{s\,Required} = A_{sf} + A_{s2} = 1\,474\ \text{mm}^2 + 1\,381\ \text{mm}^2 = 2\,855\ \text{mm}^2$$

$$\text{No. of Rebars} = \frac{2855\text{mm}^2}{1000\text{mm}^2} = 2.86$$

Try 3Ø35mm

$$A_{S\,Provided} = 3\,000\ \text{mm}^2$$

$$b_{Required} = 40 \times 2 + 10 \times 2 + 3 \times 35 + 2 \times 35 = 275\text{mm} < 300mm\ Ok.$$

- Check $A_{S\,Provided}$ with minimum steel area permitted by the ACI Code:

$$A_{s\,minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

$$A_{s\,minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{414} \times 300 \times 482 = 489\ \text{mm}^2 < A_{S\,Provided}\ \text{Ok.}$$

- Check the $A_{S\,Provided}$ with the maximum steel area permitted by ACI Code:

$$\rho_w = \frac{A_{S\,Provided}}{b_w d}\ ?\ \rho_{w\,max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$

$$\rho_w = \frac{3000}{300 \times 482} = 20.7 \times 10^{-3}\ ?\ \rho_{w\,max} = 0.85^2 \frac{21}{414} \frac{0.003}{0.003 + 0.004} + \frac{1\,474\ \text{mm}^2}{300 \times 482}$$

$$\rho_w = 20.7 \times 10^{-3} < \rho_{w\,max} = 15.7 \times 10^{-3} + 10.2 \times 10^{-3} = 25.9 \times 10^{-3}\ \text{Ok.}$$

- Check the assumption of $\emptyset = 0.9$:
  - Compute "a":

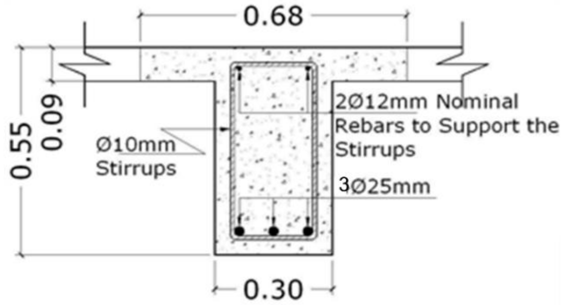
$$a = \frac{(A_s - A_{sf})f_y}{0.85 f_c'(b_w)} = \frac{(3\,000 - 1\,474) \times 414}{0.85 \times 21 \times 300} = 118\ \text{mm}$$

  - Compute steel stain based on the following relations:

$$c = \frac{a}{\beta_1} = \frac{118\ \text{mm}}{0.85} = 139\ \text{mm}$$

$$\epsilon_t = \frac{d - c}{c}\epsilon_u = \frac{482 - 139}{139} \times 0.003 = 7.40 \times 10^{-3}$$

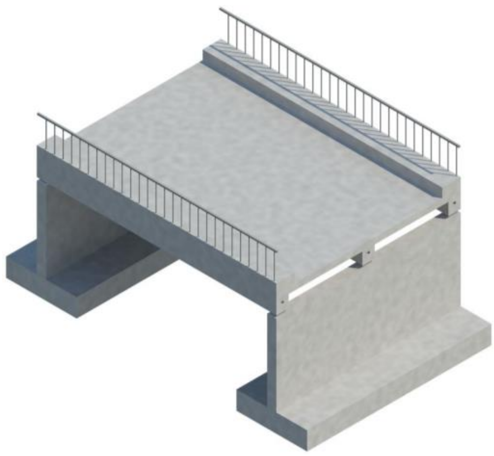
  - As $\epsilon_t > 0.005$, then $\emptyset = 0.9$ Ok.
- Draw the Section Details:



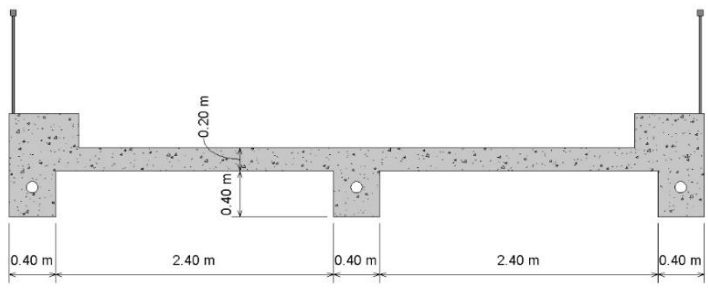
Example 4.9-3

For a pedestrian bridge indicated in ***Figure*** 4.9-7 below, a structural designer includes sleeve with diameter of 100mm for communication and electrical cables. Design for flexural the central supporting beam. In your design, assume that:

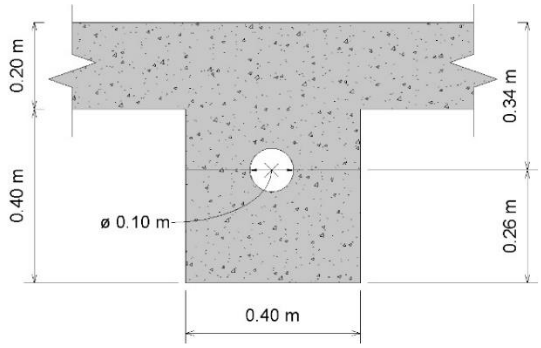
- Materials strength are $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.
- Two layers of reinforcement with a bar diameter of 20mm for longitudinal reinforcement,
- Rebar with a diameter of 10mm for stirrups,
- $W_{Superimposed} = 20\ kN/m$, not including beam own weight, $W_{Live} = 12\ kN/m$

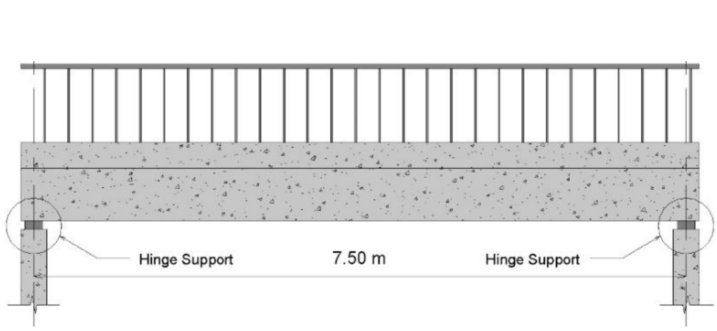


3D view.



Cross sectional view.





Callout view. Elevation view.

Figure 4.9-7: A pedestrian bridge for Example 4.9-3.

Solution

- Compute factored load $\boldsymbol{W_u}$:

  Assuming that selfweight of the beam flange has been included in the superimposed dead load of $W_{Superimposed} = 20\, kN/m$, therefore what should be included as a selfweight would include stem selfweight only.

  $$W_{Selfweight} = 0.4 \times 0.4 \times 24 = 3.84 \frac{kN}{m}$$

  Reducing in selfweight due to pipe conduit has been conservatively neglected. The total dead load would be:

  $$W_{Dead} = W_{Selfweight} + W_{Superimposed} = 3.84 + 20 = 23.8 \frac{kN}{m}$$

  The factored uniformly distributed load that acting on the beam would be:

  $$W_u = \max(1.4W_D, 1.2W_D + 1.6W_L) = \max(1.4 \times 23.8, 1.2 \times 23.8 + 1.6 \times 12) = 47.8 \frac{kN}{m}$$

  With the indicated simple supports, the maximum factored moment, M_u, at beam mid-span would be:

  $$M_u = \frac{W_u l^2}{8} = \frac{47.8 \times 7.50^2}{8} = 336\, kN.m$$

- Compute $\mathrm{M_n}$:

  $$\mathrm{M_n} = \frac{\mathrm{M_u}}{\emptyset}$$

  where $\emptyset$ will be assumed 0.9 to be checked later. This assumption is generally satisfied in the design of T section.

  $$M_n = \frac{336}{0.9} = 373\, kN.m$$

- Beam effective depth, d:

  With adopting of two layers of $No.20$ for longitudinal reinforcement and $No.10$ for stirrup reinforcement, the effective depth, d, would be:

  $$d = 600 - 40 - 10 - 20 - \frac{25}{2} \approx 517\, mm$$

- Effective flange width:

  With referring to Figure 4.9-8:

  $$b = \mathrm{b_w} + minimum \left[\frac{S_{w\,left}}{2}\ or\ 8h\ or \frac{l_n}{8}\right] + minimum \left[\frac{S_{w\,right}}{2}\ or\ 8h\ or \frac{l_n}{8}\right]$$

  $$b = 0.4 + \min\left(\left(\frac{2.4}{2}\right), (8 \times 0.2), \left(\frac{7.5}{8}\right)\right) + \min\left(\left(\frac{2.4}{2}\right), (8 \times 0.2), \left(\frac{7.5}{8}\right)\right) = 2.275\ \mathrm{m}$$

- Check type of section:

  Check to see if the stress block would be in the flange or extends to the web:

  $$\mathrm{M_n} = 373\, kN.m \ll 0.85\mathrm{f_c'}\,\mathrm{h_f b}\left(\mathrm{d} - \frac{\mathrm{h_f}}{2}\right) = \frac{0.85 \times 28 \times 200 \times 2.275 \times 10^3 \times \left(517 - \frac{200}{2}\right)}{10^6}$$
  $$= 4516\, kN.m$$

  Then $\mathrm{a} < \mathrm{h_f}$ and the section behaves as a rectangular section with dimensions of b by d.

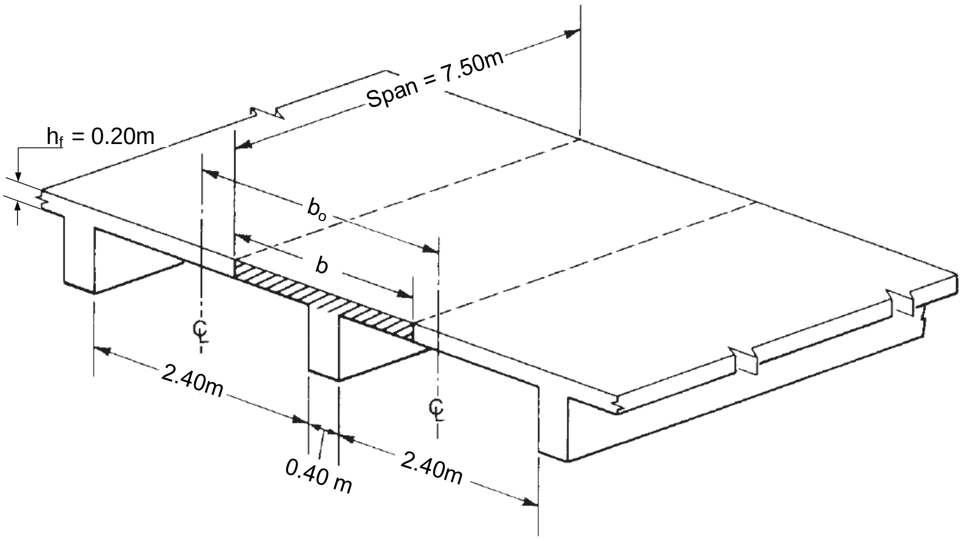


Figure 4.9-8: Flange computation parameters for Example 4.9-3.

- Effect of conduit hole:
  According to aforementioned argument, stress block is located at flange; therefore, the conduit hole has no effect on beam strength, as it is located at the tension zone that completely neglected in traditional concrete theory.
- Required reinforcement ratio, $\rho_{Required}$:

$$\rho_{\text{Required}} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f'_c bd^2}}}{1.18 \times \frac{f_y}{f'_c}} = \frac{\left(1 - \sqrt{1 - 2.36 \times \left(\frac{373 \times 10^6}{28 \times (2.275 \times 10^3) \times 517^2}\right)}\right)}{\left(1.18 \times \frac{420}{28}\right)}$$
$$= 1.48 \times 10^{-3}$$

$A_{s\ Required} = 1.48 \times 10^{-3} \times (2.275 \times 10^3) \times 517 = 1741\ mm^2$

- Check with $A_{s\ minmum}$:
  With conservative neglecting of the conduit hole, minimum required reinforcement, $A_{s\ minimum}$ would be:
  $\because f'_c < 31\ MPa$

$$A_{\text{s minimum}} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 400 \times 517 = 689\ mm^2 < A_{s\ Required}\ \therefore Ok.$$

