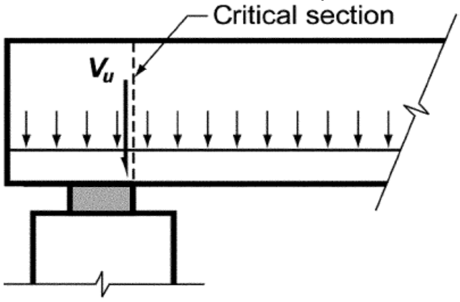
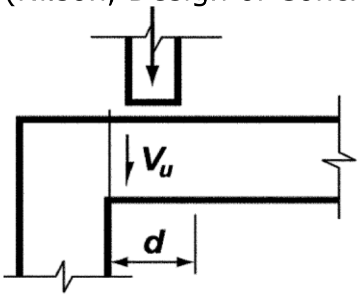
- For the figure below, the critical section for computing of V_u is at the face of support as loads are not applied at or near the top of the member (Nilson, Design of Concrete Structures, 14th Edition, 2010).



- For the figure below, the critical section for computing of V_u is at the face of support as concentrated load occurs within a distance "d" from the face of support (Nilson, Design of Concrete Structures, 14th Edition, 2010).



5.3 SHEAR STRENGTH PROVIDED BY CONCRETE V_c

5.3.1 Upper Bound of Concrete Compressive Strength, f_c', in Estimating V_c

- According to ***article 22.5.3.1*** of ACI code, except for ***article 22.5.3.2***, related to prestressed beams and joist construction, ***the value of*** $\sqrt{f_c'}$ ***used to calculate*** V_c ***shall not exceed 8.3 MPa***.
- The above statement is because of a lack of test data and practical experience with concretes having compressive strengths greater than 70 MPa.

5.3.2 Plain Concrete Beams

- As the load increases in such a beam, a tension crack will form where the tensile stresses are largest, and it will immediately cause the beam to fail.
- Except for beams of very unusual proportions, the largest tensile stresses are those caused at the outer fiber by bending alone, at the section of maximum bending moment. In this case, shear has little, if any, influence on the strength of a beam.

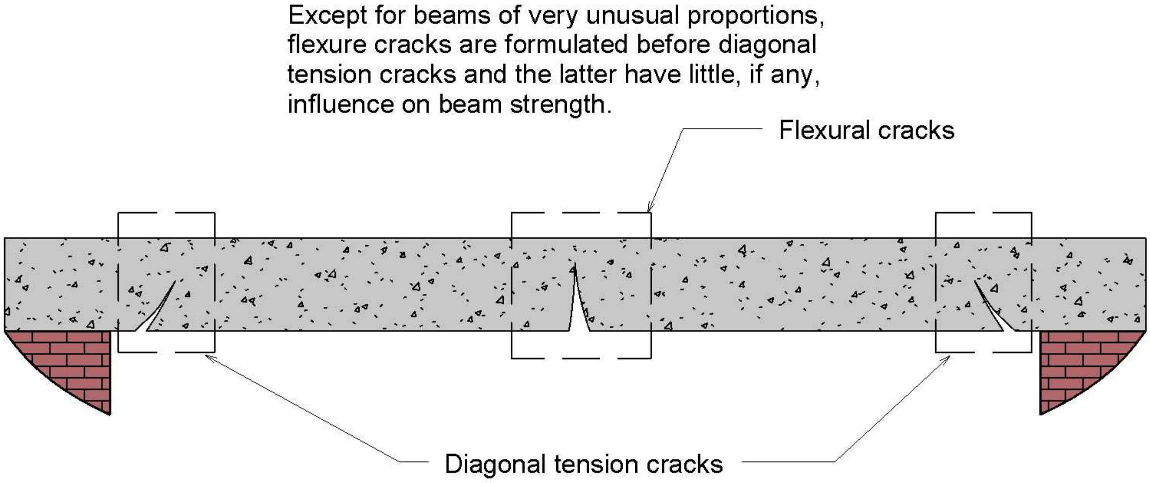


Figure 5.3-1: Behavior of plain concrete beams.

5.3.3 Reinforced Concrete Beams without Shear Reinforcement

- For beams designed properly for flexure, diagonal cracks may propagate faster than flexural cracks, and shear aspects may govern the beam failure.

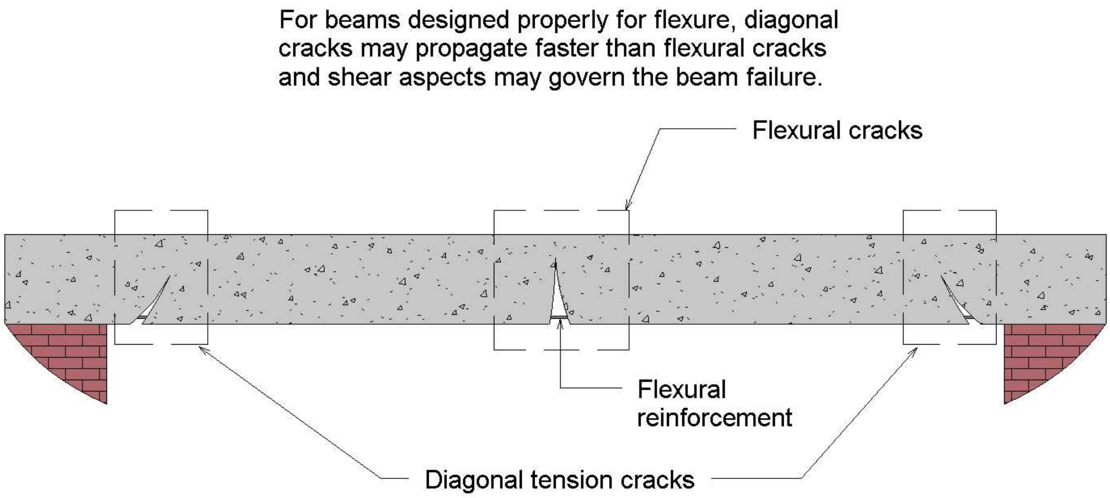


Figure 5.3-2: Behavior of a beam reinforced for flexure only.

- For concrete beams ***reinforced for flexure only***, shear force required ***to initiates diagonal cracks in web-shear cracks region***, or ***to propagate cracks in a flexure-shear region*** can be estimated from relation below, ***Article 22.5.5.1*** of (ACI318M, 2014):

$$V_c = 0.17\lambda\sqrt{f_c'}\,b_w d \qquad \textbf{Eq. 5.3-1}$$

where:

λ is the lightweight modification factor that taken from ***Table 5.3-1*** below, Table 19.2.4.2 of (ACI318M, 2014).

Table 5.3-1: Modification factor λ, Table 19.2.4.2 of (ACI318M, 2014).

Concrete	Composition of aggregates	λ
All-lightweight	Fine: ASTM C330M Coarse: ASTM C330M	0.75
Lightweight, fine blend	Fine: Combination of ASTM C330M and C33M Coarse: ASTM C330M	0.75 to 0.85[1]
Sand-lightweight	Fine: ASTM C33M Coarse: ASTM C330M	0.85
Sand-lightweight, coarse blend	Fine: ASTM C33M Coarse: Combination of ASTM C330M and C33M	0.85 to 1[2]
Normalweight	Fine: ASTM C33M Coarse: ASTM C33M	1

[1]Linear interpolation from 0.75 to 0.85 is permitted based on the absolute volume of normalweight fine aggregate as a fraction of the total absolute volume of fine aggregate.

[2]Linear interpolation from 0.85 to 1 is permitted based on the absolute volume of normalweight coarse aggregate as a fraction of the total absolute volume of coarse aggregate.

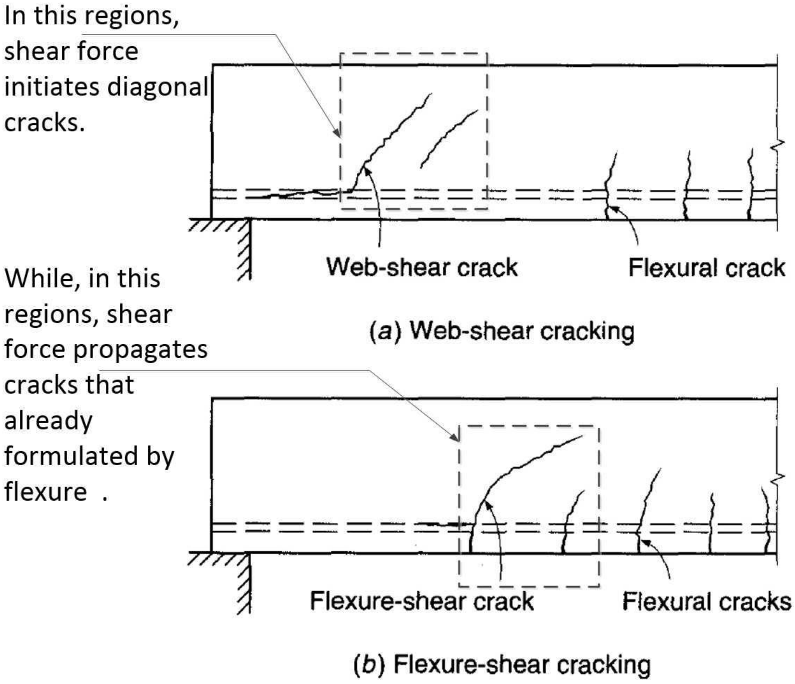


Figure 5.3-3: Diagonal tension cracking in reinforced concrete beams.

- With referring to ***Figure 5.3-3*** above, it is useful to note that the Eq. 5.3-1 is ***more suitable flexure-shear crack*** and ***relatively conservative for web-shear cracks***. A more accurate relation has been presented in ***Article 5.8*** of this chapter.
- In spite of its conservative nature in the web-shear crack region, in practice, most of the beams are usually designed based on Eq. 5.3-1.
- For solid circular members, the area used to compute V_c shall be taken as shown in ***Figure 5.3-4*** (***Article 22.5.2.2*** of ACI Code).

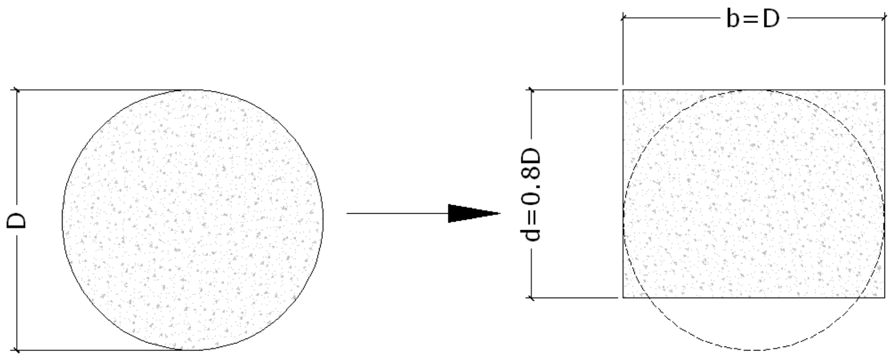


Figure 5.3-4: Effective area for shear in solid circular sections.

5.3.4 Beams Reinforced for Shear

- As for flexural behavior, current ACI code permits formation of web-shear cracks and flexure-shear cracks when beams are reinforced for shear and diagonal tension.
- With shear reinforcements, that resist propagation of web-shear cracks, the free body diagram for one side of crack at failure stage would be as shown ***Figure 5.3-5*** below.

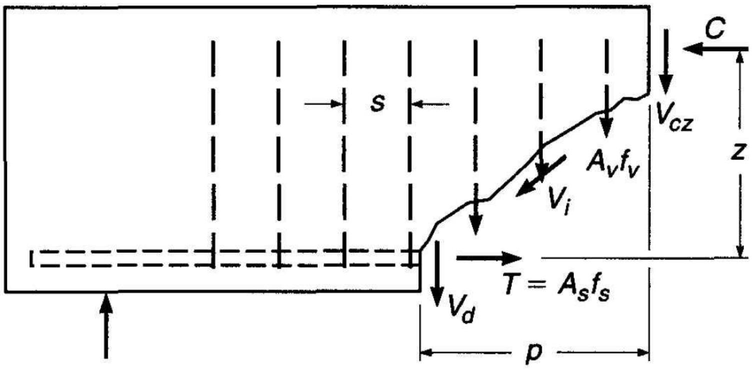


Figure 5.3-5: Forces at a diagonal crack in a beam with vertical stirrups.

where
A_vf_v is shear force resisted by each stirrup, will be discussed in detail in ***Article 5.4.2*** of this chapter,
V_{cz} shear force resisted by uncracked concrete portion,
V_i shear force resisted by the interlocking of concrete on two sides of the crack,
V_d shear force resisted by longitudinal rebars, dowel action,

- From equilibrium in vertical direction,

$$V_{ext} = V_{cz} + V_d + V_{iy} + V_s \qquad \textbf{Eq. 5.3-2}$$

- ***Empirically*** and ***conservatively*** current ACI code assumes that:

$$V_{cz} + V_d + V_{iy} \approx V_c = 0.17\lambda\sqrt{f_c'}\, b_w d \qquad \textbf{Eq. 5.3-3}$$

- Therefore, in the current ACI code, the relation:

$$V_c = 0.17\lambda\sqrt{f_c'}\, b_w d \qquad \textbf{Eq. 5.3-4}$$

has two roles:
  - It is used ***rationally*** to estimate the shear force that either initiates web-shear cracks or propagate flexure-shear cracks.
  - It is used ***empirically*** to estimate the order for summation of V_{cz}, V_d, and V_{iy}.

5.4 Shear Strength Provided by Shear Reinforcement V_s

5.4.1 Type of Shear Reinforcement

- Several types and arrangements of shear reinforcement permitted by ACI are illustrated in ***Figure 5.4-1*** (Kamara, 2005) (Page 12-6).

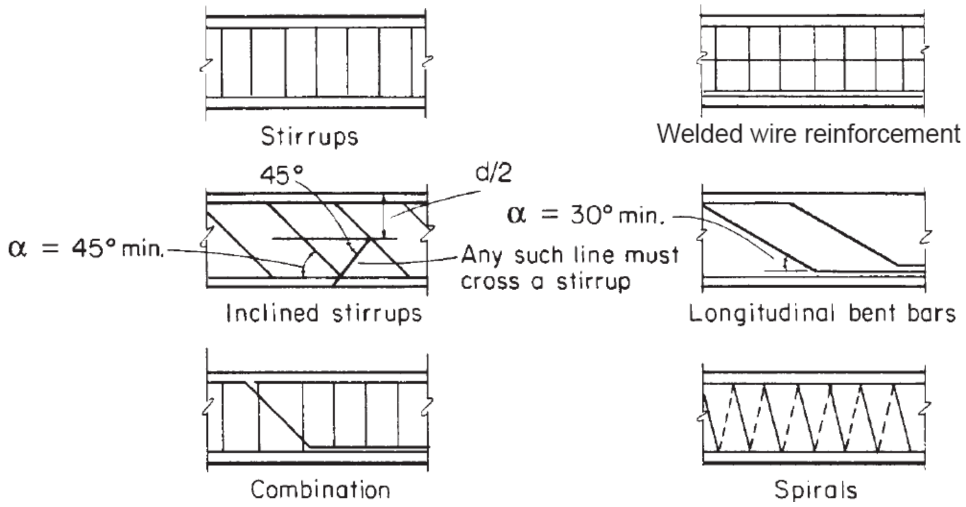


Figure 5.4-1: Types of shear reinforcement.

- Spirals, circular ties, or hoops are explicitly recognized as types of shear reinforcement starting with the 1999 code (Kamara, 2005) (Page 12-6).
- Vertical stirrups are the most common type of shear reinforcement.
- Inclined stirrups and longitudinal bent bars are rarely used as they require a special care during placement in the field.
- U-shaped bars similar to those presented in ***Figure 5.4-2*** below are the most common, although multiple-leg stirrups such as shown are sometimes necessary.

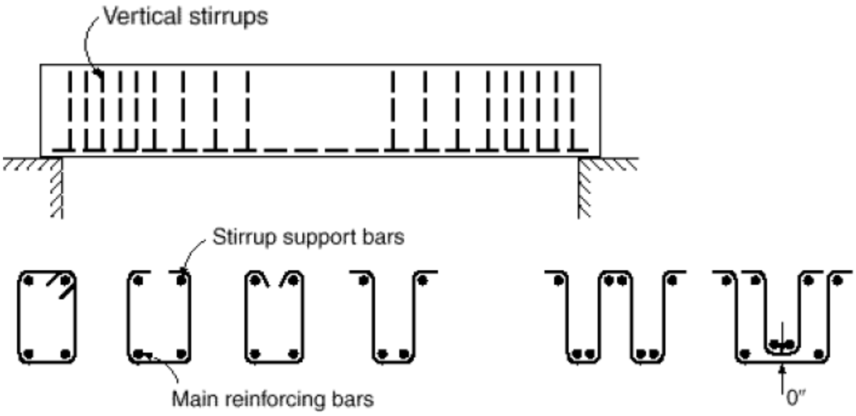


Figure 5.4-2: U stirrups shear reinforcement.

5.4.2 Theoretical Spacing between Vertical Stirrups

- Theoretical spacing for vertical stirrups can be related to other design parameters based on following relations:

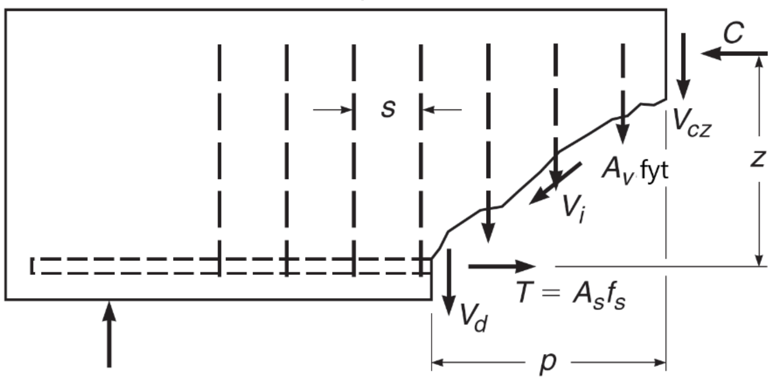


Figure 5.4-3: Forces at a diagonal crack in a beam with vertical stirrups, reproduced for convenience.

$$V_s = \text{Fore per each stirrup} \times \text{No. of stirrups through the inclined crack}$$

$$V_s = \left(A_v \times f_{yt}\right)_{\text{Fore per each stirrup}} \times \left(\frac{p}{s}\right)_{\text{No.of stirrups through the inclined crack}}$$

where:

$A_v =$ area of shear reinforcement $= \frac{\pi \phi^2_{Stirrups}}{4} \times$ No. of Legs

- If the crack is assumed to have an angle of 45 degree with the horizon, then p can be computed approximately based on following relation:
  $p \approx d$
  Then:
  $$V_s = \frac{A_v f_{yt} d}{s}$$ **Eq. 5.4-1**
  Above relation that suitable for analysis purpose, can be solved for s to be more suitable for design purpose:
  $$s = \frac{A_v f_{yt} d}{V_s} \blacksquare$$ **Eq. 5.4-2**
- In addition to this theoretical spacing for shear reinforcement, ACI Code also includes many other nominal requirements that related to shear reinforcement. ACI practical procedure for shear design has been summarized in article below.

5.5 SUMMARY OF PRACTICAL PROCEDURE FOR SHEAR DESIGN

5.5.1 Essence of the Problem

- Generally, beam dimensions (b and h) are determined based on considerations other than shear and diagonal tension requirements.
- Then, in a shear problem, the designer deals with a beam that has pre-specified dimensions and main unknowns in the design problem are the shear reinforcement (if needed) and its details that can be summarized as follows:
  - The diameter of shear reinforcement.
  - Spacing (for economic aspect, a beam may be divided to sub-regions with different shear reinforcements) for shear reinforcements.
  - Anchorage requirements for shear reinforcements.
- The detailed procedure for each one of the above three unknowns will be discussed below.

5.5.2 Bar Diameter for Stirrups and Stirrups Support Bars

- As was previously discussed in Chapter 4, bar diameters that used for shear reinforcements usually include 10mm, or 13mm.
- A Bar diameter of 16mm rarely used as shear reinforcement.
- Where no top bars are required for flexure, stirrups support bars must be used. These are usually about the same diameter as the stirrups themselves (Nilson, Design of Concrete Structures, 14th Edition, 2010).

5.5.3 Spacing for Shear Reinforcements

Computing of required spacing can be summarized as follows:

- Draw the shear force diagram based on factored load and span length, and divide the diagram into the three distinct regions shown in ***Figure 5.5-1*** (Kamara, 2005) (Page 12-9):

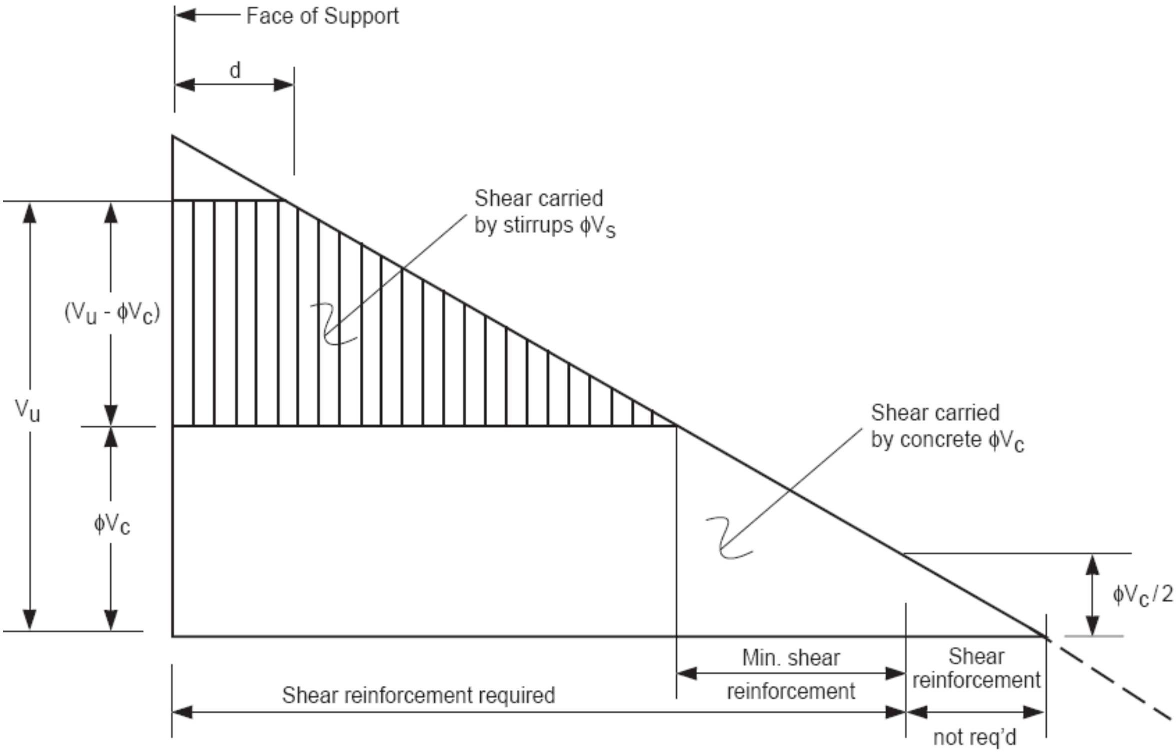


Figure 5.5-1: Three distinguish regions of shear force diagram.

- Based on ***Table 5.5-1***, compute the required spacing for each one of the regions shown above (if shear reinforcement is required for this region) (Kamara, 2005) (Page 12-8):

Table 5.5-1: ***ACI provisions for shear design.***

Region	$V_u \leq \emptyset \frac{V_c}{2}$	$\emptyset \frac{V_c}{2} < V_u \leq \emptyset V_c$	$\emptyset V_c \leq V_u$
V_s	None	None	$= \frac{V_u - \emptyset V_c}{\emptyset} \leq 0.66\sqrt{f_c'} b_w d$ Else, change beam dimensions.
$S_{Theoretical}$	None	None	$= \frac{A_v f_{yt} d}{V_s}$
$S_{for\ Av\ minimum}$ (9.6.3.3)	None	minimum $\left(\frac{A_v f_{yt}}{0.062\sqrt{f_c'} b_w} \text{ or } \frac{A_v f_{yt}}{0.35 b_w}\right)$	minimum $\left(\frac{A_v f_{yt}}{0.062\sqrt{f_c'} b_w} \text{ or } \frac{A_v f_{yt}}{0.35 b_w}\right)$
$S_{maximum}$ (9.7.6.2.2)	None	Minimum $\left[\frac{d}{2} \text{ or } 600\text{mm}\right]$	$V_s \leq 0.33\sqrt{f_c'} b_w d$ Minimum $\left[\frac{d}{2} \text{ or } 600\text{mm}\right]$ $V_s > 0.33\sqrt{f_c'} b_w d$ Minimum $\left[\frac{d}{4} \text{ or } 300\text{mm}\right]$
$S_{Required}$	None	Minimum $\left[S_{for\ Av\ minimum}, S_{maximum}\right]$	Minimum $\left[S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}\right]$

- Notes on A_{vmin}:
  According to ***Article 9.6.3.1***, for cases presented in Table below, $A_{v\,minimum}$ is not required even with $\emptyset \frac{V_c}{2} < V_u \leq \emptyset V_c$:

Table 5.5-2: Cases where $A_{v\,\min}$ is not required if $0.5\phi Vc < Vu \leq \phi Vc$, Table 9.6.3.1 of (ACI318M, 2014).

Beam type	Conditions
Shallow depth	$h \leq 250$ mm
Integral with slab	$h \leq$ greater of $2.5t_f$ or $0.5b_w$ and $h \leq 600$ mm
Constructed with steel fiber-reinforced normalweight concrete conforming to 26.4.1.5.1(a), 26.4.2.2(d), and 26.12.5.1(a) and with $f_c' \leq 40$ MPa	$h \leq 600$ mm and $V_u \leq \phi 0.17\sqrt{f_c'} b_w d$
One-way joist system	In accordance with 9.8

5.5.4 Anchorage Requirement for Shear Reinforcements

5.5.4.1 Design Assumptions Regarding to Anchorage

Above design is based on assumption that the stirrups will yield at ultimate load. This will be true only if the stirrups are well anchored.

5.5.4.2 General Anchor Requirements

- Generally, the upper end of the inclined crack approach very closed to the compression face of the beam. Thus, the portion of the stirrups shown shaded in ***Figure 5.5-2*** must be able to anchor the stirrups.

