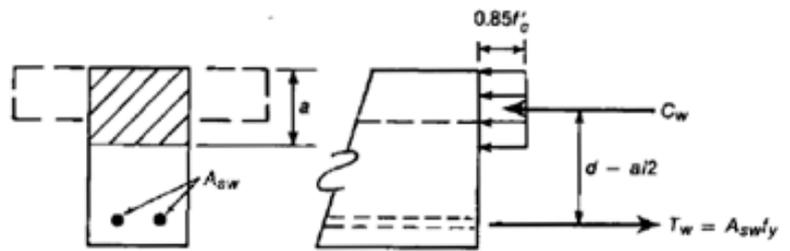


For this moment "M_{n2}", the section can be designed as a rectangular section with dimensions of b_w and d:



$$\rho_{\text{Required}} = \frac{1 - \sqrt{1 - 2.36 \frac{M_{n2}}{f'_c b_w d^2}}}{1.18 \times \frac{f_y}{f'_c}} = 11.4 \times 10^{-3} \Rightarrow A_{s2} = \rho_{\text{Required}} b_w d = 1881 \text{ mm}^2$$

Then:

$$A_{s \text{ Required}} = A_{sf} + A_{s2} = 7236 \text{ mm}^2 \Rightarrow \text{No. of Rebars} = 8.84$$

$$\text{Try } 9\emptyset 32\text{mm} \Rightarrow A_{s \text{ Provided}} = 7371 \text{ mm}^2$$

- Check A_{S Provided} with minimum steel area permitted by the ACI Code:

$$A_{s \text{ minimum}} = \frac{0.25 \sqrt{f'_c}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d \Rightarrow A_{s \text{ minimum}} = 578 \text{ mm}^2 < A_{s \text{ Provided}} \text{ Ok.}$$

- Check the A_{S Provided} with the maximum steel area permitted by ACI Code:

$$\rho_w = \frac{A_{s \text{ Provided}}}{b_w d} \leq \rho_w \text{ max} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$

$$\rho_w = 44.7 \times 10^{-3} \leq \rho_w \text{ max} = 21.7 \times 10^{-3} + 32.5 \times 10^{-3}$$

$$\rho_w = 20.7 \times 10^{-3} < \rho_w \text{ max} = 54.2 \times 10^{-3} \text{ Ok.}$$

- Check the assumption of $\phi = 0.9$:

- Compute "a":

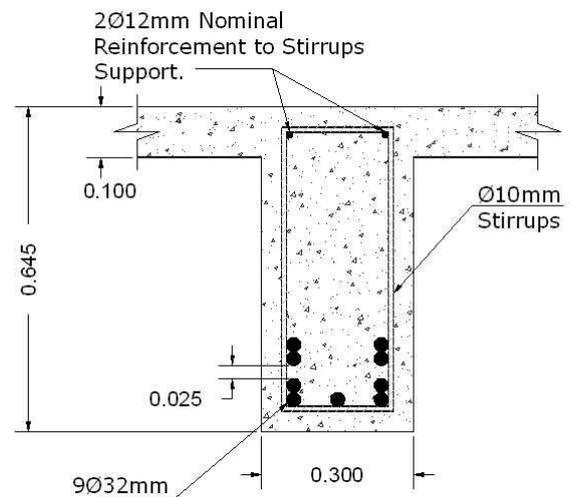
$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f'_c (b_w)} = 113 \text{ mm}$$

- Compute steel strain based on the following relations:

$$c = \frac{a}{\beta_1} = 133 \text{ mm} \Rightarrow \epsilon_t = \frac{d - c}{c} \epsilon_u = 9.41 \times 10^{-3}$$

- As $\epsilon_t > 0.005$, then $\phi = 0.9$ Ok.

- Draw the Section Details:



Problem 4.9-1

A reinforced concrete T-beam is to be designed for tension reinforcement. The beam width is 250mm and total depth of 490mm. The flange thickness is 100mm and its effective width has been computed to be 900mm. The applied total factored moment is 300kN.m

Assume that the designer intends to use:

- $f_y = 414 \text{ Mpa}, f'_c = 21 \text{ Mpa}$
- $\emptyset 28\text{mm}$ for longitudinal reinforcement and $\emptyset 10\text{mm}$ for stirrups.
- Two layers of reinforcement.

Answers

- Compute of Required Nominal Flexure Strength M_n :

$$M_n = \frac{M_u}{\phi} = 333 \text{ kN.m}$$

where ϕ will be assumed 0.9 to be checked later.

- Check if this section can be design with a compression block in section flange or extend to section web based on following comparison:

$$M_n \leq 0.85 f'_c h_f b \left(d - \frac{h_f}{2} \right)$$

$$d = 400 \text{ mm}$$

$$M_n = 333 \text{ kN.m} > 0.85f'_c h_f b \left(d - \frac{h_f}{2} \right) = 562 \text{ kN.m}$$

- Design of a section with $a \leq h_f$:

This section can be designed as a rectangular section with dimensions of b and d .

$$\rho_{\text{Required}} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f'_c b d^2}}}{1.18 \times \frac{f_y}{f'_c}} = \frac{1 - \sqrt{1 - 2.36 \times \frac{333 \times 10^6}{21 \times 900 \times 400^2}}}{1.18 \times \frac{414}{21}} = 6.01 \times 10^{-3}$$

$$A_{S \text{ Required}} = \rho_{\text{Required}} b d = 6.01 \times 10^{-3} \times 900 \times 400 = 2164 \text{ mm}^2$$

$$A_{\text{Bar}} = 615 \text{ mm}^2$$

$$\text{No of Rebars} = \frac{2164}{615} = 3.52$$

Try 4Ø28mm

$$A_{S \text{ provided}} = 2460 \text{ mm}^2$$

$$b_{\text{Required}} = 296 \text{ mm} > 250 \text{ mm}$$

Then the reinforcement must be put in two layers as the designer is assumed.

- Check $A_{S \text{ Provided}}$ with minimum steel area permitted by the ACI Code:

$$A_{S \text{ minimum}} = \frac{0.25 \sqrt{f'_c}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d \Rightarrow A_{S \text{ minimum}} = 338 \text{ mm}^2$$

As $A_{S \text{ Provided}} > A_{S \text{ minimum}}$ Ok.

- Check the $A_{S \text{ Provided}}$ with the maximum steel area permitted by ACI Code:

$$\rho_w = \frac{A_{S \text{ Provided}}}{b_w d} \quad ? \quad \rho_w \text{ max} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$

$$A_{sf} = \frac{0.85 f'_c h_f (b - b_w)}{f_y} = 2802 \text{ mm}^2$$

$$\rho_w = 24.6 \times 10^{-3} \quad ? \quad \rho_w \text{ max} = 15.7 \times 10^{-3} + 28.0 \times 10^{-3}$$

$$\rho_w = 24.6 \times 10^{-3} \ll \rho_w \text{ max} = 43.7 \times 10^{-3} \text{ Ok.}$$

- Check the assumption of $\phi = 0.9$:

- Compute "a":

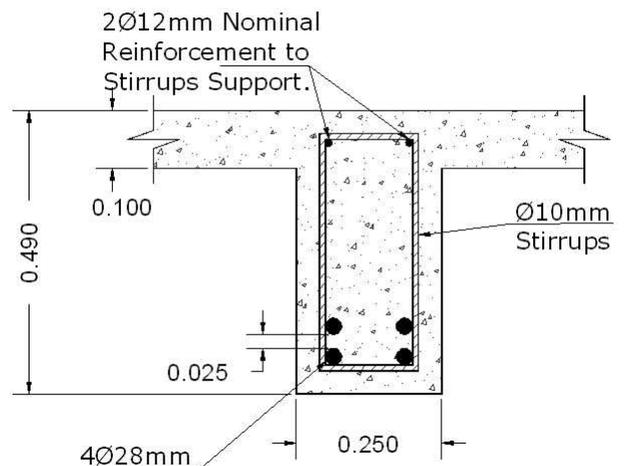
$$\sum F_x = 0 \Rightarrow a = 63.4 \text{ mm}$$

- Compute steel strain based on the following relations:

$$c = \frac{a}{\beta_1} = 74.6 \text{ mm} \Rightarrow \epsilon_t = 13.1 \times 10^{-3}$$

- As $\epsilon_t > 0.005$, then $\phi = 0.9$ Ok.

- Draw the Section Details:



4.10 ANALYSIS OF BEAMS WITH IRREGULAR SECTIONS

4.10.1 Basic Concepts

- Beams having shapes other than rectangular and T-shaped cross sections are common, **particularly in structures using precast elements**.
- The approach for the analysis of such beams is based on applications of basic principles (**compatibility**, **stress-strain relations**, and **equilibrium equations**).
- To avoid problems related to unsymmetrical bending:
 - All beams will be assumed to have an axis of symmetry.

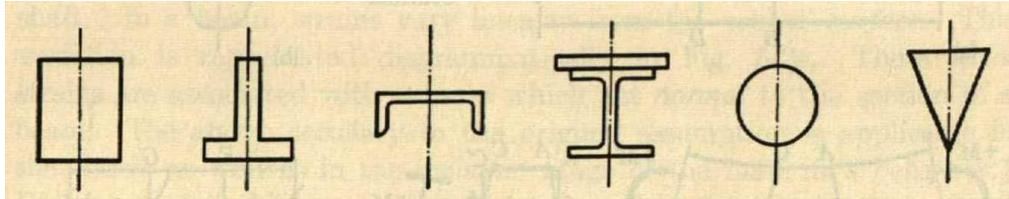


Figure 4.10-1: Different sections with axes of symmetry.

- All loads will be assumed to act through symmetrical plane.

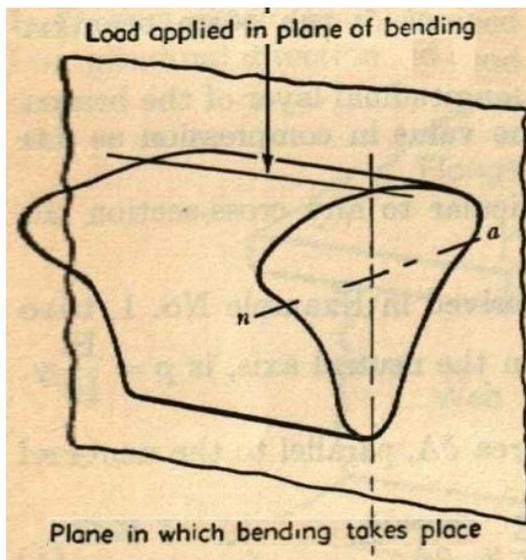


Figure 4.10-2: A beam with loads acting in symmetry plane.

4.10.2 Examples

Example 4.10-1

The cross-section in **Figure 4.10-3** below is sometimes referred to as an **inverted T girder**. Check if proposed section satisfies ACI requirements and then find its design moment (ϕM_n). In your solution, assume that:

$f'_c = 21$ MPa and $f_y = 420$ MPa

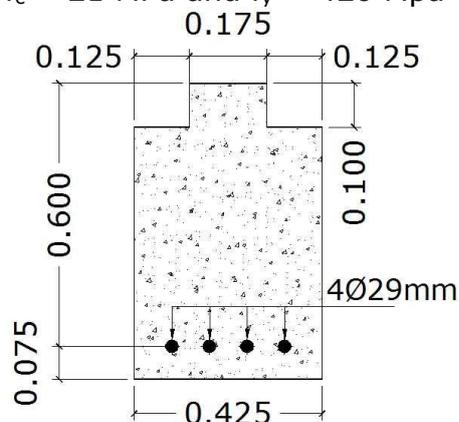


Figure 4.10-3: Inverted T girder for Example 4.10-1.

Solution

- Check type of failure:
Based on compatibility conditions, and based on the definition of maximum reinforcement area as the area that produces a tensile strain of 0.004 at failure state, the following strain distribution can be concluded:

Based on triangles similarities, following relation for c_{max} can be concluded:

$$c_{max} = \frac{\epsilon_u}{\epsilon_u + 0.004} d$$

$$= \frac{0.003}{0.003 + 0.004} d$$

$$\Rightarrow c_{max} = 0.429 d \blacksquare$$

As this relation is derived with considering of strain distribution only, then it is applicable for general shapes.

$$c_{max} = 0.429 \times 600 = 257 \text{ mm}$$

Using Whitney block concept,

$$a_{maximum} = \beta_1 c_{maximum} = 0.85 \times 257 = 218 \text{ mm}$$

Based on equilibrium condition,

$$\Sigma F_x = 0 \Rightarrow 0.85 \times 21 \times (175 \times 100 + 118 \times 425)$$

$$= 420 \times A_s \text{ Maximum} \Rightarrow A_s \text{ Maximum}$$

$$= 2875 \text{ mm}^2$$

$$A_s \text{ Provided} = 4 \times \frac{\pi \times 29^2}{4} = 2640 \text{ mm}^2 < A_s \text{ maximum}$$

$\therefore Ok.$

- Check $A_s \text{ Minimum}$:

As divergence from rectangular section is limited to the upper part only, then the minimum reinforcement area can be computed based on traditional relation:

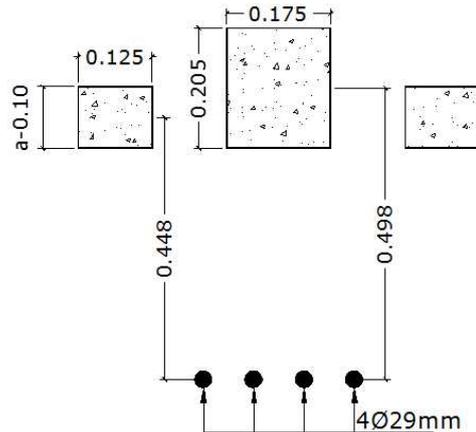
$$A_s \text{ Minimum} = \frac{1.4}{420} \times 425 \times 600 = 850 \text{ mm}^2 < A_s \text{ Provided} \therefore Ok.$$

- Compute M_n :

Let $a \leq 100$:

$$\Sigma F_x = 0 \Rightarrow 0.85 \times 21 \times 175 \times a = 4 \times 660 \times 420 \Rightarrow a = 354 \text{ mm}$$

$> 100 \text{ Not Ok}$



$$\Sigma F_x = 0 \Rightarrow 0.85 \times 21$$

$$\times (175 \times a + 2 \times (125 \times (a - 100)))$$

$$= 4 \times 660 \times 420$$

$$0.85 \times 21 \times (175 \times a + 250 \times a - 25000)$$

$$= 4 \times 660 \times 420$$

$$(425 \times a - 25000) = 62117.6 \Rightarrow a = 205$$

$$\Sigma M_{\text{about Reinforcement}} = 0$$

$$M_n = 0.85 \times 21$$

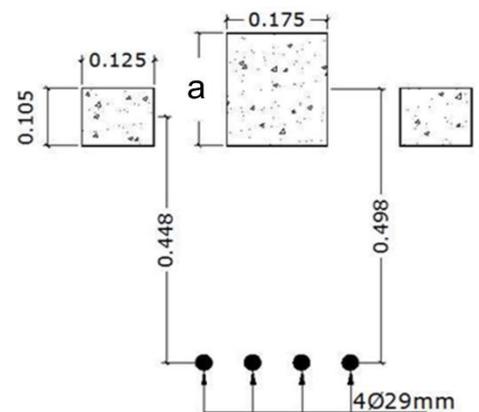
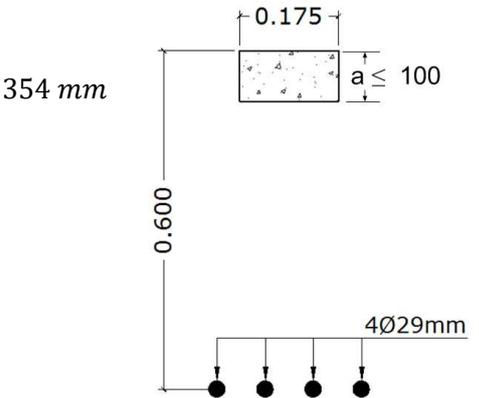
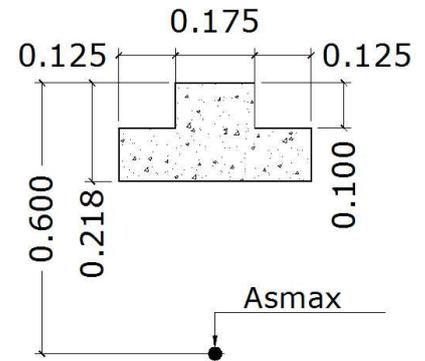
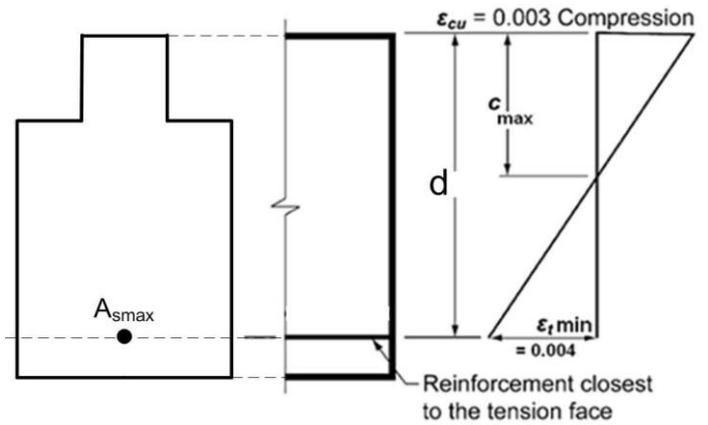
$$\times (205 \times 175 \times 498$$

$$+ 2(125 \times 105 \times 448)) = 529 \text{ kN.m}$$

- Compute ϕ :

- Compute "a":

$$a = 205 \text{ mm}$$



- Compute steel stain based on the following relations:
 $c = \frac{a}{\beta_1} = \frac{205}{0.85} = 241 \text{ mm} \Rightarrow \epsilon_t = \frac{d - c}{c} \epsilon_u = \frac{600 - 241}{241} \times 0.003 = 4.47 \times 10^{-3}$
- Then ϕ should be computed based on following relation:
 $\phi = 0.483 + 83.3\epsilon_t \Rightarrow 0.483 + 83.3 \times 4.47 \times 10^{-3} = 0.855$

- Compute ϕM_n :
 $\phi M_n = 0.855 \times 529 = 452 \text{ kN.m} \blacksquare$

Example 4.10-2

For the simply supported beam with a trapezoidal section that shown in Figure 4.10-4 below, and based on flexure strength of the given section find required beam depth (h) and width (b) that are necessary to support the applied loads.

In your solution, assume that:

- Beam selfweight could be neglected.
- $A_s = 510 \text{ mm}^2$ for $\phi 25 \text{ mm}$ rebars.
- $f'_c = 28 \text{ MPa}$ and $f_y = 420 \text{ MPa}$.

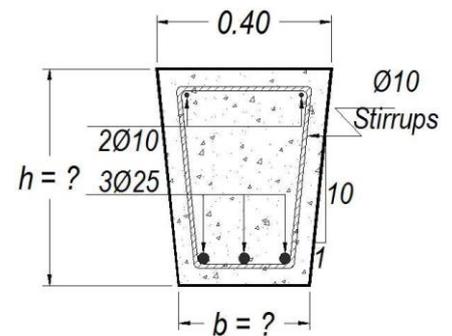
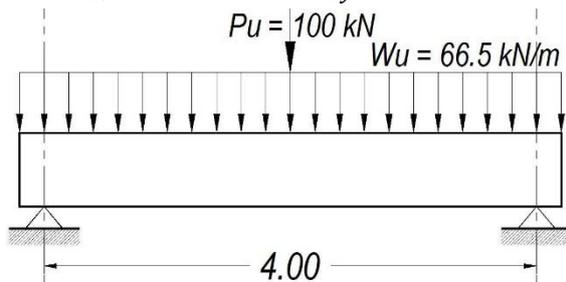


Figure 4.10-4: Trapezoidal beam for Example 4.10-2.

Solution

Compute depth of compression block "a":

$$\Sigma F_x = 0$$

$$0.85 \times 28 \left(\frac{400 + (400 - 0.2a)}{2} \right) \times a = 1530 \times 420$$

$$(800 - 0.2a) \times a = 54000$$

$$0.2a^2 - 800a + 54000 = 0$$

Solve for a:

$$a = 68.7 \text{ mm}$$

Compute effective depth "d":

$$M_u = \frac{W_u l^2}{8} + \frac{P_u l}{4} = \frac{66.5 \times 4.0^2}{8} + \frac{100 \times 4}{4}$$

$$= 233 \text{ kN.m}$$

Let $\phi = 0.9$ to be checked later:

$$M_n = \frac{233}{0.9} = 259 \text{ kN.m}$$

$$259 \times 10^6 = 0.85 \times 28 \times \left(386 \times 68.7 \times \left(d - \frac{68.7}{2} \right) + \frac{7 \times 68.7}{2} \times 2 \times \left(d - \frac{68.7}{3} \right) \right)$$

$$d = 437 \text{ mm}$$

Check ϕ :

$$a = 68.7 \text{ mm} \Rightarrow c = \frac{68.7}{0.85} = 80.8 \text{ mm}$$

$$\epsilon_t = \frac{d - c}{c} \epsilon_u = \frac{437 - 80.8}{80.8} \times 0.003 = 0.0132 > 0.005$$

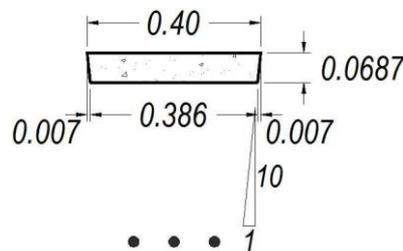
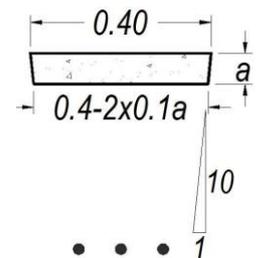
then:

$$\phi = 0.9$$

$$\therefore h = 437 + \frac{25}{2} + 10 + 40 = 500 \text{ mm} \blacksquare$$

$$b = 400 - \frac{1}{10} \times 500 \times 2 = 300 \text{ mm} \blacksquare$$

Typical Section



Example 4.10-3

In a trail to reduce the cost of beam through reducing of concrete on tension side, a structural designer has been proposed section shown in Figure 4.10-5 below to be used through the length of beam show.

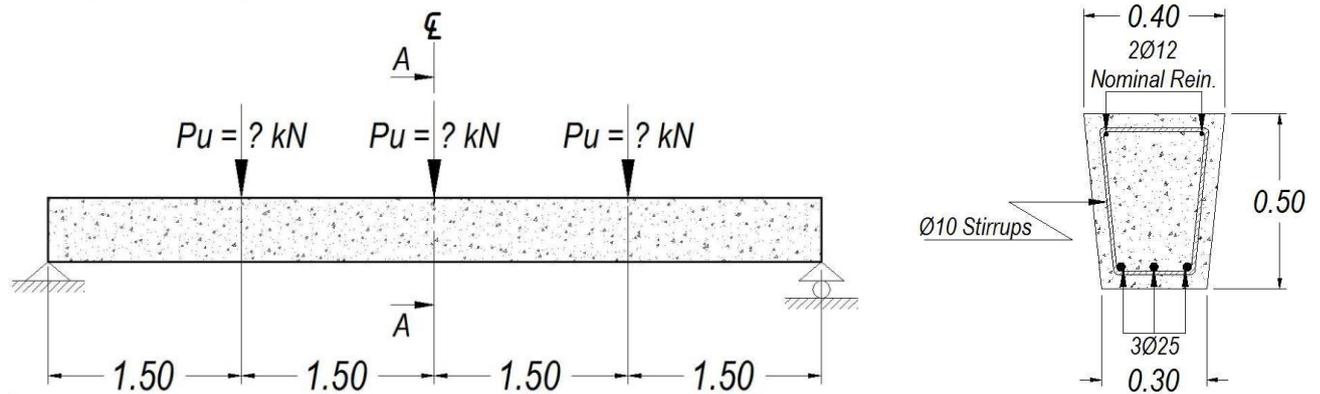


Figure 4.10-5: Trapezoidal beam for Example 4.10-3.