- Check with $\rho_{maximum}$:

$$\rho_{\text{w max}} = 0.85\beta_1\ \frac{f'_c}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$

$$A_{sf} = \frac{0.85 f'_c h_f (b - b_w)}{f_y} = \frac{\left(0.85 \times 28 \times 200 \times \left((2.275 \times 10^3) - 400\right)\right)}{420} = 21250\ mm^2$$

$$\rho_{w\ maximum} = 0.85 \times 0.85 \times \left(\frac{28}{420}\right) \times \left(\frac{0.003}{0.003 + 0.004}\right) + \left(\frac{21250}{400 \times 517}\right) = 123 \times 10^3$$

$$\rho_{w\ Required} = \frac{1741}{400 \times 517} = 8.42 \times 10^{-3}$$

$\rho_{w\ maximum} \gg \rho_{w\ Required}\ \therefore Ok.$

  As expect, the Tee section has very high ductility level and would fail as a tension control section. This implicitly indicates that ***the assumption of*** $\phi = 0.9$ ***is valid and there is no need to be checked explicitly***.
- Details of the section:

$$No.of\ Reabrs = \frac{1741}{\frac{\pi \times 20^2}{4}} = 5.54$$

  Therefore use $6No.20$ in two layers as indicated in below.

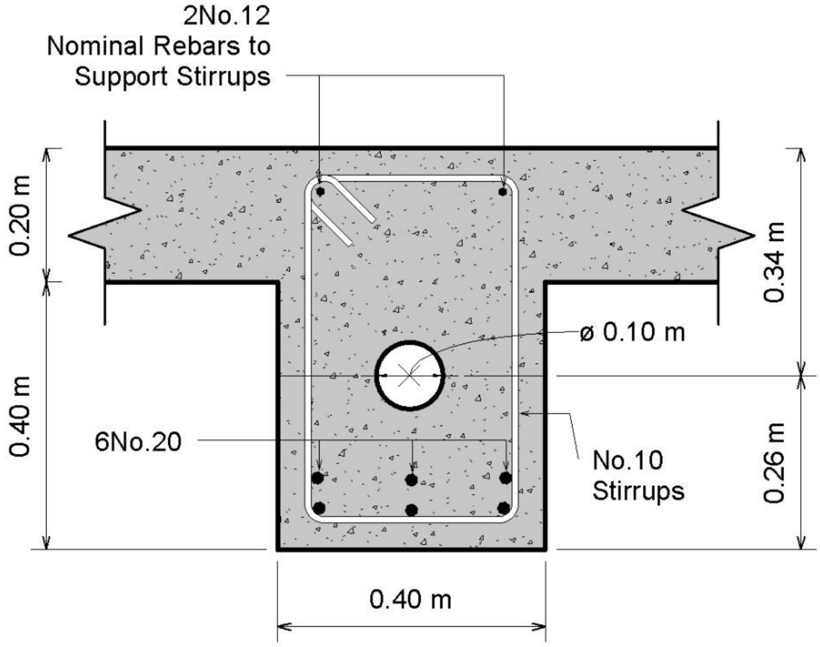


Figure 4.9-9: Final detailed section for Example 4.9-3.

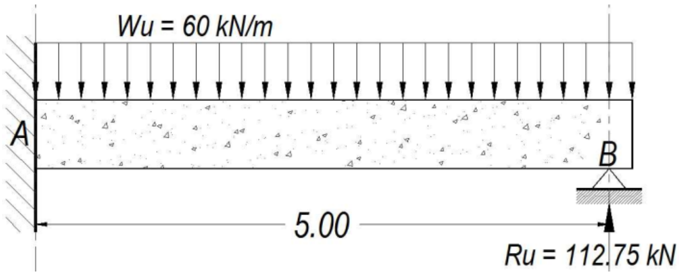
Example 4.9-4

The vertical reaction at end B of an indeterminate propped cantilever beam has been computed as shown in ***Figure 4.9-10*** below:

- Design flexure reinforcement for section at support end (A).
- Check adequacy if same amount of flexure reinforcement calculated at end (A) is used for section of maximum positive bending moment.

Assume that:

- Beam selfweight is neglected.
- Use a rebar of $\phi 25mm$ with A_{bar} of 510 mm^2 and stirrups of $\phi 10mm$.
- $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.