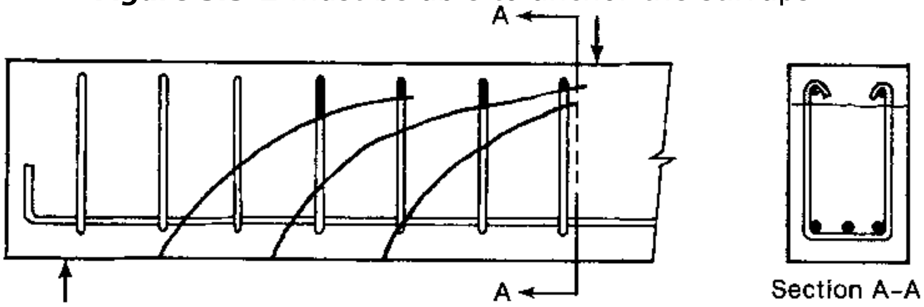


Figure 5.5-2: General requirements for anchorage of stirrups.

- ACI general anchor requirement can be summarized in ***Figure 5.5-3***.

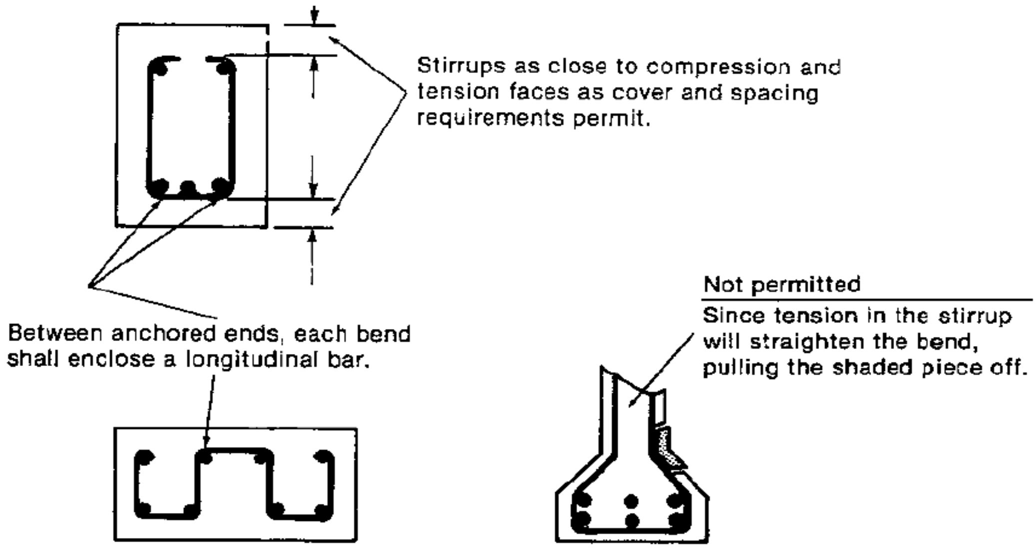


Figure 5.5-3: General requirements for anchorage of stirrups, continued.

- According to anchorage requirements, stirrups may be classified into the following two types.

5.5.4.3 Open Stirrups

- They may take any one of the shapes indicated in ***Figure 5.5-4***.
- As shown in ***Figure 5.5-5***, anchorage of an open stirrup depends on using standard hooks at the corners of the stirrups supporting rebars.

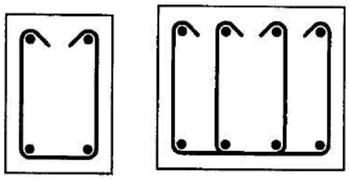


Figure 5.5-4: Open stirrups.

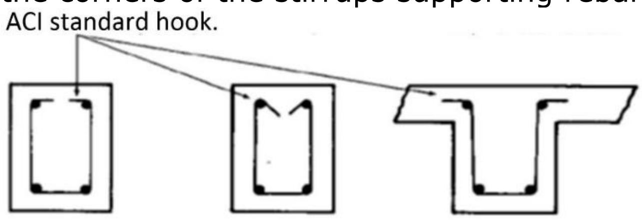
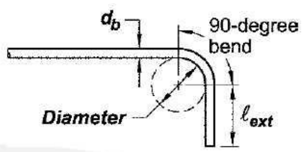
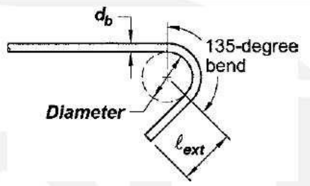
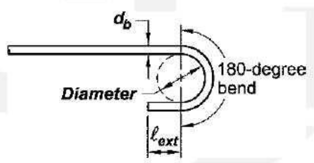


Figure 5.5-5: Standard hook anchorage for open stirrups.

- Minimum inside bend diameters and standard hook geometry ***for stirrups, ties, and hoops*** are presented in ***Table 5.5-3***.

Table 5.5-3: Minimum inside bend diameters and standard hook geometry for stirrups, ties, and hoops, Table 25.3.2 of (ACI318M, 2014).

Type of standard hook	Bar size	Minimum inside bend diameter, mm	Straight extension[1] ℓ_{ext}, mm	Type of standard hook
90-degree hook	No. 10 through No. 16	$4d_b$	Greater of $6d_b$ and 75 mm	
	No. 19 through No. 25	$6d_b$	$12d_b$	
135-degree hook	No. 10 through No. 16	$4d_b$	Greater of $6d_b$ and 75 mm	
	No. 19 through No. 25	$6d_b$		
180-degree hook	No. 10 through No. 16	$4d_b$	Greater of $4d_b$ and 65 mm	
	No. 19 through No. 25	$6d_b$		

[1]A standard hook for stirrups, ties, and hoops includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

- According to ACI (25.7.1.3b), for No. 19, through No. 25 stirrups with f_{yt} greater than 280 MPa, a standard stirrup hook around a longitudinal bar plus an embedment between mid-height of the member and the outside end of the hook equal to or greater than $0.17 d_b\, f_{yt}\, /(\lambda \sqrt{f_c'})$.

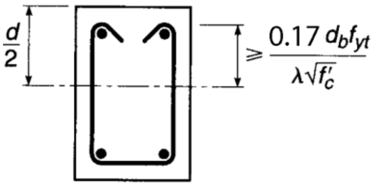


Figure 5.5-6: Embedment length for open stirrups with for No. 19, through No. 25 stirrups with f_{yt} greater than 280 MPa.

- This requirement has been included as it is not possible to bend a No. 19, No. 22, or No. 25 stirrup tightly around a longitudinal.

5.5.4.4 Closed Stirrups

- Its typical shapes are shown ***Figure 5.5-7***.

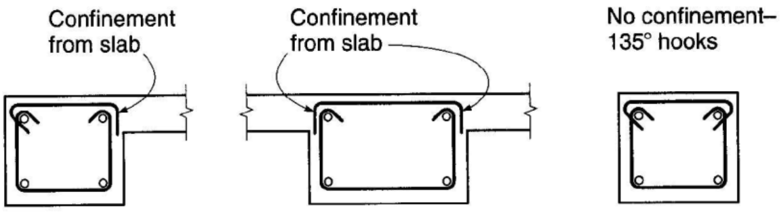


Figure 5.5-7: Typical closed stirrups.

- It may be taking the form of closed tie shown in ***Figure 5.5-8***.
- Closed stirrups or closed ties should be used for:
  - For beams with compression reinforcements.
  - For members subjected to torsion.

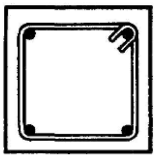


Figure 5.5-8: Tie reinforcement.

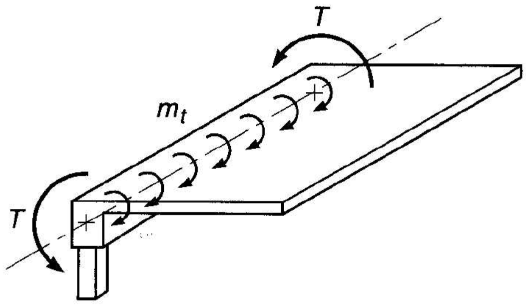


Figure 5.5-9: Beam subjected to torsion where closed stirrups should be adopted.

  - For reversals stresses.

5.5.4.5 Spliced Stirrup

- According to article ***25.7.1.7*** of (ACI318M, 2014), ***except where used for torsion*** or ***integrity reinforcement***, closed stirrups are permitted to be made using pairs of U-stirrups spliced to form a closed unit where lap lengths are at least $1.3l_d$.
- In members with a total depth of at least 450 mm, such splices with $A_b f_{yt} \leq\ 40\ kN$ per leg shall be considered adequate if stirrup legs extend the full available depth of member.

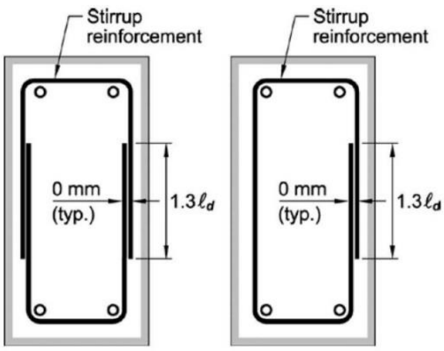


Figure 5.5-10: Closed stirrup configurations.

- The development length may be defined as ***the length of embedment necessary to develop the full tensile strength of the bar***. It will be discussed in details in ***Chapter 6***.
- Its approximate value can be computed from Table below:

Table 5.5-4: Simplified tension development length in bar diameters l_d/d_b for uncoated bars and normalweight concrete

		No. 6 (No. 19) and Smaller[a]			No. 7 (No. 22) and Larger		
		f'_c, psi			f'_c, psi		
	f_y, ksi	4000	5000	6000	4000	5000	6000
(1) Bottom bars							
Spacing, cover and ties as per Case *a* or *b*	40	25	23	21	32	28	26
	50	32	28	26	40	35	32
	60	38	34	31	47	42	39
Other cases	40	38	34	31	47	42	39
	50	47	42	39	59	53	48
	60	57	51	46	71	64	58
(2) Top bars							
Spacing, cover and ties as per Case *a* or *b*	40	33	29	27	41	37	34
	50	41	37	34	51	46	42
	60	49	44	40	62	55	50
Other cases	40	49	44	40	62	55	50
	50	62	55	50	77	69	63
	60	74	66	60	92	83	76

Case *a*: Clear spacing of bars being developed or spliced $\geq d_b$, clear cover $\geq d_b$, and stirrups or ties throughout l_d not less than the Code minimum.

Case *b*: Clear spacing of bars being developed or spliced $\geq 2d_b$, and clear cover not less than d_b.

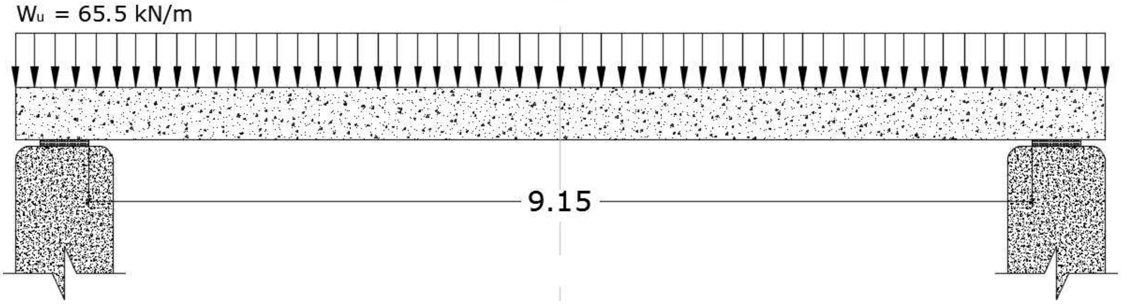
[a]ACI Committee 408 recommends that the values indicated for bar sizes No. 7 (No. 22) and larger be used for all bar sizes.

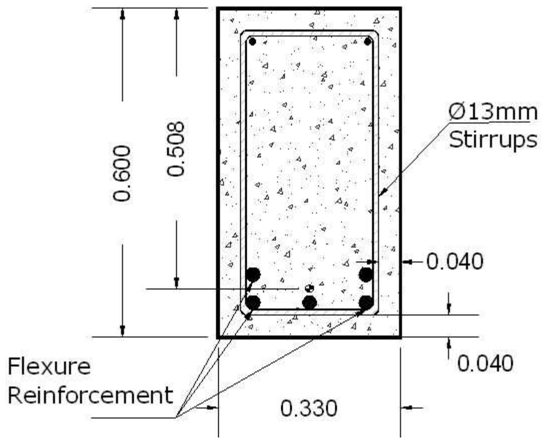
5.6 BASIC DESIGN EXAMPLES

Example 5.6-1

Check adequacy of proposed size and determine required spacing of vertical stirrups for a 9.15m span simply supported beam with following data:

$b_w = 330\text{mm},\ d = 508\text{mm},\ f_c' = 21\text{ MPa},\ f_{yt} = 275\text{ MPa},\ W_u = 65.5\frac{\text{kN}}{\text{m}}$



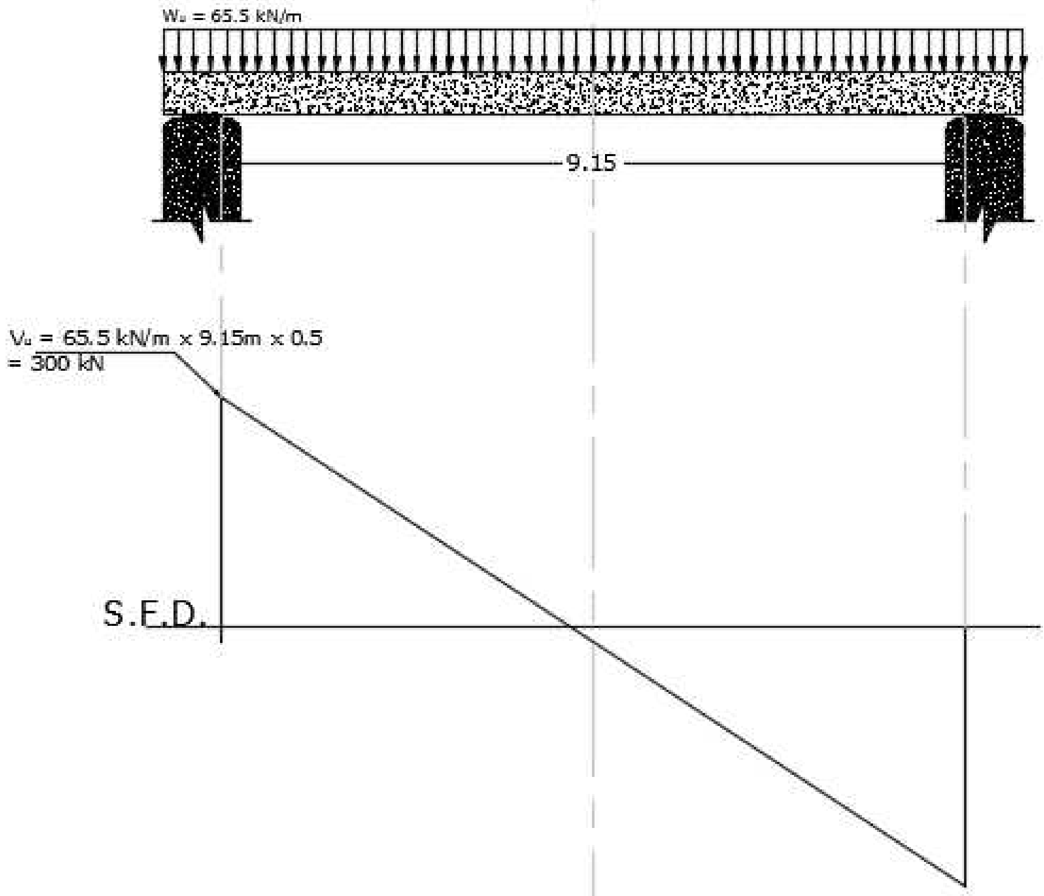


Proposed beam section.

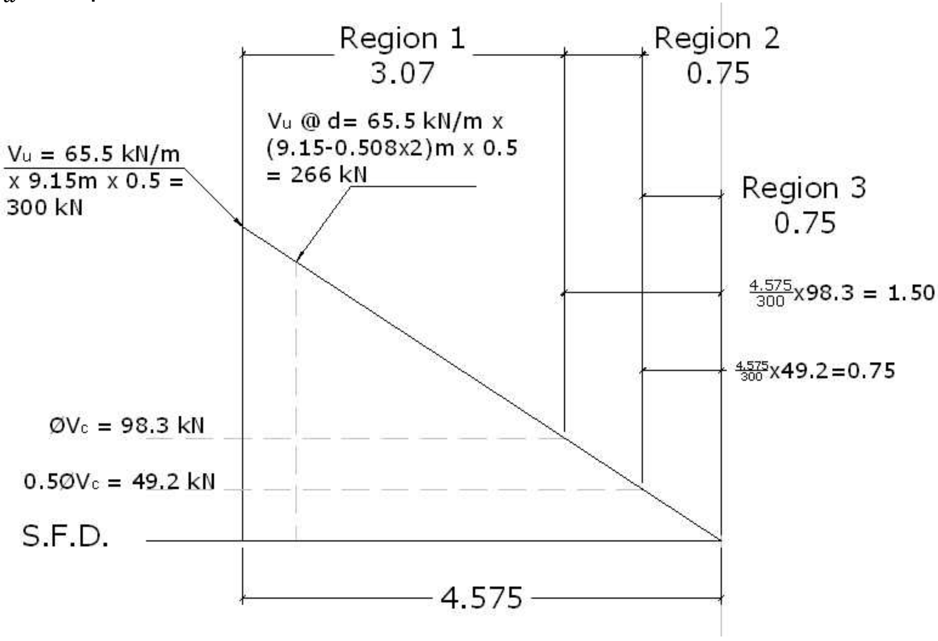
Figure 5.6-1: Simply supported beam for Example 5.6-1.

Solution

- Regarding to bar diameter for stirrups, the proposed diameter of 13mm is common and accepted one.
- Draw the shear force diagram for the beam:



- Compute of Shear Strength Provided by Concrete V_c:
  $V_c = 0.17\lambda\sqrt{f_c'}\, b_w d$
  As $\lambda = 1.0$ for normal weight concrete, then:
  $V_c = 0.17\sqrt{f_c'}\, b_w d = 0.17 \times \sqrt{21}\,\frac{N}{mm^2} \times 330mm \times 508mm = 131\,000\ N = 131\ kN$
  $\emptyset V_c = 0.75 \times 131\ kN = 98.3\ kN$
- Based on value of $\emptyset V_c$ divide the shear force diagram into three regions indicated below. As all limitations of article (**9.4.3.2**) are satisfied, then sections located less than a distance "d" from face of support shall be permitted to be designed for V_u computed at a distance "d".



To compute shear force at distance "d" from face of supported any one of the following two approaches can be adopted:

- Based on differential equations of equilibrium:

From mechanics of materials, to satisfy equilibrium of an infinitesimal element, following differential equations should be satisfied:

$$w = \frac{dV}{dx}$$

$$V = \frac{dM}{dx}$$

The first equation indicates that the load value, w, represents the slope for shear diagram while the second equation indicates that the value of shear force represents the slope of the bending moment diagram. It is useful to note that both equations are consistent in units.

From the first equation:

$$dV = wdx$$

Integrate to obtain

$$V_2 - V_1 = \int_1^2 w\, dx$$

Or

$$V_{u\,@\,distance\ d} = \int_{From\ face\ of\ support}^{To\ a\ distance\ d\ from\ face\ of\ support} w\, dx \; + V_{u\,@\,face\ of\ support}$$

It is worthwhile to note that the above ***finite integral is equal to area under load diagram from face of support to a distance "d" from face of support***.

$$V_{u\,@\,distance\ d} = (-65.5 \times 0.508 \; + 300) \approx 266\ kN$$

- Based on Symmetry

From problems that have symmetry, shear force at distance "d" can be determined based on following relation:

$V_{u\ at\ distance\ d\ from\ face\ of\ support} = \frac{1}{2} \times W_u(l_n - 2d)$

where l_n is the clear span measured from face to face of supports.

$V_{u\ at\ distance\ d\ from\ face\ of\ support} = \frac{1}{2} \times 65.5 \times (9.15 - 2 \times 0.508) = 266\ kN$

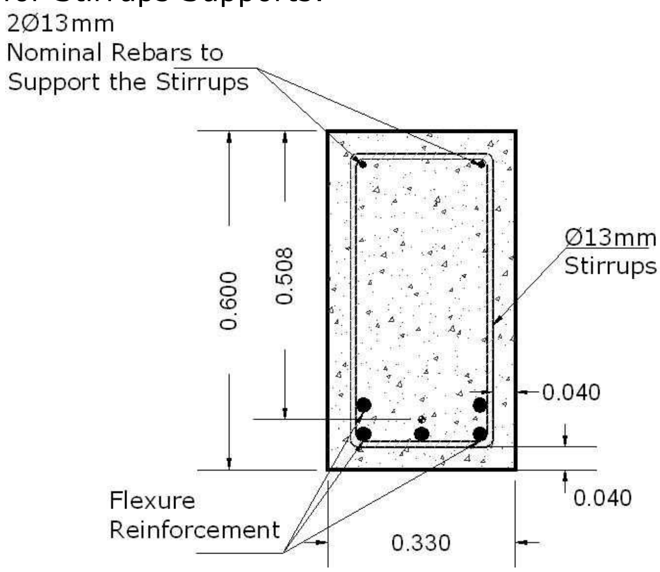
- Compute stirrups spacing for each region based on the table presented below: Try U Shape stirrups of 13mm diameter, then A_v will be:

$A_v = \frac{\pi \times 13^2}{4} \times 2 = 265\ mm^2$

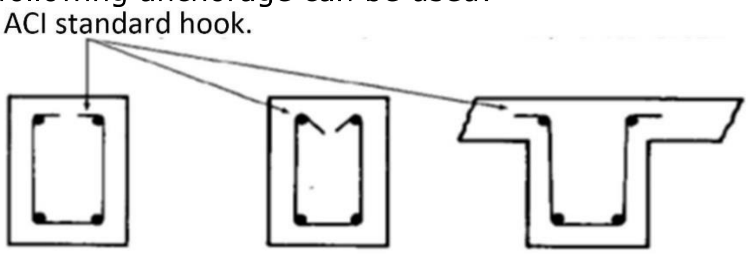
Stirrups Spacing for Example 5.6-1

Region	$V_u \leq \emptyset \frac{V_c}{2}$	$\emptyset \frac{V_c}{2} < V_u \leq \emptyset V_c$	$\emptyset V_c \leq V_u$
V_s	None	None	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d$ $\frac{266 - 98.3}{0.75}\ ?\ 0.66 \times \sqrt{21} \times 330 \times 508$ $224kN < 507\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	None	None	$= \frac{A_v f_{yt} d}{V_s} = \frac{265 \times 275 \times 508}{224\ 000} = 165\ mm$
$S_{for\ Av\ minimum}$	None	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $= 630\ mm$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ (\frac{265 \times 275}{0.062\sqrt{21} \times 330}\ or\ \frac{265 \times 275}{0.35 \times 330})$ $minimum\ (777\ or\ 630)$ $= 630\ mm$
$S_{maximum}$	None	$Minimum\ \left[\frac{d}{2}\ or\ 600mm\right]$ $= 254\ mm$	$V_s \leq 0.33\sqrt{f_c'}b_w d$ $224kN \leq 0.33\sqrt{21} \times 330 \times 508$ $224\ kN \leq 254\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right]$ $Minimum\ \left[\frac{508}{2}\ or\ 600mm\right] = 254\ mm$
			$V_s > 0.33\sqrt{f_c'}b_w d$ $Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	None	$Minimum\ [S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [630\ mm, 254\ mm]$ = 254 mm Use $\emptyset 13mm$ @ $250mm$	$Minimum\ \left[\begin{matrix} S_{Theoretical}, \\ S_{for\ Av\ minimum}, S_{maximum} \end{matrix}\right]$ $Minimum\ \left[\begin{matrix} 165\ mm, \\ 630\ mm, 254\ mm \end{matrix}\right]$ $= 165\ mm$ Use $\emptyset 13mm$ @ $150mm$

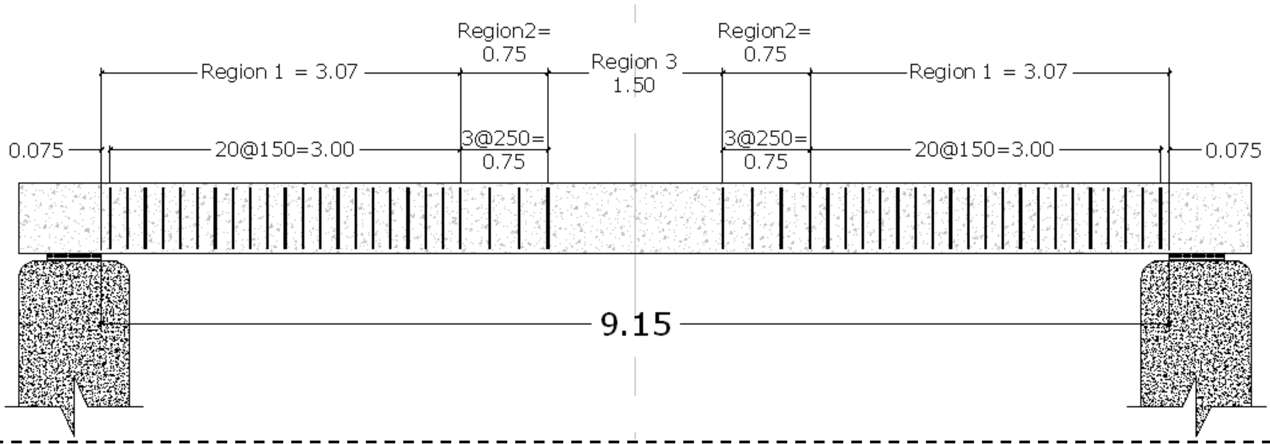
- Selecting of Nominal Reinforcement for Stirrups Supports:
  As no top bars are required for flexure, stirrups support bars must be used. These are usually about the same diameter as the stirrups themselves (Nilson, Design of Concrete Structures, 3th Edition , 2003) (Page 180).