Typical Section

Check the adequacy of proposed section to ACI flexure requirements and then computed the maximum factored point load (P_u) that can be supported by the beam based on flexural strength.

In your solution, assume that:

- Beam selfweight can be neglected.
- $f'_c = 21 \text{ MPa}, f_y = 420 \text{ MPa}$
- $A_{Bar} = 510 \text{ mm}^2$ for $\phi 25\text{mm}$

Solution

- Check type of failure:

Based on compatibility conditions, and based on definition of A_{smax} as the reinforcement area that produce a tensile strain of 0.004 at failure state:

$$c_{max} = \frac{\epsilon_u}{\epsilon_u + 0.004} d = \frac{0.003}{0.003 + 0.004} d = 0.429 d$$

$$d = 500 - 40 - 10 - \frac{25}{2} = 437 \text{ mm} \Rightarrow c_{max} = 0.429 \times 437 = 187 \text{ mm}$$

Using Whitney block concept,

$$a_{maximum} = \beta_1 c_{maximum} = 0.85 \times 187 = 159 \text{ mm}$$

$$\theta = \tan^{-1} \frac{50}{500} = 5.71^\circ$$

$$x = 400 - 2 \times \tan 5.71 \times 159 = 368 \text{ mm}$$

$$0.85 \times 21 \times \left(\frac{400 + 368}{2} \right) \times 159 = 420 \times A_{s \text{ Maximum}}$$

$$A_{s \text{ Maximum}} = 2595 \text{ mm}^2$$

$$A_{s \text{ Provided}} = 3 \times 510 = 1530 \text{ mm}^2 < A_{s \text{ maximum}}$$

$\therefore Ok.$

- Compute M_n :

Whitney block depth, a , could be computed based on following relation:

$$\Sigma F_x = 0$$

$$0.85 \times 21 \times \left(400a - 2 \times a \times \frac{a \times \tan 5.71}{2} \right) = 1530 \times 420$$

$$0.1a^2 - 400a + 36000 = 0$$

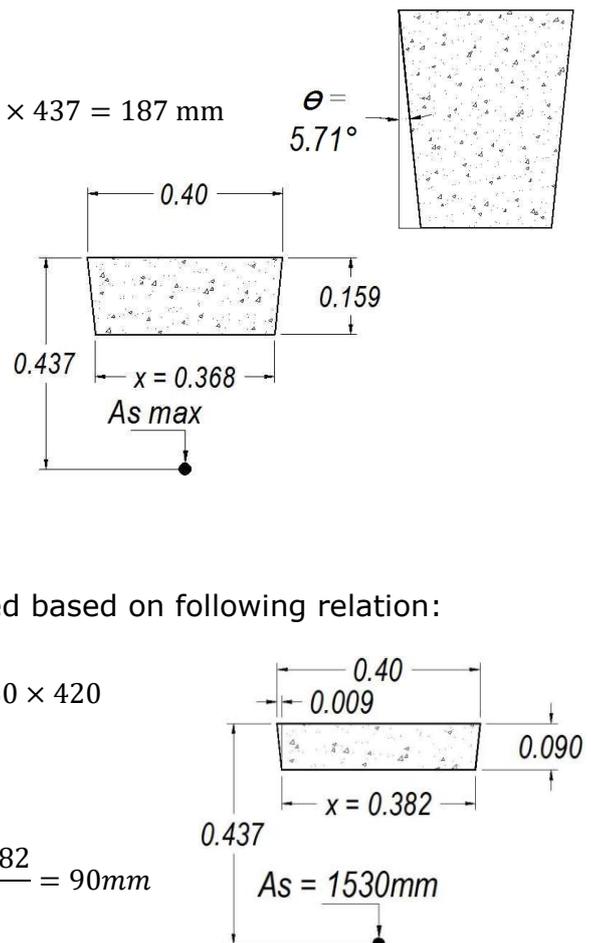
$$a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = \frac{+400 \pm \sqrt{400^2 - 4 \times 0.1 \times 36000}}{2 \times 0.1} = \frac{400 \pm 382}{0.2} = 90 \text{ mm}$$

$$\Sigma M_{\text{about steel center}} = 0$$

$$M_n = 0.85 \times 21 \times \left[382 \times 90 \times \left(437 - \frac{90}{2} \right) + 2 \times \frac{1}{2} \times 9 \times 90 \times \left(437 - \frac{90}{3} \right) \right]$$

$$M_n = 0.85 \times 21 \times [13.5 \times 10^6 + 0.330 \times 10^6] = 247 \text{ kN.m}$$



- Compute ϕ :
 - Compute "a":
 $a = 90 \text{ mm}$
 - Compute steel stain based on the following relations:

$$c = \frac{a}{\beta_1} = \frac{90}{0.85} = 106 \text{ mm} \Rightarrow \epsilon_t = \frac{d - c}{c} \epsilon_u = \frac{437 - 106}{106} \times 0.003 = 9.37 \times 10^{-3}$$
- Then:
 $\phi = 0.9$
- Compute ϕM_n :
 $\phi M_n = 0.9 \times 247 = 222 \text{ kN.m}$
- Compute P_u :

$$M_u = \frac{P_u \times 6}{4} + P_u \times 1.5 = 222 \Rightarrow P_u = 74 \text{ kN} \blacksquare$$

Example 4.10-4

Based on flexure strength for beam shown in Figure 4.10-6 below, what is the maximum factored uniformly distributed load "W_u" that can be supported?

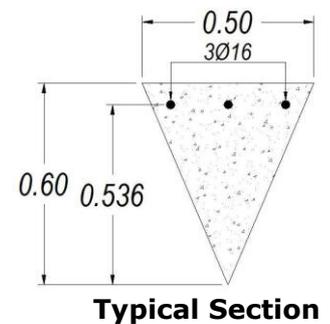
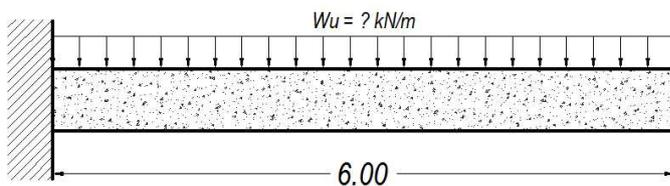


Figure 4.10-6: Beam with triangular section for Example 4.10-4.

In your solution, assume that:

- Beam selfweight can be.
- $f'_c = 21 \text{ MPa}, f_y = 420 \text{ MPa}$
- $A_{Bar} = 200 \text{ mm}^2$ for $\phi 16 \text{ mm}$
- Neglect the checking for A_s minimum.

Solution

- Check type of failure:
Based on compatibility conditions, and based on definition of A_{smax} as the reinforcement area that produce a tensile strain of 0.004 at failure state:

$$c_{max} = \frac{\epsilon_u}{\epsilon_u + 0.004} d = \frac{0.003}{0.003 + 0.004} d = 0.429 d \blacksquare$$

$$d = 536 \text{ mm} \Rightarrow c_{max} = 0.429 \times 536 = 230 \text{ mm}$$

Using Whitney block concept,

$$a_{maximum} = \beta_1 c_{maximum} = 0.85 \times 230 = 196 \text{ mm}$$

Based on triangles similarities,

$$\frac{x}{196} = \frac{500}{600} \Rightarrow x = 163 \text{ mm}$$

$$\Sigma F_x = 0$$

$$0.85 \times 21 \times \left(\frac{196 \times 163}{2} \right) = 420 \times A_s \text{ Maximum} \Rightarrow A_s \text{ Maximum} = 679 \text{ mm}^2$$

$$A_s \text{ Provided} = 3 \times 200 = 600 \text{ mm}^2 < A_s \text{ maximum} \therefore Ok.$$

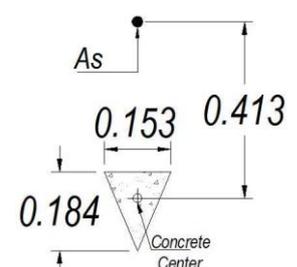
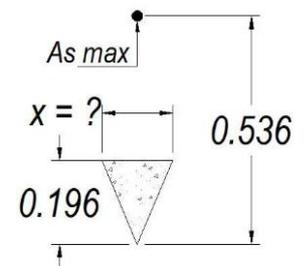
- Compute M_n :
Triangle base for a height of "a" could be computed based on following relation:

$$\frac{x}{a} = \frac{500}{600} \Rightarrow x = 0.833 a$$

$$\Sigma F_x = 0$$

$$0.85 \times 21 \times \left(\frac{0.833a^2}{2} \right) = 600 \times 420 \Rightarrow a = 184 \text{ mm}$$

$$\Sigma M_{\text{about concrete center}} = 0 \Rightarrow M_n = 600 \times 420 \times 413 = 104 \text{ kN.m}$$



Compute ϕ :

- Compute "a":

$$a = 184 \text{ mm}$$

- Compute steel stain based on the following relations:

$$c = \frac{a}{\beta_1} = \frac{184}{0.85} = 216 \text{ mm} \Rightarrow \epsilon_t = \frac{d - c}{c} \epsilon_u = \frac{536 - 216}{216} \times 0.003 = 4.44 \times 10^{-3}$$

- Then ϕ should be computed based on following relation:

$$\phi = 0.483 + 83.3\epsilon_t = 0.483 + 83.3 \times 4.44 \times 10^{-3} = 0.853$$

- Compute ϕM_n :

$$\phi M_n = 0.853 \times 104 = 88.7 \text{ kN.m} \blacksquare$$

- Compute W_u :

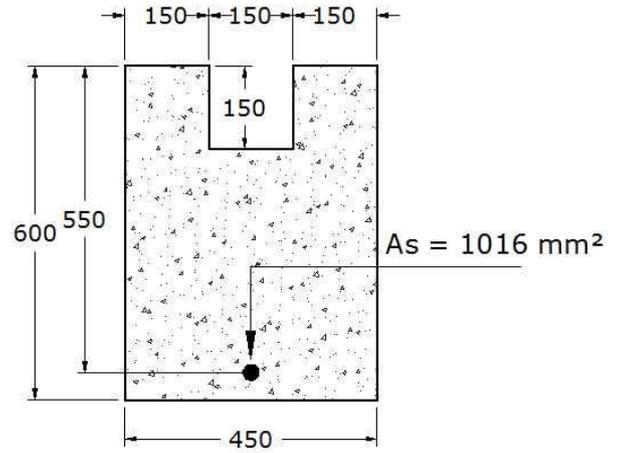
$$88.7 = \frac{W_u \times 6^2}{2} \Rightarrow W_u = 4.93 \frac{\text{kN}}{\text{m}} \blacksquare$$

4.10.3 Problems for Solution

Problem 4.10-1

Check adequacy of the indicated section for ACI requirements of maximum and minimum steel areas and compute its design bending strength if it is satisfied for ACI requirements.

$f'_c = 25 \text{ MPa}, f_y = 400 \text{ MPa}$



Answers

- Check for A_{smax} and A_{smin} :

$c_{max} = 0.429 d = 0.429 \times 550 = 236 \text{ mm}$

$a_{maximum} = 0.85 \times 236 = 201 \text{ mm}$

$\Sigma F_x = 0$

$0.85 \times 25 \times (2 \times 150 \times 150 + (201 - 150) \times 450) = A_{s maximum} \times 400$

$A_{s maximum} = 3610 \text{ mm}^2 > A_s \text{ Ok}$

$A_{s minimum} = \frac{1.4}{400} \times 450 \times 550 = 866 \text{ mm}^2 < A_s \text{ Ok.}$

- Compute M_n :

Assume $a \leq 150$:

$0.85 \times 25 \times 2 \times 150 \times a = 400 \times 1016 \Rightarrow a = 63.7 \text{ mm} < 150 \text{ Ok}$

$\Sigma M_{about T} = 0$

$M_n = 0.85 \times 25 \times 300 \times 63.7 \left(550 - \frac{63.7}{2} \right) = 210 \text{ kN.m}$

- Compute ϕM_n :

- a. Compute "a":

$a = 63.7 \text{ mm}$

- b. Compute steel stain based on the following relations:

$c = \frac{a}{\beta_1} = \frac{63.7}{0.85} = 74.9 \text{ mm} \Rightarrow \epsilon_t = \frac{d - c}{c} \epsilon_u = \frac{550 - 74.9}{74.9} \times 0.003 = 19.0 \times 10^{-3}$

- c. As $\epsilon_t \geq 0.005$ then:

$\phi = 0.9$

$\phi M_n = 0.9 \times 210 = 189 \text{ kN.m}$

Problem 4.10-2

Check adequacy of the indicated section for ACI requirements of maximum and minimum steel areas and compute its design bending strength if it is satisfied for ACI requirements.

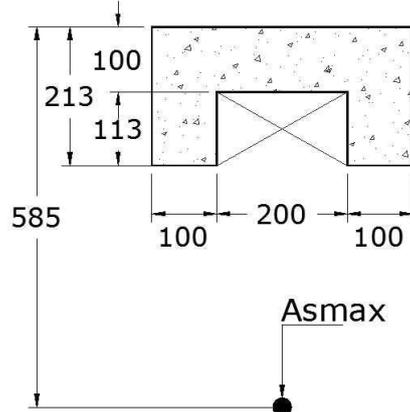
$f'_c = 20 \text{ MPa}, f_y = 400 \text{ MPa}$

Answers

- Check for A_{smax} and A_{smin} :

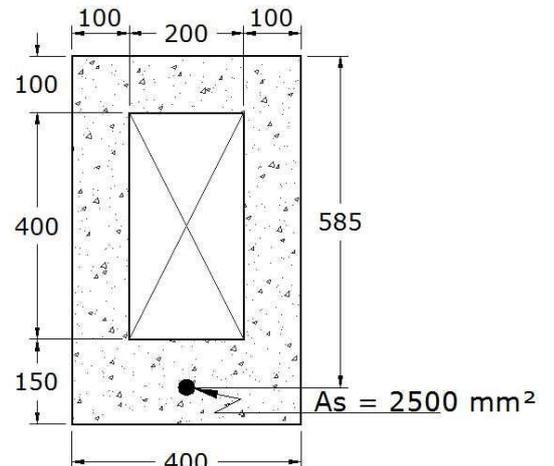
$c_{max} = 0.429 d = 0.429 \times 585 = 251 \text{ mm}$

$a_{maximum} = 213$



$\Sigma F_x = 0$

$0.85 \times 20 \times (400 \times 100 + 113 \times 100 \times 2) = A_{s Maximum} \times 400$



$$A_{s \text{ maximum}} = 2660 \text{ mm}^2 > A_s \text{ Ok.}$$

$A_{s \text{ min}}$ could be conservatively computed based on following relation:

$$A_{s \text{ minimum}} = \frac{1.4}{400} \times 400 \times 585 = 819 \text{ mm}^2 < A_s \text{ Ok.}$$

- Compute M_n :

Assume that $a \leq 100$:

$$\Sigma F_x = 0$$

$$0.85 \times 20 \times 400 \times a = 400 \times 2500$$

$$a = 147 \text{ mm} > 100 \text{ Not Ok.}$$

$$0.85 \times 20 \times (200 \times 100 + 2 \times 100 \times a) = 400 \times 2500$$

$$a = 194 \text{ mm}$$

$$\Sigma M_{\text{about } T} = 0$$

$$M_n = 0.85 \times 20 \times \left(200 \times 100 \times \left(585 - \frac{100}{2} \right) + 2 \times 100 \times 194 \times \left(585 - \frac{194}{2} \right) \right) = 503 \text{ kN.m}$$

- Compute ϕM_n :

- a. Compute "a":

$$a = 194 \text{ mm}$$

- b. Compute steel stain based on the following relations:

$$c = \frac{a}{\beta_1} = \frac{194}{0.85} = 228 \text{ mm}$$

$$\epsilon_t = \frac{d - c}{c} \epsilon_u = \frac{585 - 228}{228} \times 0.003 = 4.70 \times 10^{-3}$$

- c. Then ϕ should be computed based on following relation:

$$\phi = 0.483 + 83.3 \epsilon_t$$

$$\phi = 0.483 + 83.3 \times 4.70 \times 10^{-3} = 0.875$$

$$\phi M_n = 0.875 \times 503 = 440 \text{ kN.m}$$

4.11 USING STAAD PRO SOFTWARE FOR FLEXURAL ANALYSIS AND DESIGN OF RC BEAMS*

In general, most of software, including of STAAD Pro, have been prepared to design problems with pre-specified dimensions where analysis and design processes are simulated as pure iterative. Therefore, only sections analysis and design (with pre-specified dimensions) are presented in this article.

4.11.1 Design of a Singly Reinforced Concrete Beam with a Rectangular Shape

STAAD Pro steps for analysis and design of simply supported beams have been presented in this article with referring for Example 4.4-2 that, for convenient, has been represented in Figure 4.11-1 below. Following data have been adopted for this design:

- Concrete of $f_c' = 30 \text{ MPa}$.
- Steel of $f_y = 420 \text{ MPa}$.
- A width of 300mm and a height of 430mm (these dimensions have been determined based on deflection considerations).
- Rebar of No. 25 for longitudinal reinforcement.
- Rebar of No. 10 for stirrups.
- Single layer of reinforcement.

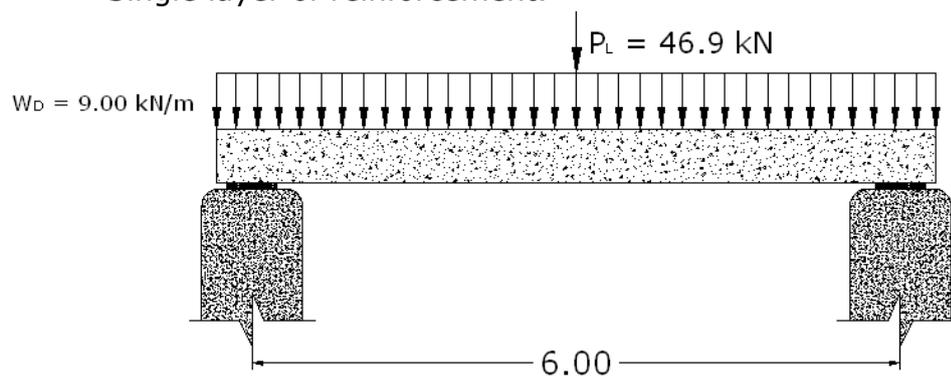


Figure 4.11-1: Simply supported bridge for Example 4.4-2, represented for convenient.

Solution

4.11.1.1 Type of Structure and Units

- Based on interactive box presented in Figure 4.11-2 below, select suitable **Structure Type** and suitable **Units** to be adopted in simulation of the beam.
- A Structure can be defined as an assemblage of elements. STAAD is capable of analyzing and designing structures consisting of frame, plate/shell and solid elements. Almost any type of structure can be analyzed by STAAD.
 - a. A **SPACE** structure, which is a three dimensional framed structure with loads applied in any plane, is the most general.
 - b. A **PLANE** structure is bound by a global X-Y coordinate system with loads in the same plane.
 - c. A **TRUSS** structure consists of truss members who can have only axial member forces and no bending in the members.
 - d. A **FLOOR** structure is a two or three-dimensional structure having no horizontal (global X or Z) movement of the structure [FX, FZ & MY are restrained at every joint]. The floor framing (in global X-Z plane) of a building is an ideal example of a FLOOR structure.
- Specification of the correct structure type reduces the number of equations to be solved during the analysis. This results in a faster and more economical solution for the user. The degrees of freedom associated with frame elements of different types of structures is illustrated in Figure 4.11-3.
- The beam of this example is simulated as a plane structure. Using a suitable structure type saving computer resources and avoid stability problems related to some structures, for example plane trusses, when simulated with a three dimensional model.

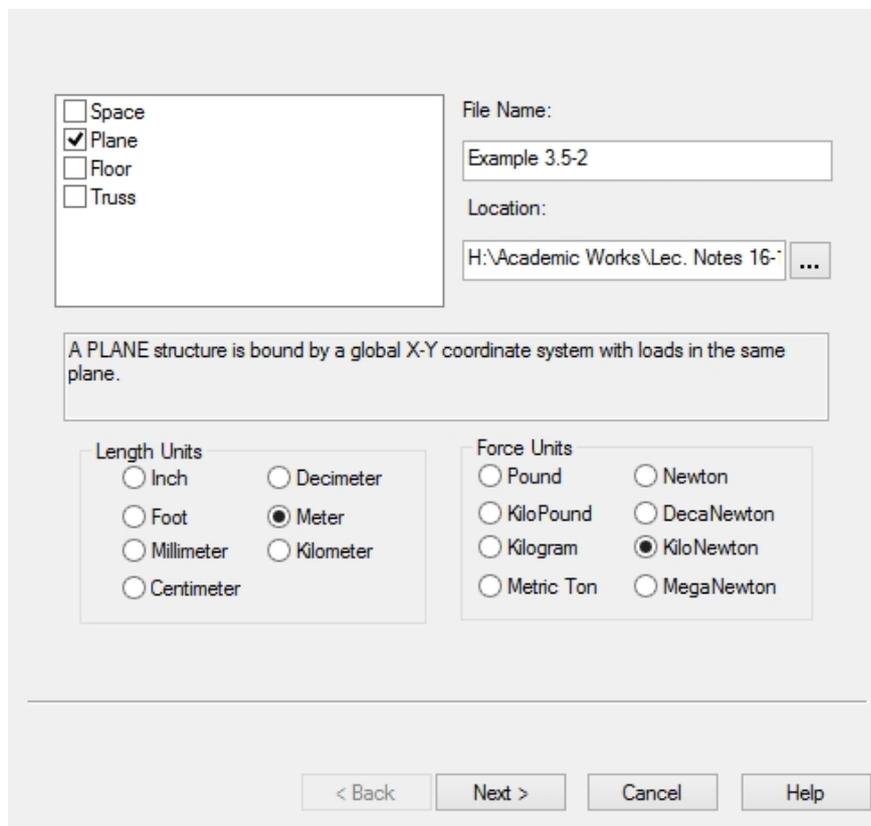


Figure 4.11-2: Different structure types in STAAD environment.