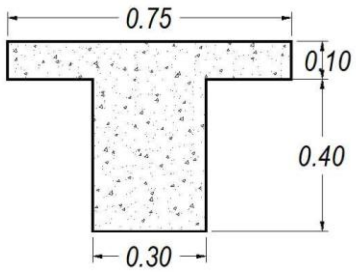
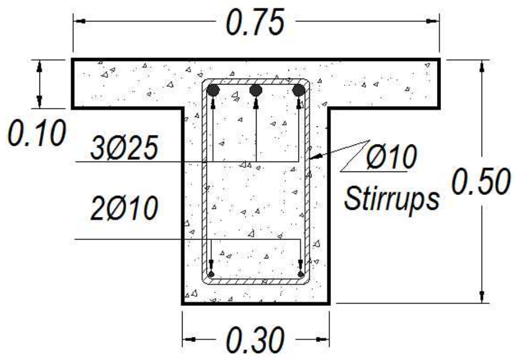


Figure 4.9-10: Propped cantilever beam Example 4.9-4. **Beam Section**

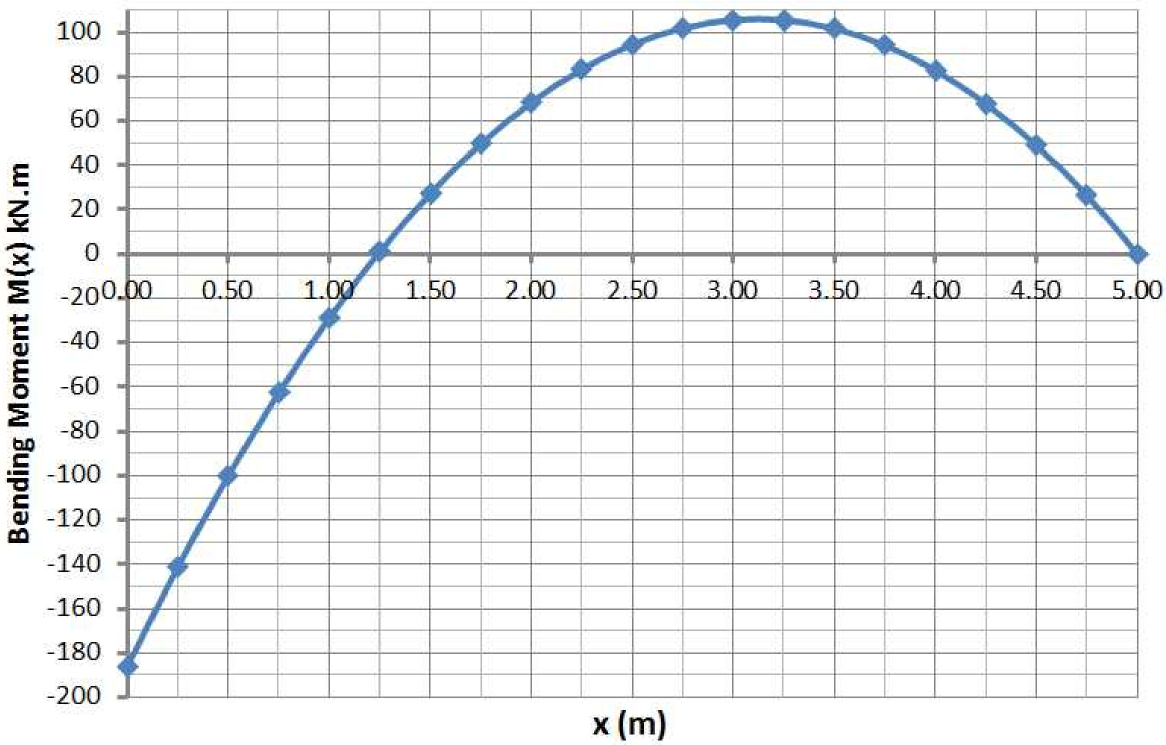
Solution

- **Design flexure reinforcement for section at support end (A):**
  In negative region, section behaves as rectangular section as flange is already cracked.
  - Compute M_n:
    $$M_u = -60 \times 5.00 \times \frac{5.00}{2} + 112.75 \times 5 = -186\ kN.m$$
    Assume ϕ to be 0.9 to be checked later:
    $$M_n = \frac{186}{0.9} = 207$$
  - Compute ρ:
    $$d_{one\ layer} = 500 - 40 - 10 - \frac{25}{2} = 438$$
    $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_n}{f_c' bd^2}}}{1.18 \times \frac{f_y}{f_c'}} = \frac{1 - \sqrt{1 - 2.36\frac{207 \times 10^6}{28 \times 300 \times 438^2}}}{1.18 \times \frac{420}{28}} = 9.33 \times 10^{-3}$$