- Anchorage Requirement for Shear Reinforcements:
  If one assumes that no compression reinforcement is required for this beam, any one of following anchorage can be used:



- Final stirrup spacing would be as indicated in below:



Example 5.6-2

Re-design ***Example 5.6-1*** but with using same spacing along beam span. Then compare the two designs.

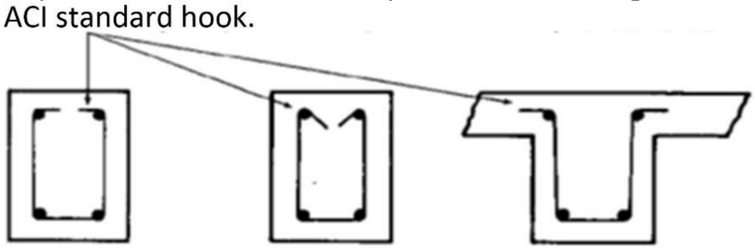
Solution

It practices, structural designers may use the same spacing along beam span. This spacing should be computed based on maximum shear force and can be used in other regions where shear forces are less than the force that used in design.

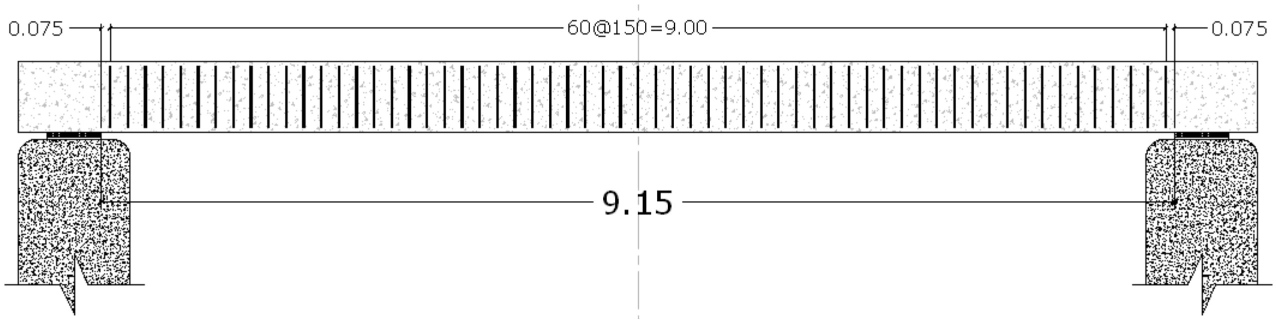
- Compute V_u:
  As all limitations of article (**9.4.3.2**) are satisfied, then sections located less than a distance "d" from face of support shall be designed for V_u computed at a distance "d".
  $$V_u = \frac{\left[65.5\frac{kN}{m} \times (9.15 - 2 \times 0.508)m\right]}{2} = 266\ kN$$
- Compute Concrete Shear Strength V_c:
  $V_c = 0.17\lambda\sqrt{f_c'}\, b_w d$
  With $\lambda = 1.0$ for normal weight concrete:
  $$V_c = 0.17\sqrt{f_c'}\, b_w d = 0.17 \times \sqrt{21}\,\frac{N}{mm^2} \times 330mm \times 508mm = 131\,000\ N = 131\ kN$$
  $\emptyset V_c = 0.75 \times 131\ kN = 98.3\ kN$
  $\because V_u = 266\ kN > \emptyset V_c = 98.3\ kN$
  Then the beam will be designed based of provisions of $V_u > \phi V_c$.

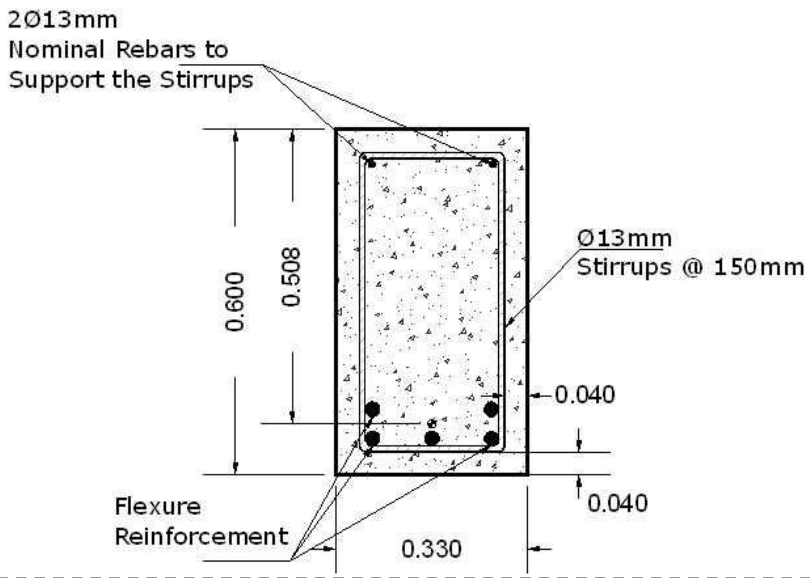
Shear Spacing Design of Example 5.6-2	
Region	$\emptyset V_c \le V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d$ $\frac{266 - 98.3}{0.75}\ ?\ 0.66 \times \sqrt{21} \times 330 \times 508 \Longrightarrow 224kN < 507\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{265 \times 275 \times 508}{224\,000} = 165\ mm$
S for Av minimum	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ \left(\frac{265 \times 275}{0.062\sqrt{21} \times 330}\ or\ \frac{265 \times 275}{0.35 \times 330}\right) = minimum\ (777\ or\ 630)$ $= 630\ mm$
$S_{maximum}$	$V_s \le 0.33\sqrt{f_c'}b_w d$ $224kN \le 0.33\sqrt{21} \times 330 \times 508 \Longrightarrow 224\ kN \le 254\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right] \Longrightarrow Minimum\ \left[\frac{508}{2}\ or\ 600mm\right] = 254\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d \Longrightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [165\ mm, 630\ mm, 254\ mm] = 165\ mm$ **Use $\emptyset 13mm$ @ $150mm$**

- Anchorage Requirement for Shear Reinforcements:
  As for previous example, if one assumes that no compression reinforcement is required for this beam, any one of following anchorage can be used:



- Comparison between two designs:
  Required Number of Stirrups for the more accurate design of Example 5.6-1 is:
  $$\text{No. of Stirrups} = \left[\left(\frac{3.0}{0.150}+1\right)+\frac{0.75}{0.250}\right]\times 2 = 48 \text{ U Stirrups}$$
  Required Number of Stirrups for the simplified design of Example 5.6-2 is:
  $$\text{No. of Stirrups} = \left(\frac{9.0}{0.150}+1\right) = 61 \text{ U Stirrups}$$
  Then dividing the beam into three regions and design of each region for its shear force can save 13 stirrups.





Example 5.6-3

Design Region 1 and Region 2 of floor beam indicated in ***Figure 5.6-2*** for shear. The beam has a width of 375mm and an effective depth of 775mm. Assume that the designer intends to use:

- $f_c' = 27.5\ \text{MPa}$.
- $f_{yt} = 414\ \text{MPa}$.
- Stirrups of 10mm diameter ($A_{Bar} = 71\text{mm}^2$).

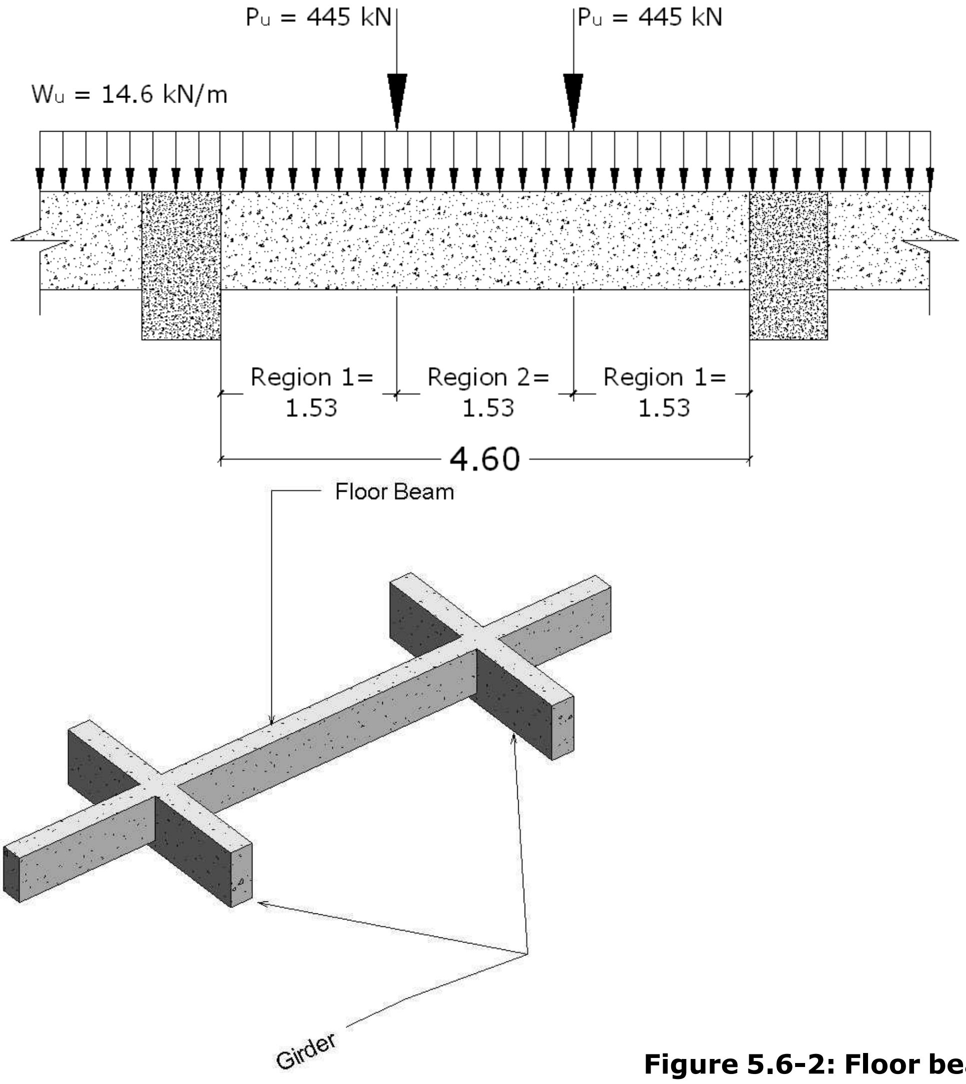


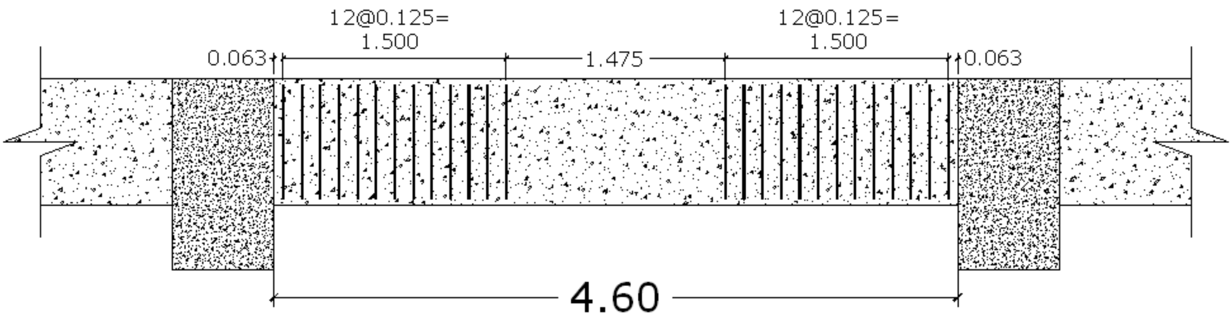
Figure 5.6-2: Floor beam for Example 5.6-3.

Solution

- Shear Reinforcement for Region 1:
  - Compute factored shear force V_u:
    As girder is deeper than floor beam, then all ACI limitations are satisfied and the shear force for Region 1 can be determined at distance "d" from face of support.
    $$V_u = 14.6\frac{\text{kN}}{\text{m}} \times (4.6\text{m} - 2 \times 0.775\text{m}) \times \frac{1}{2} + 445\ \text{kN} = 467\ \text{kN}$$
  - Shear strength of concrete V_c:
    $$V_c = 0.17\lambda\sqrt{f_c'}\,b_w d$$
    with $\lambda = 1.0$ for normal weight concrete:
    $$V_c = 0.17\sqrt{f_c'}\,b_w d = 0.17 \times \sqrt{27.5}\frac{\text{N}}{\text{mm}^2} \times 375\text{mm} \times 775\text{mm} = 259\ \text{kN}$$
  - Stirrups spacing:
    $\emptyset V_c = 0.75 \times 259\ \text{kN} = 194\ \text{kN}$
    $\because V_u = 467\ \text{kN} > \emptyset V_c = 194\ \text{kN}$
    Then, shear reinforcement must be used and its spacing can be computed from Table below:
    $A_v = 71 \times 2 = 142\ \text{mm}^2$

Shear Spacing Design of Example 5.6-3 for Region 1	
Region	$\emptyset V_c \leq V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d$ $\frac{467-194}{0.75}$? $0.66 \times \sqrt{27.5} \times 375 \times 775$ $364 < 1\,006\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{142 \times 414 \times 775}{364\,000} = 125\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ (\frac{142 \times 414}{0.062\sqrt{27.5} \times 375}\ or\ \frac{142 \times 414}{0.35 \times 375})$ $minimum\ (482\ or\ 448)$ $= 448\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'}b_w d$ $364kN \leq 0.33\sqrt{27.5} \times 375 \times 775$ $364kN\ kN \leq 503\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right]$ $Minimum\ \left[\frac{775}{2}\ or\ 600mm\right] = 387\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d$ $Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [125\ mm, 448\ mm, 387\ mm]$ $= 125\ mm$ ***Use*** $\boldsymbol{\emptyset 10mm\ @\ 125mm}$

- Shear Reinforcement for Region 2:
  - Factored shear force V_u:
    Due to symmetry
    $V_u = \left(14.6\frac{kN}{m} \times 1.53m\right) \times \frac{1}{2} = 11.1\ kN$
  - Shear strength of concrete V_c:
    According to simplified equation of the code, concrete shear force is constant along span of prismatic beam. Therefore, concrete shear strength of Region 2 would be equal to that of Region 1.
    $V_c = 259\ kN$
    $\emptyset V_c = 0.75 \times 259\ kN = 194\ kN$
    $\because \frac{\emptyset V_c}{2} = \frac{194\ kN}{2} = 97\ kN > V_u$
    Then, no shear reinforcement is required for Region 2.
- Anchorage
  As nothing is mentioned about longitudinal reinforcement, then one cannot select between closed or open stirrups.

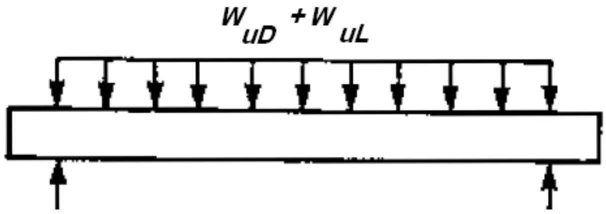


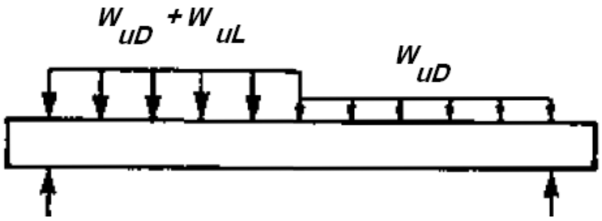
Example 5.6-4

For a simply supported beam, that has a clear span of 6m, design 10mm U stirrup at a mid-span section. In your design, assume that load pattern must be included and assume:

- $f_c' = 21\ MPa, f_{yt} = 420\ MPa$
- $h = 500, d = 450\ mm, b_w = 300mm$
- $W_{ud} = 60\frac{kN}{m}$ (Including Beam Selfweight) and $W_{ul} = 200\frac{kN}{m}$

Solution





- Compute V_u

  Although the dead load is always present over the full span, the live load may act over the full span as shown or over a part of span as shown in below.

  Based on influence line for shear at mid-span of simply supported beam, the maximum effect of live load occurs when this load acting on one half of beam span as indicated in above. Therefore, for design case when load pattern is important, shear force must be computed based on partial loading of one half of beam span:

$$V_{u\text{ @ mid span}} = 0.0_{\text{Shear due to } W_D} + \frac{W_{uL}L}{8}_{\text{Shear Due to LL on half of Beam Span}} = \frac{W_{uL}L}{8}$$

$$= \frac{200\frac{kN}{m} \times 6m}{8} = 150\ kN$$

- Compute V_c

$$V_c = 0.17\sqrt{f_c'}\,b_w d = 0.17 \times \sqrt{21}\frac{N}{mm^2} \times 300mm \times 450mm = 105\ kN \Rightarrow \emptyset V_c = 0.75 \times 105\ kN = 78.8\ kN$$

- Stirrups Design

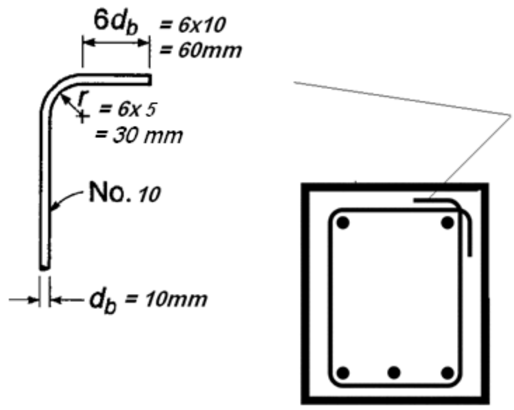
  As

  $V_u > \emptyset V_c$

  then shear stirrups is designed as presented in Table below.

Shear Spacing Design of Example 5.6-4	
Region	$\emptyset V_c \leq V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d$ $\frac{150 - 78.8}{0.75}$? $0.66 \times \sqrt{21} \times 300 \times 450 \Rightarrow 94.9\ kN < 408\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{157 \times 420 \times 450}{94.9 \times 10^3} = 313\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ (\frac{157 \times 420}{0.062\sqrt{21} \times 300}\ or\ \frac{157 \times 420}{0.35 \times 300}) \Rightarrow minimum\ (774\ or\ 628) = 628\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'}b_w d$ $94.9\ kN \leq 0.33\sqrt{21} \times 300 \times 450$ $94.9\ kN < 204\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right] \Rightarrow Minimum\ \left[\frac{450}{2}\ or\ 600mm\right] = 225\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d \Rightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [313\ mm, 628\ mm, 225\ mm] = 225\ mm$ ***Use*** $\emptyset 10mm\ @\ 225mm$

- Stirrups Details
  As movable live load is a reversal load, then closed stirrup must be used here as shown in the figure below.



Example 5.6-5

For the roof system shown in ***Figure 5.6-3*** below, design shear reinforcement for a typical floor beam and a typical girder.

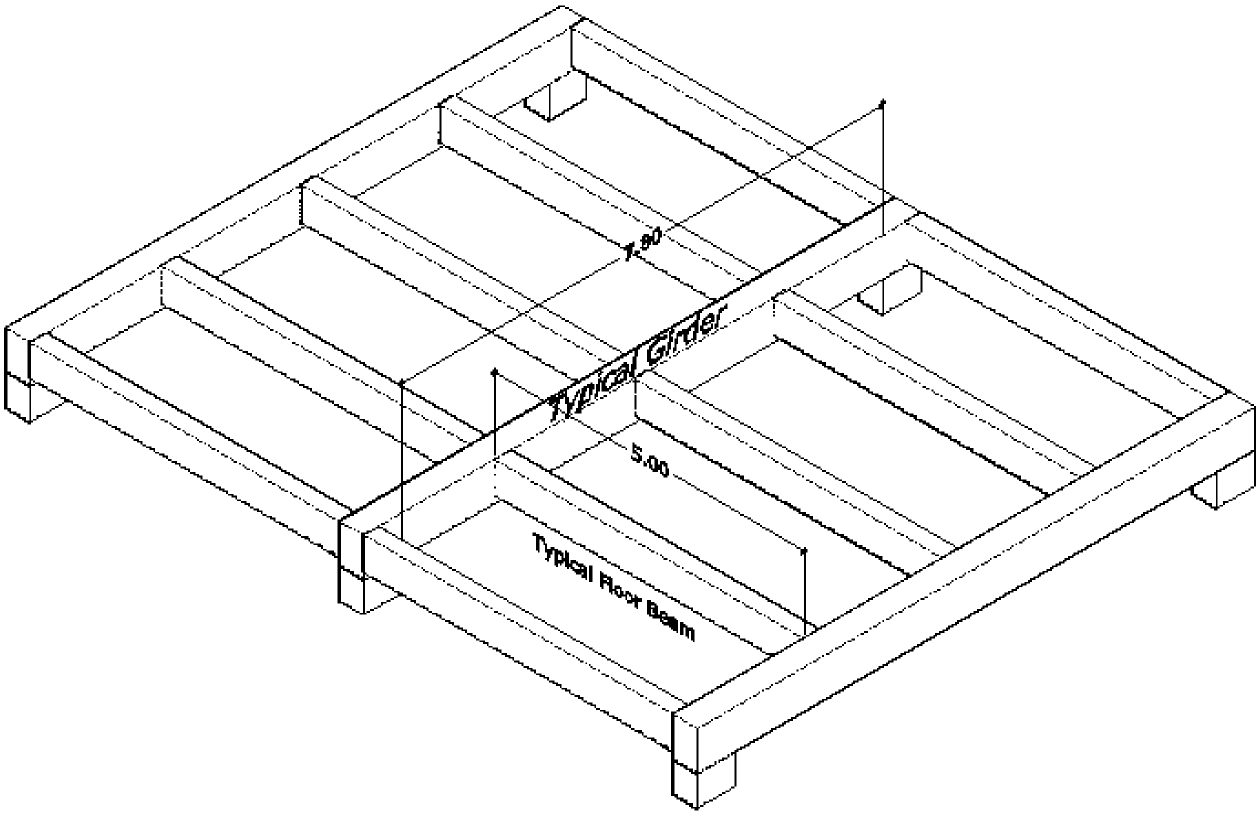


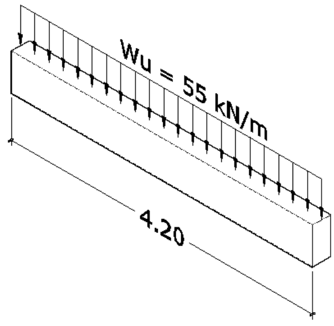
Figure 5.6-3: Roof system for Example 5.6-5.

In your design, assume that:

- $f_c' = 21\ MPa$ and $f_{yt} = 420\ MPa$.
- Floor beams have b = 250mm, h = 450mm, and d = 400mm and subjected to a uniformly distributed factored load of W_u = 55 kN/m transferred from the supported slab.
- Girders have b = 400mm, h = 600mm, and d = 520mm.
- Selfweight of floor beams and girders should be included in your design.
- Try 10mm U stirrups for the floor beam and 12mm U stirrups for the girder.

Solution

- Design Shear Reinforcement for Floor Beam:
  - Computing of Vu:
    As the girder is deeper than the floor beam, then critical section for the floor beam can be taken at distance "d" from face of support (girder in this case).



$$W_u = 55\frac{kN}{m} + \left((0.45 \times 0.25\ m^2) \times 24\frac{kN}{m^3}\right) \times 1.2 = 58\frac{kN}{m}$$

$$V_{u\ @\ d\ from\ face\ of\ support} = \left(58\frac{kN}{m} \times (5.0 - 0.4 \times 2)m\right) \times \frac{1}{2} = 122\ kN$$

- Compute V_c:

$$V_c = 0.17\sqrt{f_c'}\, b_w d = 0.17 \times \sqrt{21}\frac{N}{mm^2} \times 250mm \times 400mm = 77.9\ kN$$

$$\emptyset V_c = 0.75 \times 77.9\ kN = 58.4\ kN$$

- Design of Shear Reinforcement:

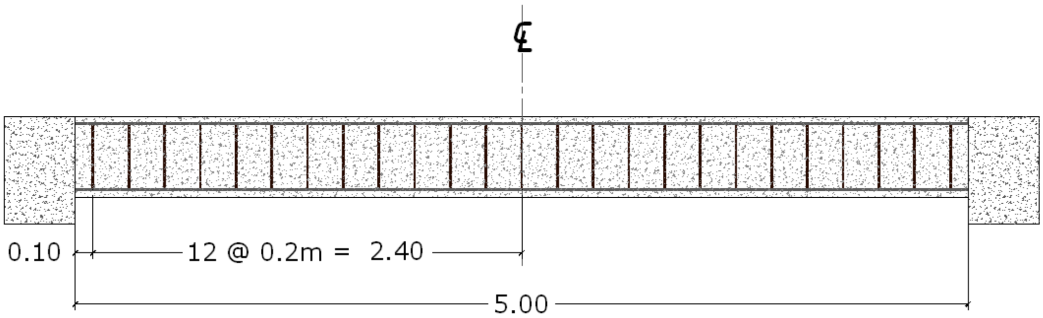
As

$V_u > \emptyset V_c$

Then shear reinforcement must be designed based on zone 1 (see the table below).