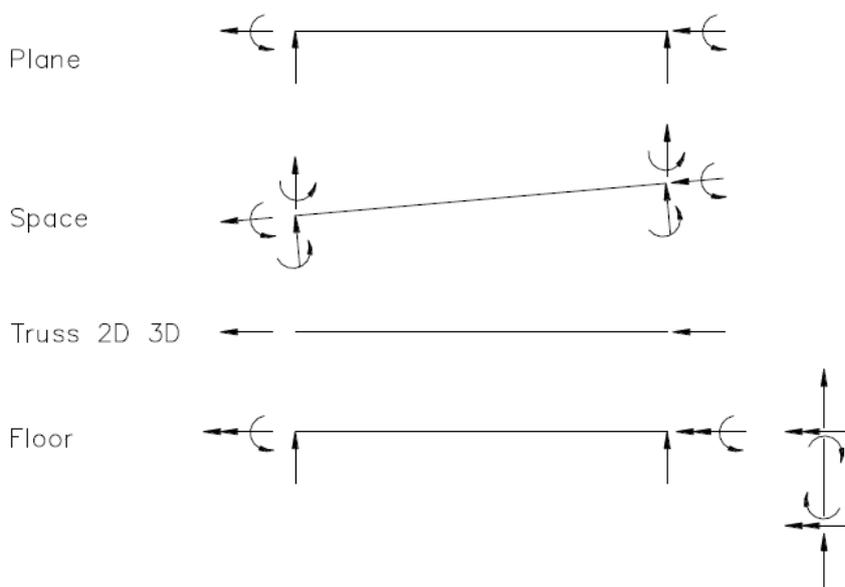


Figure 4.11-3: Degrees of freedom associated with frame elements of different types of structures.

4.11.1.2 STAAD Pages for a Sequential Work

- Workflow in STAAD environment has prepared in form of pages. When these pages are followed, the model would be complete and ready for execution.
- According to STAAD software, the preparation process is called **Modeling** and indicated with icon below:



Main modeling pages in STAAD environment are presented in Figure 4.11-5 Design parameters should be defined in the Modeling stage. Each pages are explained briefly in below.

4.11.1.3 Setup Page

In the Setup page, the user can input all information related to Job based on interactive box indicated Figure 4.11-4.

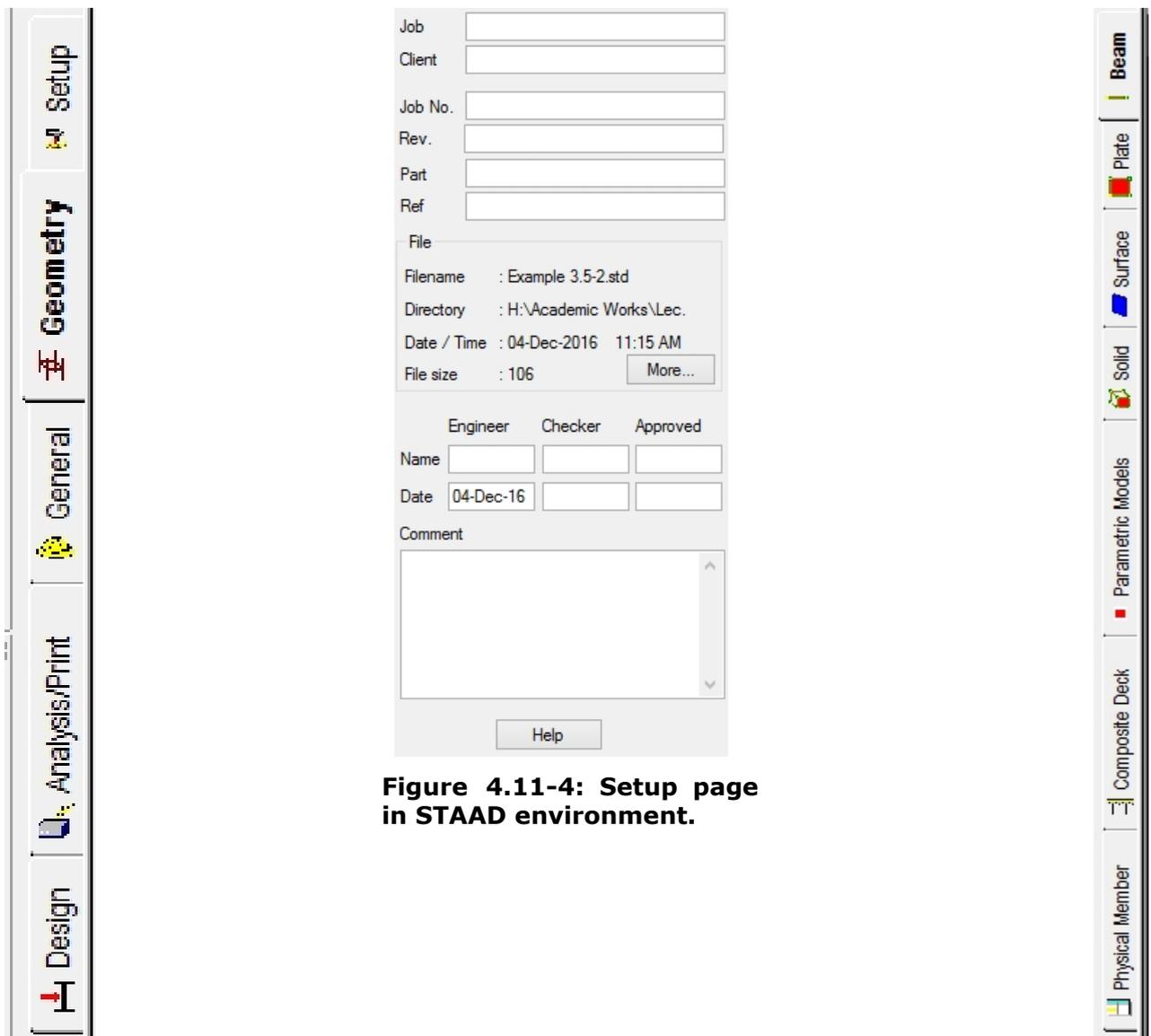


Figure 4.11-4: Setup page in STAAD environment.

Figure 4.11-5: Main modeling pages in STAAD environment.

Figure 4.11-6: Geometry Page in STAAD environment.

4.11.1.4 Geometry of the Beam

- Geometry Page has sub-pages indicated in Figure 4.11-6 above.
- STAAD software starts with definition of **Nodes** to prepare the geometry of the structure. In skeleton structures, nodes have been physically defined and located at ends of member.
- For the beam of this article, two nodes with coordinates below have been generated.

Node	X m	Y m	Z m
1	0.000	0.000	0.000
2	6.000	0.000	0.000
3			

N1

N2

- After definition of nodes, use Add Beam icon  to draw the beam that connecting between nodes.

N1

E1

N2

4.11.1.5 Definition of Material Prosperities

- Definition of new concrete properties has been presented in Figure 4.11-7 below.
- Regarding to shear modulus, G , based on mechanic of materials, one can show that:

$$G = \frac{E}{2(1 + \nu)}$$

4. Give an indicative name.

5. Elastic properties of concrete. These properties affect analysis process only in defining of concrete stiffness.

According to (ACI318M, 2014), article 19.2.2, modulus of elasticity, E_c , for concrete can be estimated based on following correlation:

- For values of w_c between 1440 and 2560 kg/m³

$$E_c = w_c^{1.5} 0.043 \sqrt{f'_c} \text{ (in MPa)}$$

- For normal weight concrete

$$E_c = 4700 \sqrt{f'_c} \text{ (in MPa)}$$

At stresses lower than about $0.7f'_c$, Poisson's ratio for concrete falls within the limits of 0.15 to 0.20.

Figure 4.11-7: Definition of material properties in STAAD environment.

4.11.1.6 Properties of the Beam

- Use page General and correspond sub-pages indicated below to defined and assign:
 - Beam Section,
 - Material,
 - Supports.

- Beam Section:

Use subpage **Property** to defined and assign beam section.

- Define section properties as indicated in steps of Figure 4.11-9 below.
- Defined beam section can now be assigned to pertained member based on steps of Figure 4.11-10 below.



Figure 4.11-8: General page and corresponding subpages in STAAD environment.

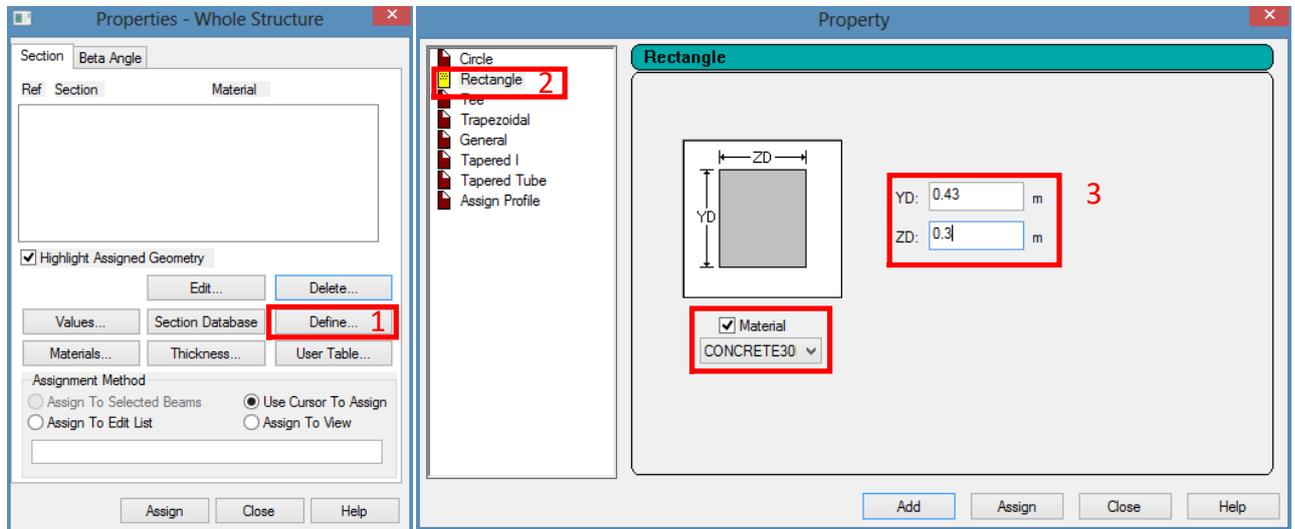
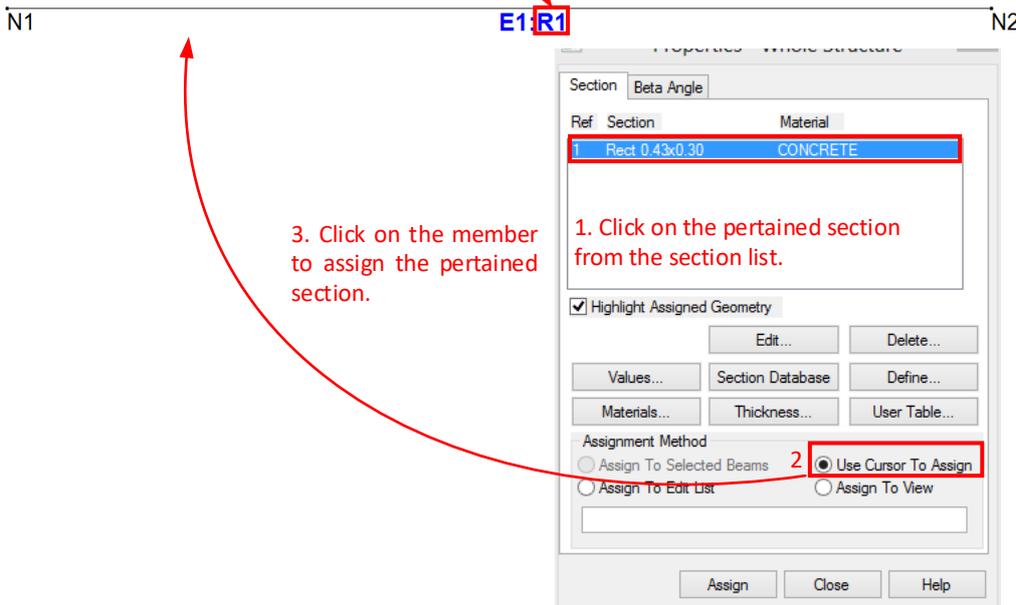


Figure 4.11-9: Steps to define section properties in STAAD environment.

4. Member property, including section and material indicated after assignment process.

Figure 4.11-10: Steps to section assignment in STAAD environment.



3. Click on the member to assign the pertained section.

1. Click on the pertained section from the section list.

4.11.1.7 Definition and Assignment of the Supports

- Supports can be defined as indicated in steps below.

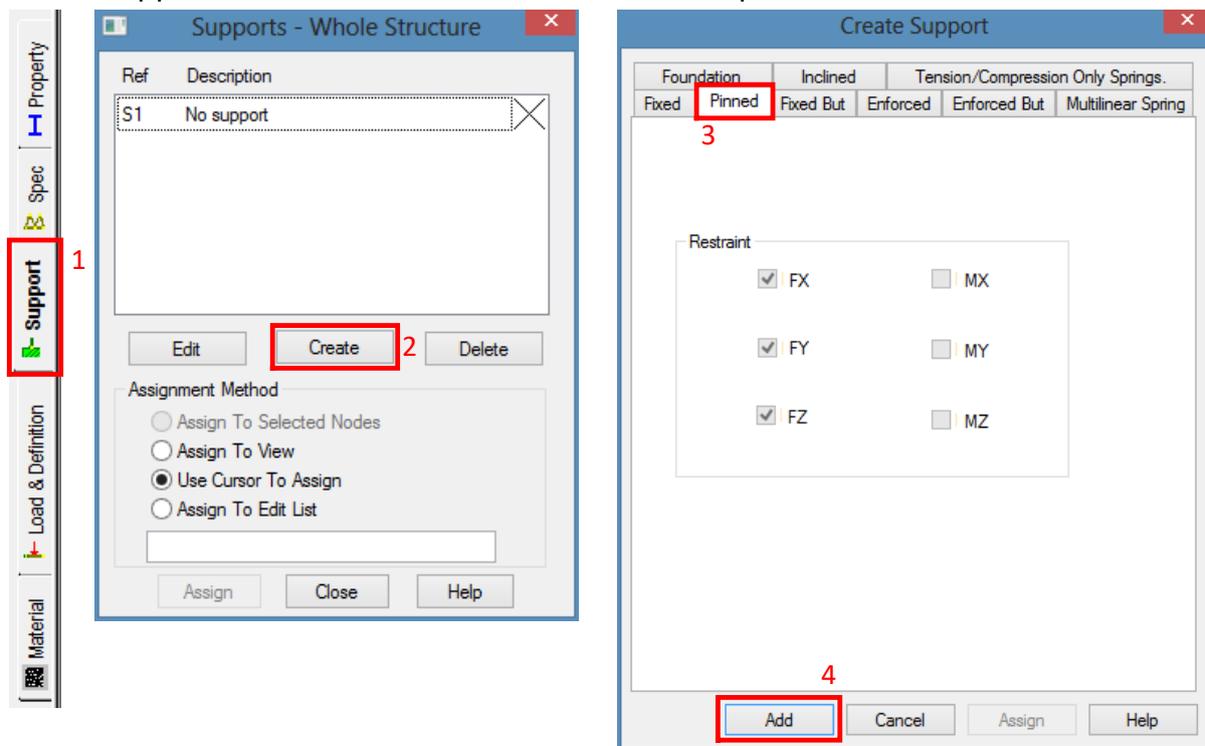


Figure 4.11-11: Steps to define supports in STAAD environment.

- For beam linear analysis, axial forces are already neglected and therefore there is no difference between hinge support and roller support form point of view.
- Defined supports can be assigned to related nodes based on following steps:

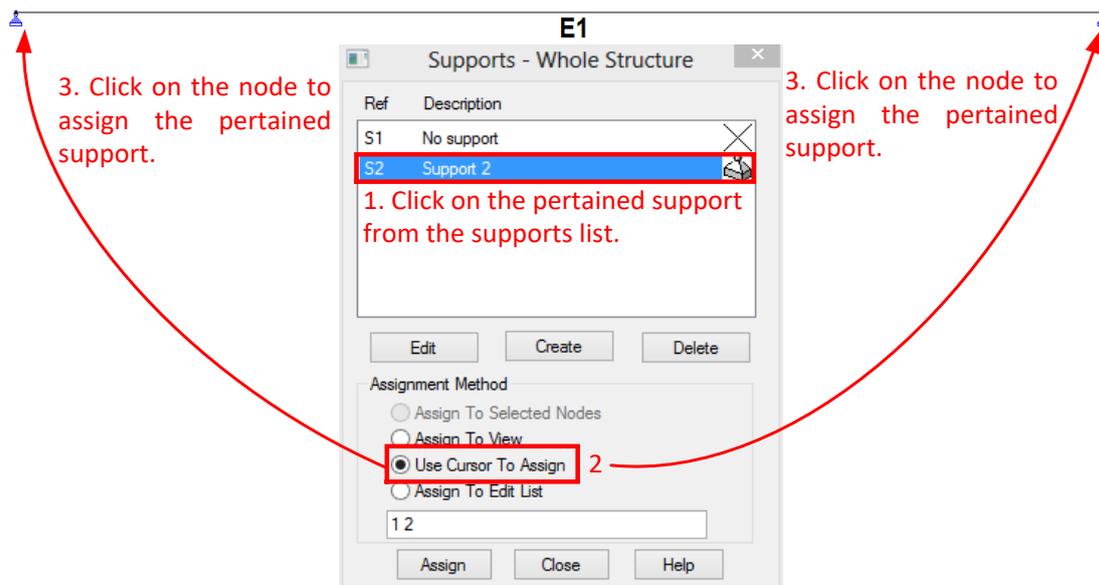


Figure 4.11-12: Steps to assign supports in STAAD environment.

4.11.1.8 Definition of Basic Load Cases, and Load Combinations

4.11.1.8.1 Definition Loads Cases

Basic load cases, namely **Dead** and **Live** can be defined based on following steps:

The screenshot displays the 'Load & Definition' window. On the left sidebar, the 'Load & Definition' icon is highlighted with a red box and labeled '1'. The main tree view on the left shows 'Definitions' expanded, with 'Load Cases Details' selected and highlighted with a red box and labeled '2'. The right-hand configuration panel is divided into two sections. The top section, labeled '4', shows the configuration for 'Dead Load' (Number 1, Loading Type: Dead). The bottom section, labeled '5', shows the configuration for 'Live Load' (Number 2, Loading Type: Live). The 'Add...' button at the bottom of the main panel is highlighted with a red box and labeled '3'. The configuration panel also includes a 'Toggle Load' checkbox, an 'Assignment Method' section with radio buttons for 'Assign To Selected Entities', 'Use Cursor To Assign', 'Assign To View', and 'Assign To Edit List', and 'Assign', 'Close', and 'Help' buttons at the bottom.

Figure 4.11-13: Definition of basic load cases.

4.11.1.8.2 Definition of Load Values and Assign them to Related Members

- Selfweight can be defined and assigned based on following steps.

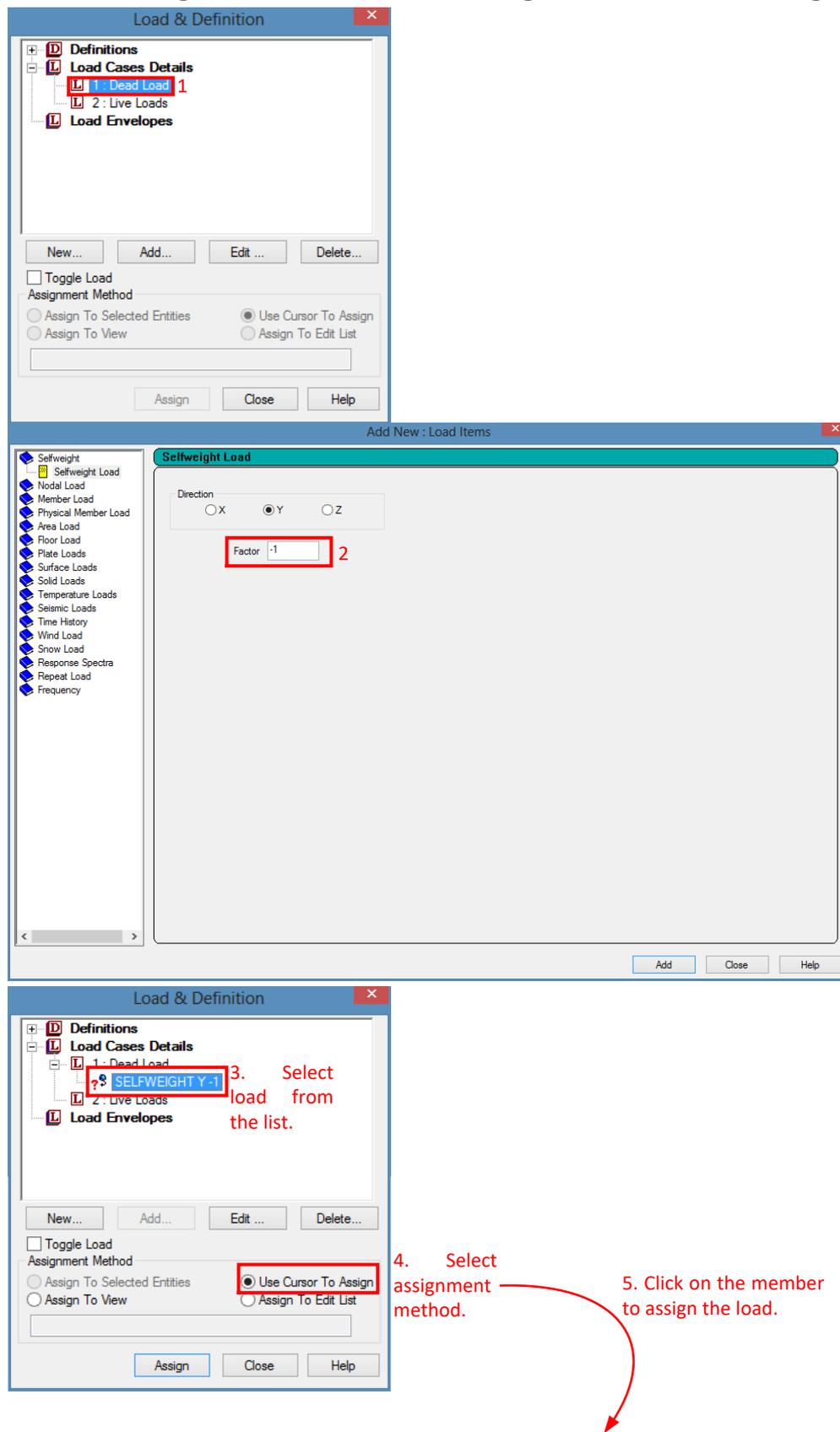


Figure 4.11-14: Definite and assignment of beam selfweight.

- In previous STAAD versions, definition of selfweight implicitly assign it to the whole structure.
- For beams, selfweight is determined based on material density and beam cross sectional dimensions.

- Regarding to the superimposed dead load of $W_{D\ Superimposed}$ of $9.00\ kN/m$, and point live load P_L of $46.9\ kN$, except for loads definition that presented in below all other steps are similar to above steps for selfweight definition and assignment.