- Check $\rho_{Maximum}$:
  $\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85^2 \times \frac{28}{420} \times \frac{0.003}{0.007} = 20.6 \times 10^{-3} > \rho\ Ok.$
- Compute As:
  $A_s = 9.33 \times 10^{-3} \times 438 \times 300 = 1226\ mm^2$
  $No.of\ rebars = \frac{1226}{510} = 2.4$
  Try $3\phi25$.
- Check b:
  $b_{Required} = 40 \times 2 + 10 \times 2 + 25 \times 3 + 25 \times 2 = 225 < 300\ Ok.$
- Check A_{smin}:
  Since the span is a statically indeterminate span and $f_c' < 31.4\ MPa$, then A_{smin} will be:
  $A_{smin} = \frac{1.4}{fy} b_w d = \frac{1.4}{420} \times 300 \times 438 = 438\ mm^2 < A_{s\ provided}\ Ok.$
- Check ϕ Assumption:
  $a = \frac{3 \times 510 \times 420}{0.85 \times 28 \times 300} = 90\ mm \Rightarrow c = \frac{a}{\beta_1} = \frac{90}{0.85} = 106\ mm$
  $\epsilon_t = \frac{438 - 106}{106} \times 0.003 = 9.40 \times 10^{-3}$
  $> 5 \times 10^{-3} Ok.$
- Draw final section. Drawing shown is a preliminary one as instead of adding two rebars with nominal diameter for lower side, specific amount of positive reinforcement should be extended into support region according to ACI code requirements. This aspect will be discussed in Chapter 5 of the course.



- **Check adequacy if same amount of flexure reinforcement calculated at end (A) is used for section of maximum positive bending moment:**
  - Intuitively one can conclude that reinforcement computed for the negative region would be adequate when used on the bottom side for the positive moment. This is due to the facts that:
    - As will be discussed in ***Chapter 11***, the positive moment is lower than the negative moment for regular spans that subjected to uniformly distributed loads.
    - The flange is effective in the positive region while it is neglected in the negative region.
  - Compute the maximum positive moments:
    The maximum positive moment could either be computed by:
    $R_{u\ left} = 60 \times 5.0 - 112.75 = 187.25\ kN$
    $M(x) = -186.25 + 187.25x - \frac{60x^2}{2}$
    $\frac{dM}{dx} = 187.25 - 60x$
    $x_{maximum} = 3.121$
    $M(3.121) = -186.25 + 187.25 \times 3.121 - \frac{60 \times 3.121^2}{2}$
    $M_{u+ve\ Max} = 106\ kN.m$
    Or by:
    $\Sigma F_y = 0_{for\ right\ side}$
    $60x = 112.75$
    $x = 1.88\ m\ from\ right\ support$
    $M_{u\ +ve\ maximum} = 112.75 \times 1.88 - 60 \times \frac{1.88^2}{2} = 106\ kN.m$



- Check $\rho_{w\max.}$:

$$\rho_{wmax} = \rho_{max} + \frac{A_{sf}}{b_w d}$$

$$A_{sf} = \frac{0.85 \times 28 \times (750 - 300) \times 100}{420} = 2550\ mm^2$$

$$\rho_{wmax.} = 20.6 \times 10^{-3} + \frac{2550}{300 \times 438} = 40.0 \times 10^{-3} \Longrightarrow \rho_w = \frac{3 \times 510}{300 \times 438} = 11.6 \times 10^{-3} < \rho_{w\,max.}\ Ok.$$

- Check A_{smin}:

$$A_{smin} = \frac{1.4}{fy} b_w d = \frac{1.4}{420} \times 300 \times 438 = 438\ mm^2 < A_{s\,provided}\ Ok.$$

- Compute M_n:

Assume $a \leq h_f$:

$$a = \frac{3 \times 510 \times 420}{0.85 \times 28 \times 750} = 36mm < 100mm\ Ok.$$

$$M_n = (3 \times 510 \times 420) \times \left(438 - \frac{36}{2}\right) = 270\ kN.m$$