Stirrups Design of Example 5.6-5 (Floor Beam)	
Region	$\emptyset V_c \le V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d$ $\frac{122 - 58.4}{0.75}$? $0.66 \times \sqrt{21} \times 250 \times 400 \Rightarrow 84.8\ kN < 302\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{157 \times 420 \times 400}{84.8 \times 10^3} = 311\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ (\frac{157 \times 420}{0.062\sqrt{21} \times 250}\ or\ \frac{157 \times 420}{0.35 \times 250}) \Rightarrow minimum\ (928\ or\ 754)$ $= 754\ mm$
$S_{maximum}$	$V_s \le 0.33\sqrt{f_c'}b_w d$ $84.8\ kN \le 0.33\sqrt{21} \times 250 \times 400 \Rightarrow 84.8\ kN \le 151\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right] \Rightarrow Minimum\ \left[\frac{400}{2}\ or\ 600mm\right] = 200\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d$ $Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [311\ mm,\ 754\ mm,\ 200\ mm] = 200\ mm$ **Use $\emptyset 10mm\ @\ 200mm$**

- Draw of Stirrups:



- Design of Shear Reinforcement for Girder:
  - Compute of Vu:

Forces acting on the girder are summarized in the figure below. Shear force, R_u, transfers from floor beams to the supporting girder can be computed as follows:

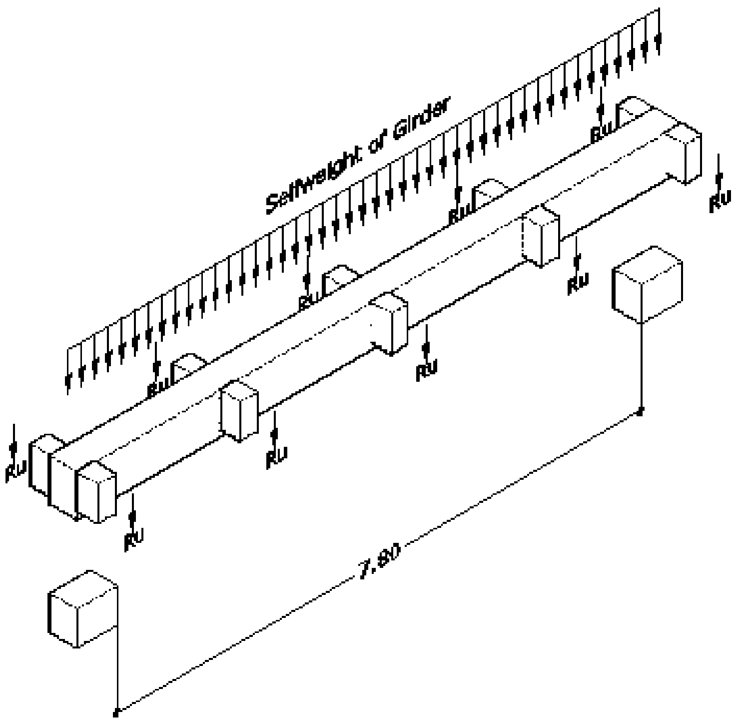
$$R_u = 58\frac{kN}{m} \times \frac{5m}{2} = 145\ kN$$

Shear force due to girder selfweight is

$$V_{u\ Due\ to\ Girder\ Selfweight} = \left((0.6 \times 0.4)m^2 \times 24\frac{kN}{m^3} \times (7.8 - 0.52 \times 2)m \times \frac{1}{2}\right) 1.2 = 23.4\ kN$$

Therefore, the total factored shear force would be:

$$V_u = \left((3 \times 145\ kN_{Reactions\ from\ 3\ floor\ beam}) \times 2_{Two\ faces}\right)\frac{1}{2} + 23.4\ kN = 458\ kN$$



- Compute V_c:

  $V_c = 0.17\sqrt{f_c'}\,b_w d = 0.17 \times \sqrt{21}\frac{N}{mm^2} \times 400mm \times 520mm = 162\ kN$

  $\emptyset V_c = 0.75 \times 162\ kN = 121\ kN$
- Design of Shear Reinforcement:

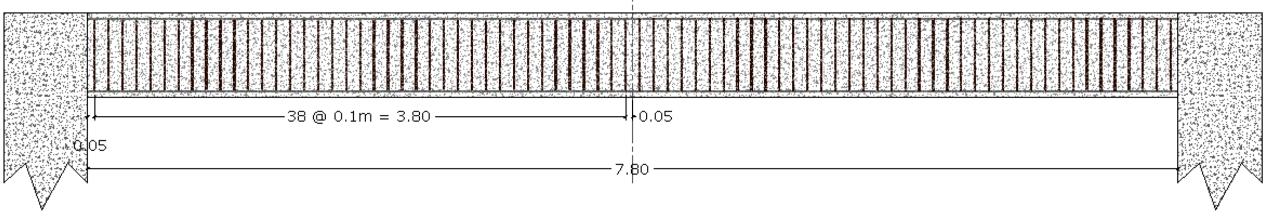
  As

  $V_u > \emptyset V_c$

  then, shear reinforcement is designed as indicated in the table below.

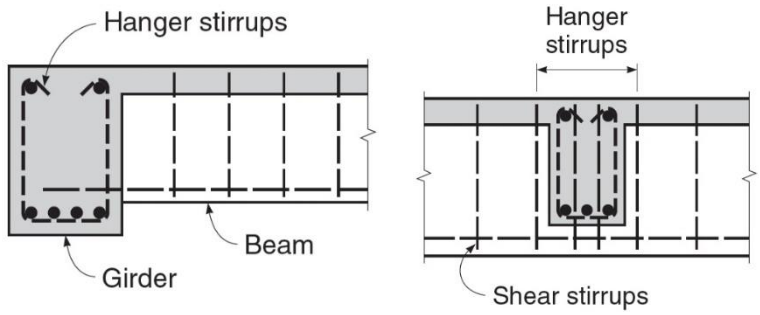
Stirrups Design of Example 5.6-5 (Girder Design)	
Region	$\emptyset V_c \le V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d \Rightarrow \frac{458-121}{0.75}\ ?\ 0.66 \times \sqrt{21} \times 400 \times 520$ $\Rightarrow 449\ kN < 629\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{226 \times 420 \times 520}{449 \times 10^3} = 110\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ (\frac{226 \times 420}{0.062\sqrt{21} \times 400}\ or\ \frac{226 \times 420}{0.35 \times 400}) \Rightarrow minimum\ (835\ or\ 678)$ $= 678\ mm$
$S_{maximum}$	$V_s \le 0.33\sqrt{f_c'}b_w d$
	$V_s > 0.33\sqrt{f_c'}b_w d$ $449\ kN > 0.33\sqrt{21} \times 400 \times 520 \Rightarrow 449\ kN > 314\ kN$ $Minimum\ \left[\frac{d}{4}\ or\ 300mm\right] \Rightarrow Minimum\ \left[\frac{520}{4}\ or\ 300mm\right] = 130\ mm$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $\Rightarrow Minimum\ [110\ mm,\ 678mm,\ 130\ mm]$ $= 110\ mm$ ***Use*** $\boldsymbol{\emptyset 12mm @ 100mm}$

- Draw stirrups for the girder.

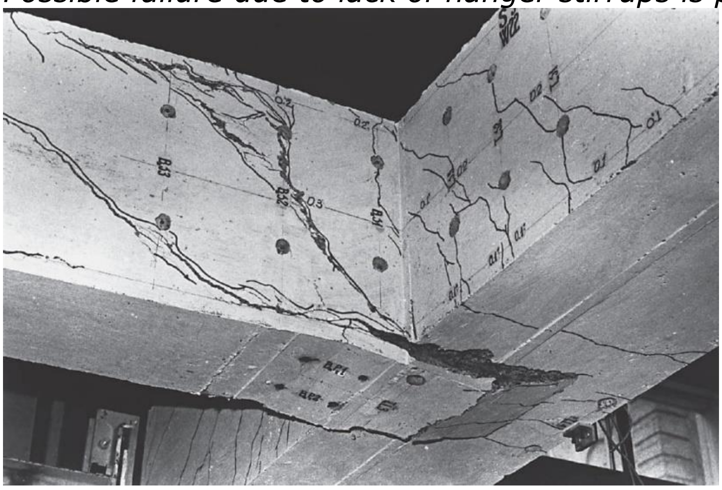


Important Notes

- *It is useful to note that shear forces in this example have been determined based on assumption of equal shear at beam-ends. More accurate assumption will be discussed later when we study the analysis and design of slabs and continuous beams.*
- *Hanger Stirrups:*
  - *Proper detailing of steel in the region of beam-to-girder connection such a joint requires the use of well-anchored "hanger" stirrups in the girder, as shown in below:*



  - *The hanger stirrups are required in addition to the normal girder stirrups.*
  - *Possible failure due to lack of hanger stirrups is presented in below:*

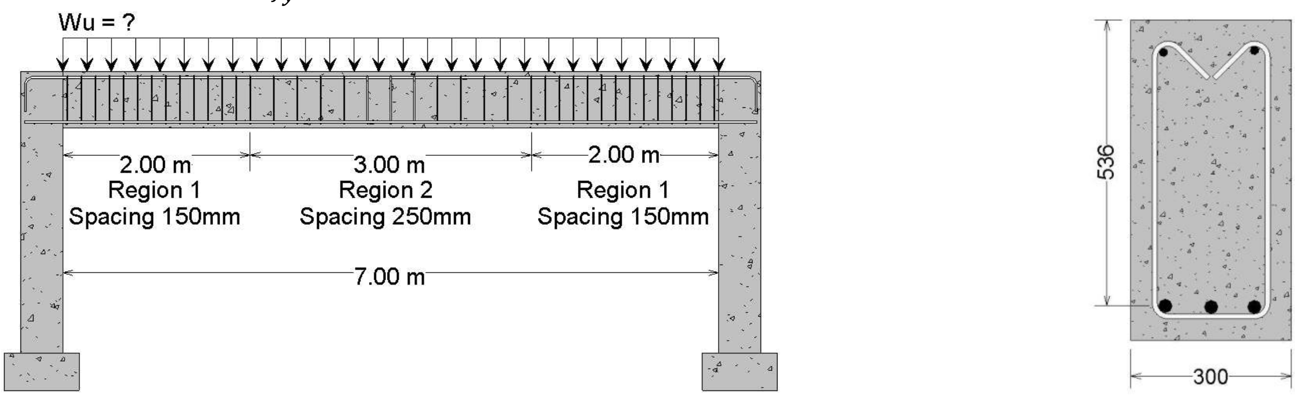


  - *Design of hanger stirrups is out of our scope, for more information about their design see* (Darwin, Dolan, & Nilson, 2016)*, page 557.*

Example 5.6-6

For the singly reinforced beam of the portal frame shown in ***Figure 5.6-4*** below, a designer has proposed to use open U stirrups with diameter of 10mm and with indicated spacing for shear reinforcement of the beam.

- Is using of open U stirrups justified according to ACI requirements? Explain your answer.
- Based on proposed spacing and beam shear strength, what is the maximum uniformly factored load W_u that could be applied? In your solution assume $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.



Longitudinal Sectional View. **Beam Cross Section**

Figure 5.6-4: Frame for Example 5.6-6.

Solution

- Using of Open U Stirrups:
  As the beam is singly reinforced and with assuming that it is not subjected to torsion nor to reversal loads, then using of open U stirrups is justified according to ACI code.
- Maximum Uniformly Distributed Load $\boldsymbol{W_u}$:
  Based on Shear Strength of Region 1:

$V_c = 0.17 \times 1.0 \times \sqrt{28} \times 300 \times 536 = 145\ kN$

$A_v = \frac{\pi \times 10^2}{4} \times 2 = 157\ mm^2$

$V_s = \frac{A_v f_y d}{s} = \frac{157 \times 420 \times 536}{150} = 236\ kN < 0.33 \times 1.0 \times \sqrt{28} \times 300 \times 536 = 281\ kN$

$S_{maximum} = minimum\left(\frac{536}{2}, 600\right) = 268\ mm > 150mm \therefore Ok.$

$\phi V_n = 0.75 \times (145 + 236) = 286\ kN$

$V_u = \frac{W_u \times (7.0 - 2 \times 0.536)}{2} = 286 \Rightarrow W_u = 96.5 \frac{kN}{m}$

Based on Shear Strength of Region 2:

$V_s = \frac{A_v f_y d}{s} = \frac{157 \times 420 \times 536}{250} = 141\ kN$

$S_{maximum} = 268\ mm > 250mm \therefore Ok.$

$\phi V_n = 0.75 \times (145 + 141) = 214\ kN$

$V_u = \frac{W_u \times 3}{2} = 214$

$W_u = 143 \frac{kN}{m}$

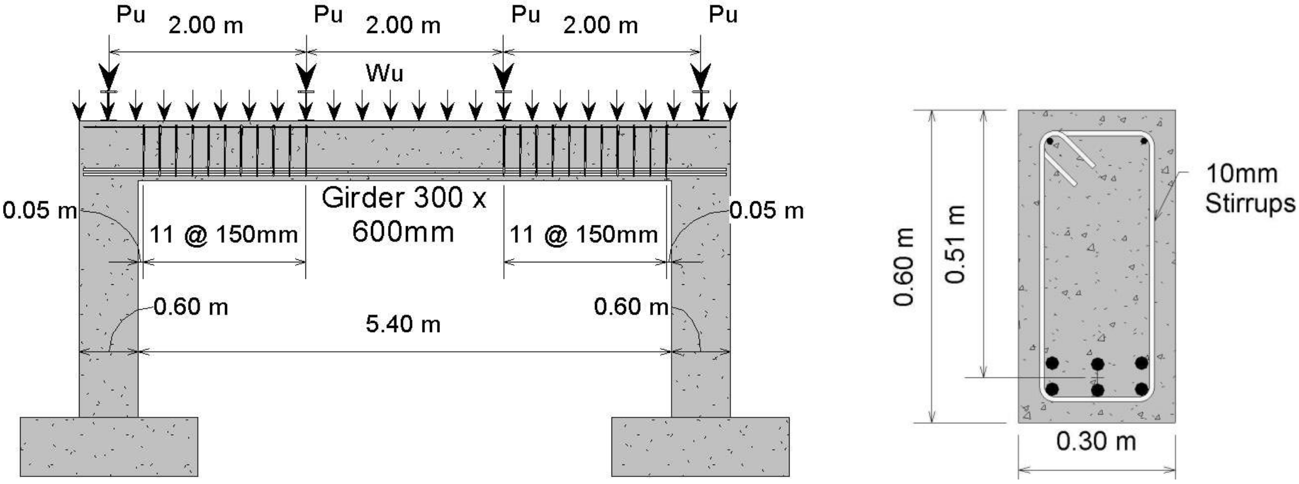
Finally,

$W_u = minimum\ (96.5, 143) = 96.5 \frac{kN}{m}$ ■

Example 5.6-7

For a frame shown in ***Figure 5.6-5*** below, based on shear capacity of Girder 300x600, what are maximum values for point load "P_u", and distributed load "W_u" that can be supported by the beam?

In your solution, assume that selfweight could be neglected, $f'_c = 28\ MPa$ and $f_y = 420\ MPa$.



Longitudinal Sectional View. **Girder Section.**

Figure 5.6-5: Frame for Example 5.6-7.

Solution

Distributed load "W_u" could be computed from middle region where no shear reinforcement are used:

$V_u = \frac{\phi V_c}{2} = \frac{1}{2} \times \left(0.75 \times 0.17 \times \sqrt{28} \times 300 \times 510\right) \Rightarrow V_u = \frac{\phi V_c}{2} = 51.6\ kN$

$V_u = \frac{W_u \times 2.00}{2} = 51.6\ kN \Rightarrow W_u = 51.6\ kN$ ■

Point load "Pu" could be computed from support regions where stirrups of $\phi 10 \ @ \ 150mm$ are used.

$$V_s = 2 \times \frac{\pi \times 10^2}{4} \times 420 \times \frac{510}{150} = 224\ kN \Rightarrow \mathrm{V_s} = 224 < \ 0.66\sqrt{28} \times 300 \times 510 = 534\ kN \therefore Ok.$$

$$\because \mathrm{V_s} = 224 < \ 0.33\sqrt{28} \times 300 \times 510 = 267\ kN \Rightarrow \mathrm{S} = 150\mathrm{mm} < \mathrm{Minimum}\ \left[\frac{510}{2} \mathrm{or}\ 600\right]$$

$$\mathrm{S} = 150\mathrm{mm} < 255\ \mathrm{mm} \therefore Ok.$$

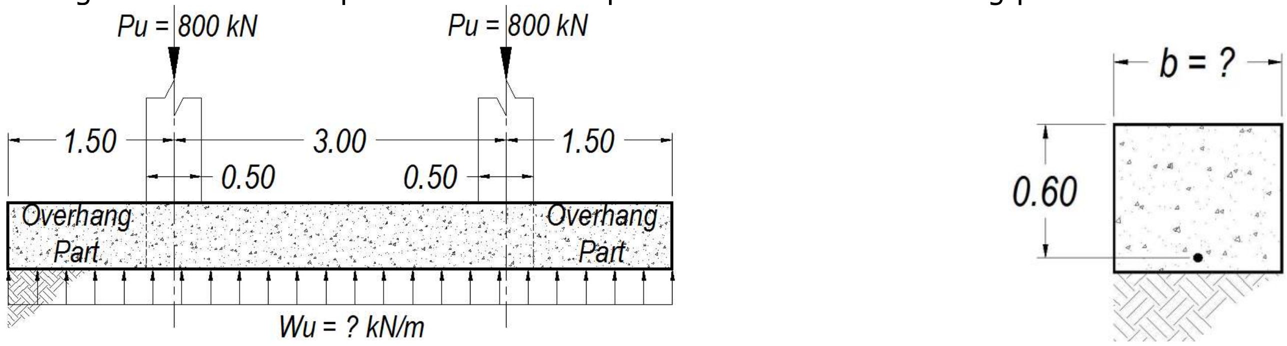
$$V_c = \ 0.17 \times \sqrt{28} \times 300 \times 510 = 138\ kN \Rightarrow V_u = 0.75 \times (138 + 224) = 272\ kN$$

$$(W_u \times (5.4 - 0.51 \times 2) + 2P_u) = \ 2 \times 272 \Rightarrow (51.6\ \times (5.4 - 0.51 \times 2) + 2P_u) = \ 2 \times 272$$

$$\Rightarrow P_u = 159\ kN\ \blacksquare$$

Example 5.6-8

For beam shown in ***Figure 5.6-6*** below, select beam width such that concrete shear strength would be adequate for shear requirements in the overhang parts.



Logitudinal veiw **A Section in Overhang Region**

Figure 5.6-6: Foundation for Example 5.6-8.

In your solution, assume that:

- Beam selfweight can be neglected.
- $f_c' = 21\ MPa$

Solution

$$W_u = \frac{800 \times 2}{6} = 267\frac{kN}{m} \Rightarrow V_{u\ @\ d\ fram\ face\ of\ support} = 267\frac{kN}{m}(1.5 - 0.25 - 0.6)m = 174\ kN$$

$$V_u = \frac{\phi V_c}{2} \Rightarrow 174000\ N = \frac{1}{2}\left(0.75\left(0.17\sqrt{21} \times b \times 600\right)\right) \Rightarrow b = 993\ mm$$

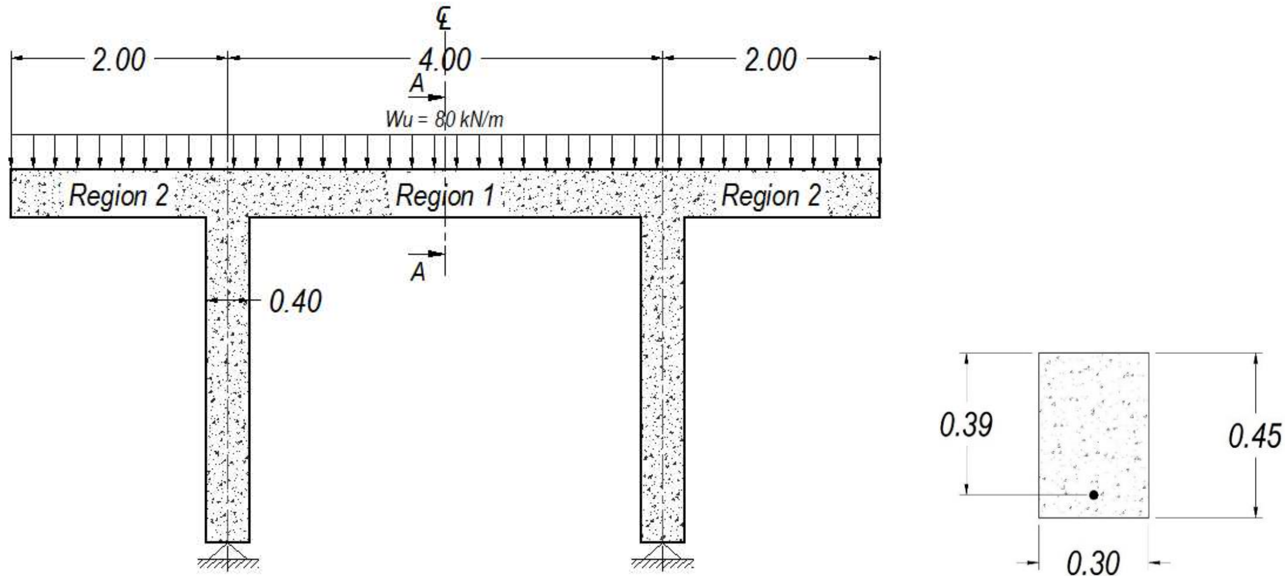
Say

$b = 1000\ mm$ ■

Example 5.6-9

For the frame shown in Figure 5.6-7 below,

- Design Region 1 for shear according ACI requirements.
- Is shear reinforcement for Region 1 adequate for Region 2?



Elevation view. **Section A-A**

Figure 5.6-7: Frame for Example 5.6-9.

In your solution, assume that:

- $f_c' = 21\ MPa,\ f_y = 420\ MPa$
- No.10 for stirrups.

Solution

Region 1:

$V_{u\ for\ Region\ 1} = 80\frac{kN}{m}(4 - 0.4 - 2 \times 0.39)m \times \frac{1}{2} = 113\ kN$

$\phi V_c = 0.75 \times 0.17 \times \sqrt{21} \times 300 \times 390 = 68.4\ kN < V_u$

Shear Spacing Design of Region 1	
Region	$\emptyset V_c \leq V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d \Rightarrow \frac{113 - 68.4}{0.75}\ ?\ 0.66 \times \sqrt{21} \times 300 \times 390$ $59.5kN < 354\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{157 \times 420 \times 390}{59.5 \times 10^3} = 432\ mm$
S for Av minimum	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w}) \Rightarrow minimum\ (\frac{157 \times 420}{0.062\sqrt{21} \times 300}\ or\ \frac{157 \times 420}{0.35 \times 300})$ $minimum\ (773\ or\ 628) = 628\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'}b_w d \Rightarrow 59.5kN \leq 0.33\sqrt{21} \times 300 \times 390 \Rightarrow 59.5kN \leq 177\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right] \Rightarrow Minimum\ \left[\frac{390}{2}\ or\ 600mm\right] = 195\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d \Rightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [432\ mm, 628\ mm, 195\ mm] = 195\ mm$ **Use $\emptyset 10mm\ @\ 175mm$**

Region 2:

$V_{u\ for\ Region\ 2} = 80\frac{kN}{m}\left(2.0 - \frac{0.4}{2} - 0.39\right)m = 113\ kN$

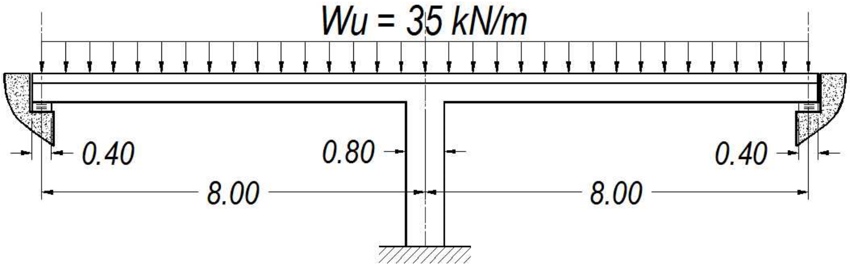
$\because\ V_{u\ for\ Region\ 2} = V_{u\ for\ Region\ 1}$

Therefore, the shear reinforcement for Region 1 is adequate for Region 2.

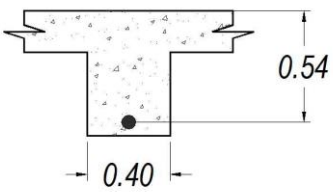
Example 5.6-10

Design for shear the most critical region of pedestrian bridge shown in Figure 5.6-8 below. In your solution, assume that:

- Shear force at interior support to be increased by 15%.
- Beam selfweight could be neglected.
- U stirrups with 10mm diameter.
- $f_c' = 28\ MPa\ f_{yt} = 420\ MPa$



Longitudinal Section



Typical Cross Section

Figure 5.6-8: Pedestrian bridge for Example 5.6-10.

Solution

- Design Shear Force:
  As will be discussed in design of ***one-way slabs*** and ***continuous beams***, according to ACI code, the most critical shear for continuous beams occurs at the exterior face of first interior support with a shear force of 15% greater than average shear force for simple beams.

$$V_{u\ @\ face\ of\ support} = 1.15\frac{W_u l_n}{2}, l_n = 8.0 - \frac{0.8}{2} - \frac{0.4}{2} = 7.4\ m$$

$$V_{u\ @\ face\ of\ support} = 1.15\frac{\left(35\frac{kN}{m} \times 7.4m\right)}{2} = 149\ kN$$

As all related conditions are satisfied, then shear at distance "d" could be used in beam design.

$$V_{u\ @\ distance\ d\ from\ face\ of\ support} = 149\ kN - 35\frac{kN}{m} \times 0.54m = 130\ kN$$

- Concrete Shear Strength:

$$\phi V_c = \phi(0.17\lambda\sqrt{f_c'}\, b_w d) = \phi(0.17 \times 1 \times \sqrt{28} \times 400 \times 540) = 146\ \text{kN}$$

$$\because \emptyset\frac{V_c}{2} < V_u \leq \emptyset V_c$$

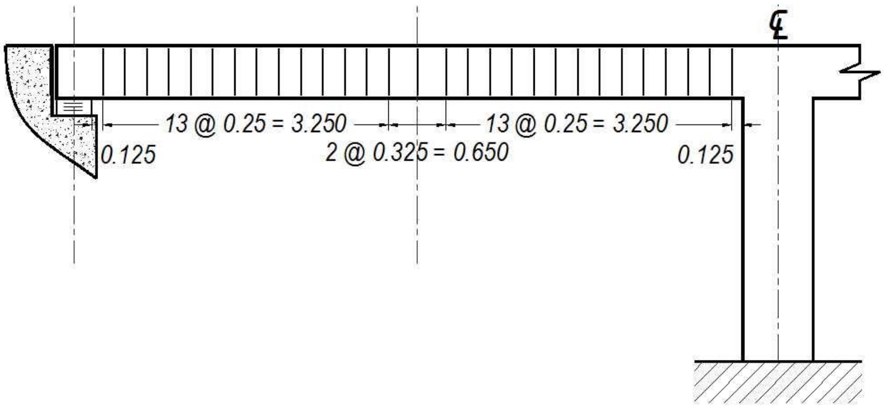
then only nominal shear reinforcement is required.