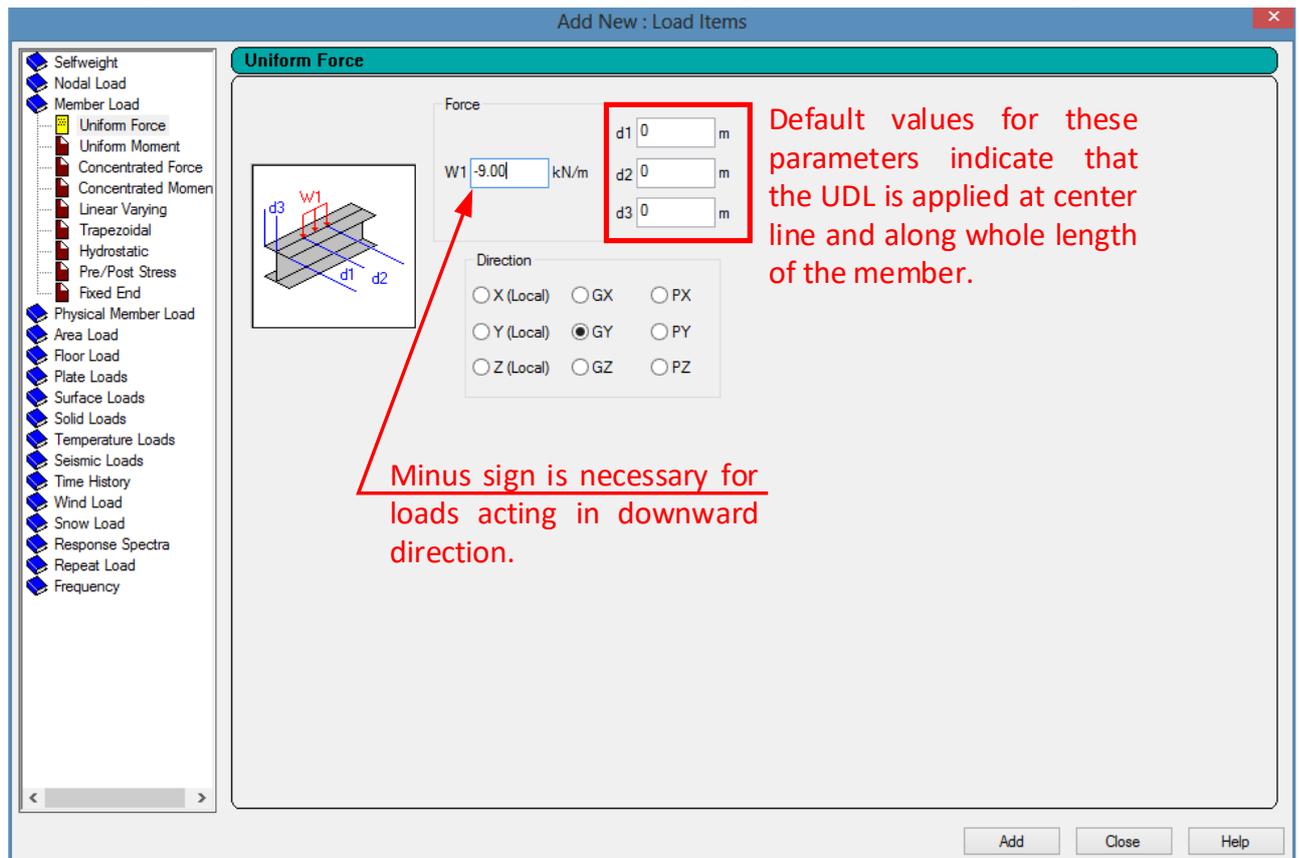


Figure 4.11-15: Definition of uniformly distributed superimposed dead load.

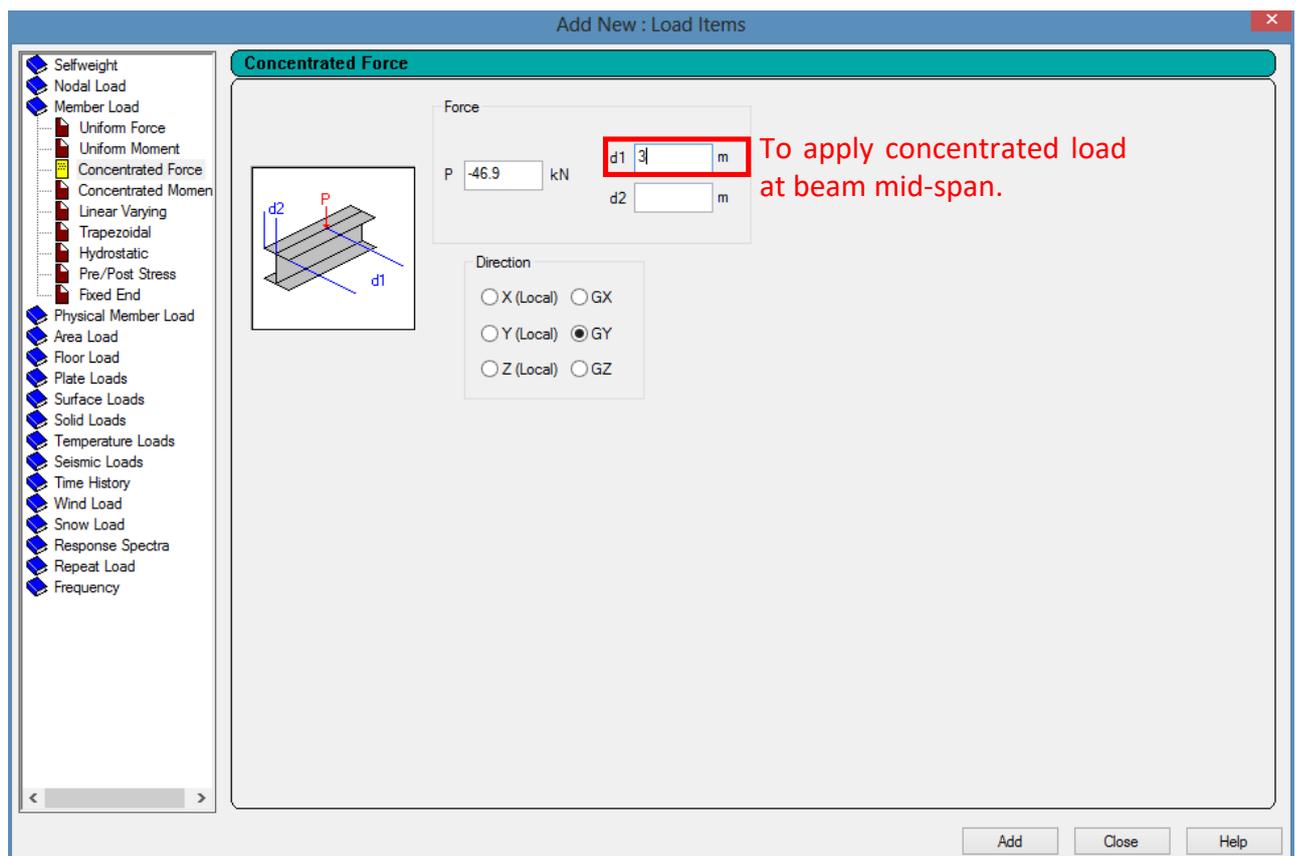


Figure 4.11-16: Definition of concentrated live load at beam mid-span.

4.11.1.8.3 Definition of Load Combinations

As indicated in steps below, load combinations have been generated automatically according to requirements of ACI code.

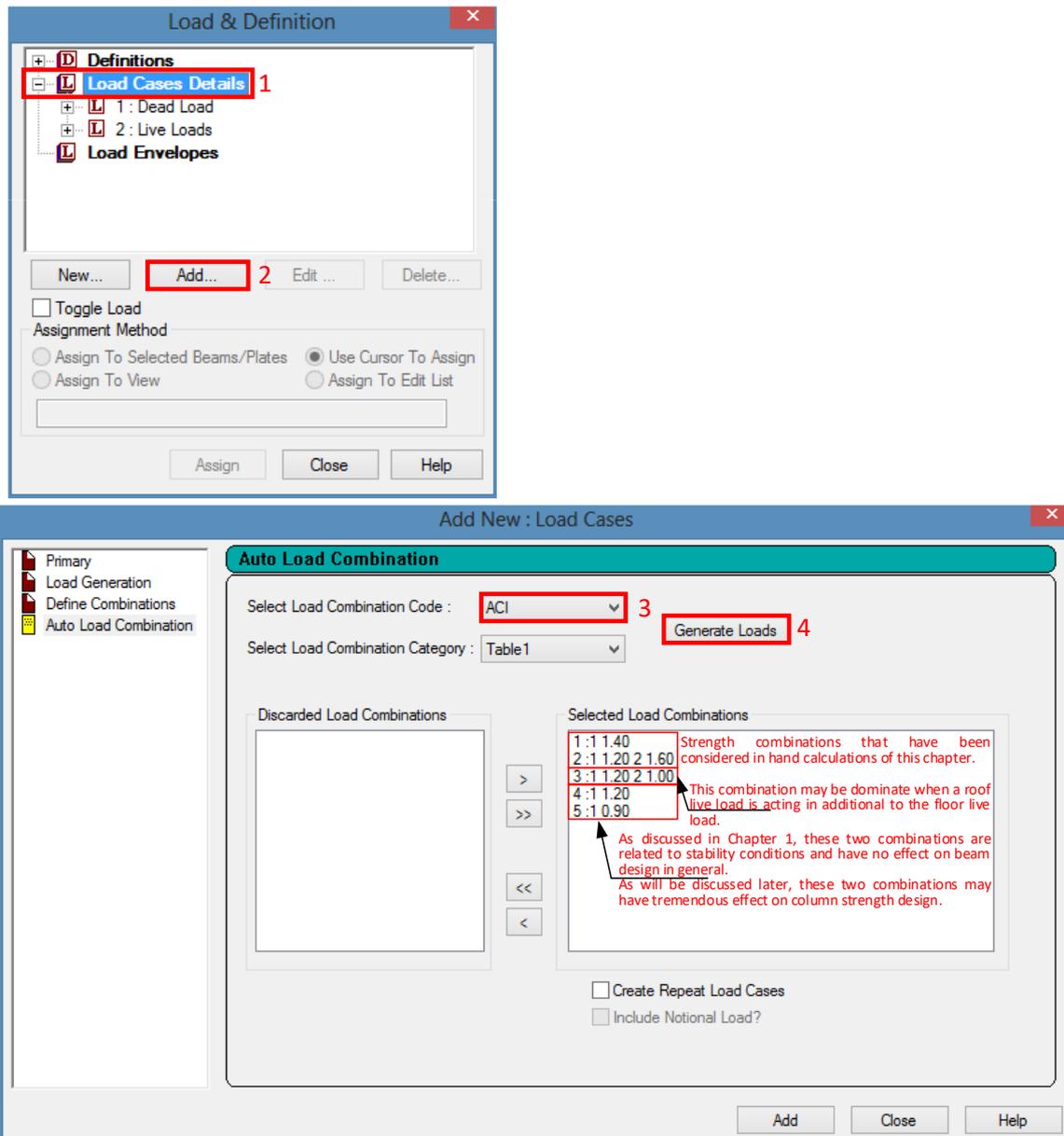


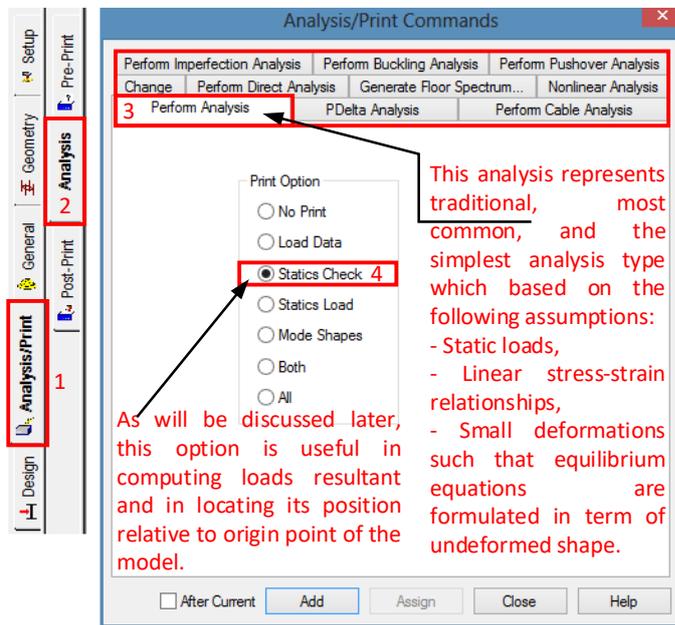
Figure 4.11-17: Steps for automatic generation of load combinations according to ACI code.

4.11.1.9 Definition of Analysis Type

- Traditional linear elastic analysis has been defined based on steps presented in Figure 4.11-18 below.
- When term "**Analysis**" is used in STAAD environment, it refers to a traditional elastic analysis that usually adopted in engineering practice.

4.11.1.10 Review of Input File and Execute the Analysis

- Access of input file is one of the main feature for STAAD software. The input file is a structure program with specific start and end and that executed in a sequential form.
- The input file is similar to codes of common programming languages like Basic, Fortran, and Matlab.
- STAAD input is so useful to describe problem in a concise form.
- To access to model input file in STAAD environment, just click on icon 
- Relation between STAAD command and corresponding GUI is presented in Figure 4.11-19 below.



Different analysis capabilities offered by STAAD software.

Figure 4.11-18: Steps for definition of analysis type.

Node	X m	Y m	Z m
1	0.000	0.000	0.000
2	6.000	0.000	0.000

· N1

· N2

Space
 Plane
 Floor
 Truss

```

1 STAAD PLANE
2 START JOB INFORMATION
3 ENGINEER DATE 04-Dec-16
4 END JOB INFORMATION
5 INPUT WIDTH 79
6 UNIT METER KN
7 JOINT COORDINATES
8 1 0 0 0; 2 6 0 0;
9 MEMBER INCIDENCES
10 1 1 2;
    
```

Job:
 Client:
 Job No.:
 Rev.:
 Part:
 Ref:
 File
 Filename: Example 3.5-2.std
 Directory: H:\Academic Works\Lec.
 Date / Time: 04-Dec-2016 11:15 AM
 File size: 106
 Engineer: Checker: Approved:
 Name:
 Date: 04-Dec-16
 Comment:

After column 79 in a command row, the command details are neglected. In this aspect, STAAD language is similar FORTRAN language.

Length Units
 Inch Decimeter
 Foot Meter
 Millimeter Kilometer
 Centimeter

Force Units
 Pound Newton
 KiloPound DecaNewton
 Kilogram KiloNewton
 Metric Ton MegaNewton

· N1 ————— E1 ————— · N2

Beam	Node A	Node B
1	1	2

Figure 4.11-19: Relation between STAAD commands and corresponding GUI.

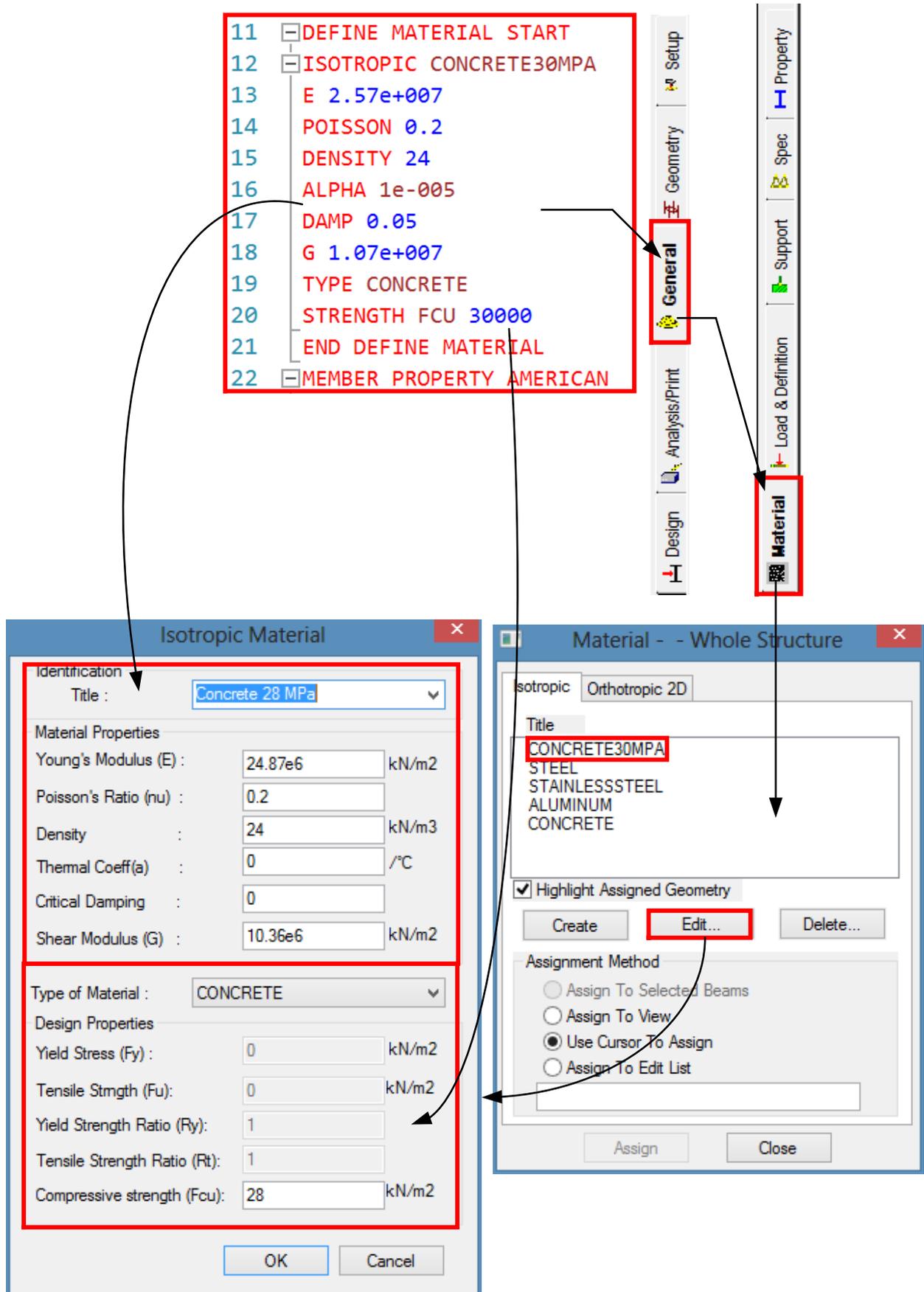


Figure 4.11-19: Relation between STAAD commands and corresponding GUI, continue.

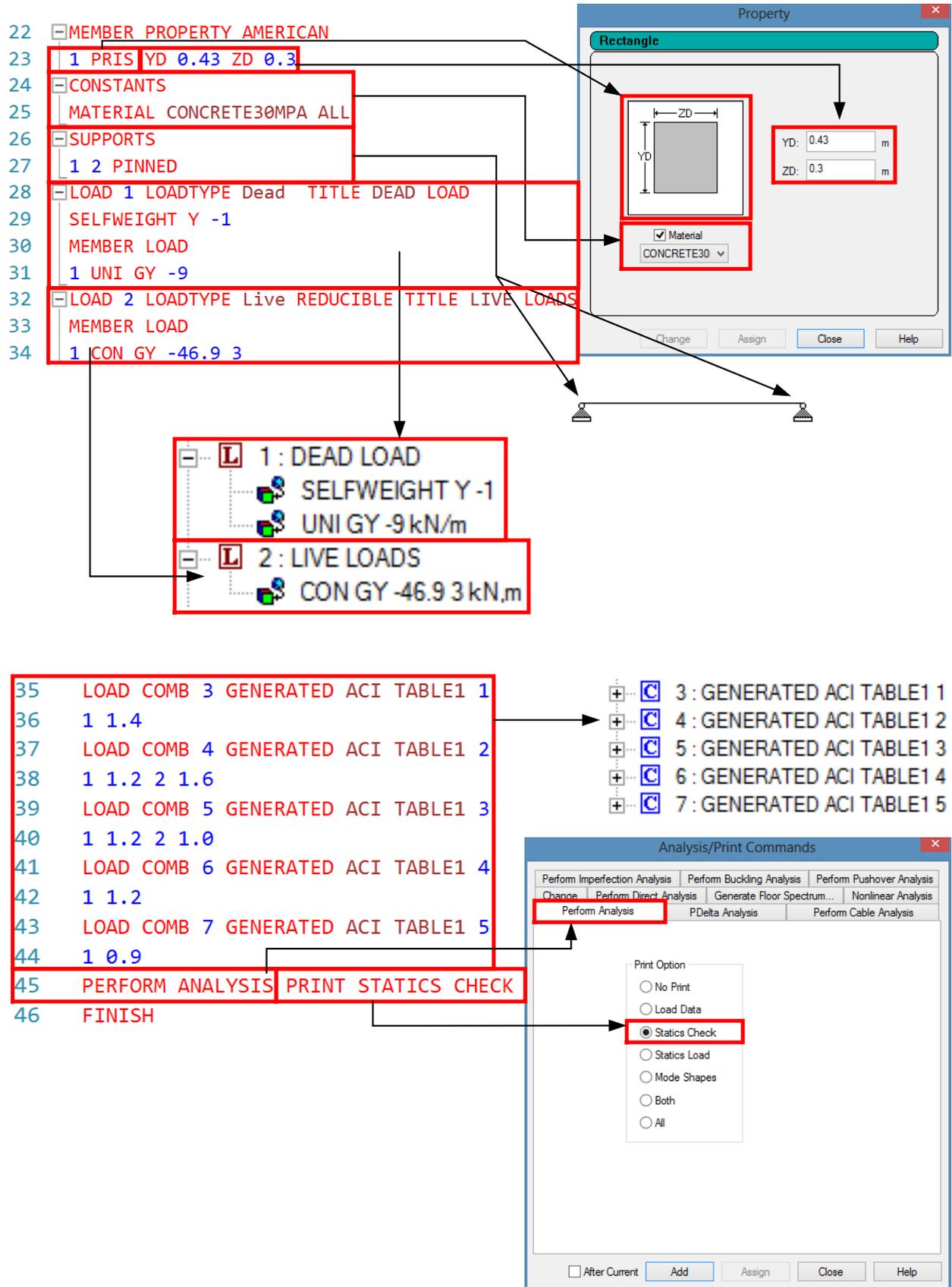


Figure 4.11-19: Relation between STAAD commands and corresponding GUI, continue.