- Compute ϕ:

$$a = 36\ mm \Longrightarrow c = \frac{a}{\beta_1} = \frac{36}{0.85} = 42.3\ mm$$

$$\epsilon_t = \frac{438 - 42.3}{42.3} \times 0.003 = 28.1 \times 10^{-3} > 5 \times 10^{-3} Ok.$$

- ϕM_n:

$$\phi M_n = 0.9 \times 270 = 243\ kN.m > M_u\ Ok.$$

Example 4.9-5

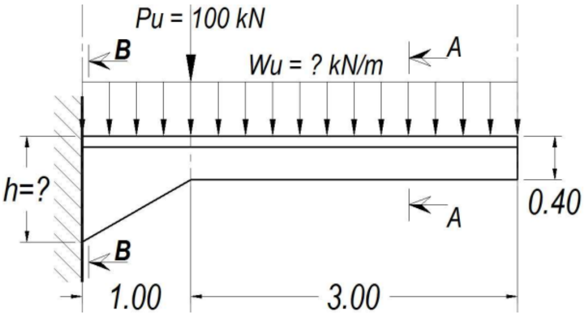
A structural designer has proposed dimensions and reinforcement for the cantilever T-beam shown in Figure 4.9-11 below.

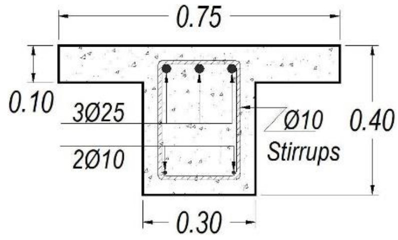
Based on flexure strength for the beam at Section A-A and at Section B-B find:

- Maximum factored uniform load (W_u) that could be supported by the beam.
- Minimum beam depth (h) for Section B-B.

In your solution, assume that:

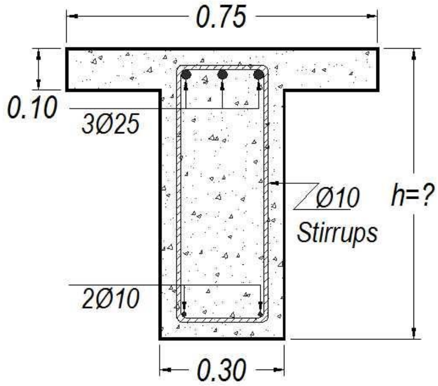
- Beam selfweight could be neglected.
- As = 510mm^2 for $\phi 25mm$ rebars.
- $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.





Sec. A-A

Figure 4.9-11: Cantilever T-beam for Example 4.9-5.



Sec. B-B

Solution

As flange is on the tension side, then the beam can be analyzed as a rectangular section except in computing $A_{s\,minimum}$ where flange should be considered.

Find Wu:

$$d = 400 - 40 - 10 - \frac{25}{2} = 338mm, A_s = 3 \times 510 = 1530\ mm^2$$

$$\rho_{provided} = \frac{1530}{300 \times 338} = 15.0 \times 10^{-3}$$

$$\rho_{maximum} = 0.85^2 \times \frac{28}{420} \times \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} > \rho_{provided}\ Ok$$

For this statically determinate span with a flange in tension, minimum flexure reinforcement should be computed based on:

$$A_{s\,min} = minimum \left(\frac{0.25\sqrt{f_c'}}{f_y} bd, \frac{0.50\sqrt{f_c'}}{f_y} b_w d\right)$$

As

$b = 750\ mm > 2b_w = 2 \times 300 = 600\ mm$

then, the second term governs.

$$A_{s\,min} = \frac{0.50\sqrt{28}}{f_y} b_w d = \frac{0.50\sqrt{28}}{420} \times 300 \times 338 = 639\ mm^2 < A_s \therefore Ok.$$

$$M_n = 15.0 \times 10^{-3} \times 420 \times 300 \times 338^2 \times \left(1 - 0.59 \times \frac{15.0 \times 10^{-3} \times 420}{28}\right) = 187\ kN.m$$

Check ϕ:

$$a = \frac{1530 \times 420}{0.85 \times 28 \times 300} = 90\ mm \Rightarrow c = \frac{90}{0.85} = 106\ mm \Rightarrow \epsilon_t = \frac{d-c}{c}\epsilon_u = \frac{338 - 106}{106} \times 0.003$$
$$= 0.0065 > 0.005$$

then:

$\phi = 0.9$

$M_u = \phi M_n = 0.9 \times 187 = 168\ kN.m$

$$M_u = \frac{W_u l^2}{2} \Rightarrow W_u = \frac{2M_u}{l^2} = \frac{2 \times 168}{3^2} = 37.3 \frac{kN}{m} \blacksquare$$

Find beam depth "d":

$$M_u = \frac{37.3 \times 4^2}{2} + 100 \times 1.0 = 398\ kN.m$$

$$\Sigma F_x = 0 \Rightarrow a = \frac{1530 \times 420}{0.85 \times 28 \times 300} = 90mm$$

Compute M_n (assume that $\phi = 0.9$, to be checked later):

$M_n = \frac{398}{0.9} = 442\ kN.m \Rightarrow 442 \times 10^6 = 1530 \times 420 \times \left(d - \frac{90}{2}\right) \Rightarrow d = 733\ mm$

Check ϕ:

$a = 90\ mm \Rightarrow c = \frac{90}{0.85} = 106\ mm \Rightarrow \epsilon_t = \frac{d-c}{c}\epsilon_u = \frac{733-106}{106} \times 0.003 = 0.0177 > 0.005$

then:

$\phi = 0.9$

Beam depth:

$h = 733 + \frac{25}{2} + 10 + 40 = 796\ mm$

Use

$h = 800\ mm$ ■

4.9.4 Problems for Solution

A T-beam having a span of 6.0 m, a web thickness of 300mm, and an overall depth of 645 mm. The beams spacing is 1.2m center to center and the slab thickness is 100 mm. Design this beam for flexure to carries a total factored moment of 1300 kN.m.

Assume that the designer intends to use:

- $f_y = 400$ Mpa $f_c' = 28$ Mpa
- $\emptyset 32$mm for longitudinal reinforcement ($A_{Bar} = 819 mm^2$) and $\emptyset 10$mm for stirrups.
- Two layers of reinforcement.

Answers

- Compute of Required Nominal Flexure Strength M_n:

  $M_n = \frac{M_u}{\emptyset} = 1\,444$ kN.m

  where $\emptyset$ will be assumed 0.9 to be checked later.
- Compute the effective flange width "b":

  $b = b_w + minimum\ \left[\frac{s_w}{2}\ or\ 8h\ or\ \frac{l_n}{8}\right] \times 2$

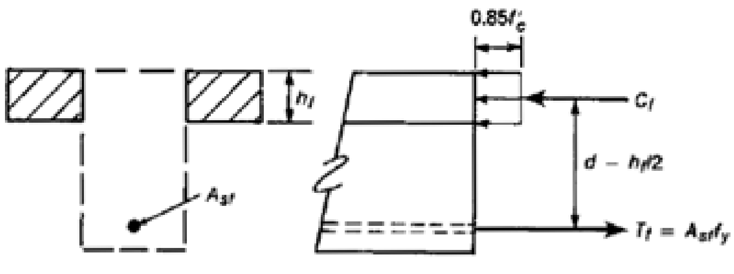
  $b = 300 + minimum\ \left(\frac{1200-300}{2}\ or\ 8 \times 100\ or\ \frac{6000}{8}\right) \times 2$

  $= 300 + minimum\ (450\ or\ 800\ or 750) \times 2 = 300 + 450 \times 2 = 1200\ mm$
- Check if this section can be design with a compression block in section flange or extend to section web based on following comparison:

  $M_n ?\ 0.85 f_c' h_f b \left(d - \frac{h_f}{2}\right)$

  $d = 550$mm

  $M_n = 1\,444\ kN.m > 0.85 f_c' h_f b \left(d - \frac{h_f}{2}\right) = 1\,428\ kN.m$
- Design of a section with $a > h_f$:
  - Compute the nominal moment that can be supported by flange overhangs:



    $M_{n1} = 0.85 f_c' h_f (b - b_w)(d - \frac{h_f}{2}) = 1\,071\ kN.m$

    Steel reinforcement for this part will be:

    $A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y} = 5\,355\ mm^2$
  - Compute the remaining nominal strength that must be supported by section web:

    $M_{n2} = M_n - M_{n1} = 373\ kN.m$