- Required Shear Reinforcement:

$$A_v = \frac{\pi \times 10^2}{4} \times 2 = 157\ mm^2$$

Shear reinforcement for Example 5.6-10	
Region	$\emptyset\frac{V_c}{2} < V_u \leq \emptyset V_c$
V_s	None
$S_{Theoretical}$	None
$S_{for\ Av\ minimum}$	$\text{minimum}\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ \text{or}\ \frac{A_v f_{yt}}{0.35 b_w})$ $\text{minimum}\ (\frac{157 \times 420}{0.062 \times \sqrt{28} \times 400}, \frac{157 \times 420}{0.35 \times 400}) \Rightarrow \text{minimum}\ (502, 471) = 471\ mm$
$S_{maximum}$	$\text{Minimum}\ \left[\frac{d}{2}\ \text{or}\ 600\text{mm}\right] = 270$
$S_{Required}$	$\text{Minimum}\ \begin{bmatrix}471,\\270\end{bmatrix} = 270\ mm$ ***Use U Stirrups ϕ10mm @ 250mm***

- Reinforcement Drawings:



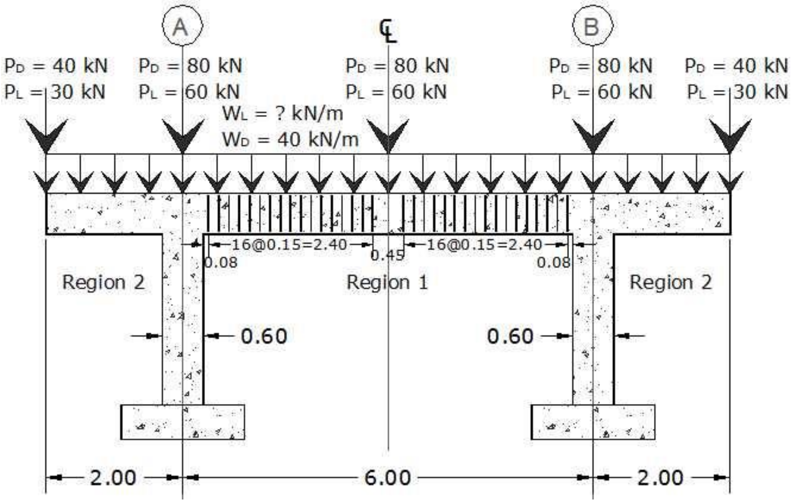
Example 5.6-11

For the frame that shown in Figure 5.6-9 below.

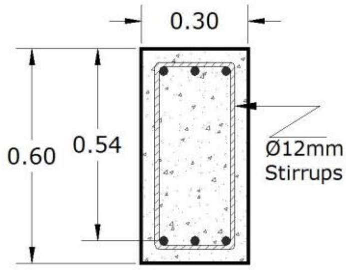
- Based on shear reinforcement that proposed for Region 1, what is maximum uniform distributed live load "W_L" that could be supported?
- Is shear reinforcement that proposed for Region 1 adequate when used in Region 2?

In your solution, assume that:

- U stirrups with 12mm diameter.
- $f_c' = 28\ MPa\ f_{yt} = 420\ MPa$
- W_u = 1.2D + 1.6L



Longitudinal Section



Typical Cross Section.

Figure 5.6-9: Frame for Example 5.6-11.

Solution

- Based on shear reinforcement that proposed for Region 1, the maximum uniform distributed live load "W_L" that could be supported would be:

$$A_v = \frac{\pi \times 12^2}{4} \times 2 = 226\ mm^2 \Longrightarrow V_s = \frac{A_v f_{yt} d}{s} = \frac{226 \times 420 \times 540}{150} = 342\ kN$$

$$V_c = \left(0.17\lambda\sqrt{f_c'}\, b_w d\right) = 0.17 \times \sqrt{28} \times 300 \times 540 = 146\ kN \Longrightarrow \phi V_n = \phi(V_c + V_s)$$

$$= 0.75 \times (146 + 342) = 366\ kN$$

$$V_{u\,@\,face\ of\ support} = \phi V_n = 366\ kN$$

$$P_u = 1.2 \times 80 + 1.6 \times 60 = 192\ kN$$

$$(W_u \times (6.0 - 0.6 - 0.54 \times 2) + 192) \times \frac{1}{2} = 366 \Longrightarrow W_u = 125 \frac{kN}{m}$$

$$W_D = 40 + (0.6 \times 0.3 \times 24) = 44.3\ kN$$

$$W_u = 125 = 1.2 \times 44.3 + 1.6 \times W_L \Longrightarrow W_l = 44.9\ kN\ ■$$

- Check if the shear reinforcement that proposed for Region 1 is adequate when used in Region 2?

$$P_u = 1.2 \times 40 + 1.6 \times 30 = 96.0\ kN$$

$$V_{u\ at\ d} = 125 \times \left(2.0 - \frac{0.6}{2} - 0.54\right) + 96.0 \Longrightarrow V_{u\ at\ d} = 241\ kN < \phi V_n\ \therefore Ok.\ ■$$

5.7 PROBLEMS FOR SOLUTION ON BASIC SHEAR ASPECTS

Problem 5.7-1

A reinforced concrete beam with a rectangular cross section is reinforced for moment only and subjected to a shear V_u of 40.0 kN. Beam width b=300mm and effective depth d=184mm, $f_c' = 21\text{MPa}$ and $f_y = 414\text{MPa}$. Is beam satisfactory for shear?

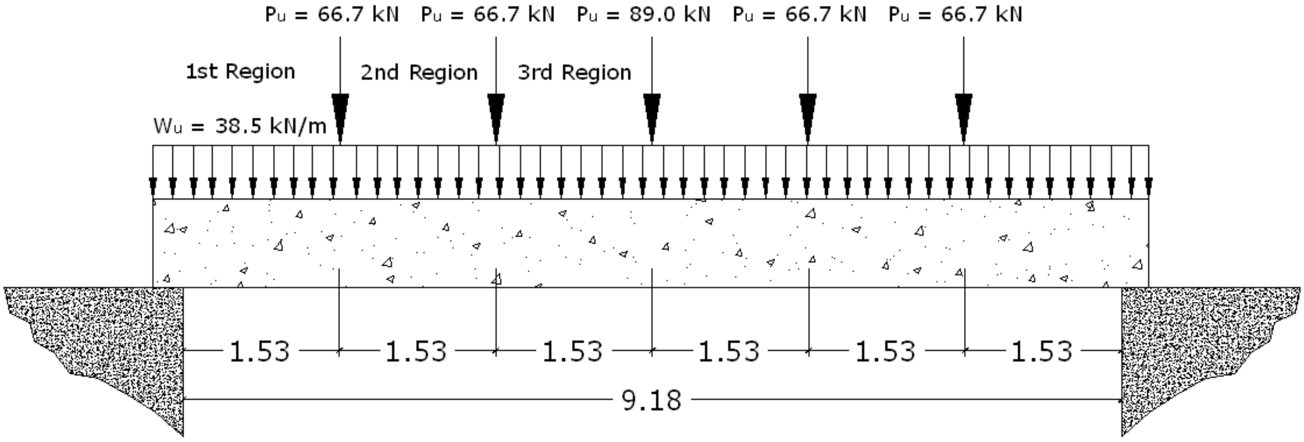
Answers

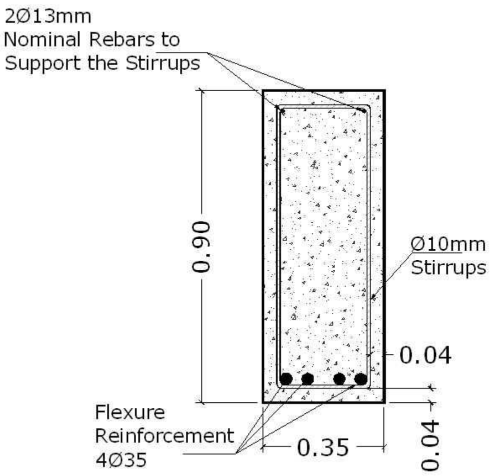
$V_c = 43.0, \frac{1}{2}\emptyset V_c \,?\, V_u, \quad \because \frac{1}{2}\emptyset V_c < V_u$

Then shear reinforcement is required for this beam. As no shear reinforcement is provided, then the beam is inadequate for shear.

Problem 5.7-2

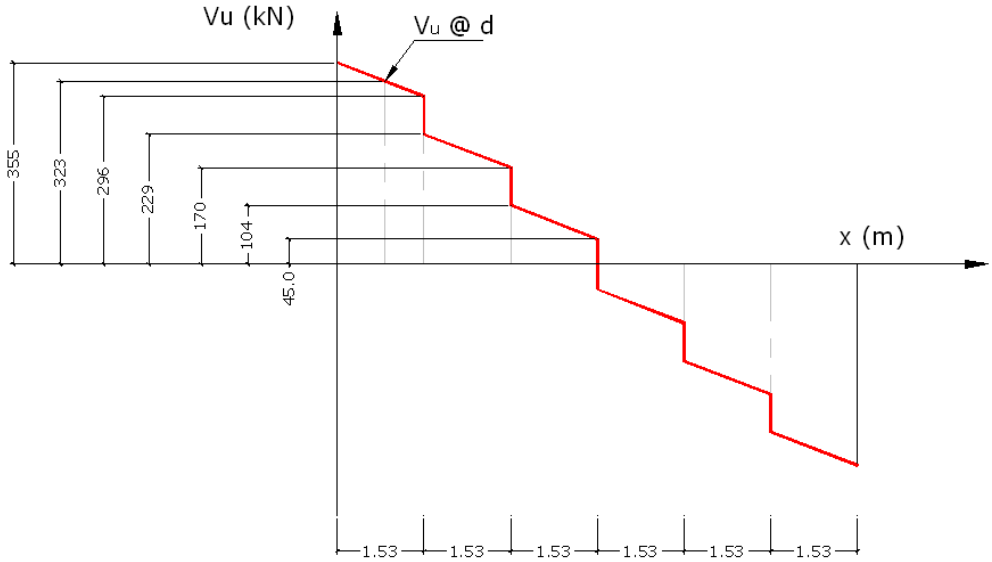
For beam shown below, design single-loop stirrups. The loads shown are factored loads. Use $f_c' = 21\text{MPa}$ and $f_y = 414\text{MPa}$. The uniformly load includes the beam selfweight.





Answers

Draw the shear force diagram:



$d = 833\ mm$

Shear Design for 1st Region:

$V_{u\ @\ d\ from\ Face\ of\ Support} = 323\ kN$

$\emptyset V_c = 170\ kN$

$A_v = 157\ mm^2$

Stirrups Design of Problem 5.7-2 (Region 1)	
Region	$\emptyset V_c \leq V_u$
V_s	$\frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d \Rightarrow 204kN < 882\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = 266\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w}) \Rightarrow minimum\ (654\ or\ 531) = 531\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'}b_w d \Rightarrow 204\ kN \leq 441\ kN$ $Minimum\ \left[\frac{833}{2}\ or\ 600mm\right] = 416\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d \Rightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}] \Rightarrow Minimum\ [266mm, 531\ mm, 416\ mm]$ $= 266\ mm$ Use $\emptyset 10mm$ @ $250mm$

Shear Design for 2nd Region:

Stirrups Design of Problem 5.7-2 (Region 2)	
Region	$\emptyset V_c \leq V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d \Rightarrow 78.7kN < 882\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = 688\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w}) \Rightarrow minimum\ (654\ or\ 531) = 531\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'}b_w d \Rightarrow 204\ kN \leq 441\ kN \Rightarrow Minimum\ \left[\frac{833}{2}\ or\ 600mm\right] = 416\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d \Rightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [688\ mm, 531\ mm, 416\ mm] = 416mm$ Use $\emptyset 10mm$ @ $400mm$

Shear Design for 3rd Region:

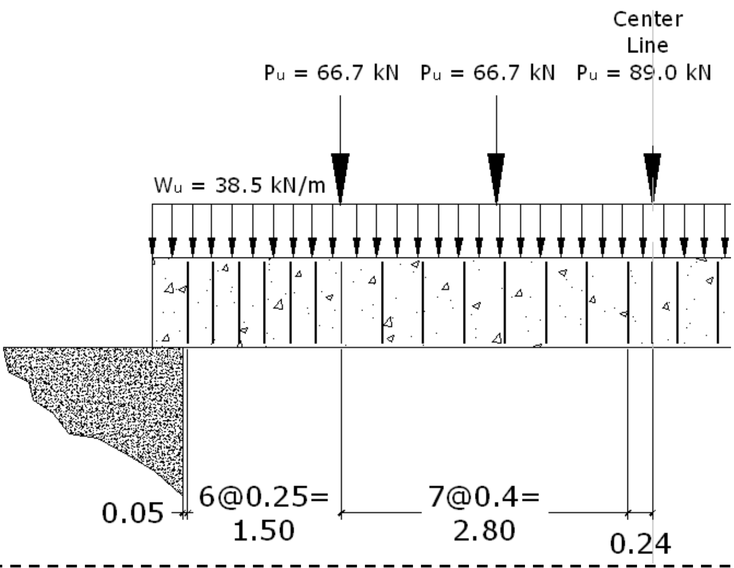
$\because V_u = 104\ kN < \emptyset V_c = 170\ kN$

Then, only nominal requirement is required for 2nd Region:

$S_{Required} = Minimum\ [531\ mm, 416\ mm]$

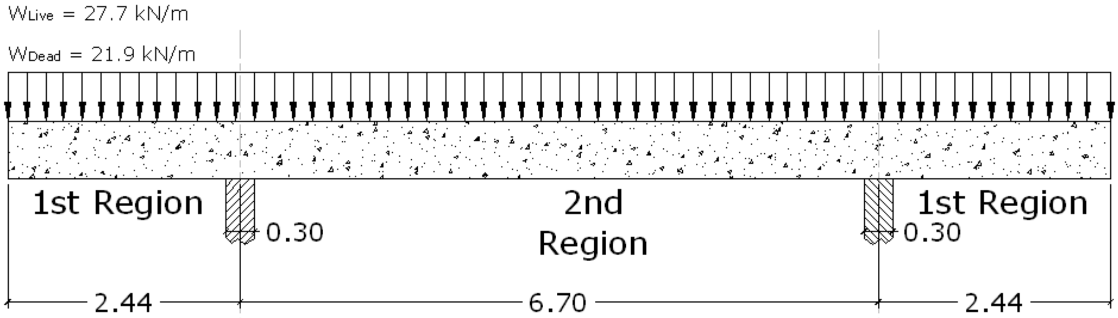
$S_{Required} = 416mm$

$\Rightarrow$ Use $\emptyset 10mm$ @ $400mm$



Problem 5.7-3

Design stirrups for the beam shown. Service loads are 21.9 kN/m dead load (including beam selfweight) and 27.7 kN/m live load. Beam width "b" is 325mm and effective depth "d" is 600mm for both top and bottom reinforcement. Use $f_c' = 21\text{MPa}$ and $f_y = 414\text{MPa}$. Use 10mm U Stirrups.



Answers

Computed the factored load:

$W_u = \text{maximum of } [1.4 \text{ Dead or } 1.2 \text{ Dead } + 1.6 \text{ Live}]$

$W_u = \text{maximum of } \left[31.0\frac{\text{kN}}{\text{m}} \text{ or } 70.6\frac{\text{kN}}{\text{m}}\right] = 70.6\frac{\text{kN}}{\text{m}}$

Shear Design for Region 1:

$V_{u\,@\,d} = 70.6(2.44 - 0.15 - 0.6) = 119\ kN$

$\text{Ø}V_c = 0.75 \times 152 \text{ kN} = 114 \text{ kN}$

$A_v = 157 \text{ mm}^2$

Summary of stirrups design for this region is given in Table below.

Shear Design for Region 2:

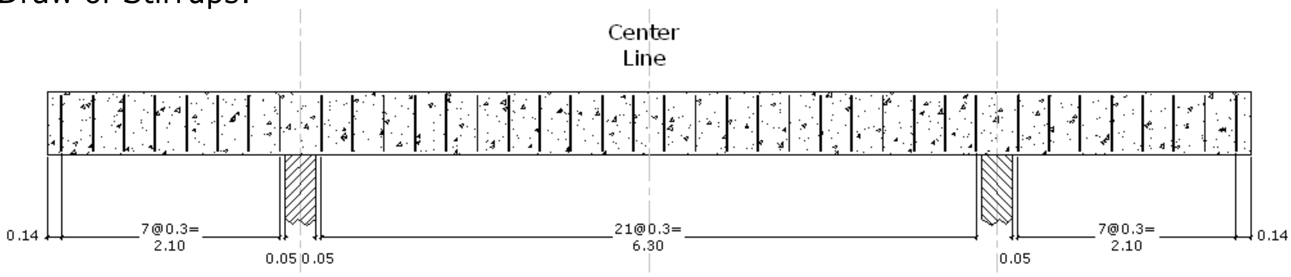
$V_{u\,@\,d} = 70.6\ (6.7 - 0.15 \times 2 - 0.6 \times 2) \times \frac{1}{2} = 184\ kN$

$\text{Ø}V_c = 114 \text{ kN}$

$A_v = 157 \text{ mm}^2$

Summary of stirrups design for this region is given in the table below.

Draw of Stirrups:



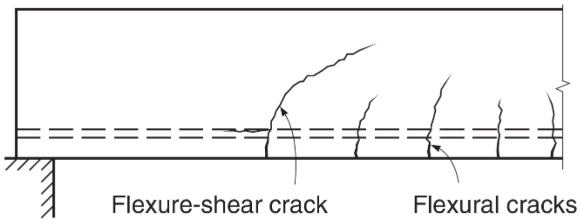
Stirrups Design of (Region 1)	
Region	$\text{Ø}V_c \le V_u$
V_s	$= \frac{V_u - \text{Ø}V_c}{\text{Ø}} = 0.66\sqrt{f_c'}b_w d$ $6.67kN < 590\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = 5847\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w}) \Rightarrow minimum\ (704\ or\ 571) = 571\ mm$
$S_{maximum}$	$V_s \le 0.33\sqrt{f_c'}b_w d \Rightarrow 6.67kN \le 295\ kN \Rightarrow Minimum\ \left[\frac{d}{2}\ or\ 600mm\right]$ $Minimum\ \left[\frac{600}{2}\ or\ 600mm\right] = 300\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d \Rightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}] \Rightarrow Minimum\ [5847\ mm, 571\ mm, 300\ mm]$ $= 300\ mm$ **Use $\text{Ø}10mm\ @\ 300\ mm$**

Stirrups Design of (Region 2)	
Region	$\emptyset V_c \leq V_u$
V_s	$\frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c{}'}b_w d \Longrightarrow 93.3kN < 590\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = 418\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c{}'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w}) \Longrightarrow minimum\ (704\ or\ 571) = 571\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c{}'}b_w d \Longrightarrow 93.3kN \leq 295\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right] \Longrightarrow Minimum\ \left[\frac{600}{2}\ or\ 600mm\right] = 300\ mm$
	$V_s > 0.33\sqrt{f_c{}'}b_w d \Longrightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [418\ mm, 571\ mm, 300\ mm] = 300mm$ Use $\emptyset 10mm\ @\ 300\ mm$

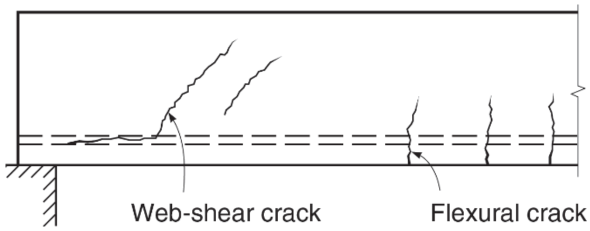
5.8 *SHEAR DESIGN BASED ON THE MORE DETAILED RELATION FOR V_c

5.8.1 Basic Concepts

- As discussed in Article 5.3 above, there are two types of shear or diagonal tension cracks:
  - First Type (Flexure-shear Crack): Shear cracks of this type occur after formation of flexural cracks and growth form the end of flexural cracks.



  - Second Type (Web-shear crack): Shear cracks of this type occur in region with small bending moments and form mainly due to applied shear force.



- For flexure-shear cracks, value of V_c represents shear force that is required to expand preexisting flexural cracks. While for web-shear crack, V_c represents shear force required to initiate web cracks and has a value greater than that required for expands preexisting flexural cracks.
- Factors Affecting V_c:
  - Based on above definition, it is evident that the shear value at which diagonal cracks developed or/and propagate depends on the ratio of shear force to bending moment. This ratio can be expressed in terms of $V_u d/Mu$.
  - It can also be shown that increasing values of reinforcing ratio ρ_w have a beneficial effect in that they increase the shear at which diagonal cracks develop. This is so because larger amount of longitudinal steel results in smaller and narrower flexural tension cracks prior to the formation of diagonal cracks, leaving a larger area of uncracked concrete available to resist shear.
- Based on above reasoning, ACI offers Table 5.8-1 below to simulate the effects of $V_u d/Mu$ and ρ_w on concrete cracking shear strength.
- Expression (b) in Table 5.8-1 limits V_c near points of inflection.

Table 5.8-1: Detailed method for calculating V_c, Table 22.5.5.1 of the code.

V_c		
Least of (a), (b), and (c):	$\left(0.16\lambda\sqrt{f_c'}+17\rho_w\frac{V_u d}{M_u}\right)b_w d$ [1]	(a)
	$(0.16\lambda\sqrt{f_c'}+17\rho_w)b_w d$	(b)
	$0.29\lambda\sqrt{f_c'}b_w d$	(c)

[1] M_u occurs simultaneously with V_u at the section considered.

5.8.2 Detailed versus Simplified Relations for V_c

- In simplified equation of Article 5.3, the second term in expressions (a) and (b) of ***Table 5.8-1*** have been assumed equals $0.01\lambda\sqrt{f_c}$ and use V_c equal to $V_c = 0.17\lambda\sqrt{f_c'}\ b_w d$
  This simplified relation has been used in solutions of previous examples and problems.
- It is useful to note that the simplified equation has been derived based assumption of low $\frac{(V_u d)}{M_u}$ and low ρ_w that lead to a second term that has a small value of $(0.01\lambda\sqrt{f_c'})$. Therefore, it gives an accurate estimation of V_c in regions with large moment but gives an underestimation (conservative value) in regions with small moment.

5.8.3 Which Relation Should be Adopted

Use of more detailed or simplified ACI relations can be summarized with refers to Figure 5.8-1, Figure 5.8-2, and Figure 5.8-3 below.

- Simple Span with Uniformly Distributed Load:

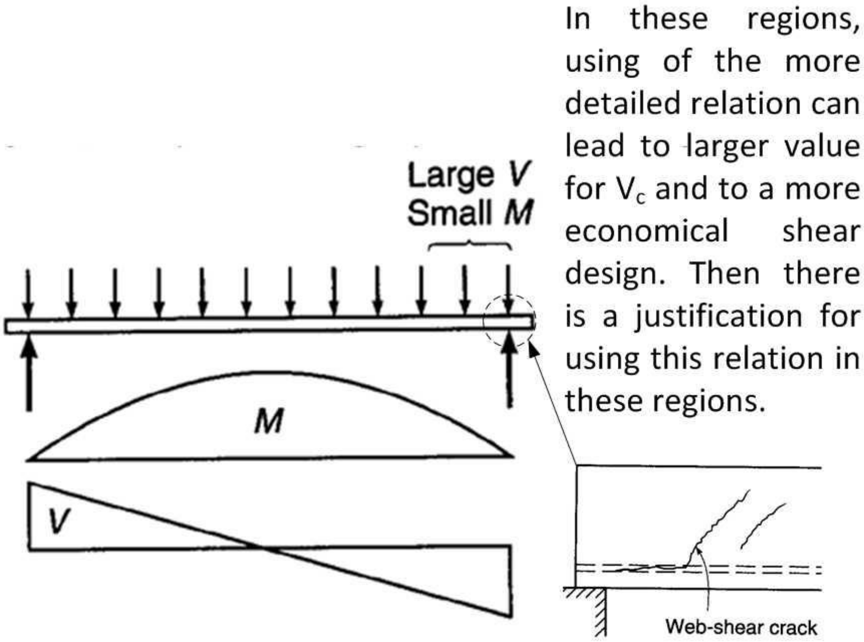


Figure 5.8-1: Simple span with uniformly distributed loads.

- Simple Span with a Concentrated Load at Mid-span:

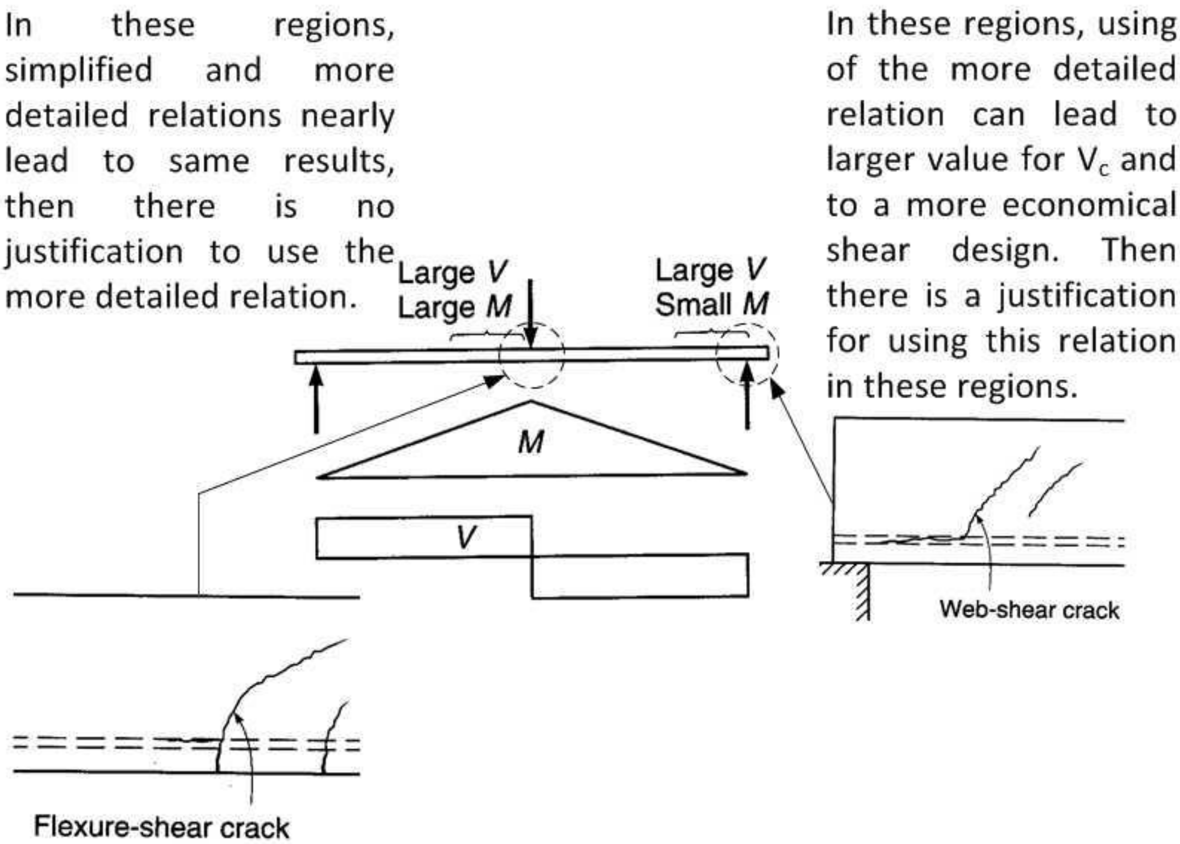


Figure 5.8-2: Simple span with a point load.