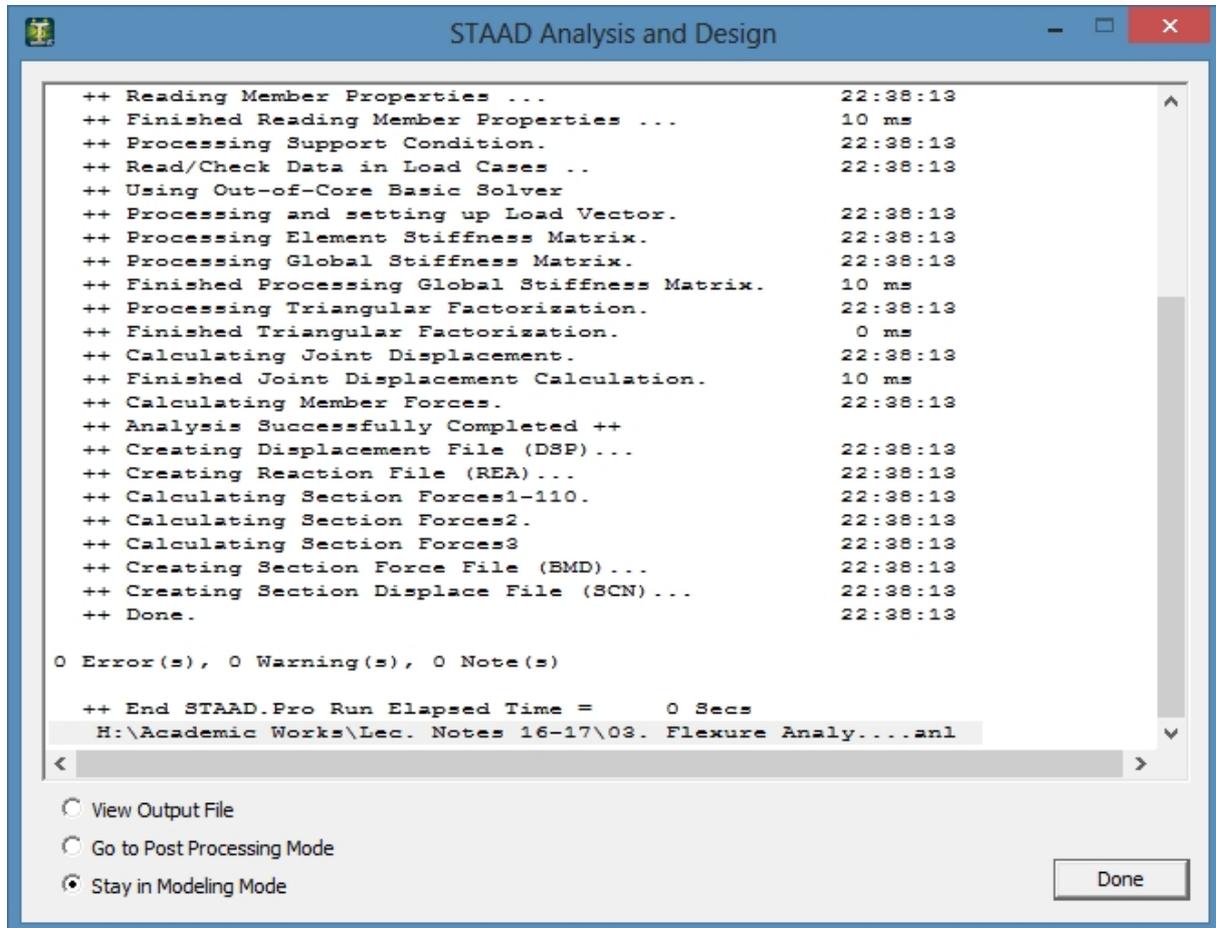
4.11.1.11 Run the Analysis

- After completing the preparation part of the input file, it can be executed as indicated in Figure 4.11-20



Figure 4.11-20: Executing of input file.

- As indicated in below, STAAD analysis engine indicate that input file contains no warning and no error.



4.11.1.12 Post Processing

- After structural analysis process, one can review internal moments and shear forces before the design process.
- Reviewing of analysis results starts from change the mode for a Modeling mode with icon of  To post-processing mode with icon of .
- Transformation from **Modeling** to **Post Processing** can also be done through **Mode** page indicated in Figure 4.11-21.

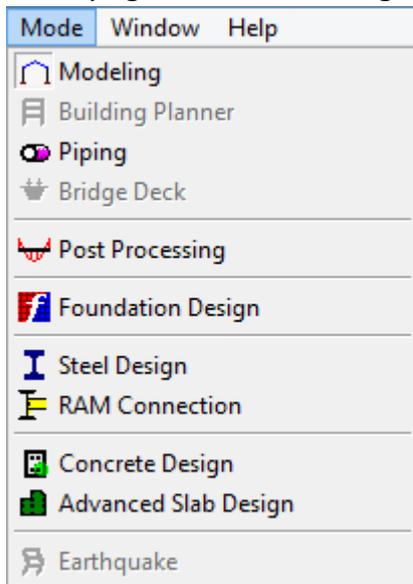


Figure 4.11-21: Mode page to transform from Modeling stage to Post Processing stage.

- Then, one should select load cases and/or load combinations that he intends to review their results. Usually all load cases and combinations are selected as indicated in Figure 4.11-22.

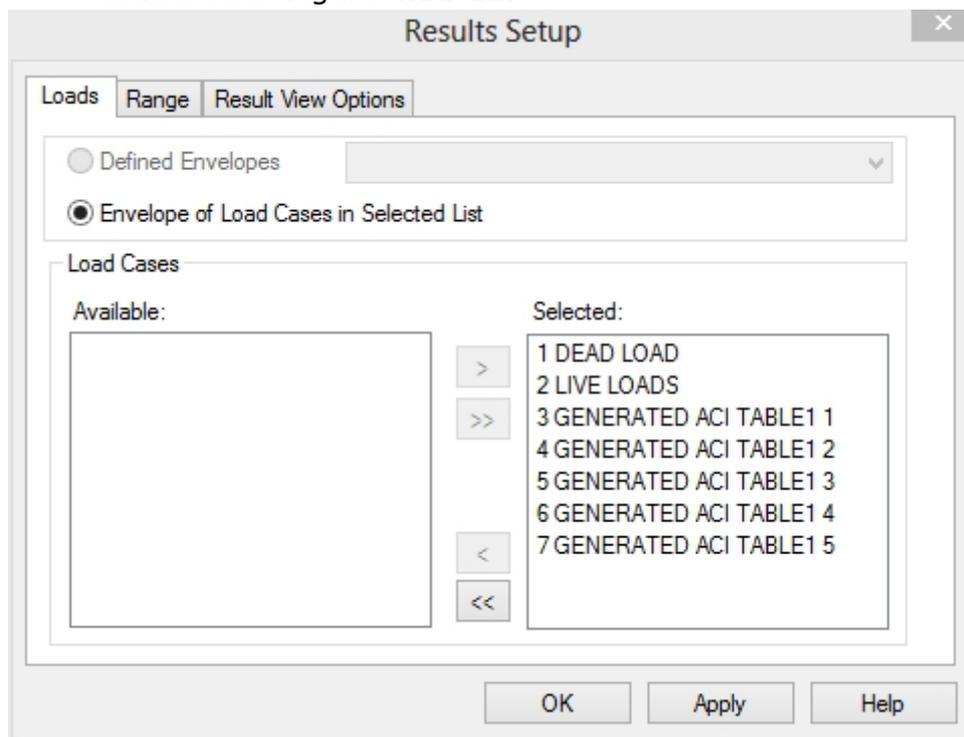
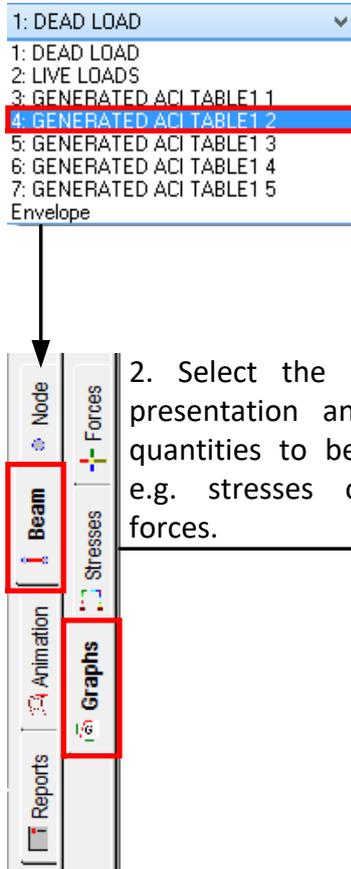


Figure 4.11-22: Selection of load cases and load combinations to review their results.

- The most powerful methods to review of problems with single element have been presented in Figure 4.11-23.



1. From load menu select the most critical load combination. Load combination of GENERATED ACI TABLE 2 is the combination of $U = 1.2D + 1.6L$ That governs most of beam designs in the course.

2. Select the method of presentation and type of quantities to be reviewed, e.g. stresses of internal forces.

3. The shear force and bending moment diagrams would be:

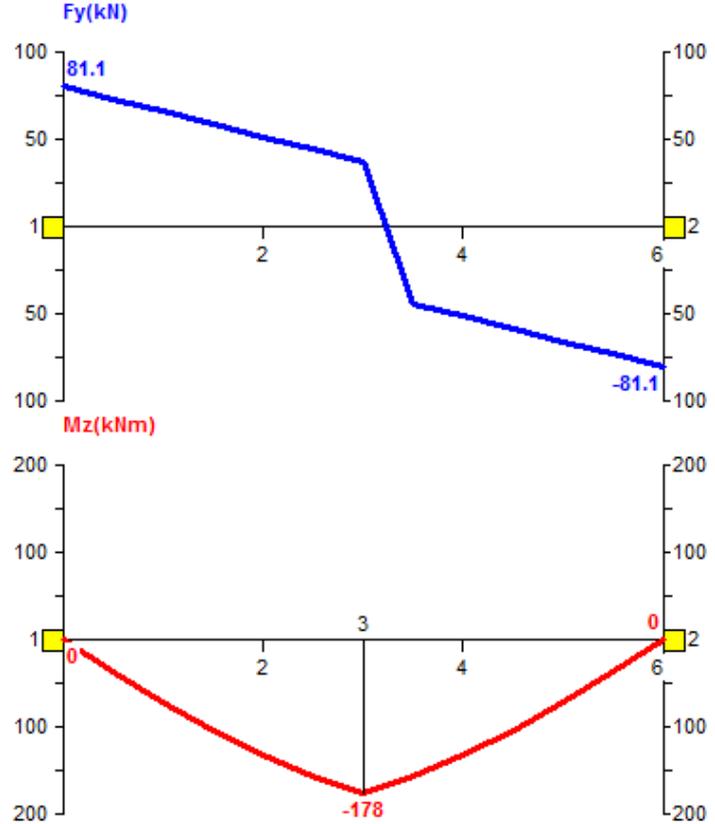


Figure 4.11-23: Selection of load combination, method of presentation, and nature of quantiles to review their analysis results in STAAD environment.

- It is useful to note that the sign convention adopted for bending moments differs from that adopted in analytical solutions. In a more systematic formulation, including STAAD formulation, bending moment is considered positive when produces tensile stresses on side with positive Y, see Figure 4.11-24 below.

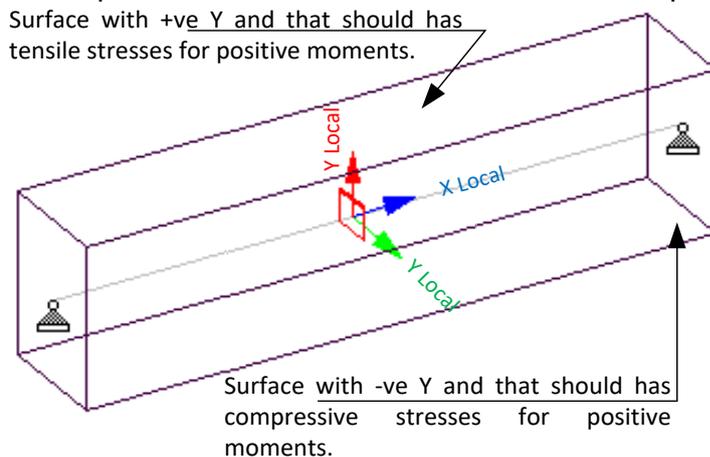


Figure 4.11-24: Definition of positive local moment, M_z , in STAAD environment.

- Based on hand calculations, factored design moment would be:
 - Moment due to Dead Loads:

$$W_{\text{Selfweight}} = 0.43\text{m} \times 0.3\text{m} \times 24 \frac{\text{kN}}{\text{m}^3} = 3.1 \frac{\text{kN}}{\text{m}}$$

$$W_{\text{Dead}} = 9.00 \frac{\text{kN}}{\text{m}} + 3.10 \frac{\text{kN}}{\text{m}} = 12.1 \frac{\text{kN}}{\text{m}}$$

$$M_{\text{Dead}} = \frac{12.1 \frac{\text{kN}}{\text{m}} \times 6.0^2 \text{m}^2}{8} = 54.5 \text{ kN.m}$$

- Moment due to Live Load:

$$M_{\text{Live}} = \frac{46.9 \text{kN} \times 6.0 \text{m}}{4} = 70.4 \text{ kN.m}$$

- Factored Moment M_u :

$$M_u = \text{Maximum of } (1.4M_D \text{ or } 1.2M_D + 1.6M_L)$$

$$M_u = \text{Maximum of } [1.4 \times 54.5 \text{ or } (1.2 \times 54.5 + 1.6 \times 70.4)] =$$

$$M_u = \text{Maximum of } [76.3 \text{ or } 178] = 178 \text{ kN.m} = M_{u \text{ from STAAD}} \therefore \text{Ok.}$$

- While factored shear force would be:

$$V_D = \frac{12.2 \times 6.0}{2} = 36.6 \text{ kN}$$

$$V_L = \frac{46.9}{2} = 23.5 \text{ kN}$$

- The factored shear force would be:

$$V_u = 1.2 \times V_D + 1.6V_L = 1.2 \times 36.6 + 1.6 \times 23.5 = 81.5 \text{ kN}$$

$$\approx V_u @ \text{ center of support computed by STAAD}$$

4.11.1.13 Design Process

- In STAAD environment, design process is starting by selecting **Design** from main pages and then select **Concrete** from sub-pages as indicated in **Figure 4.11-26** below.
- In general, the design process consists from three basic steps indicated **Figure 4.11-25** below. Each step has been discussed in some details in sub-articles below.

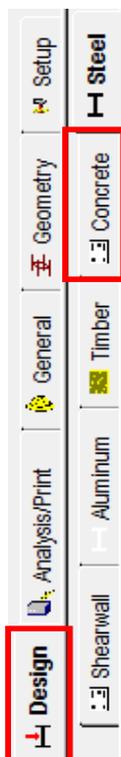


Figure 4.11-26: Starting design process through selecting Design from main pages and selecting Concrete from sub-pages.

4.11.1.13.1 Step 1: Select Parameters

- In this step, from the list of "**Available Parameters**" indicated **Figure 4.11-27** below, the user can select a list of "**Selected Parameters**" that pertinent to the design problem.

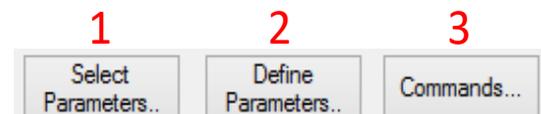


Figure 4.11-25: Basic three steps in design process.

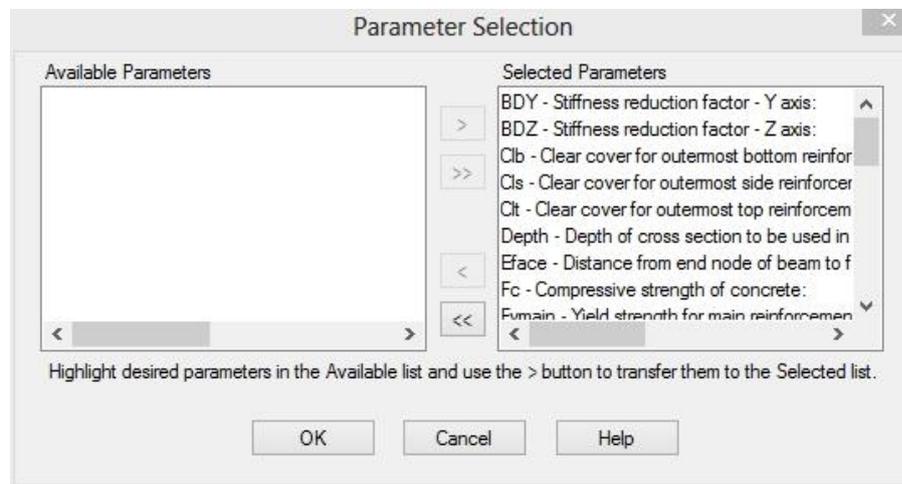


Figure 4.11-27: List of parameters in STAAD Pro software for design of RC members.

- To know which parameters should be selected, one should review definition and default values for the parameters pertinent to beam design.

4.11.1.13.1.1 Beam Dimensions

Beam dimensions can be defined using the two parameters presented in **Table 4.11-1** below. With these parameters, the user can adopt in the design process sections other than those that adopted in analysis process.

Table 4.11-1: Dimension Parameters.

Parameter Name	Default Value	Description
<u>DEPTH</u>	YD	Depth of concrete member. This value defaults to YD as provided under MEMBER PROPERTIES.
<u>WIDTH</u>	ZD	Width of concrete member. This value defaults to ZD as provided under MEMBER PROPERTIES.

4.11.1.13.1.2 Reinforcement Covers

Rebar covers have been defined with referring to **Table 4.11-2** below. STAAD uses metric in cover conversions; therefore, a cover of 1.5 inch is equivalent to 38mm. To be compatible with metric version of the ACI, the cover should be rounded to 40mm.

Table 4.11-2: Rebar covers.

Parameter Name	Default Value	Description
<u>CLB</u>	1.5 in. for beams	Clear cover for bottom reinforcement.
<u>CLS</u>	1.5 in.	Clear cover for side reinforcement.
<u>CLT</u>	1.5 in. for beams	Clear cover for top reinforcement.

4.11.1.13.1.3 Material Properties

Material properties related to the design process have been defined with referring to **Table 4.11-3** below. These material properties, including f_c' , f_y , f_{yt} , and λ . Default values for dimensional properties have been defined based on the imperial unit system and they would be transformed based on a metric conversion when metric system is adopted.

Table 4.11-3: Material properties in the design process.

Parameter Name	Default Value	Description
<u>FC</u>	4,000 psi	Compressive strength of concrete.
<u>FYMAIN</u>	60,000 psi	Yield stress for main reinforcing steel.
<u>FYSEC</u>	60,000 psi	Yield stress for secondary steel.
<u>LWF</u>	1.0	Modification factor, λ , for lightweight concrete as specified in ACI.

4.11.1.13.1.4 Rebar Size

- STAAD Pro. not only computes required reinforcement areas, $A_{s\text{ Required}}$, but also offers reinforcement distributions including number of rebars, spacing between rebars, and number of reinforcement layers. Therefore, preferable rebar size should be proposed by the user through design parameters indicated in **Table 4.11-4** below.

- To enforce STAAD to adopt a single bar diameter, the user should adopt same bar diameter for MAXMAIN and MINMAIN.
- When using metric units for ACI design, provide values for these parameters in actual 'mm' units instead of the bar number. The following metric bar sizes are available: 6 mm, 8 mm, 10 mm, 12 mm, 16 mm, 20 mm, 25 mm, 32 mm, 40 mm, 50 mm and 60 mm.

Table 4.11-4: Parameters for proposing a preferable bar sizes in STAAD environment.

Parameter Name	Default Value	Description
<u>MAXMAIN</u>	#18 bar	Maximum main reinforcement bar size.
<u>MINMAIN</u>	#4 bar	Minimum main reinforcement bar size (Number 4 - 18).
<u>MINSEC</u>	#4 bar	Minimum secondary reinforcement bar size (Number 4 - 18)

4.11.1.13.1.5 Design Sections

- Design parameters related to number and location of design sections have been presented in **Table 4.11-5** below.
- Through the parameter of NSECTION, the user can determine the number of sections that should be adopted in the design process. STAAD distributes these sections uniformly along member span.
- After locating of the sections, STAAD computes factored forces, e.g. M_u and V_u , at each section and then computes required reinforcement accordingly.
- STAAD analytical model replaces actual physical members that has definite depth with analytical lines that, by default, are located at centroid of members. Therefore, to determine shear force at face of supports where it has its physical meaning, the user should adopt the parameters of SFACE and EFACE that have been defined and interpreted with referring to **Table 4.11-5** and **Figure 4.11-28** below.

Table 4.11-5: Number and locations of design sections in STAAD environment.

Parameter Name	Default Value	Description
<u>NSECTION</u>	12	Number of equally spaced sections to be considered in finding critical moments for beam design. NSECTION should have no member list since it applies to all members. The minimum value allowed is 12, the maximum is 20. If more than one NSECTION entered, then highest value is used.
<u>EFACE</u>	0.0	Face of support location at end of beam. If specified, the shear force at end is computed at a distance of EFACE +d from the end joint of the member.
<u>SFACE</u>	0.0*	Face of support location at start of beam. If specified, the shear force at start is computed at a distance of SFACE +d from the start joint of the member.

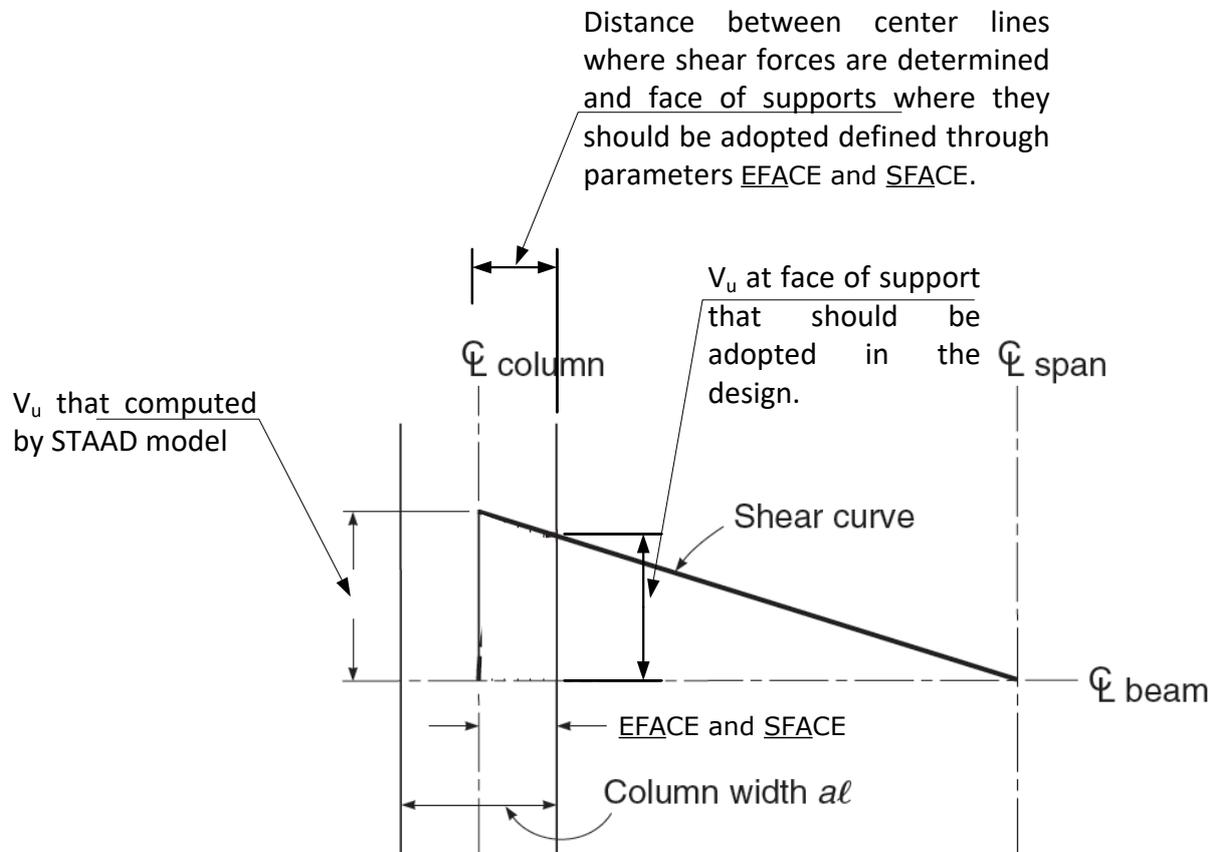


Figure 4.11-28: Interpretation of STAAD parameters EFACE and SFACE.

- Assuming pads that have width of 300mm the EFACE and SFACE would be:

$$EFACE = SFACE = \frac{0.300}{2} = 0.150 \text{ m}$$
- It will be discussed in **Chapter 5, Article 5.2.2** the design shear force can be determined at distance "d" from face of support when three conditions below are satisfied:
 - Support reaction, in direction of applied shear, introduces compression into the end regions of member.
 - Loads are applied at or near the top of the member.
 - No concentrated load occurs between face of support and location of critical.

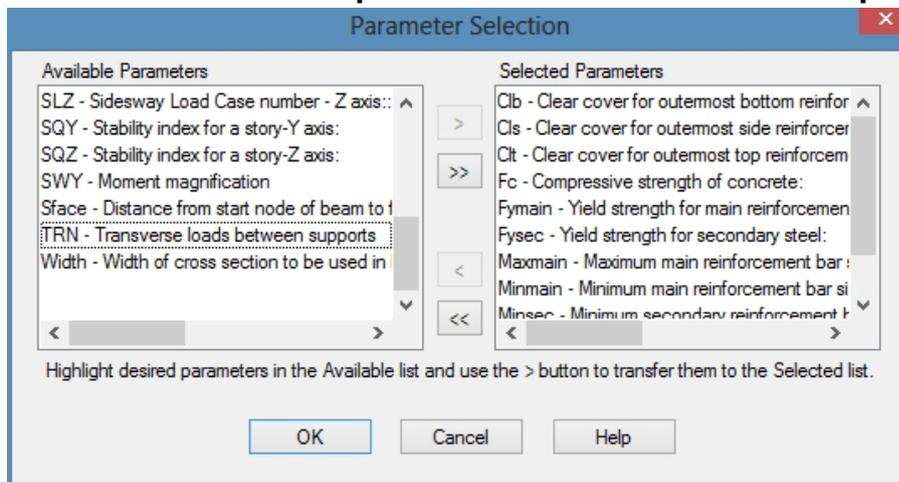
4.11.1.13.1.6 Detail Level of Outputs

- Finally, based on TRACK parameter, STAAD offers three different levels of output detail that indicated in

Table 4.11-6: Three different levels of output details

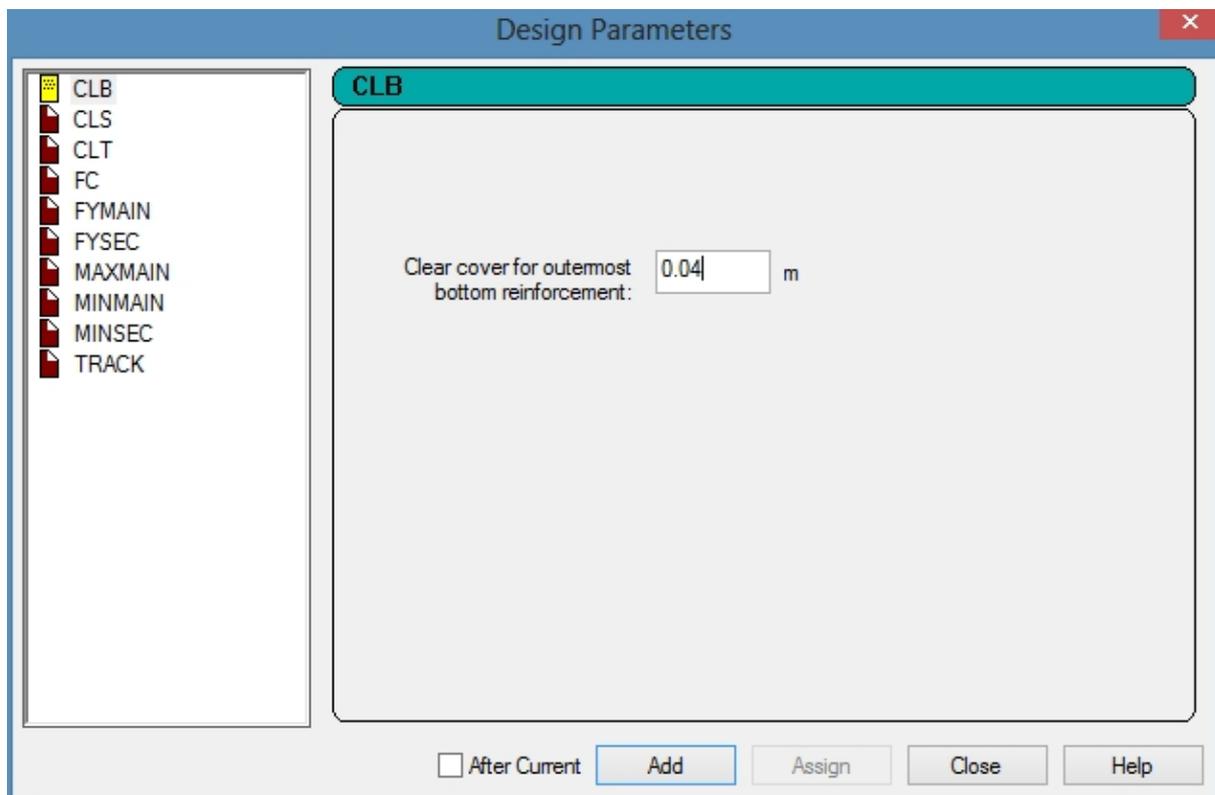
Parameter Name	Default Value	Description
<u>TRACK</u>	0.0	Beam Design: 0.0 = Critical moment will not be printed out with beam design report. 1.0 = Critical moment will be printed out with beam design report 2.0 = Print out required steel areas for all intermediate sections specified by <u>NSECTION</u> .

- Based on above discussion, it is clear that only following parameters should be selected, as values other than their default values should be assigned:
 - Reinforcement covers,
 - Material properties,
 - Rebar size,
 - Design sections for shear,
 - Detail level of outputs.
- These parameters have been selected as indicated in Table 4.11-7 below.

Table 4.11-7: Selected parameters from available list of parameters.

4.11.1.13.2 Step 2: Define Parameters

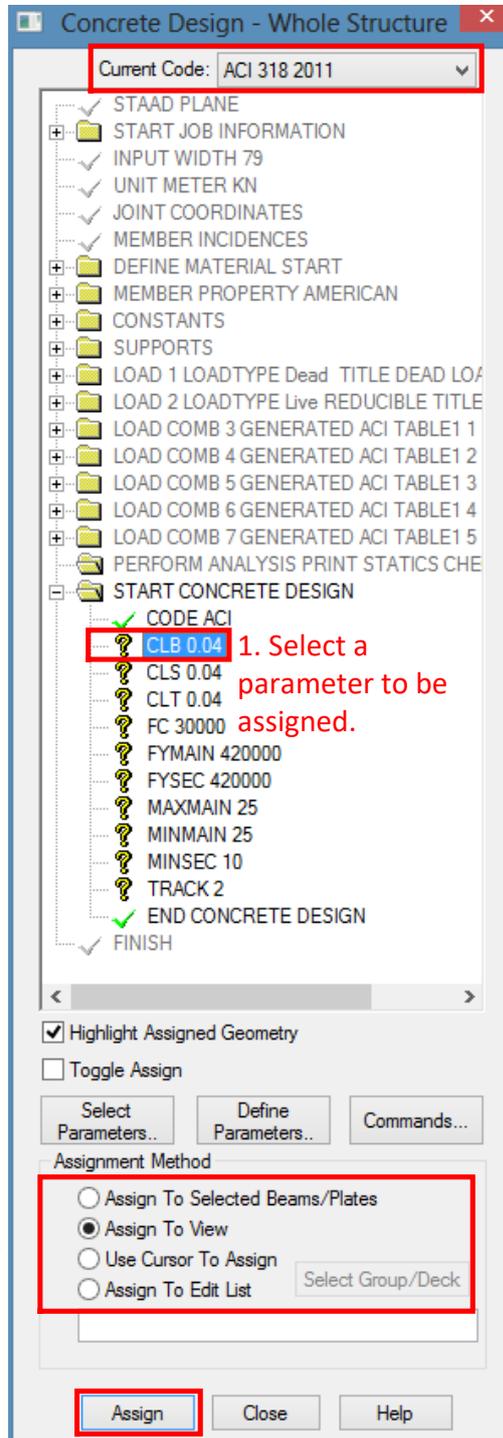
- Referring to Figure 4.11-25 above, the second step of design process in STAAD environment, is to assign values differ from the default values for the design parameters that have been selected in **Step 1** above. As an example, consider **Figure 4.11-29** below where a value of **40mm**, or **0.04m**, for the **Clear cover for the outmost bottom reinforcement**.

**Figure 4.11-29: Assign the Clear cover for the outmost bottom reinforcement.**

- In the same approach over values can be assigned to other design parameters as indicated in **Figure 4.11-30** below.
- As they not assigned to a specific member yet, all parameters are noted with question mark "?". Steps indicated in **Figure 4.11-31** below can be adopted to assign a parameter, e.g. CLB 0.04, to a specific member.
- From pulldown list indicated in **Figure 4.11-31** below. Unfortunately, **ACI 318-14 not included yet in the STAAD environment**.

Figure 4.11-30: Assigned values for other pertinent design parameters.

- START CONCRETE DESIGN
- ✓ CODE ACI
- ? CLB 0.04
- ? CLS 0.04
- ? CLT 0.04
- ? FC 30000
- ? FYMAIN 420000
- ? FYSEC 420000
- ? MAXMAIN 25
- ? MINMAIN 25
- ? MINSEC 10
- ? TRACK 2
- ✓ END CONCRETE DESIGN



Design code adopted in the design process.

1. Select a parameter to be assigned.

2. Select a suitable assignment method.

3. Press Assign to assign the selected parameter to the specific member(s).

Figure 4.11-31: Assign a design parameter to the pertinent member.

4.11.1.13.3 Step 3: Commands

- In the third and final step, based on interactive box indicated in **Figure 4.11-32** below, the user can inform the software to design a specific member as a beam or as a column. As indicated in the interactive box, **according to STAAD, the beam is the member that should be designed for flexure, shear, and torsion.**
- When including DESIGN BEAM command, the design list would be as indicated in **Figure 4.11-33** below. This command can be assigned to the specific member in a method similar to that discussed above.

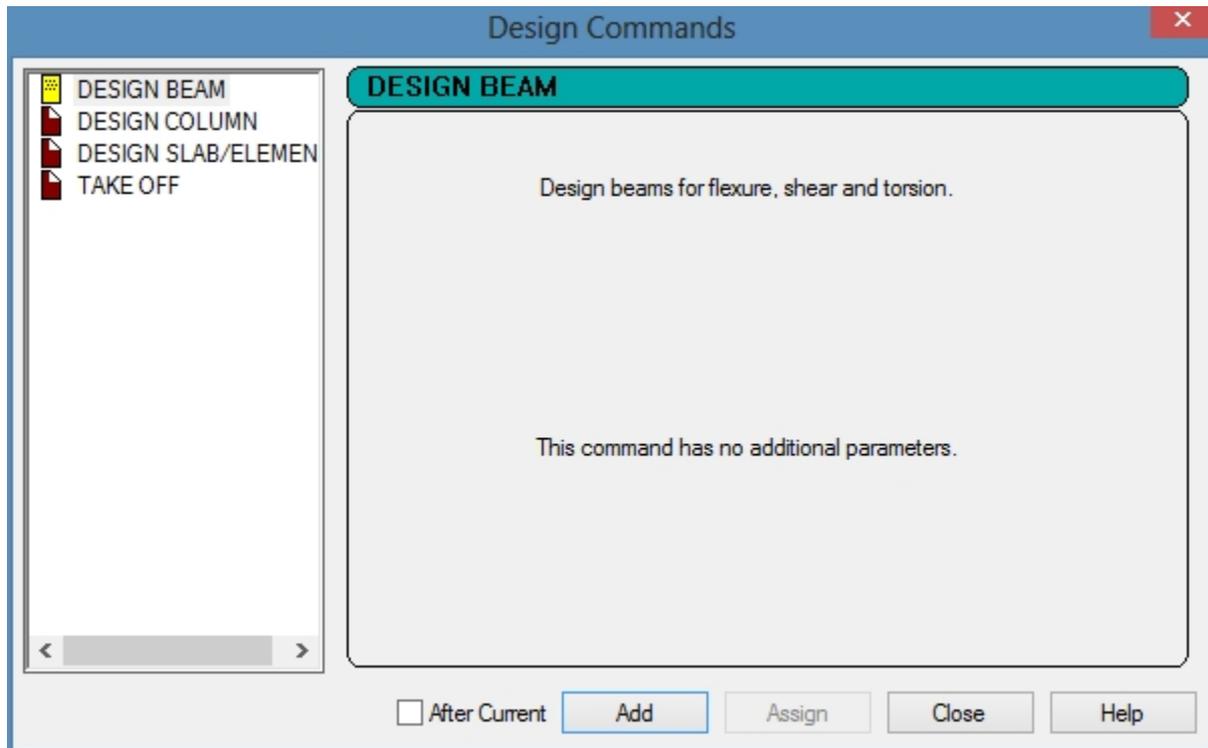


Figure 4.11-32: Commands interactive box in STAAD environment.

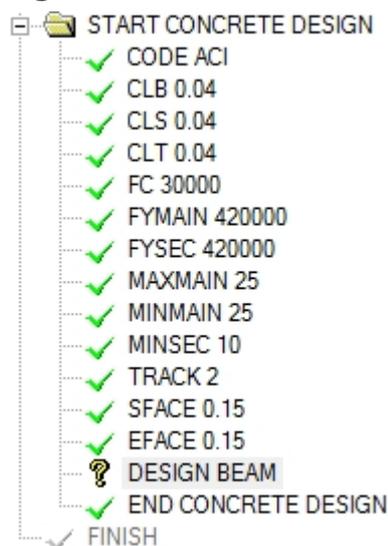


Figure 4.11-33: Updated design list after including DESIGN BEAM command.

4.11.1.13.4 Load Combinations that Adopted in the Design Process

- Design should be done in terms of load combinations only, therefore all basic design cases including DEAD LOAD and LIVE LOAD should be excluded from the list of design loads.
- In STAAD environment, selecting of the loads that should be including in the design process can be executed with **Load List** command based on steps indicated in **Figure 4.11-34** below.

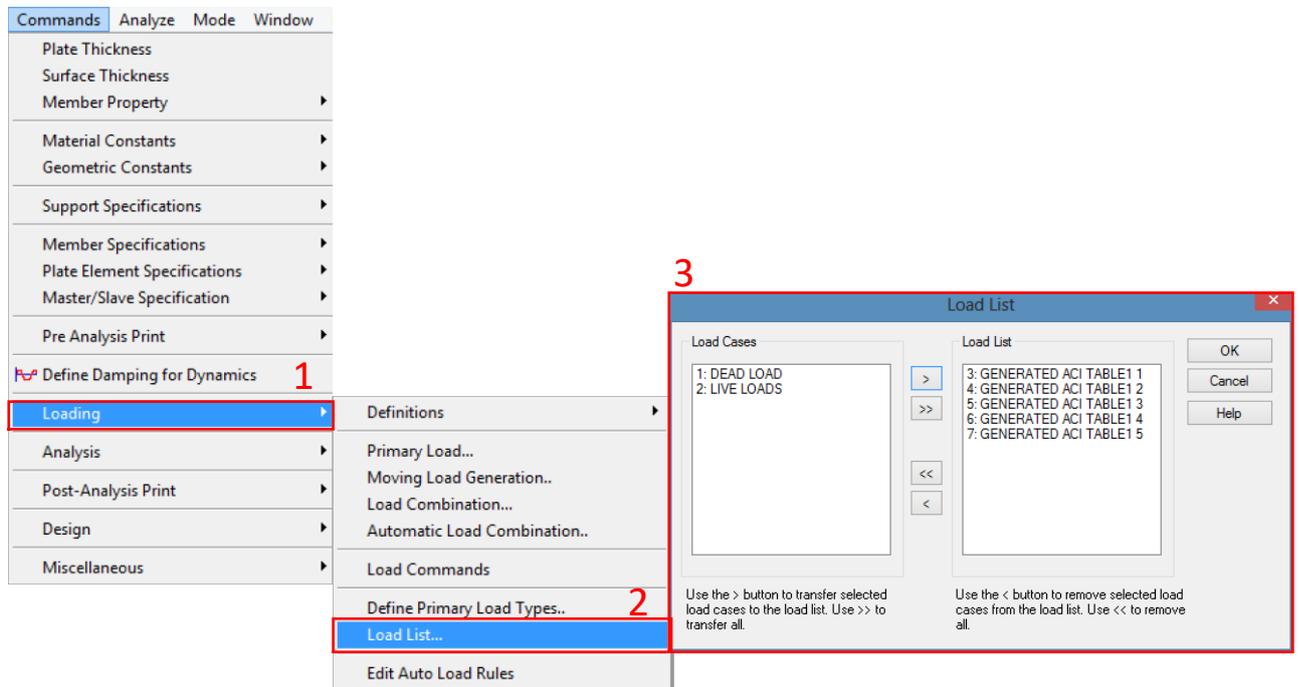


Figure 4.11-34: Steps to select loads that should be adopted in the design process using Load List command.

4.11.1.13.5 Input File with Design Parameters

In input file, the design parameters that have been defined and assigned using GUI above are presented in **Figure 4.11-35** below.

```

47  START CONCRETE DESIGN
48  CODE ACI
49  CLB 0.04 ALL
50  CLS 0.04 ALL
51  CLT 0.04 ALL
52  FC 30000 ALL
53  FYMAIN 420000 ALL
54  FYSEC 420000 ALL
55  MAXMAIN 25 ALL
56  MINMAIN 25 ALL
57  MINSEC 10 ALL
58  TRACK 2 ALL
59  SFACE 0.15 ALL
60  EFACE 0.15 ALL
61  END CONCRETE DESIGN
62  FINISH

```

Figure 4.11-35: Design parameters in the STAAD input file.

4.11.1.13.6 Run Model and Review of Design Results

After completion of definition, and assignment of all design parameters that related to the design process, the STAAD model can be executed or run in the way that discussed in **Article 4.11.1.11**.

Design results are indicated **Figure 4.11-36** through **Figure 4.11-39** below. Flexural and shear designs have been discussed in below and compared with those obtained based on hand calculations.

4.11.1.13.6.1 Design for Flexure

Design forces computed by STAAD have been compared with those of hand calculations presented in **Article 4.4**. Required reinforcement ratio based on hand calculation is:

$$d_{\text{for One Layer}} = 430 - 40 - 10 - \frac{25}{2} = 368 \text{ mm}$$

$$\rho_{\text{Required}} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f'_c b d^2}}}{1.18 \times \frac{f_y}{f'_c}} = \frac{1 - \sqrt{1 - 2.36 \frac{198 \times 10^6 \text{ N}\cdot\text{mm}}{30 \times 300 \times 368^2}}}{1.18 \times \frac{400}{30}} = 13.6 \times 10^{-3}$$

≈ RHO = 0.0130 from STAAD

The maximum and minimum reinforcement ratios according to hand calculations would be:

$$\rho_{\text{max}} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$

$$\beta_1 = 0.85 - \frac{30 - 28}{7} \times 0.05 = 0.836 > 0.65 \text{ Ok}$$

$$\rho_{\text{max}} = 0.85 \times 0.836 \frac{30}{400} \frac{0.003}{0.003 + 0.004} = 22.8 \times 10^{-3} \approx \text{RHOMX} = 0.0223 \text{ from STAAD}$$

$$A_{s \text{ minimum}} = \frac{1.4}{f_y} b_w d \Rightarrow \rho_{\text{Minimum}} = \frac{A_{s \text{ Minimum}}}{b_w d} = \frac{1.4}{f_y} = \frac{1.4}{420} = 0.00333 = \text{RHOMN from STAAD}$$

ACI 318-11 BEAM NO. 1 DESIGN RESULTS						
=====						
LEN - 6000. MM FY - 420. FC - 30. MPA, SIZE - 300. X 430. MMS						
LEVEL	HEIGHT (MM)	BAR INFO	FROM (MM)	TO (MM)	ANCHOR STA END	
1	62.	3 - 25MM	0.	6000.	YES	YES

CRITICAL POS MOMENT= 177.88 KN-MET AT 3000.MM, LOAD 4						
REQD STEEL= 1434.MM2, RHO=0.0130, RHOMX=0.0223 RHOMN=0.0033						
MAX/MIN/ACTUAL BAR SPACING= 250./ 50./ 88. MMS						
REQD. DEVELOPMENT LENGTH = 989. MMS						

Cracked Moment of Inertia Iz at above location = 85985.2 cm^4						

Figure 4.11-36: Details of flexure design at the most critical section.

REQUIRED REINF. STEEL SUMMARY :

SECTION (MM)	REINF STEEL (+VE/-VE) (SQ. MM)	MOMENTS (+VE/-VE) (KNS-MET)	LOAD (+VE/-VE)
0.	0./ 0.	0./ 0.	4/ 7
500.	286./ 0.	39./ 0.	4/ 0
1000.	557./ 0.	74./ 0.	4/ 0
1500.	811./ 0.	105./ 0.	4/ 0
2000.	1045./ 0.	133./ 0.	4/ 0
2500.	1256./ 0.	157./ 0.	4/ 0
3000.	1443./ 0.	178./ 0.	4/ 0
3500.	1256./ 0.	157./ 0.	4/ 0
4000.	1045./ 0.	133./ 0.	4/ 0
4500.	811./ 0.	105./ 0.	4/ 0
5000.	557./ 0.	74./ 0.	4/ 0
5500.	286./ 0.	39./ 0.	4/ 0
6000.	0./ 0.	0./ 0.	3/ 4

Figure 4.11-37: Design Summary for different design sections.

4.11.1.13.6.2 Design for Shear

As will be discussed in **Chapter 5, Article 5.2.2**, design shear force can be determined at distance "d" from face of support when three conditions below are satisfied:

- Support reaction, in direction of applied shear, introduces compression into the end regions of member.
- Loads are applied at or near the top of the member.
- No concentrated load occurs between face of support and location of critical.

As all these conditions are satisfied, therefore the design shear force, V_u , can be determined at distance d from face of support.

$$V_u @ \text{distance } d = \frac{1}{2} \left(1.2 \times 12.1 \times \left(6 - 2 \times \left(\frac{0.300}{2} + 0.368 \right) \right) + 1.6 \times 46.9 \right) = 73.6 \text{ kN} \approx V_u \text{ in STAAD}$$

According to ACI code, concrete shear strength, V_c , is:

$$V_c = 0.17\lambda\sqrt{f'_c} b_w d = (0.17 \times 1.0 \times \sqrt{30} \times 300 \times 368) \times \frac{1}{1000} = 102.7 \text{ kN} \approx V_c \text{ from STAAD}$$

Based on V_u and V_c , required shear force that should be supported by shear reinforcement can be determined accordingly.

$$\therefore V_u = \phi(V_c + V_s) \Rightarrow \therefore V_s = \frac{V_u - \phi V_c}{\phi} = \frac{73.6 - \frac{102.7}{0.75}}{0.75} = 0.0$$

Therefore, no theoretical reinforcement are required and only nominal reinforcement of **Article 11.5.5.1** should be adopted:

$$\therefore V_s \leq 0.33\sqrt{f'_c} b_w d$$

$$\therefore S_{\text{maximum}} = \text{Minimum} \left[\frac{d}{2} \text{ or } 600\text{mm} \right] = \min \left(\frac{368}{2}, 600 \right) = 184 \text{ mm}$$

B E A M N O . 1 D E S I G N R E S U L T S - S H E A R

```

AT START SUPPORT - Vu= 68.18 KNS Vc= 104.66 KNS Vs= 0.00 KNS
Tu= 0.00 KN-MET Tc= 3.9 KN-MET Ts= 0.0 KN-MET LOAD 4
NO STIRRUPS ARE REQUIRED FOR TORSION.
REINFORCEMENT FOR SHEAR IS PER CL.11.5.5.1.
PROVIDE 10 MM 2-LEGGED STIRRUPS AT 184. MM C/C FOR 2112. MM
AT END SUPPORT - Vu= 68.18 KNS Vc= 104.66 KNS Vs= 0.00 KNS
Tu= 0.00 KN-MET Tc= 3.9 KN-MET Ts= 0.0 KN-MET LOAD 4
NO STIRRUPS ARE REQUIRED FOR TORSION.
REINFORCEMENT FOR SHEAR IS PER CL.11.5.5.1.
PROVIDE 10 MM 2-LEGGED STIRRUPS AT 184. MM C/C FOR 2112. MM
    
```

Figure 4.11-38: Detailed design for shear reinforcement.