- Continuous Span with Uniformly Distributed:

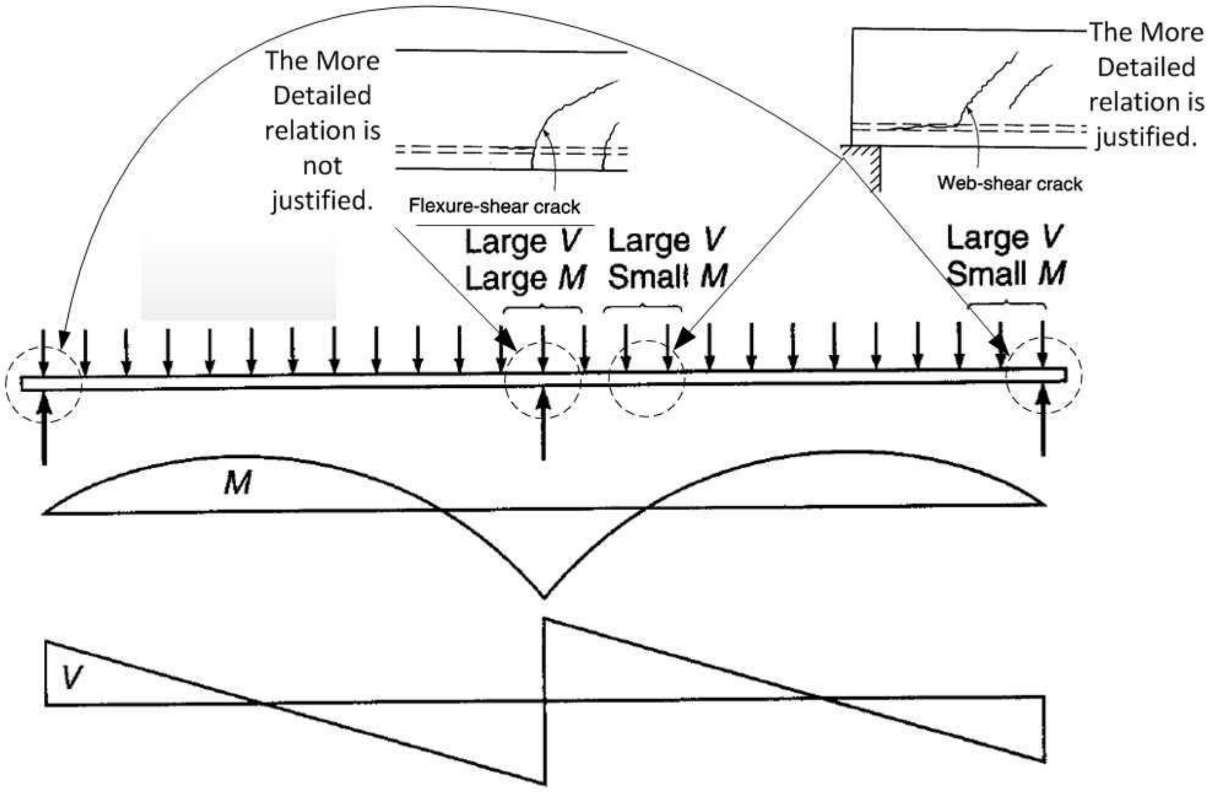


Figure 5.8-3: Continuous span with uniformly distributed loads.

5.8.4 Examples

Example 5.8-1

Based on a statically indeterminate analysis, shear force and bending moment have been computed and drawn for the continuous beam shown in ***Figure 5.8-4*** below. For this beam, compute V_c based on simplified relation and more detailed relation at exterior and interior supports. Assume that $f_c' = 21\ MPa$ and $f_y = f_{yt} = 420\ MPa$.

Based on flexural design following values have been determined:

b = 300mm, d= 535mm, h = 600mm.

$\rho_{-ve} = 19.4 \times 10^{-3}, \rho_{+ve} = 10.6 \times 10^{-3}$

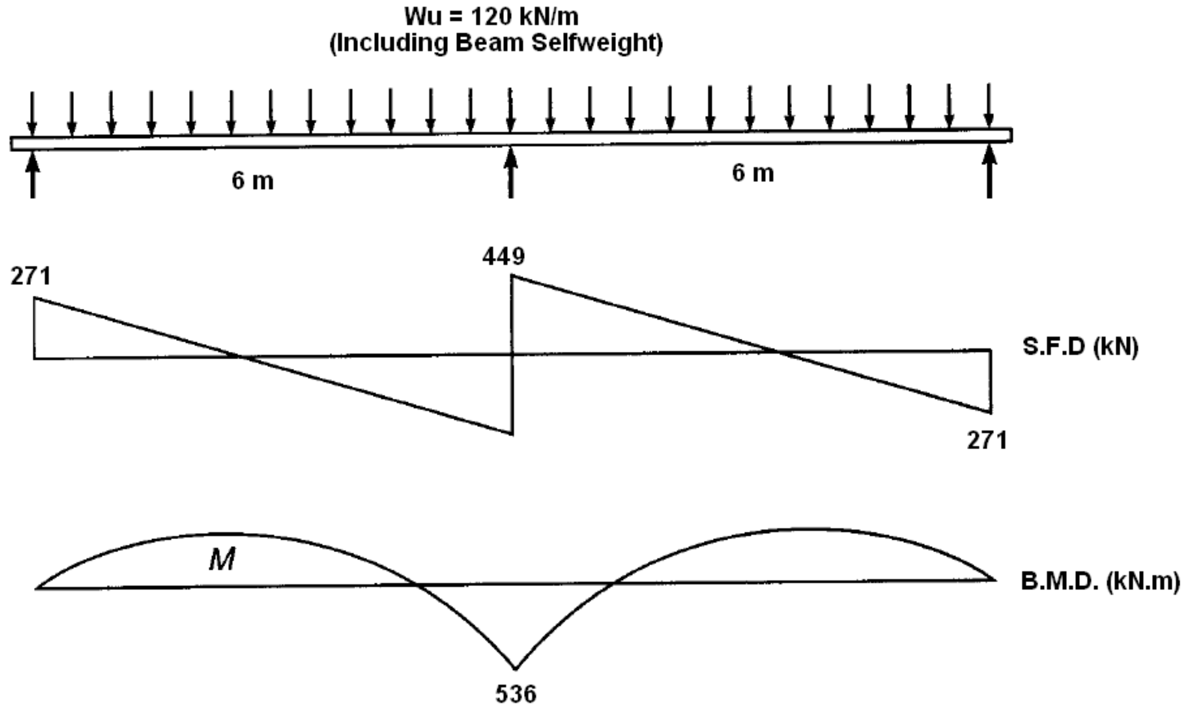


Figure 5.8-4: Continuous beam for Example 5.8-1 with its shear force and bending moment diagrams.

Solution

At Exterior Support

Based on Simplified Relation:

$V_c = 0.17\lambda\sqrt{f_c'}\, b_w d \Rightarrow V_c = 0.17\sqrt{21} \times 300 \times 535 = 0.779\, MPa \times 300 \times 535 = 125\, kN$

Based on the more detailed relation:

$$V_c = \left(0.16\lambda\sqrt{f_c'} + 17\rho_w \frac{(V_u d)}{M_u}\right) b_w d \le 0.29\lambda\sqrt{f_c'} b_w d$$

For inflection points that have zero moment, $\frac{(V_u d)}{M_u}$ taken equal to 1.0, then:

$V_c = \left(0.16\sqrt{21} + 17 \times (10.6 \times 10^{-3}) \times 1.0\right)300 \times 535\ ?\ 0.29\lambda\sqrt{f_c'} b_w d$

$V_c = (0.733\, MPa + 0.180\, MPa)300 \times 535\ ?\ 0.29\lambda\sqrt{f_c'} b_w d$

$V_c = (0.913\, MPa)300 \times 535\ ?\ 0.29\lambda\sqrt{f_c'} b_w d \Rightarrow V_c = 147\, kN < 213\, kN \text{ Ok.} \Rightarrow V_c = 147\, kN$

Increase percentage due to use of the more detailed relation:

$$Increase\ Percentage = \frac{147 - 125}{125} \times 100\% = 17.6\, \%$$

At Interior Support

Based on Simplified Relation:

$V_c = 0.17\lambda\sqrt{f_c'}\, b_w d \Rightarrow V_c = 0.17\sqrt{21} \times 300 \times 535 = 0.779\, MPa \times 300 \times 535 = 125\, kN$

Based on the more detailed relation:

$$V_c = \left(0.16\lambda\sqrt{f_c'} + 17\rho_w \frac{(V_u d)}{M_u}\right) b_w d \le 0.29\lambda\sqrt{f_c'} b_w d$$

$$\frac{(V_u d)}{M_u} = \left(\frac{449000\, N \times 535\, mm}{536 \times 10^6\, N.mm}\right) = 0.448 < 1.0\ Ok.$$

$V_c = \left(0.16\sqrt{21} + 17 \times (19.4 \times 10^{-3}) \times 0.448\right)300 \times 535\ ?\ 0.29\lambda\sqrt{f_c'} b_w d$

$V_c = (0.733\, MPa + 0.147\, MPa)300 \times 535\ ?\ 0.29\lambda\sqrt{f_c'} b_w d \Rightarrow V_c = 141\, kN < 213\, kN \text{ Ok.}$

$$Increase\ Percentage = \frac{141 - 125}{125} \times 100\% = 12.8\, \%$$

As it is expected, using of the more detailed ACI equation is more useful in regions of large shear and small moment (regions of inflection points).

Example 5.8-2

For the simply supported beam shown in ***Figure 5.8-5*** below, make a complete shear design use same spacing of 10mm U stirrups along beam span. In your design assume that:

- Beam selfweight can be neglected,
- $f_c' = 28\, MPa$ and $f_y = f_{yt} = 420\, MPa$,
- Beam has dimensions of d = 550 mm and b = 300mm,
- V_c must be computed from the more detailed ACI relation. Use the same V_c value along beam span.
- Steel reinforcement area for positive moment has been computed to be $2835\, mm^2$.

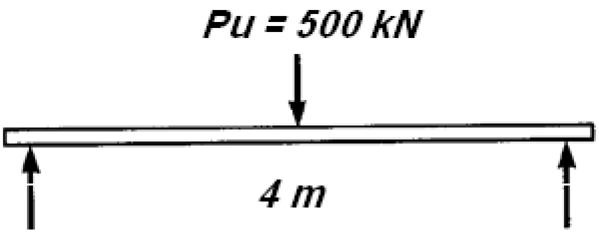
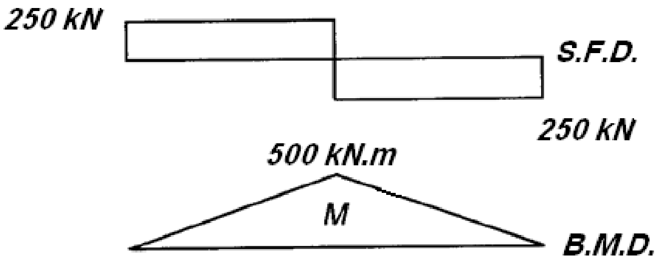


Figure 5.8-5: Simply supported beam for Example 5.8-2.

Solution

- Compute of V_c
  As same V_c must be used along beam span, therefore V_c must be computed based on a region of large shear and large moment (under concentrated load in this example) to obtain a value that is conservative along beam span.



$$V_c = \left(0.16\lambda\sqrt{f_c'} + 17\rho_w \frac{(V_u d)}{M_u}\right) b_w d \leq 0.29\lambda\sqrt{f_c'} b_w d$$

$$\rho_w = \frac{2835\ mm^2}{550 \times 300\ mm^2} = 17.2 \times 10^{-3}$$

$$\frac{V_u d}{M_u} = \frac{(250\ kN \times 0.55m)}{500\ kN.m} = 0.275 < 1.0\ \therefore Ok.$$

$$V_c = \left(0.16\sqrt{28} + 17 \times 17.2 \times 10^{-3} \times \frac{(250\ kN \times 0.55m)}{500\ kN.m}\right) 300 \times 550\ ?\ 0.29\sqrt{28} \times 300 \times 550$$

$$V_c = (0.847\ MPa + 0.080\ MPa)300 \times 550\ ?\ 0.29\sqrt{28} \times 300 \times 550 \Longrightarrow V_c = 153\ kN\ <\ 253\ kN$$

As $V_c < V_u$, therefore shear must be designed based on region of theoretical and nominal reinforcement.

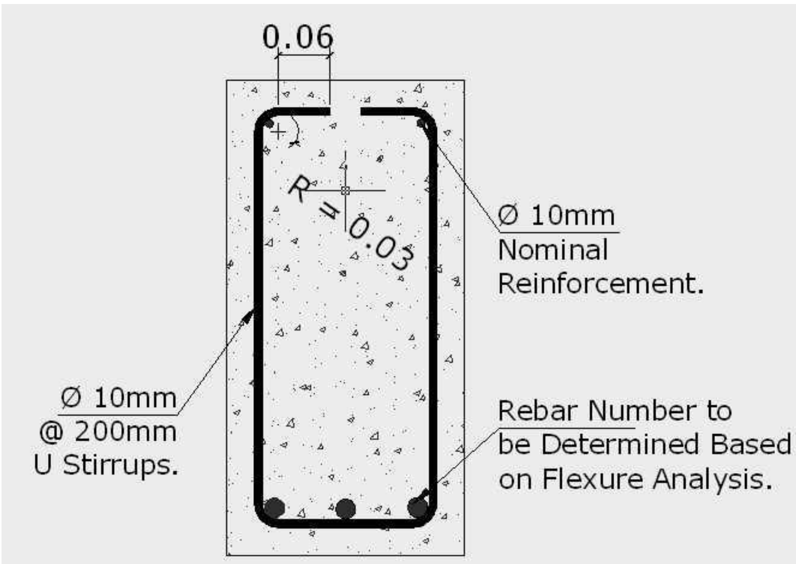
- Shear Design:

  Shear design is summarized in Table below.

Stirrups Design of Example 5.8-2	
Region	$ØV_c \leq V_u$
V_s	$= \frac{V_u - ØV_c}{Ø}\ ?\ 0.66\sqrt{f_c'}b_w d \Longrightarrow \frac{250 - 0.75 \times 153}{0.75}\ ?\ 0.66\sqrt{28} \times 300 \times 550$ $180kN <\ 576\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{157\ mm^2 \times 420 \frac{N}{mm^2} \times 550mm}{180\ 000\ N} = 201\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ (\frac{157\ mm^2 \times 420 \frac{N}{mm^2}}{0.062\sqrt{28} \times 300}\ or\ \frac{157\ mm^2 \times 420 \frac{N}{mm^2}}{0.35 \times 300})$ $minimum\ (700\ or\ 628) = 628\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'}b_w d \Longrightarrow 180kN \leq 0.33\sqrt{28} \times 300 \times 550$ $180\ kN < 288\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right] \Longrightarrow Minimum\ \left[\frac{550}{2}\ or\ 600mm\right] = 225\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d \Longrightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [201\ mm, 628\ mm, 225\ mm] = 201\ mm$ ***Use*** $\mathbf{Ø10mm\ @\ 200\ mm}$

- Drawing and Details of Stirrups:

  The longitudinal section through beam cannot be drawn in this example, as nothing has been mentioned about supports widths. Cross section for this beam is shown below:



5.9 *SHEAR DESIGN WITH EFFECTS OF AXIAL LOADS

5.9.1 Scope

- The beams considered in the preceding sections were subjected to shear and flexure only.
- Reinforced concrete beams may also be subjected to axial forces, acting simultaneously with shear and flexure, due to a variety of causes. These include:
    - External axial loads,
    - Longitudinal prestressing,
    - Restraint forces introduced as a result of shrinkage of the concrete or temperature changes.
- Axial forces due to prestressing are out of our scope where this article deals only with non-prestressed members.
- As for members without axial forces, the ACI codes offers simplified and detailed relations to simulate the effect of axial forces on shear strength of concrete. Only simplified equations are considered in this article.

5.9.2 Effects of Axial Forces on Shear Strength of Concrete

- The main effect of axial load is to modify the diagonal cracking load of the member.
- It was shown in Article 5.1 that diagonal tension cracking will occur when the principal tensile stress in the web of a beam, resulting from combined action of shear and bending, reaches the tensile strength of the concrete.
- It is clear that the introduction of longitudinal force, which modifies the magnitude and direction of the principal tensile stresses, may significantly alter the diagonal cracking load. ***Axial compression will increase the cracking load, while axial tension will decrease it***.

5.9.3 Members with Compressive Axial Forces

- According to ***Article 22.5.6.1*** of the ACI code, for nonprestressed members with axial compression, V_c shall be calculated by:

$$V_c = 0.17\left(1 + \frac{N_u}{14A_g}\right)\lambda\sqrt{f_c'}b_w d$$ **Eq. 5.9-1**

where
    - N_u is the factored axial force normal to cross section occurring simultaneously with V_u; ***to be taken as positive for compression*** , it is expressed in unit of "N",
    - A_g is gross area of concrete section, mm^2. For a hollow section, A_g is the area of the concrete only and does not include the area of the void(s).
- From equation above, it is clear that the term of $N_u/14A_g$ represents the increasing in concrete shear strength, V_c, due to existing of the compressive axial force N_u.

5.9.4 Member with Significant Axial Tensile Forces

- According code commentary, R22.5.7.1, the term "significant" is adopted to recognize that judgment is required in deciding whether axial tension needs to be considered. Axial tension often occurs due to volume changes, but the levels may not be detrimental to the performance of a structure with adequate expansion joints and minimum reinforcement.
- According to ***Article 22.5.7.1*** of the code, for nonprestressed members with ***significant axial tension***, V_c shall be calculated by:

$$V_c = 0.17\left(1 + \frac{N_u}{3.5A_g}\right)\lambda\sqrt{f_c'}b_w d \geq 0$$ **Eq. 5.9-2**

where
    - N_u is the factored axial force normal to cross section occurring simultaneously with V_u; ***to be taken as negative for tension***, it is expressed in unit of "N",
    - A_g is gross area of concrete section, mm^2. For a hollow section, A_g is the area of the concrete only and does not include the area of the void(s).

- From equation above, it is clear that the term of $N_u/3.5A_g$ represents the decreasing in concrete shear strength, V_c, due to existing of the tensile axial force N_u.
- According commentary ***Article R22.5.7.1***, ***it may be desirable to design shear reinforcement to resist the total shear if there is uncertainty about the magnitude of axial tension***.

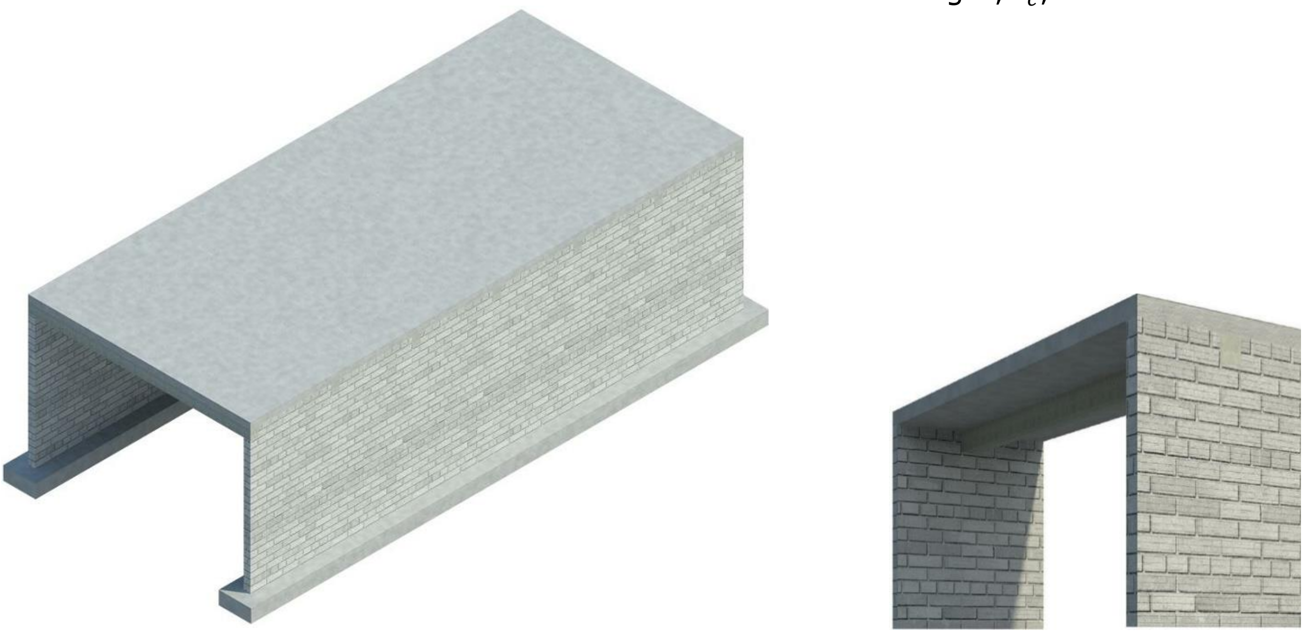
5.9.5 Comparing between Effect of Axial Compression and Axial Tension

Comparing between effect of axial compression, $N_u/14A_g$, and the effect of axial tension, $N_u/3.5A_g$, one concludes that the code is more conservative in estimating decreasing in shear strength, V_c, due to tensile stresses than its increasing due to compressive stresses.

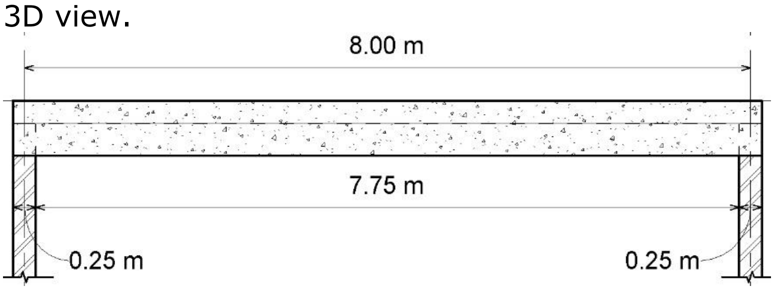
5.9.6 Examples

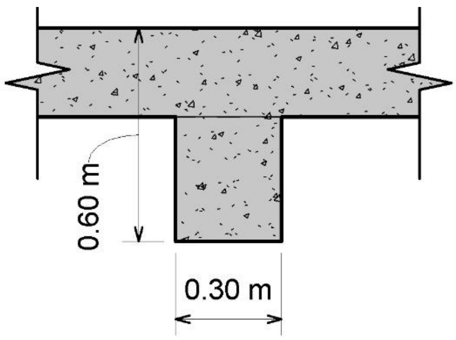
Example 5.9-1

Concrete roof slab and its supporting beams indicated in ***Figure 5.9-1*** below have been casted against and supported on brick bearing walls. The slab and beams have been concreted monolithically at a temperature of 20°C. Determine axial forces that are developed in a typical beam when the temperature decreases into 0°C or increases into 40°C then show how these forces can alter concrete shear strength, V_c, for the beam.



3D view.





Longitudinal section. Cross section.

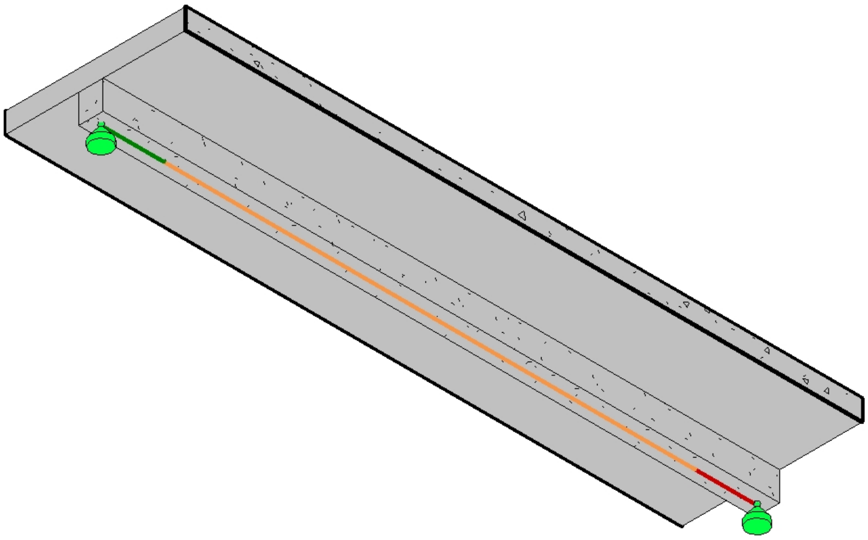
Figure 5.9-1: Roof slab and its supporting beams for Example 5.9-1.

In your analysis,

- Assume that the contact surface between beams and walls is rough enough to restrained beam movement,
- Assume that f_c' is 28 MPa and that the coefficient for thermal expansion of concrete is $\alpha_{Concrete} = 11 \times 10^{-6}\ 1/^{o}C$.
- Assume a load factor of 1.6 for forces due to temperature change.