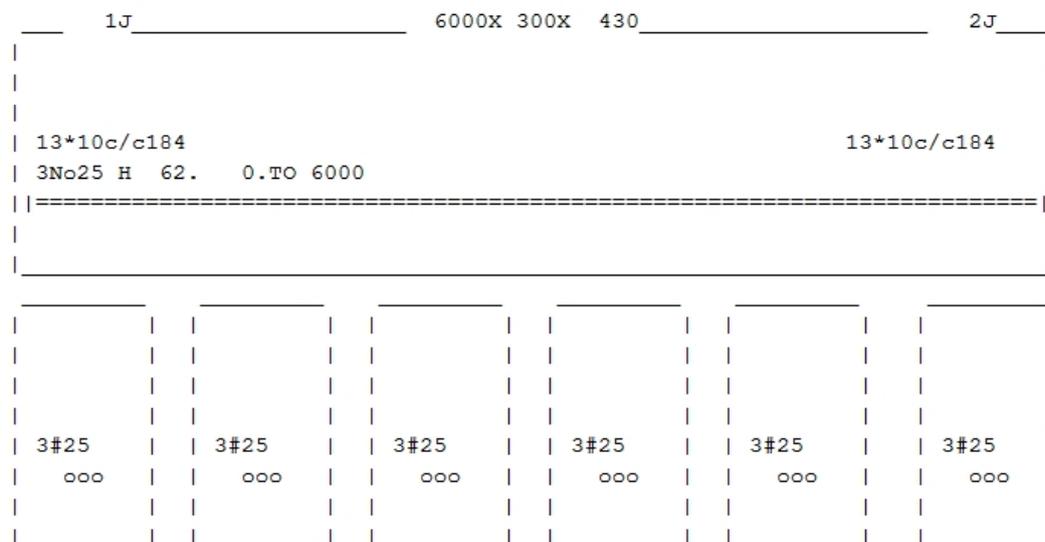


Figure 4.11-39: Longitudinal and cross sections to present required reinforcements and their distribution.

4.11.2 Design of a Doubly Reinforced Concrete Beam

This article aims to show how STAAD Pro software can be adopted for analysis and design of doubly reinforced beam. Analysis and design process are presented with referring to Example 4.7-1 on page 92. Data for this example has been represented in below for convenient:

- The beam has a simple span of 5.49m and subjected to dead load of 15.3 kN/m (including its selfweight) and to service live load of 36.0 kN/m.
- Beam dimensions are 250mm width and 500mm depth.
- $f_y = 414$ Mpa
- $f_c' = 27.5$ Mpa
- No.29 for longitudinal tension reinforcement.
- No.19 for compression reinforcement if required.
- No.10 for stirrups (it's adequacy must be checked when used as a tie).
- Two layers of tension reinforcement.

STAAD Pro input file has been prepared in same steps of Section 4.11.1 and summarized in Table 4.11-8.

Table 4.11-8: STAAD input file for Example 4.7-1.

```

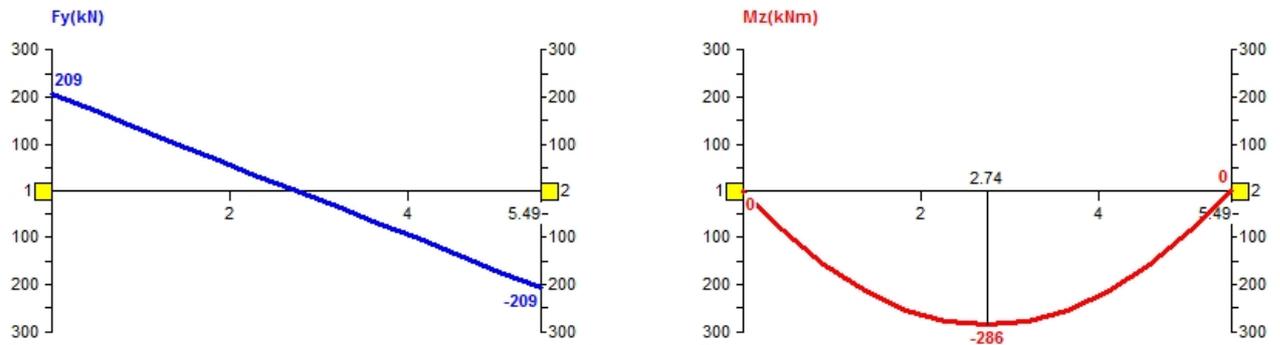
1  STAAD PLANE
2  START JOB INFORMATION
3  ENGINEER DATE 11-Feb-18
4  END JOB INFORMATION
5  INPUT WIDTH 79
6  UNIT METER KN
7  JOINT COORDINATES
8  1 0 0 0; 2 5.49 0 0;
9  MEMBER INCIDENCES
10 1 1 2;
11 DEFINE MATERIAL START
12 ISOTROPIC CONCRETE
13 E 2.17185e+007
14 POISSON 0.17
15 DENSITY 23.5616
16 ALPHA 1e-005
17 DAMP 0.05
18 TYPE CONCRETE
19 STRENGTH FCU 27579
20 END DEFINE MATERIAL
21 MEMBER PROPERTY
22 1 PRIS YD 0.5 ZD 0.25
23 CONSTANTS
24 MATERIAL CONCRETE ALL
25 SUPPORTS
26 1 2 PINNED
27 LOAD 1 LOADTYPE Dead TITLE DEAD
28 MEMBER LOAD
29 1 UNI GY -15.3
30 LOAD 2 LOADTYPE Live TITLE LIVE
31 MEMBER LOAD
32 1 UNI GY -36
33 LOAD COMB 3 Generated ACI Table1 1
34 1 1.4
35 LOAD COMB 4 Generated ACI Table1 2
36 1 1.2 2 1.6
37 LOAD COMB 5 Generated ACI Table1 3
38 1 1.2 2 1.0
39 LOAD COMB 6 Generated ACI Table1 4
40 1 1.2
41 LOAD COMB 7 Generated ACI Table1 5
42 1 0.9
43 PERFORM ANALYSIS
44 START CONCRETE DESIGN
45 CODE ACI
46 CLB 0.04 ALL
47 CLS 0.04 ALL
48 CLT 0.04 ALL
49 FC 27579.2 ALL
50 FYMAIN 414000 ALL
51 FYSEC 414000 ALL
52 MAXMAIN 25 ALL
53 MINMAIN 20 ALL
54 MINSEC 10 ALL
55 TRACK 2 ALL
56 DESIGN BEAM 1
57 END CONCRETE DESIGN
58 FINISH

```

Based on STAAD analysis factored shear force and bending moment diagrams have been determined and presented in Figure 4.11-40. Based on hand calculations, factored forces would be:

$$W_u = \max(1.4 \times 15.3, 1.2 \times 15.3 + 1.6 \times 36.0) \approx 76 \frac{\text{kN}}{\text{m}} \Rightarrow M_u = \frac{76 \times 5.49^2}{8} = 286 \text{ kN.m}$$

$$V_u = \frac{76 \times 5.49}{2} = 209 \text{ kN} = V_u \text{ from STAAD analysis}$$



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Figure 4.11-40: STAAD analysis factored shear force and bending moment diagrams for beam of Example 4.7-1.

Flexural design results are presented in Figure 4.11-41. Comparing required reinforcement from STAAD design with those of hand calculation:

$$A_s \text{ required from hand calculations} = 2527 \text{ mm}^2 > A_s \text{ from STAAD} = 2131 \text{ mm}^2$$

It is useful to note that based on 0.04m value for CLB parameter, STAAD implicitly assumes a single layer of reinforcement and adopts an overestimate for effective depth d and hence a lower estimate for required reinforcement as indicated in the aforementioned comparison. To have a more accurate analysis, CLB parameter should be modified to reflect the two layers of reinforcement: $CLB = 40 + \frac{25}{2} + \frac{25}{2} = 65 \text{ mm}$

When this value is adopted, STAAD design results will be updated to those indicated in Figure 4.11-42 to indicate that the section could not be designed as a single reinforced section and it should be designed as a doubly reinforced one. To design a doubly reinforced section, STAAD Pro RC Design Module should be adopted. This module is out of the scope of this course.

ACI 318-11		BEAM NO.		1		DESIGN RESULTS	
LEN -	5490. MM	FY -	414.	FC -	28. MPA,	SIZE -	250. X 500. MMS
LEVEL	HEIGHT (MM)	BAR INFO	FROM (MM)	TO (MM)	ANCHOR STA END		
<p>*** A SUITABLE BAR ARRANGEMENT COULD NOT BE DETERMINED. REQD. STEEL = 2131. MM2, MAX. STEEL PERMISSIBLE = 2326. MM2 MAX POS MOMENT = 286.18 KN-MET, LOADING 4</p>							

Figure 4.11-41: Details of flexure design at the most critical section Example 4.7-1.

ACI 318-11		BEAM NO.		1		DESIGN RESULTS	
LEN -	5490. MM	FY -	414.	FC -	28. MPA,	SIZE -	250. X 500. MMS
LEVEL	HEIGHT (MM)	BAR INFO	FROM (MM)	TO (MM)	ANCHOR STA END		
<p>***MEMBER FAILS IN MAX REINFORCEMENT. INCREASE MEMBER SIZE. MAX POS MOMENT = 286.18 KN-MET, LOADING 4</p>							

Figure 4.11-42: Details of flexure design at the most critical section Example 4.7-1 with updated CLB parameter.

4.11.3 Design of Tee Section

- This article presents analysis and design of T beams using STAAD Pro software.
- It has been presented with referring to beam of **Example 4.9-1** that has been represented in below. This beam is a simply supported one with span of 6.71m, and it is subjected to superimposed load of 29.2 kN, and to a live load of 14.6kN/m. Material properties are $f_y = 414 \text{ Mpa}$ and $f'_c = 21 \text{ Mpa}$. Adopted rebars are one-layer of $\varnothing 25\text{mm}$ for longitudinal reinforcement ($A_{Bar} = 510\text{mm}^2$) and $\varnothing 10\text{mm}$ for stirrups.
- As discussed previously, STAAD can deal only with sections that have predefined dimensions. Hence, flange width, b , should be determined manually based on ACI provisions, see Table 4.8-1, and feedback to the software. Based on calculations of **Example 4.9-1**, $b = 1900 \text{ mm}$.

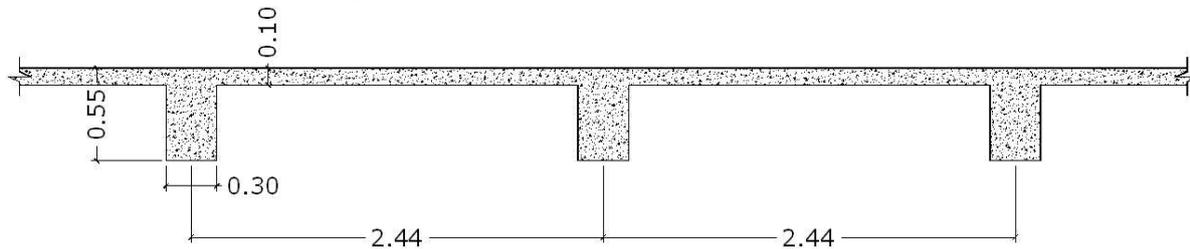


Figure 4.9-5: Floor system for Example 4.9-1. Reproduced for convenient.

- As indicated in Figure 4.11-43, T section can be defined from **General** page and from **Property** subpage.

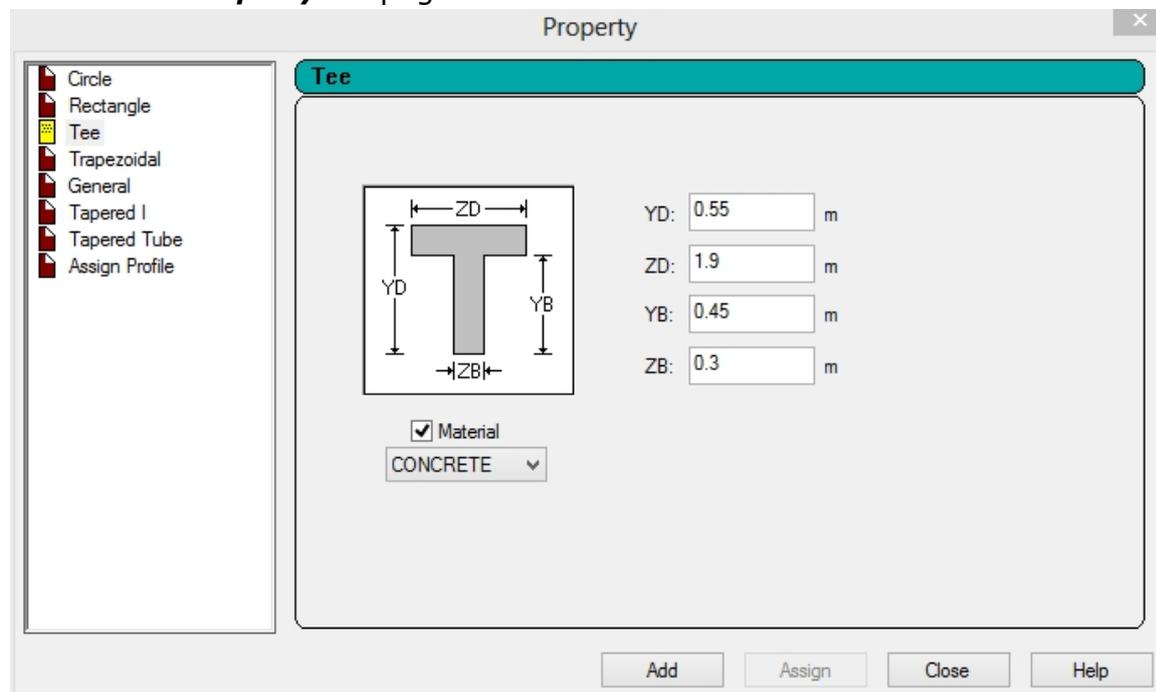


Figure 4.11-43: Definition of T section for Example 4.9-1 in STAAD environment.

- Beam rendered view indicated in Figure 4.11-44 can be reviewed from 3D Rendered View icon, .
- As discussed in Section 4.11.1.8.1, STAAD computes beam selfweight based on proposed section and material densities. In this example, it duplicates flange selfweight, which is already included in the superimposed load, see Section 4.9.2. To avoid this duplication, dead load of

$$W_{\text{Dead}} = 29.2 \frac{\text{kN}}{\text{m}} + 3.24 \frac{\text{kN}}{\text{m}} = 32.4 \frac{\text{kN}}{\text{m}}$$

that includes selfweight and superimposed dead is determined and assigned to the beam.

- Other steps and parameters can be executed and defined in same approach discussed in Section 4.11.1 and Section 4.11.2 above. STAAD input file is presented in Table 4.11-9.

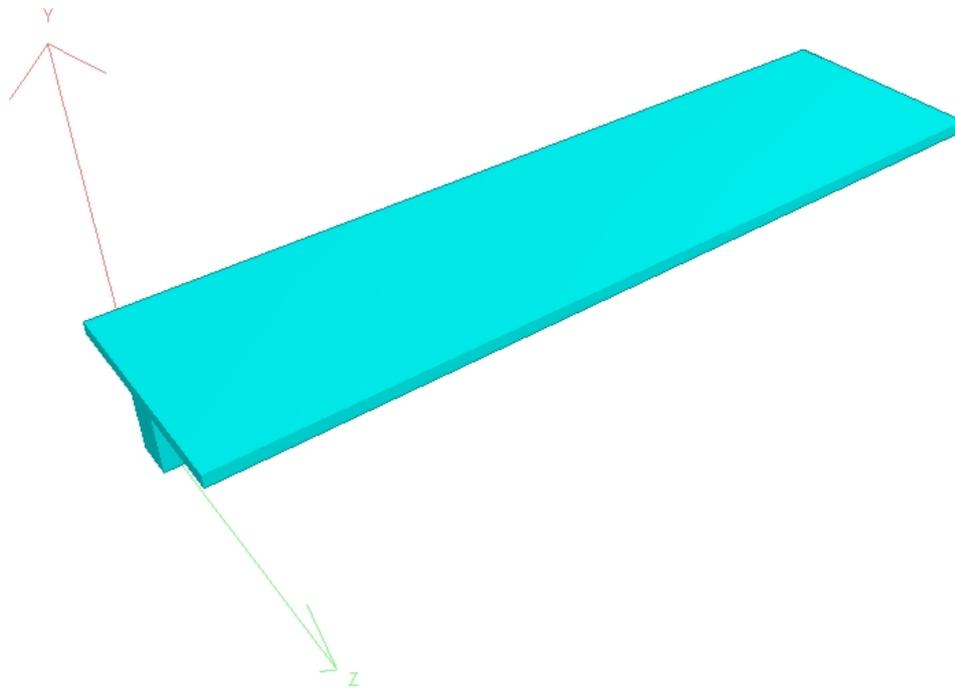


Figure 4.11-44: Render view for beam of Example 4.9-1 in STAAD environment.

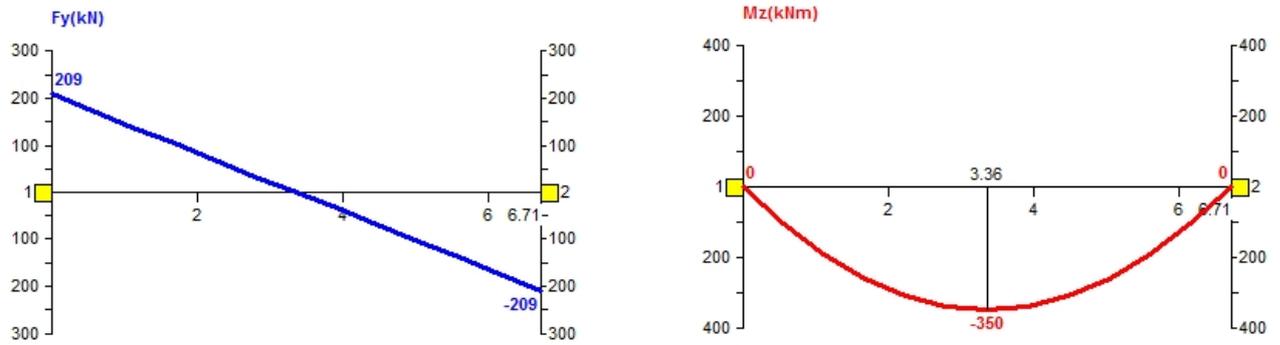
Table 4.11-9: STAAD input file for Example 4.9-1.

```

1  STAAD PLANE
2  [-] START JOB INFORMATION
3  | ENGINEER DATE 21-Feb-18
4  | END JOB INFORMATION
5  | INPUT WIDTH 79
6  | UNIT METER KN
7  [-] JOINT COORDINATES
8  | 1 0 0 0; 2 6.71 0 0;
9  [-] MEMBER INCIDENCES
10 | 1 1 2;
11 [-] DEFINE MATERIAL START
12 [-] ISOTROPIC CONCRETE
13 | E 2.17185e+007
14 | POISSON 0.17
15 | DENSITY 23.5616
16 | ALPHA 1e-005
17 | DAMP 0.05
18 | TYPE CONCRETE
19 | STRENGTH FCU 27579
20 | END DEFINE MATERIAL
21 [-] MEMBER PROPERTY
22 | 1 PRIS YD 0.55 ZD 1.9 YB 0.45 ZB 0.3
23 [-] CONSTANTS
24 | MATERIAL CONCRETE ALL
25 [-] SUPPORTS
26 | 1 2 PINNED
27 [-] LOAD 1 LOADTYPE Dead TITLE Dead Load
28 | MEMBER LOAD
29 | 1 UNI GY -32.4
30 [-] LOAD 2 LOADTYPE Live REDUCIBLE TITLE Live
31 | MEMBER LOAD
32 | 1 UNI GY -14.6
33 | LOAD COMB 3 Generated ACI Table1 1
34 | 1 1.4
35 | LOAD COMB 4 Generated ACI Table1 2
36 | 1 1.2 2 1.6
37 | LOAD COMB 5 Generated ACI Table1 3
38 | 1 1.2 2 1.0
39 | LOAD COMB 6 Generated ACI Table1 4
40 | 1 1.2
41 | LOAD COMB 7 Generated ACI Table1 5
42 | 1 0.9
43 PERFORM ANALYSIS
44 [-] START CONCRETE DESIGN
45 | CODE ACI
46 | CLB 0.04 ALL
47 | CLS 0.04 ALL
48 | CLT 0.04 ALL
49 | FC 21000 ALL
50 | FYMAIN 414000 ALL
51 | FYSEC 414000 ALL
52 | MAXMAIN 25 ALL
53 | MINMAIN 25 ALL
54 | MINSEC 10 ALL
55 | TRACK 2 ALL
56 | DESIGN BEAM 1
57 | END CONCRETE DESIGN
58 | FINISH

```

- STAAD factored shear force and bending moment diagrams are presented in Figure 4.11-45. The maximum bending moment of 350 kN.m is equal to that determined based on simple statics in **Example 4.9-1**.



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Figure 4.11-45: STAAD analysis factored shear force and bending moment diagrams for beam of Example 4.9-1.

- STAAD flexural design output is presented in Figure 4.11-46. The output indicates that required reinforcement from STAAD analysis of 2021 mm^2 is close to 1971 mm^2 that has been determined manually in **Example 4.9-1**. It also indicates that, the required number of rebars, four according to hand calculation, cannot be distributed within the available width of 300mm. This seems natural as hand calculations indicates a width of 275mm is essential to accommodate the required reinforcement. When the number of rebars increases to about five to satisfy required reinforcement according to STAAD, available width would be insufficient.

ACI 318-11 BEAM NO. 1 DESIGN RESULTS

LEN - 6710. MM FY - 414. FC - 21. MPA, SIZE - 1900. X 550. MMS
TEE BEAM ZB/YB 300.00 /450.00

LEVEL	HEIGHT (MM)	BAR INFO	FROM (MM)	TO (MM)	ANCHOR STA	ANCHOR END
-------	-------------	----------	-----------	---------	------------	------------

*** A SUITABLE BAR ARRANGEMENT COULD NOT BE DETERMINED.

REQD. STEEL = 2021. MM2, MAX. STEEL PERMISSIBLE = 2330. MM2

MAX POS MOMENT = 350.29 KN-MET, LOADING 4

Figure 4.11-46: STAAD details of flexure design at the most critical section Example 4.9-1.

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CHAPTER 5

SHEAR AND DIAGONAL TENSION IN BEAMS

5.1 BASIC CONCEPTS

5.1.1 Shear versus Flexural Failures

Due to the following points, shear, or diagonal tension, failure may be more dangerous than flexural failure:

- It has greater uncertainty in predicting,
- It is not yet fully understood, in spite of many decades of experimental research and the use of highly sophisticated analytical tools,
- If a beam without properly designed shear reinforcement is overloaded to failure, shear collapse is likely to occur suddenly, with no advance warning of distress, see **Figure 5.1-1** below.

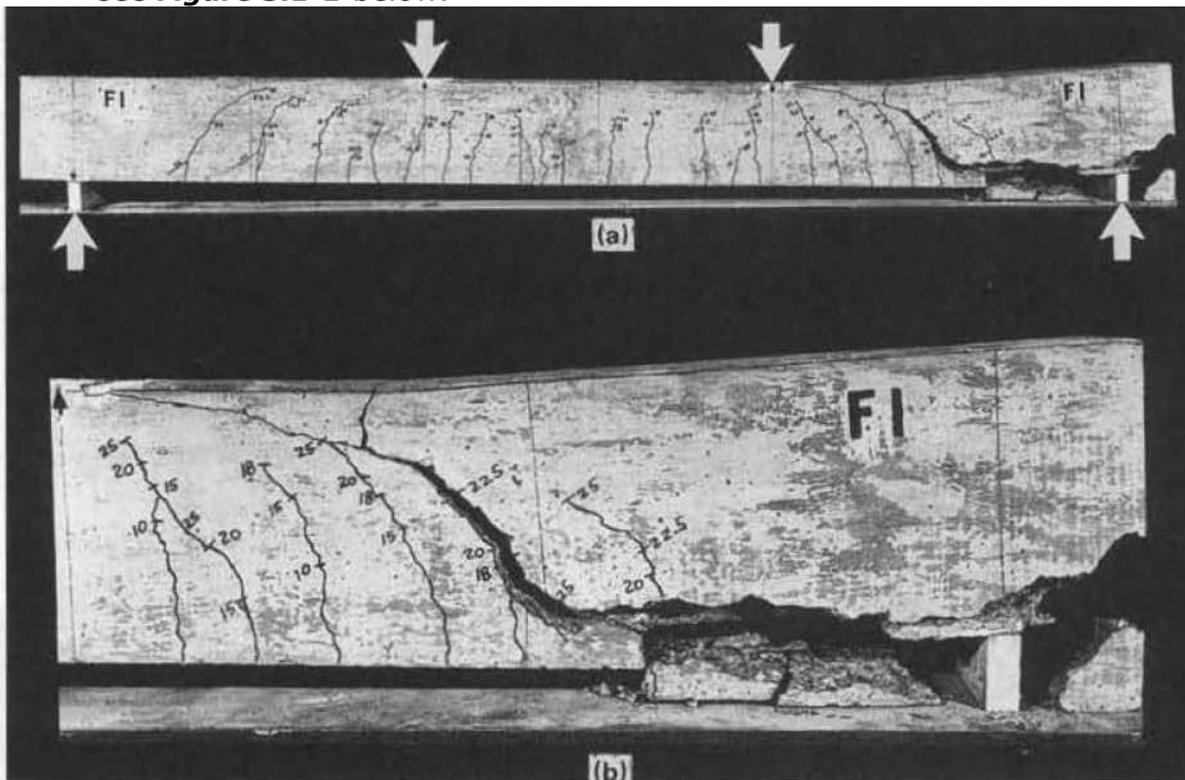


Figure 5.1-1: Shear failure of reinforced concrete beam: (a) overall view, (b) detail near right support.

5.1.2 Direct Shear versus Diagonal Tension

- It is important to realize that shear analysis and design in reinforced concrete structure are not really concerned with shear as such.
- The shear stresses in most beams are far below the direct shear strength of the concrete.
- The real concern is with diagonal tension stress, resulting from the combination of shear stress and longitudinal flexural stress.
- Difference between direct shear and diagonal tension is presented in sub article below.

5.1.2.1 Vertical and Horizontal Shears

- The simplest form of shear is the **Vertical Shear Stress** indicated in **Figure 5.1-2** below.

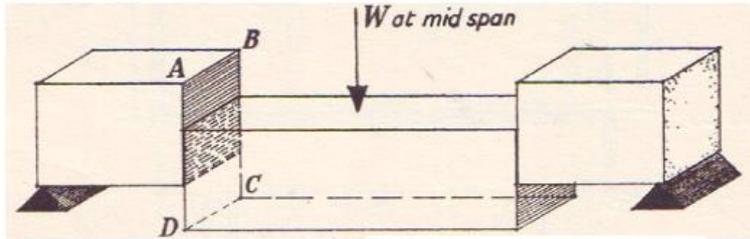


Figure 5.1-2: Vertical shear stresses.

- For homogenous beams and plain concrete beams before cracking, vertical shear stresses can be estimated from the following relation:

$$v = \frac{V \cdot Q}{Ib}$$

Eq. 5.1-1

where:

V is total shear at section,

Q is statical moment about the neutral axis of that portion of cross section lying between a line through the point in question parallel to the neutral axis and nearest face (upper or lower) of the beam,

I is the moment of inertia of cross section about neutral axis,

b is width of beam at a given point.

- Distribution of vertical shear stress along beam depth is presented in **Figure 5.1-3** below:

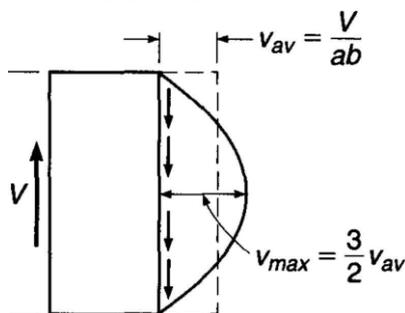


Figure 5.1-3: Shear stress distribution in homogeneous rectangular beams.

5.1.2.2 Horizontal Shear Stresses

- Referring to **Figure 5.1-4** below, imagine that a ball is placed between the two cut sections at X, because of the vertical shear action, the ball will turn in a clockwise direction.

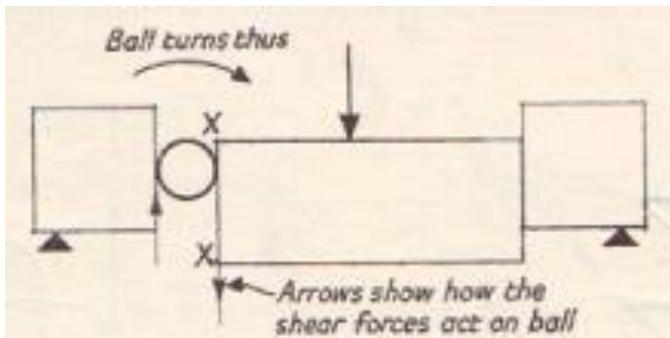


Figure 5.1-4: Conceptual view to imagine role of horizontal shear in resisting possible elemental rotation.

- Then in order to prevent turning, the cube shown below must be acted upon by horizontal forces shown in **Figure 5.1-5** below (Morgan, 1958):

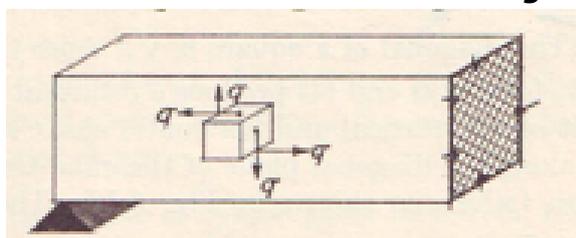


Figure 5.1-5: Horizontal shear stresses.

- These horizontal forces produce another type of shear stress called as **Horizontal Shear Stress**.
- Thus, one can conclude that the **vertical shear stress is accompanied by horizontal shear stress of equal intensity** (Morgan, 1958).

5.1.2.3 Diagonal Tension and Compression

- Force (1) in **Figure 5.1-6** below can be combined with force (3) to produce a resultant force of $q\sqrt{2}$. Similarly force (2) and (4) produce a resultant force of $q\sqrt{2}$.

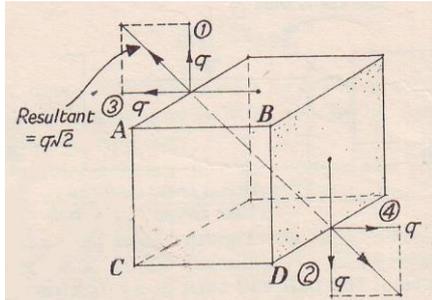


Figure 5.1-6: Diagonal tensile resultant of horizontal and vertical shear stresses.

- Thus, resultant of the vertical and horizontal shear stresses is a pull that exerted along the diagonal plane of the cube tending to cause the diagonal tension failure indicated in **Figure 5.1-7** below.

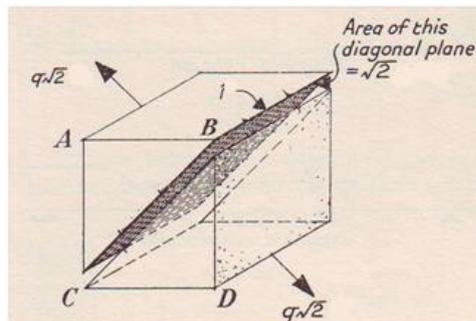


Figure 5.1-7: Diagonal tensile resultant of horizontal and vertical shear stresses, 2.

- Similarly, the vertical and horizontal shear stresses produce a compression force by combining force (2) with force (3) and force (1) with force (4) (see **Figure 5.1-8** below).

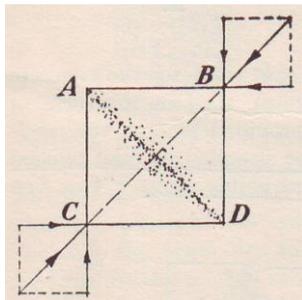


Figure 5.1-8: Diagonal compression resultant of horizontal and vertical shear stresses.

- Therefore, **whenever pure shear stress is acting on an element, it may be thought of as causing tension along one of the diagonals and compression along the other** (Popov, 1976).

5.1.2.4 Stress Trajectories

Based on the above discussion for the relation between shear stresses and corresponding diagonal stresses, **stress trajectories** in a homogeneous simply supported beam with a rectangular section are presented in **Figure 5.1-9** below.

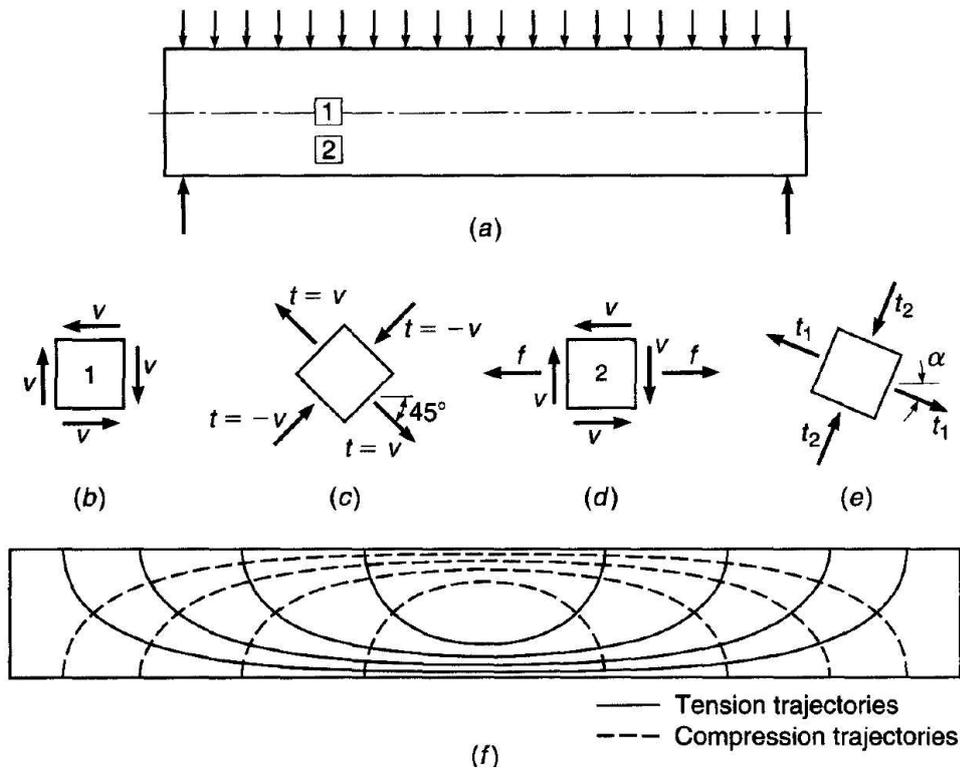


Figure 5.1-9: Stress trajectories in homogeneous rectangular beam.

5.1.2.5 Modes of Failure due to Shear or Diagonal Stresses

- Failure due to vertical shear stress is as shown in **Figure 5.1-10** below.

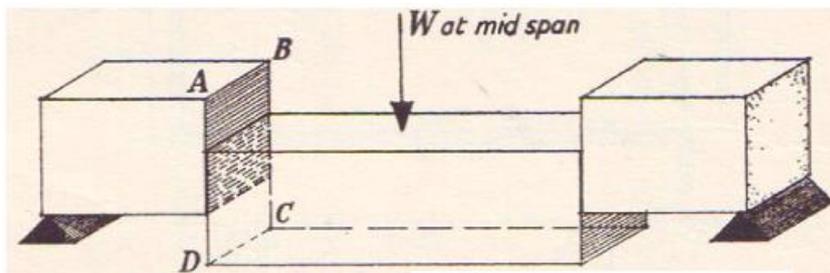


Figure 5.1-10: Mode of failure due to vertical shear stresses.

- Failure due to horizontal shear stress is as shown in **Figure 5.1-11** below.

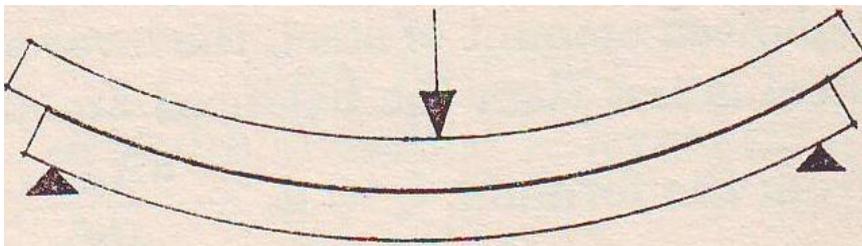


Figure 5.1-11: Mode of failure due to horizontal shear stresses.

- Failure due to diagonal compression stress is as shown in **Figure 5.1-12** below.

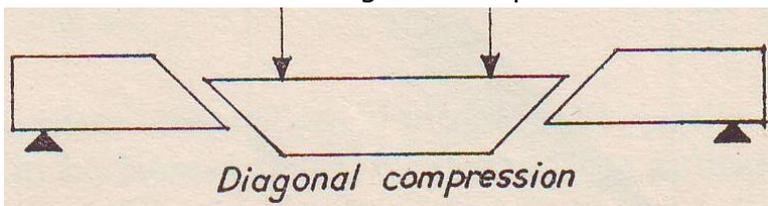


Figure 5.1-12: Mode of failure due to diagonal compression stresses.

- Failure due to diagonal tension stress is as shown in **Figure 5.1-13** below.

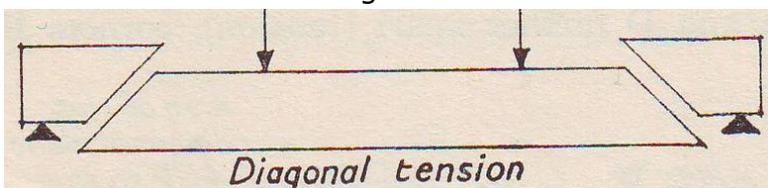


Figure 5.1-13: Mode of failure due to diagonal tensile stresses.

- It is known that concrete is at least ten times as strong in compression as it is in tension, so the typical shear failure in reinforced concrete beams is actually a diagonal tension failure.

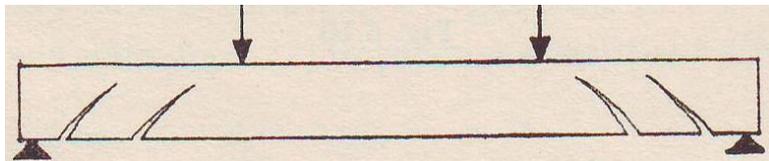


Figure 5.1-14: Cracks in concrete beams due to diagonal tension.

- When the shear stress is higher than the safe value of the concrete, steel in the form of vertical stirrups or inclined bars must be provided to take the exceed shear force.

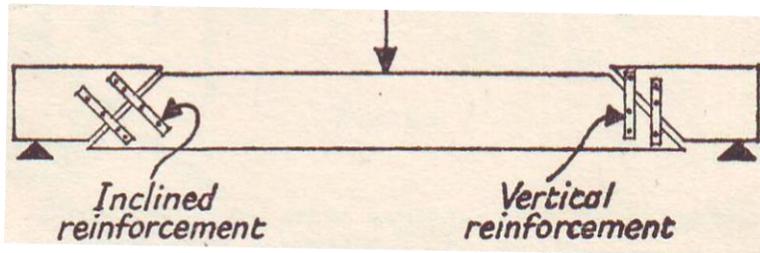


Figure 5.1-15: Conceptual view of vertical and inclined shear reinforcement.

5.1.3 Direct Shear

There are some circumstances in which consideration of direct shear is appropriate:

- The design of composite members combining precast beams with a cast-in-place top slab.



Figure 5.1-16: Composite members combining precast beams with a cast-in-place top slab.

- The horizontal shear stresses on the interface between components are important. The **shear-friction theory** is useful in this and other cases. This theory is out of our scope.

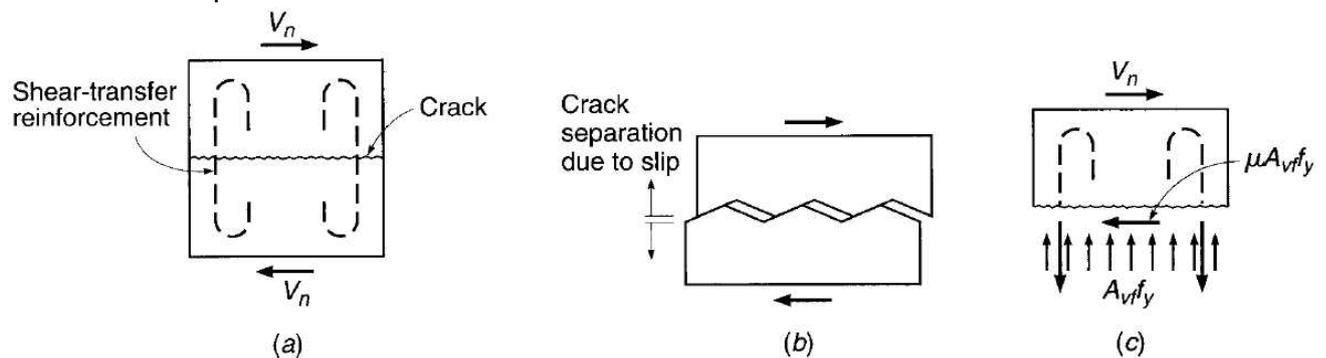


Figure 5.1-17: Basis of shear-friction design method: (a) applied shear; (b) enlarged representation of crack surface; (c) free-body sketch of concrete above crack.

5.1.4 ACI Code Provisions for Shear Design

- It is clear from the previous discussion; the problem under consideration is a problem of diagonal tension stresses.
- As the ACI Code uses the shear forces as an indication of the diagonal tension, then all design equations according to ACI Code are presented regarding shear forces.
- According to ACI Code (9.5.1.1), the design of beams for shear is to be based on the relation:

$$V_u \leq \phi V_n \tag{Eq. 5.1-2}$$

- According to the ACI Code (21.2.1), the strength reduction factor, ϕ , for shear is 0.75.
- According to article 22.5.1.1, nominal shear strength, V_n , can be computed based on the following relation:

$$V_n = V_c + V_s$$

Eq. 5.1-3

or,

$$V_u \leq V_n = \phi(V_c + V_s)$$

Eq. 5.1-4

where:

V_u is the total shear force applied at a given section of the beam due to factored loads,

V_n is the nominal shear strength, equal to the sum of the contributions of the concrete (V_c) and the steel (V_s) if present.

- Thus, according to ACI Code, the design problem for shear can be reduced to provisions for computing of V_u , V_c , and V_s if present. Each one of these quantities is discussed in some details in the articles below.

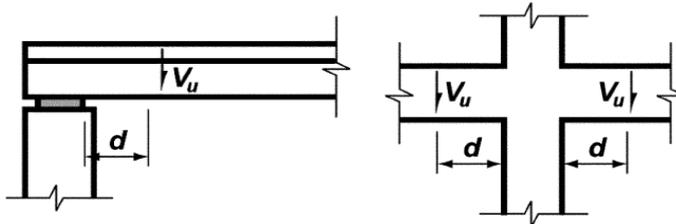
5.2 COMPUTING OF APPLIED FACTORED SHEAR FORCE V_u

5.2.1 Basic Concepts

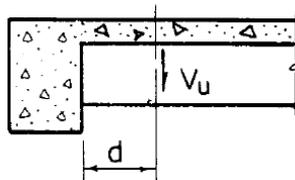
- The applied shear force can be computed based on given loads and spans.
- Generally, the applied factored shear force V_u is computed at the face of supports.
- According to ACI Code (9.4.3.2), **sections between the face of support and a critical a section located "d" from the face of support for nonprestressed shall be permitted to be designed for V_u at that critical section** if following conditions are satisfied: Discussion similar to that of classroom is preferable to add here to explain physical aspects of the three conditions below.
 - Support reaction, in the direction of applied shear, introduces compression into the end regions of the member.
 - Loads are applied at or near the top of the member.
 - No concentrated load occurs between the face of support and location of critical.

5.2.2 Examples on Computing of V_u

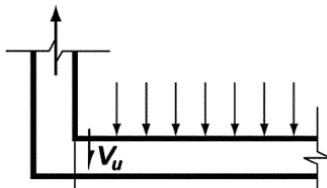
- For the figures below, critical section for computing of V_u will be taken at a distance "d" from the face of support as all above conditions are satisfied (Nilson, Design of Concrete Structures, 14th Edition, 2010). It is preferable to put these cases in groups, for example, floor beam supported on a deeper girder and a girder with same depth can be put in the same group. Besides, it is preferable that each group and corresponding figures have subtitle and caption.



- For the figure below, the critical section for computing of V_u is at distance "d" from the face of support for a floor beam supported by a deeper main girder as all above conditions are satisfied (Kamara, 2005) (Page 12-3).



- For the figure below, the critical section for computing of V_u is at the face of support as member framing into a supporting member in tension (Nilson, Design of Concrete Structures, 14th Edition, 2010) (Page 131).



- For the figure below, the critical section for computing of V_u is at the face of support if the beam is supported by a girder of similar depth (Nilson, Design of Concrete Structures, 14th Edition, 2010).