Solution

With assumption of rough surface, the analytical model for a typical supporting beam would be as indicated in below:



From strength of materials, with equating of strain due to temperature to that due to restrained forces, the relation would be:

$\because \epsilon_{Forces} = \epsilon_{Temp}.$

$\therefore \frac{\sigma}{E_C} = \alpha \Delta T$

$\sigma = E_C \alpha \Delta T$

$\frac{N}{A_g} = E_C \alpha \Delta T$ ■

With a compressive strength of f_c' of 28MPa, the modulus of elasticity of concrete, E_c in MPa would be:

$E_c = 4700\sqrt{f_c'} = 4700 \times \sqrt{28} = 24870\ MPa$

The normal force per gross area would be:

$\frac{N}{A_g} = \pm(24870 \times 11 \times 10^{-6} \times 20) = \pm 5.47\ \text{MPa}$

These stresses would be tensile when temperature decreases while it would be compressive when temperature increases. With a load factor of 1.6, the ultimate stresses due to temperature change would be:

$\frac{N_u}{A_g} = \pm 1.6 \times 5.47 = 8.75\ \text{MPa}$

When these stresses are compressive, the shear strength of concrete, V_c, would increase by 62.5% as indicated in below:

$V_c = 0.17\left(1 + \frac{N_u}{14A_g}\right)\lambda\sqrt{f_c'}b_w d = V_c = 0.17\left(1 + \frac{8.75}{14}\right)\lambda\sqrt{f_c'}b_w d = 0.17(1 + 0.625)\lambda\sqrt{f_c'}b_w d$

While, when these stresses are tensile, the shear strength of concrete, V_c, would decrease to zero as indicated in below.

$V_c = 0.17\left(1 + \frac{N_u}{3.5A_g}\right)\lambda\sqrt{f_c'}b_w d = 0.17\left(1 - \frac{8.75}{3.5}\right)\lambda\sqrt{f_c'}b_w d = 0.17(1 - 2.5)\lambda\sqrt{f_c'}b_w d < 0.0$

$\therefore Not\ Ok.$

$V_c = 0.0$

Example 5.9-2: A Portal Frame Subjected to Gravity Loads Only

Based on a statically indeterminate analysis, reactions for the portal frame shown in ***Figure 5.9-2*** above have been computed and presented as shown. Design 12mm U Stirrups for the beam BC. In your design, assume that:

- $f_c' = 21\ MPa$ and $f_y = f_{yt} = 420\ MPa$,

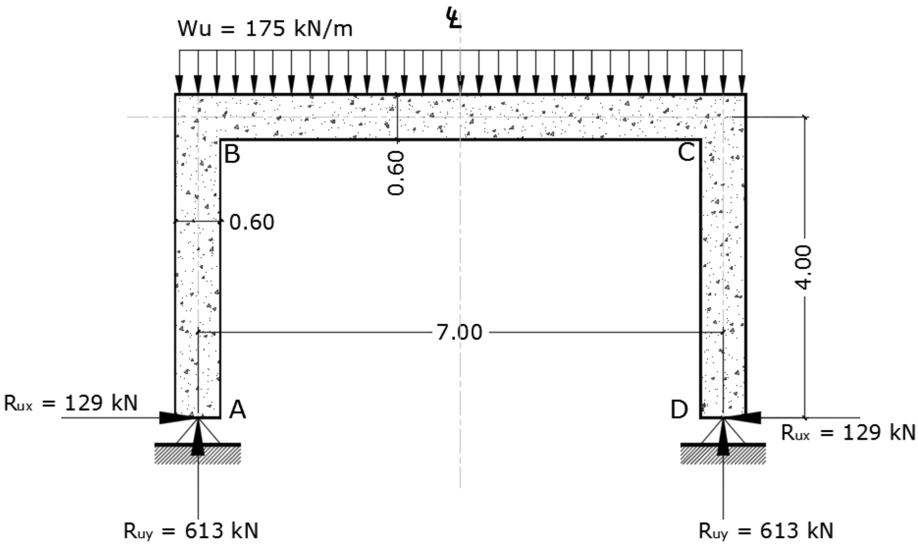
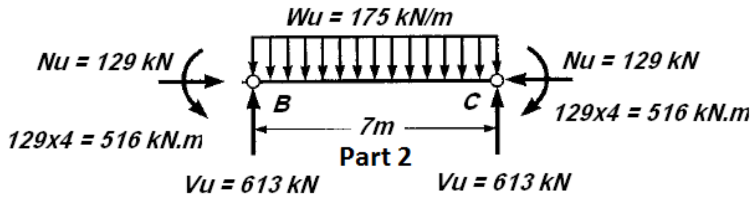


Figure 5.9-2: A portal frame subjected for gravity loads.

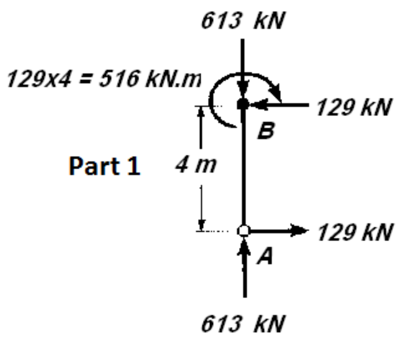
- $b_w = 300mm$, $h = 600mm$ and $d = 550mm$,
- Selfweight of the frame can be neglected,
- Effects of axial forces on concrete shear strength V_c of beam BC should be included.

Solution

- Computing of V_u:
  Applied factored shear force V_u can be computed based on any one of the following two approaches:
  - First Approach (Based on Forces Diagrams):
    - Based on simple static of column AB and beam BC, forces acting on beam BC can be determined based on indicated figure.



    - From above Figure, ***it is clear that column shear force transfer to beam axial force and column axial force transfer to beam shear***.

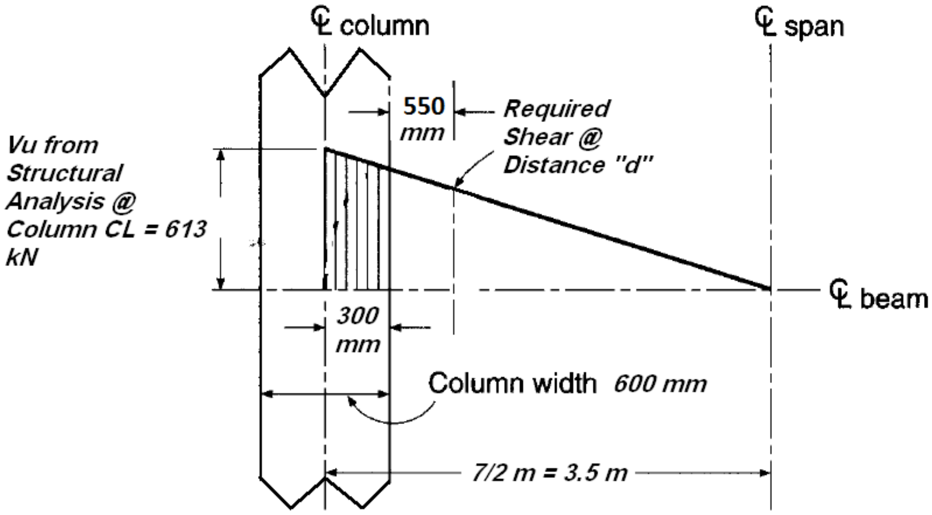


    - As the forces that computed based on structural analysis represent forces at center lines, then two transformation of beam shear force ($V_u = 613\,kN$) seems necessary to obtain required $V_{u\,@d}$. The first one transforms shear force for column center line to the face of column and the second one transforms shear force from face of column to a distance (d) from face of column as all ACI conditions are satisfied (See Figure below). These transformation can be done based on following relation:

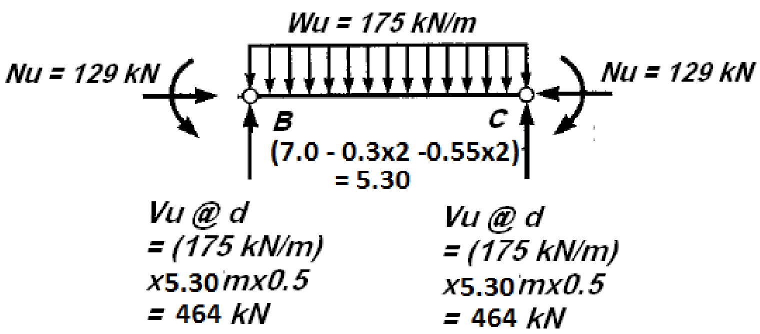
$$\because W = \frac{dV}{dx} \Longrightarrow dV = Wdx \Longrightarrow V_{u\,@\,d} - V_{@\,CL} = \int_{from\ CL}^{To\ Distance\ d\ from\ Face\ of\ Support} Wdx$$

$$V_{u\,@\,d} = \int_{from\ CL}^{To\ Distance\ d\ from\ Face\ of\ Support} Wdx \;+\; V_{@\,CL}$$

$$V_{u\,@\,d} = \left(-175\frac{kN}{m}\right) \times \left(\frac{0.6m}{2} + 0.55m\right) \;+\; 613 = 464\,kN$$



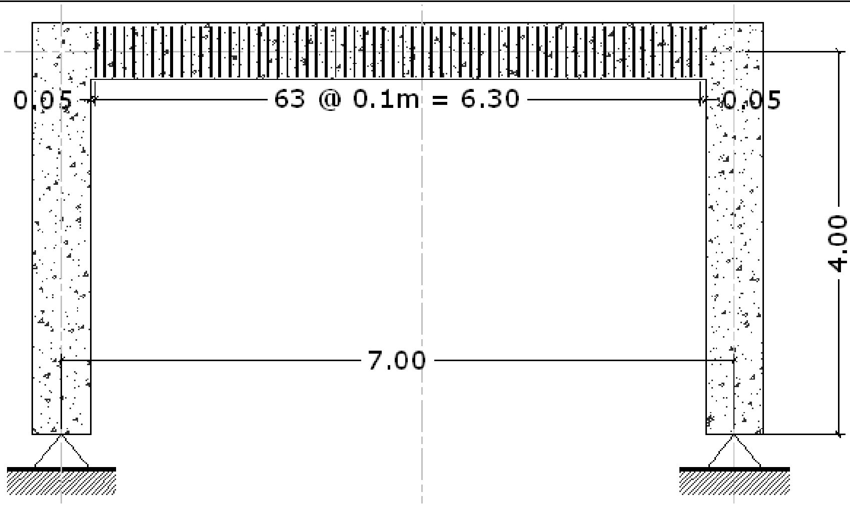
- Second Approach (Based on Symmetry):
  - For our symmetrical problem, principle of symmetry can be used to computed required shear force at distance (d) from face of support as indicated in the figure.



- Compute of V_c:
  Based on ACI code, for member with a compression axial force, concrete shear strength will be:
  $$V_c = 0.17\left(1+\frac{N_u}{14A_g}\right)\lambda\sqrt{f_c'}\,b_w d \Longrightarrow V_c = 0.17\left(1+\frac{129\,000}{14(300\times600)}\right)\times1.0\times\sqrt{21}\times300\times550$$
  $$V_c = 0.17(1+0.051)\times1.0\times\sqrt{21}\times300\times550 = 135\text{ kN}$$
  Relation above indicates that concrete shear strength, V_c, increases by about 5% due to existing of the axial compressive force, N_u, with magnitude of 129 kN. Including the strength reduction factor, ϕ, for shear, design shear strength of the concrete would be:
  $$\emptyset V_c = 0.75\times135\,kN = 101\,kN$$
- Shear Design:
  As applied shear force V_u is greater that $ØV_c$, then beam should be designed on region with theoretical and nominal shear reinforcement (See Table below):

SHEAR DESIGN OF Example 5.9-2	
Region	$\emptyset V_c \le V_u$
V_s	$=\frac{V_u-\emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d \Longrightarrow \frac{464-101}{0.75}\ ?\ 0.66\times\sqrt{21}\times300\times550$ $484\,kN < 499\,kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$=\frac{A_v f_{yt} d}{V_s} = \frac{226\times420\times550}{484\,000} = 108\,mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w}) \Longrightarrow minimum\ (\frac{226\times420}{0.062\sqrt{21}\times300}\ or\ \frac{226\times420}{0.35\times300})$ $minimum\ (1\,114\ or\ 904) = 904\,mm$
$S_{maximum}$	$V_s \le 0.33\sqrt{f_c'}b_w d$
	$V_s\ ?\ 0.33\sqrt{f_c'}b_w d \Longrightarrow V_s = 484 > 0.33\sqrt{21}\times300\times550 = 249\,kN$ $Minimum\ \left[\frac{d}{4}\ or\ 300mm\right] \Longrightarrow Minimum\ \left[\frac{550}{4}\ or\ 300mm\right] = 137\,mm$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $\Longrightarrow Minimum\ [108\,mm,\ 904\,mm,\ 137\,mm] = 108\,mm$ **Use $\emptyset 12mm\ @\ 100mm$**

- Stirrups Details:



Example 5.9-3: A Portal Frame Subjected to Gravity and Lateral Loads.

Based on a statically indeterminate analysis, reactions for the portal frame shown ***Figure 5.9-3*** below have been computed and presented as shown. Design 12mm U Stirrups for the beam BC. In your design, assume that:

- $f_c' = 21\,MPa$ and $f_{yt} = 420\,MPa$,
- $b_w = 300mm$, $h = 600mm$ and $d = 550mm$,
- Selfweight of the frame can be neglected,
- Effects of axial forces on concrete shear strength, V_c, of beam BC should be included.
- Ultimate forces to be determined based on following load combination:
  U = 1.2D + 1.6L$_r$ + 0.8W.

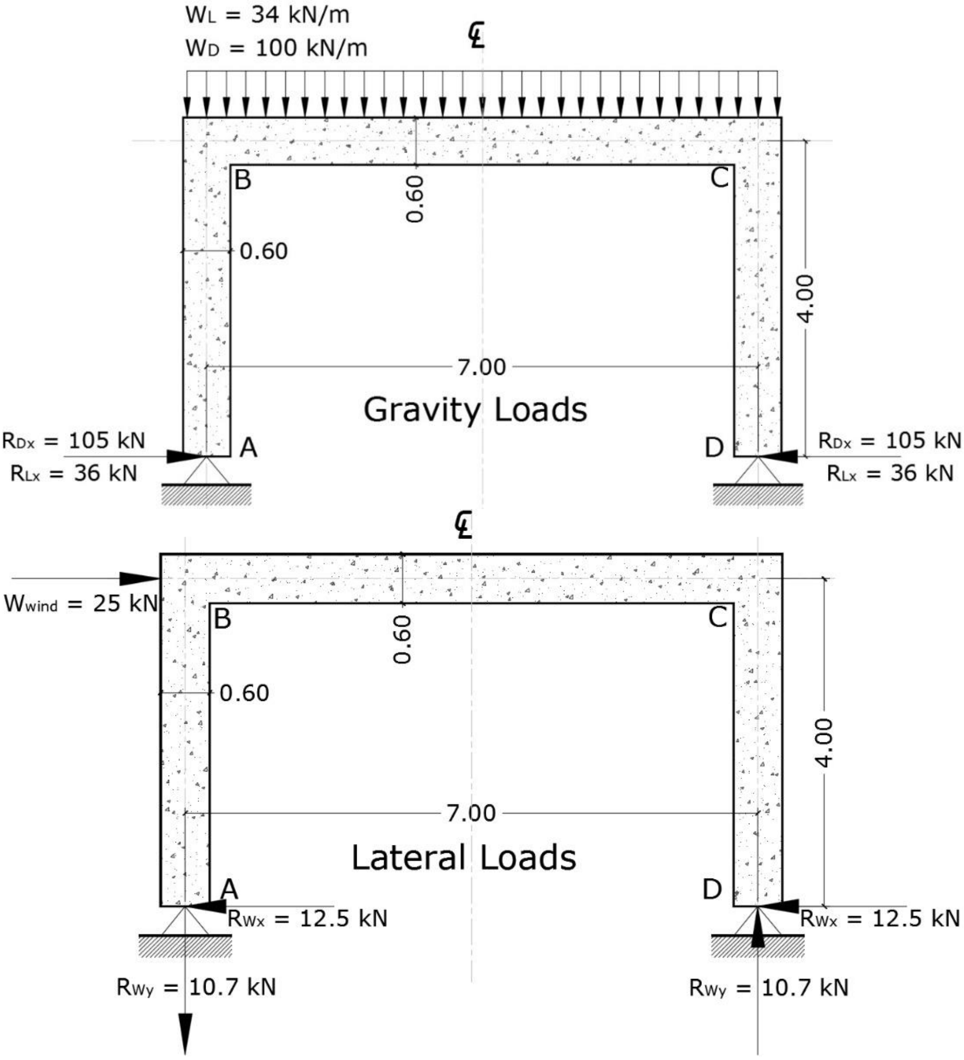


Figure 5.9-3: A portal frame subjected to gravity and lateral forces.

Solution

Compute V$_u$:

V$_u$ @ Left Support (Support B):

To determine if reaction at support is compression or tension, the resultant for R_D, R_L, and R_W should be determined:

$$R_D = (100 \times 7) \times \frac{1}{2} = 350\ kN, R_{Lr} = (34 \times 7) \times \frac{1}{2} = 119\ kN, R_w = -10.7\ kN$$

The ultimate reaction due to indicated load combination would be:

$$R_u = 1.2R_D + 1.6R_{Lr} + 0.8R_W = 1.2 \times 350 + 1.6 \times 119 - 0.8 \times 10.7 = 602\ kN$$

As the ultimate reaction is compressive, therefore shear force can be determined at distance "d" from face of support "B":

$$V_{D\ @\ d\ from\ Support\ B} = 100\frac{kN}{m}\ (7.0 - \frac{0.6}{2} \times 2 - 0.55 \times 2)m \times \frac{1}{2} = 265\ kN$$

$$V_{L\ @\ d\ from\ Support\ B} = 34\frac{kN}{m}\left(7.0 - \frac{0.6}{2} \times 2 - 0.55 \times 2\right)m \times \frac{1}{2} = 90.1\ kN$$

$V_{W\,@\,d\,from\,Support\,B} = R_{Wy} = -10.7\;kN$

$V_u = 1.2V_D + 1.6V_L + 0.8V_W$

$V_{u\,@\,d\,from\,face\,of\,Support\,B} = 1.2 \times 265 + 1.6 \times 90.1 - 0.8 \times 10.7 = 454\;kN$

V_u @ Right Support (Support C):

As all reactions (due to dead, live, and wind) are compression reactions, therefore shear force could be computed at distance "d" from face of right support "C".

$V_{D\,@\,d\,from\,Support\,C} = 100\frac{kN}{m}\;(7.0 - \frac{0.6}{2} \times 2 - 0.55 \times 2)m \times \frac{1}{2} = 265\;kN$

$V_{L\,@\,d\,from\,Support\,C} = 34\frac{kN}{m}\left(7.0 - \frac{0.6}{2} \times 2 - 0.55 \times 2\right)m \times \frac{1}{2} = 90.1\;kN$

$V_{W\,@\,d\,from\,Support\,C} = R_{Wy} = +10.7\;kN$

$V_u = 1.2V_D + 1.6V_L + 0.8V_W$

$V_{u\,@\,d\,from\,face\,of\,Support\,C} = 1.2 \times 265 + 1.6 \times 90.1 + 0.8 \times 10.7 = 471\;kN$

Critical V_u:

$V_u = maximum\;(V_{u\,@\,face\,of\,Support\,B}, V_{u\,@\,d\,from\,face\,of\,Support\,C})$

$V_u = maximum\;(454\;kN, 471\;kN) = 471\;kN$

Compute V_c:

$V_c = 0.17\left(1 + \frac{N_u}{14A_g}\right)\lambda\sqrt{f_c'}\;b_w d$

$N_D = R_{Dx} = 105\;kN, N_L = R_{Lx} = 36\;kN$

$N_W = W - R_{Wx} = 25 - 12.5 = 12.5\;kN, N_u = 1.2 \times 105 + 1.6 \times 36 + 0.8 \times 12.5 = 194\;kN$

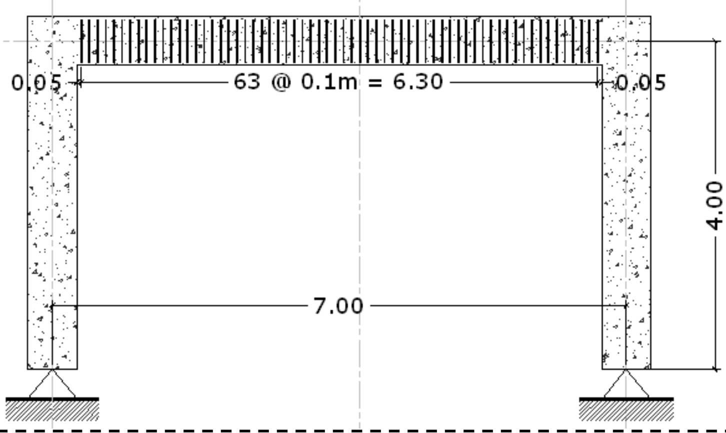
$V_c = 0.17\left(1 + \frac{194\,000}{14(300 \times 600)}\right) \times 1.0 \times \sqrt{21} \times 300 \times 550$

$= 0.17(1 + 0.077) \times 1.0 \times \sqrt{21} \times 300 \times 550 = 138\;\text{kN}$

$\phi V_c = 0.75 \times 138\;kN = 104\;kN$

Shear design for Example 5.9-3	
Region	$\emptyset V_c \leq V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d \Rightarrow \frac{471 - 104}{0.75}\;?\;0.66 \times \sqrt{21} \times 300 \times 550 \Rightarrow 489\;kN < 499\;kN\;Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{226 \times 420 \times 550}{489000} = 107\;mm$
$S_{for\;Av\;minim}$	$minimum\;(\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\;or\;\frac{A_v f_{yt}}{0.35b_w}) \Rightarrow minimum\;(\frac{226 \times 420}{0.062\sqrt{21} \times 300}\;or\;\frac{226 \times 420}{0.35 \times 300})$ $minimum\;(1\,114\;or\;904) = = 904\;mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'}b_w d$ $V_s\;?\;0.33\sqrt{f_c'}b_w d \Rightarrow V_s = 484 > 0.33\sqrt{21} \times 300 \times 550 = 249\;kN$ $Minimum\;\left[\frac{d}{4}\;or\;300mm\right] \Rightarrow Minimum\;\left[\frac{550}{4}\;or\;300mm\right] = 137\;mm$
$S_{Required}$	$Minimum\;[S_{Theoretical}, S_{for\;Av\;minimum}, S_{maximum}]$ $Minimum\;[107\;mm,\;904\;mm,\;137\;mm] = 107\;mm$ ***Use*** $\boldsymbol{\emptyset 12mm\;@\;100mm}$

- Stirrups Details:



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Contents

CHAPTER 6
BOND, ANCHORAGE, AND DEVELOPMENT LENGTH

6.1 FUNDAMENTALS OF FLEXURAL BOND

6.1.1 BOND ROLE

- An experiment to show importance of the bond:
  - If the reinforced concrete beam of ***Figure 6.1-1a*** below were constructed using plain round reinforcing bars, and, furthermore, if those bars were to be greased or otherwise lubricated before the concrete were cast, the beam would be very little stronger than if it were built of plain concrete, without reinforcement.
  - If a load were applied, as shown in ***Figure 6.1-1b***, the bars would tend to maintain their original length as the beam deflected. The bars would slip longitudinally with respect to the adjacent concrete, which would experience tensile strain due to flexure.
  - Then, ***the assumption that the strain in an embedded reinforcing bar is the same as that in the surrounding concrete***, would ***not be valid***.
- For reinforced concrete to behave as intended, it is essential that bond forces be developed on the interface between concrete and steel, such as to prevent significant slip from occurring at that interface. ***Figure 6.1-1c*** shows the bond forces that act on the concrete at the interface as a result of bending, while ***Figure 6.1-1d*** shows the equal and opposite bond forces acting on the reinforcement. It is through the action of these interface bond forces that the slip indicated in ***Figure 6.1-1b*** is prevented.

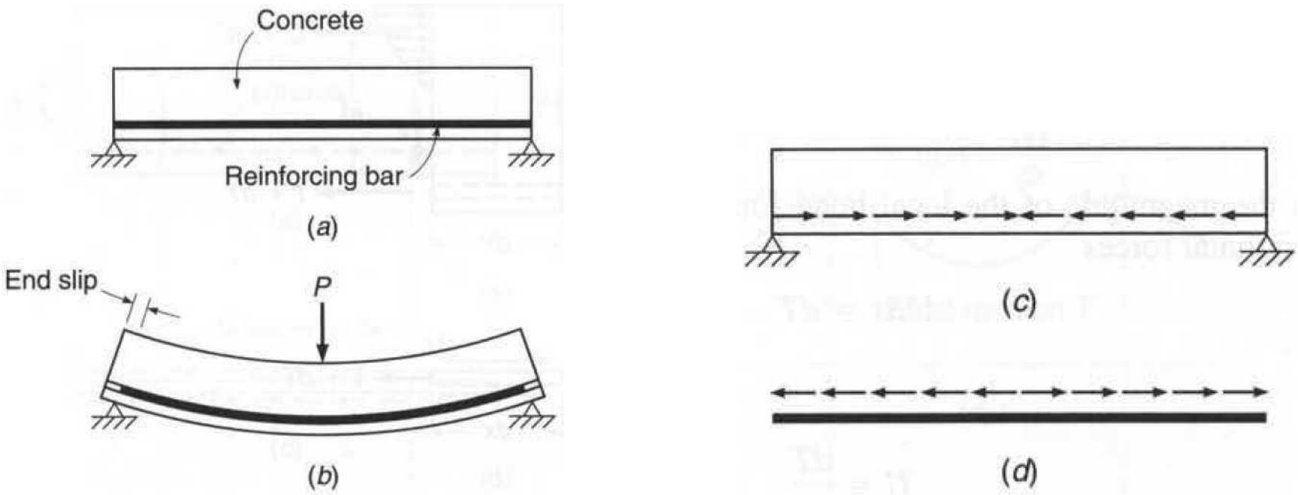


Figure 6.1-1: Bond forces due to flexure: (a) beam before loading; (b) unrestrained slip between concrete and steel; (c) bond forces acting on concrete; (d) bond forces acting on steel.

6.1.2 PLAIN AND DEFORMED REBARS

6.1.2.1 Plain Rebars

- Some years ago, when ***plain bars*** without surface deformations were used, ***initial bond strength*** was provided only by the relatively weak ***chemical adhesion*** and ***mechanical friction*** between steel and concrete. Once adhesion and static friction were overcome at larger loads, small amounts of slip led to interlocking of the natural roughness of the bar with the concrete.
- However, this natural bond strength ***is so low that in beams reinforced with plain bars***, the bond between steel and concrete was frequently broken.
- Such a beam will collapse as the bar is pulled through the concrete. To prevent this, ***end anchorage was provided***, chiefly in the form of hooks, as in ***Figure 6.1-2***.

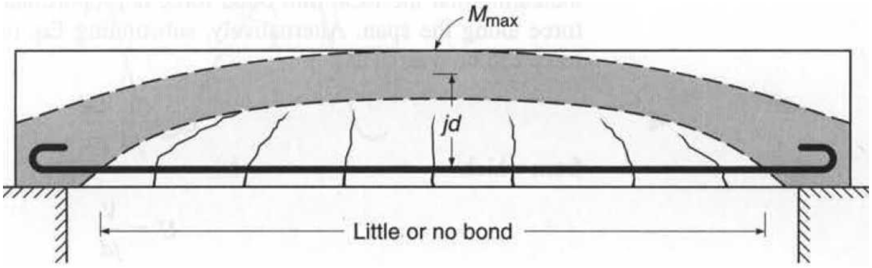


Figure 6.1-2: Tied-arch action in a beam with little or no bond.

- If the anchorage is adequate, such a beam will not collapse, even if the bond is broken over the entire length between anchorages. This is so because the member acts as a ***tied arch***, as shown in ***Figure 6.1-2***, with the uncracked concrete shown shaded representing the arch and the anchored bars the tie-rod.
- Main Disadvantage of Plain Rebars:

  In this case, over the length in which the bond is broken, bond forces are zero. This means that over the entire unbonded length the force in the steel is constant and equal to:

  $$T = \frac{M_{maximum}}{jd}$$

  As a consequence, the total steel elongation in such beams is larger than in beams in which bond is preserved, ***resulting in larger deflections*** and ***greater crack widths***.

6.1.2.2 Deformed Bars

- To improve this situation, deformed bars are now universally used in the United States and many other countries.

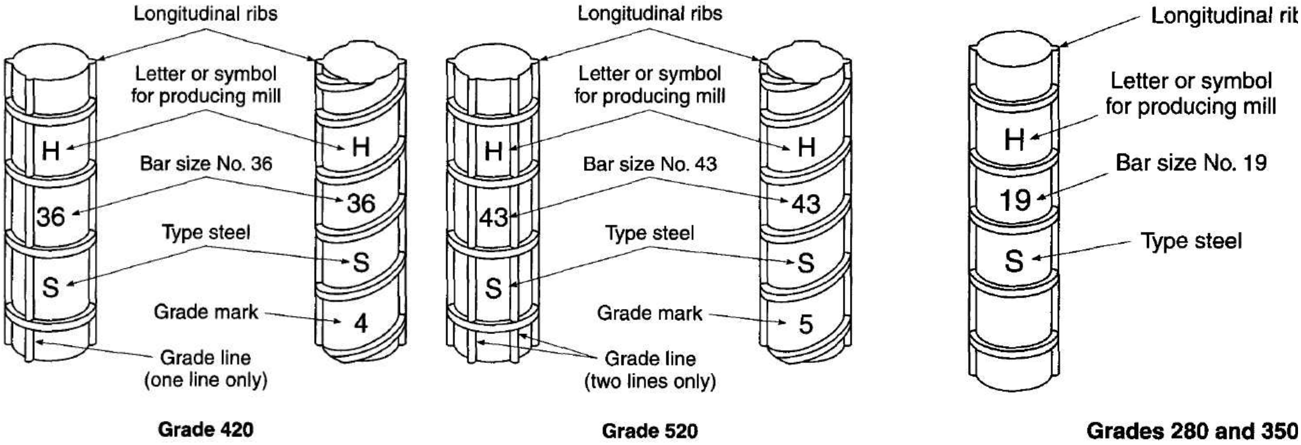


Figure 6.1-3: Marking system for reinforcing bars meeting ASTM Specifications.

6.1.3 Bond Force Based on Simple Cracked Section Analysis

- In a short piece of a beam of length dx, such as shown in ***Figure 6.1-4a***, the moment at one end will generally differ from that at the other end by a small amount dM. If this piece is isolated, and if one assumes that, after cracking, the concrete does not resist any tension stresses, the internal forces are those shown in ***Figure 6.1-4a***.

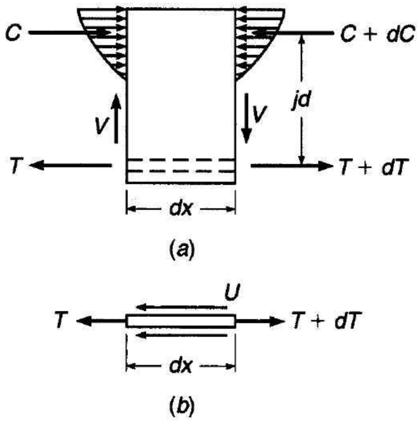


Figure 6.1-4: Forces acting on elemental length of beam: (a) free-body sketch of reinforced concrete element; (b) free-body sketch of steel element.

- Theoretical Relation for Bond Stresses:
  - The change in bending moment, dM, produces a change in the bar force:
    $$dT = \frac{dM}{jd}$$
  - If U is the magnitude of the local bond force per unit length of bar, then, by summing horizontal forces
    $$Udx \ = \ dT \Rightarrow U = \frac{dT}{dx}$$
    $$U = \frac{1}{jd}\frac{dM}{dx} \Rightarrow U = \frac{1}{jd}\,V$$
- Main Conclusions for the Relation:
  - Equation above is the "***elastic cracked section equation***" for flexural bond force, and it indicates that ***the bond force per unit length is proportional to the shear*** at a particular section, i.e., to the rate of change of bending moment.
  - Basic Assumption that Used in Relation:
    Equation assumes that ***concrete zone to be fully cracked***, with the concrete ***resisting no tension***.
- Applicability of Relation:
  The relation applies, therefore, to
  - The tensile bars in simple spans,
  - The continuous spans, either to the bottom bars in the positive bending region between inflection points or to the top bars in the negative bending region between the inflection points and the supports,
  - It does not apply to compression reinforcement.

6.1.4 ACTUAL DISTRIBUTION OF FLEXURAL BOND FORCE:

- The actual distribution of bond force along deformed reinforcing bars is ***much more complex than that represented by*** $U = \frac{1}{jd}\,V$.
- Beam with Pure Bending:
  According to $U = \frac{1}{jd}\,V$, beam with ***pure bending has no bond stresses***, <u>***but***</u> ***as the concrete fails*** to resist tensile stresses only where the actual crack is located and as between cracks, the concrete does resist moderate amounts of tension, therefore ***bond stresses distribution will be as shown*** in ***Figure 6.1-5*** below:

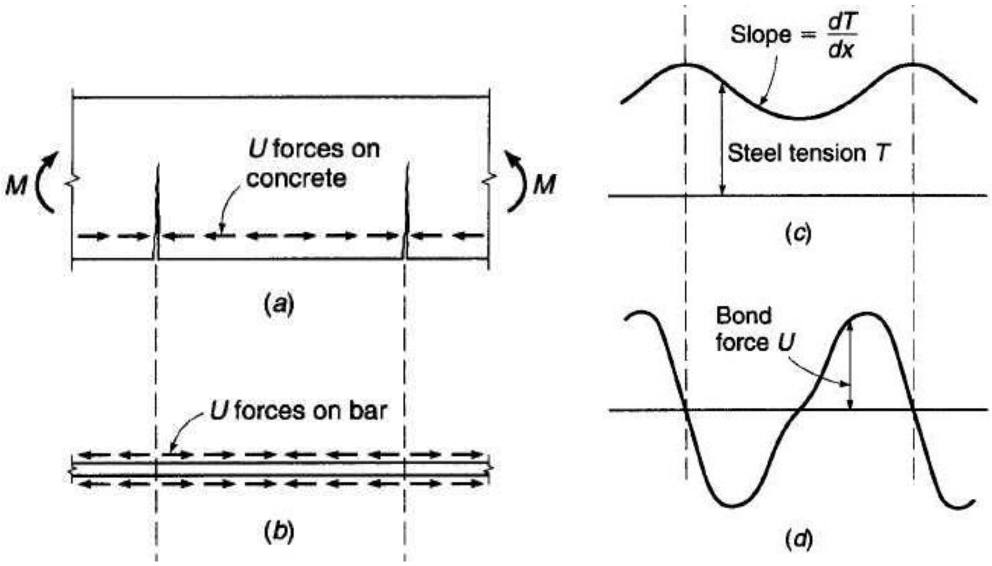


Figure 6.1-5: Variation of steel and bond forces in a reinforced concrete member subject to pure bending: (a) cracked concrete segment; (b) bond forces acting on reinforcing bar; (c) variation of tensile force in steel; (d) variation of bond force along steel.

- Beams with Bending and Shear Forces:
  - Beams are seldom subject to pure bending moment; they generally carry transverse loads producing shear and moment that vary along the span. ***Figure***

6.1-6a shows a beam carrying a distributed load. The cracking indicated is ***typical***.

- The steel force T predicted by simple cracked section analysis is proportional to the moment diagram and is as shown by the dashed line in ***Figure 6.1-6b***.
- However, the actual value of T is ***less than*** that predicted by the simple analysis everywhere except at the actual crack locations.
- In ***Figure 6.1-6c***, the bond forces predicted by the simplified theory are shown by the dashed line, and the actual variation is shown by the solid line.

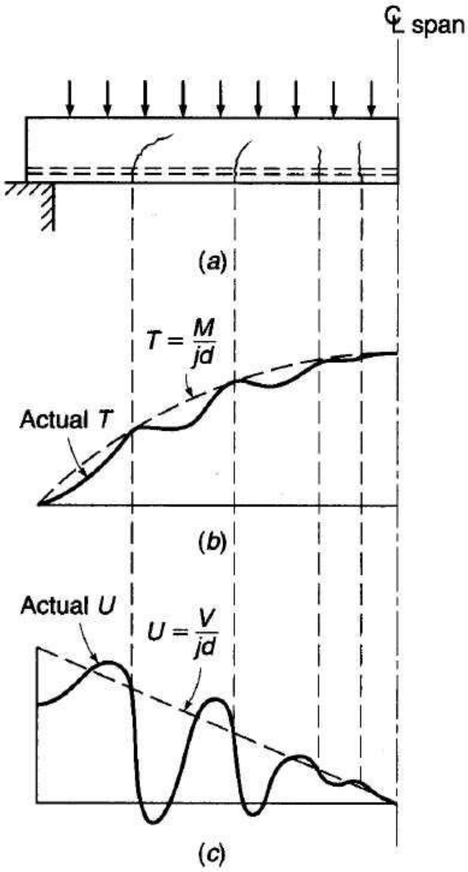


Figure 6.1-6: Effect of flexural cracks on bond forces in beam: (a) beam with flexural cracks; (b) variation of tensile force Tin steel along span; (c) variation of bond force per unit length U along span.

6.1.5 Main Conclusion About Bond Stresses

It is evident that actual bond forces in beams bear very little relation to those predicted by $U = \frac{1}{jd} V$, except in the general sense that they are highest in the regions of high shear.

6.1.6 Bond Strength

For reinforcing bars in tension, two types of bond failure have been observed:

6.1.6.1 Pullout Mode

- Occurs when ***ample confinement*** is provided by the surrounding concrete.
- It could be expected when ***relatively small-diameter bars*** are used with sufficiently large concrete cover distances and bar spacing.

6.1.6.2 Splitting Mode

- Occurs along the bar when cover, confinement, or bar spacing is insufficient to resist the lateral concrete tension resulting from the ***wedging effect*** of the bar deformations.
- It is ***more common*** in beams than direct pullout.
- It may occur either in a vertical plane as in ***Figure 6.1-7a*** or horizontally in the plane of the bars as in ***Figure 6.1-7b***.

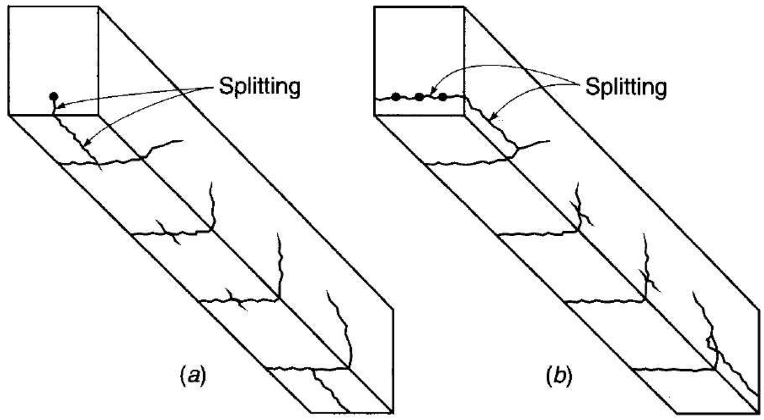


Figure 6.1-7: **Splitting of concrete along reinforcement.**

6.1.7 Concept of Development Length

- Based on above discussion, ***local failures result in small local slips*** and some widening of cracks and increase of deflections but ***will be harmless as long as failure does not propagate all along the bar, with resultant total slip***.
- This fact suggests ***the concept of development length of a reinforcing bar*** which ***could be defined as that length of embedment necessary to develop the full tensile strength of the bar, controlled by either pullout or splitting***.

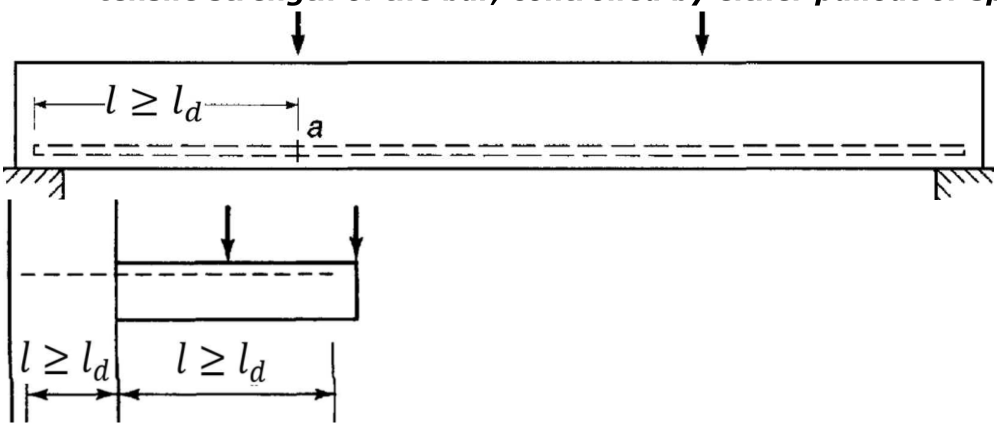


Figure 6.1-8: Concepts of development length.

6.2 ACI Provisions for Development of Reinforcement

6.2.1 Strength Reduction Factor, ϕ

- According to (ACI318M, 2014), article ***25.4.1.3***, ***the strength reduction factor*** ϕ ***is not used in the development length and lap splice length equations***.
- An allowance for strength reduction ***is already included*** in the expressions for determining development and splice lengths.

6.2.2 Maximum Value for $f_c{}'$

- According to (ACI318M, 2014), article ***25.4.1.4***, the values of $\sqrt{f_c{}'}$ used to calculate development length shall not ***exceed 8.3 MPa***. This equivalent to a $f_c{}'$ of 68.9 MPa.
- Why this limitation:
  - Tests show that the force developed in a bar in development and lap splice tests increases at a lesser rate than $\sqrt{f_c{}'}$ with increasing compressive strength.
  - Using $\sqrt{f_c{}'}$, however, is sufficiently accurate for values of $\sqrt{f_c{}'}$ up to 8.3 MPa.
  - ACI Committee 318 has chosen not to change the exponent applied to the compressive strength used to calculate development and lap splice lengths, but rather to set an upper limit of 8.3 MPa on $\sqrt{f_c{}'}$.

6.3 ACI Code Provisions for Development of Tension Reinforcement

6.3.1 Basic Equation for Development of Tension Bars

- According to ACI Code **25.4.2.3**, for deformed bars or deformed wires,

However, the product $\psi_t\psi_e$ need not be greater than 1.7.

(a) Where horizontal reinforcement is placed such that more than 300 mm of fresh concrete is cast below the development length or splice, $\psi_t = 1.3$. For other situations, $\psi_t = 1.0$.

(b) For epoxy-coated bars or wires with cover less than $3d_b$, or clear spacing less than $6d_b$, $\psi_e = 1.5$. For all other epoxy-coated bars or wires, $\psi_e = 1.2$. For uncoated and zinc-coated (galvanized) reinforcement, $\psi_e = 1.0$.

$$\ell_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}}\frac{\psi_t\psi_e\psi_s}{\left(\dfrac{c_b+K_{tr}}{d_b}\right)}\right)d_b$$

λ is modification factor to reflect the reduced mechanical properties of lightweight concrete relative to normalweight concrete of the same compressive Strength. Estimated either from Table 19.2.4.2 or from relation below:

$$\lambda = f_{ct}/(0.56\sqrt{f_c'}) \le 1.0$$

where f_{ct} is the average splitting tensile strength of lightweight concrete.

in which the confinement term $(c_b + K_{tr})/d_b$ shall not be taken greater than 2.5, and

(c) For No. 19 and smaller bars and deformed wires, $\psi_s = 0.8$. For No. 22 and larger bars, $\psi_s = 1.0$.

- Confinement term has been explained in more detailed below:

c_b = smaller of: (a) the distance from center of a bar or wire to nearest concrete surface, and (b) one-half the center-to-center spacing of bars or wires being developed, mm,

$$\left(\frac{c_b+K_{tr}}{d_b}\right) \qquad K_{tr} = \frac{40A_{tr}}{sn}$$

A_{tr} = total cross-sectional area of all transverse reinforcement within spacing s that crosses the potential plane of splitting through the reinforcement being developed, mm^2,

where n is the number of bars or wires being spliced or developed along the plane of splitting. It shall be permitted to use $K_{tr} = 0$ as a design simplification even if transverse reinforcement is present.

s = maximum spacing of transverse reinforcement within l_d center to center,

- Excess reinforcement:

  According to (ACI318M, 2014), article ***25.4.10.1***, reduction of development lengths shall be permitted by use of the ratio (As,required)/(As,provided).
- Important Notes on ACI Basic Equation:
  - Above single basic equation includes all the influences ***thus appears highly complex because of its inclusiveness***.
  - However,

  1. It does permit the designer to see the effects of all the controlling variables
  2. It allows more rigorous calculation of the required development length when it is critical.

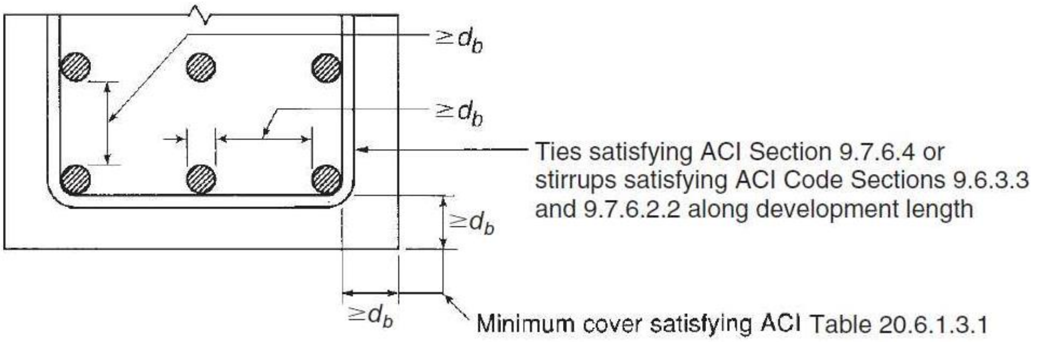
6.3.2 Simplified Equations for Development Length

- Calculation of required development length (in terms of bar diameter) by above basic equation requires that the term $(c_b + K_{tr})/d_b$ be calculated for each particular combination of cover, spacing, and transverse reinforcement.
- Alternatively, according to the Code, article ***25.4.2.2***, a simplified form of basic equation may be used in which $\mathbf{(c_b + K_{tr})/d_b}$ is set equal to ***1.5***, provided that certain restrictions are placed on cover, spacing, and transverse reinforcement. These requirements have been presented in term of confinement cases 1 and 2 that indicated in below.
- If confinement cases 1 and 2 are not satisfied, a confinement factor $\mathbf{(c_b + K_{tr})/d_b}$ of 1.0 is adopted.

Table 6.3-1: Simplified ACI Relations for Development Length (Table 25.4.2.2 of (ACI318M, 2014)).

	Spacing and cover	No. 19 and smaller bars and deformed wires ($\Psi s = 0.8$)	No. 22 and larger bars ($\Psi s = 1.0$)	
Case 1	Clear spacing of bars or wires being developed or spliced not less than d_b, clear cover not less than d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum	$\left(\frac{f_y \Psi_t \Psi_e}{2.1\lambda\sqrt{f'_c}}\right) d_b$	$\left(\frac{f_y \Psi_t \Psi_e}{1.7\lambda\sqrt{f'_c}}\right) d_b$	← $(c_b + k_{tr})/d_b = 1.5$
Case 2	or Clear spacing of bars or wires being developed or spliced not less than $2d_b$ and clear cover not less than d_b			
	Other cases	$\left(\frac{f_y \Psi_t \Psi_e}{1.4\lambda\sqrt{f'_c}}\right) d_b$	$\left(\frac{f_y \Psi_t \Psi_e}{1.1\lambda\sqrt{f'_c}}\right) d_b$	← $(c_b + k_{tr})/d_b = 1.0$

- Case 1 and Case 2 have been presented graphically in ***Figure 6.3-1*** below:



(a) Case 1.

Figure 6.3-1: Explanation of Cases 1 and 2.

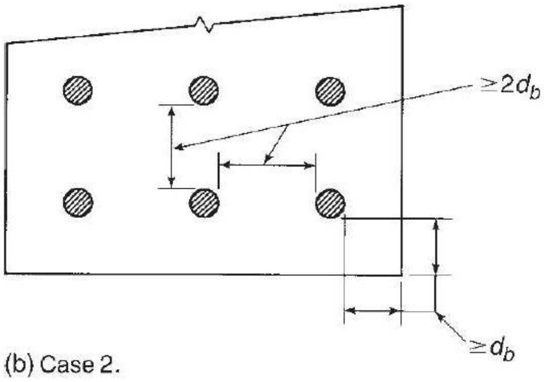


Figure 6.3-1: Explanation of Cases 1 and 2. Continue.

6.3.3 Summary of ACI Modification Factors of Deformed Bars in Tension

ACI modification factors adopted in basic equation, ***Article 6.3.1***, and simplified equations, ***Article 6.3.2***, have been summarized in ***Table 6.3-2***.

Table 6.3-2: Modification factors for development of deformed bars and deformed wires in tension, Table 25.4.2.4 of (ACI318M, 2014).

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Lightweight concrete, where f_{ct} is specified	In accordance with 19.2.4.3
	Normalweight concrete	1.0
Epoxy[1] ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement with clear cover less than $3d_b$ or clear spacing less than $6d_b$	1.5
	Epoxy-coated or zinc and epoxy dual-coated reinforcement for all other conditions	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Size ψ_s	No. 22 and larger bars	1.0
	No. 19 and smaller bars and deformed wires	0.8
Casting position[1] ψ_t	More than 300 mm of fresh concrete placed below horizontal reinforcement	1.3
	Other	1.0

[1]The product $\psi_t\psi_e$ need not exceed 1.7.

6.3.4 Further Simplified Tabular Values for Development Length

- Further simplifications are possible for the most common condition of
  - Normal density concrete ($\lambda = 1.0$),
  - Uncoated reinforcement ($\psi_e = 1.0$).
- With these simplifications, the development lengths, in terms of bar diameters, would be a function of f_c', f_y and the bar location factor ψ_t.
- Thus, development lengths are easily tabulated for the
  - Usual combinations of material strengths,
  - Bottom or top bars,
  - And for the restrictions on bar spacing, cover, and transverse steel defined.
- Results are given in ***Table 6.3-3*** below.

Table 6.3-3: Further Simplified tension development length in bar diameters l_d/d_b for uncoated bars and normal weight concrete, adopted from (Nilson, Design of Concrete Structures, 14th Edition, 2010):

		No. 6 (No. 19) and Smaller[a]			No. 7 (No. 22) and Larger		
		f'_c, MPa			f'_c, MPa		
	f_y, MPa	28	35	42	28	35	42
(1) Bottom bars							
Spacing, cover and ties as per Case *a* or *b*	280	25	23	21	32	28	26
	350	32	28	26	40	35	32
	420	38	34	31	47	42	39
Other cases	280	38	34	31	47	42	39
	350	47	42	39	59	53	48
	420	57	51	46	71	64	58
(2) Top bars							
Spacing, cover and ties as per Case *a* or *b*	280	33	29	27	41	37	34
	350	41	37	34	51	46	42
	420	49	44	40	62	55	50
Other cases	280	49	44	40	62	55	50
	350	62	55	50	77	69	63
	420	74	66	60	92	83	76

Case *a*: Clear spacing of bars being developed or spliced ≥ d_b, clear cover ≥ d_b, and stirrups or ties throughout l_d not less than the Code minimum.

Case *b*: Clear spacing of bars being developed or spliced ≥ $2d_b$, and clear cover not less than d_b.

[a]ACI Committee 408 recommends that the values indicated for bar sizes No. 7 (No. 22) and larger be used for all bar sizes.

6.3.5 Notes on the Three Different Methods

- When the confinement term, $(c_b + K_{tr})/d_b$, differs from the assumed values of 1.5 or 1.0, there would be a significant difference between values of the basic method and those of the other two methods.
- Slight differences between second and third methods are due to the unit systems. Where values for the third method have been originally prepared in US customary system.

6.3.6 ACI Lower Bound Limitation on l_d for Tension Rebars

According to ACI 25.4.2.1 l_d shall not be less than 300 mm.

6.3.7 Design Examples for Rebars in Tension

Example 6.3-1

Figure 6.3-2 below shows a beam-column joint in a continuous building frame with the following data:

- Based on frame analysis, the negative steel required at the end of the beam is 1870mm^2; two No. 36 bars are used providing A_s of 2012mm^2.
- The design will include No. 10 stirrups spaced four at 75mm, followed by a constant 125mm spacing in the region of the support, with 40mm clear cover.
- Normal weight concrete is to be used, with $f'_c = 28\ MPa$, and reinforcing bars have $f_y = 420\ MPa$.

Find the minimum distance l_d at which the negative bars can be cut off, based on development of the required steel area at the face of the column, using:

- The simplified equations,
- Further simplified tabulated values,
- The basic equation.

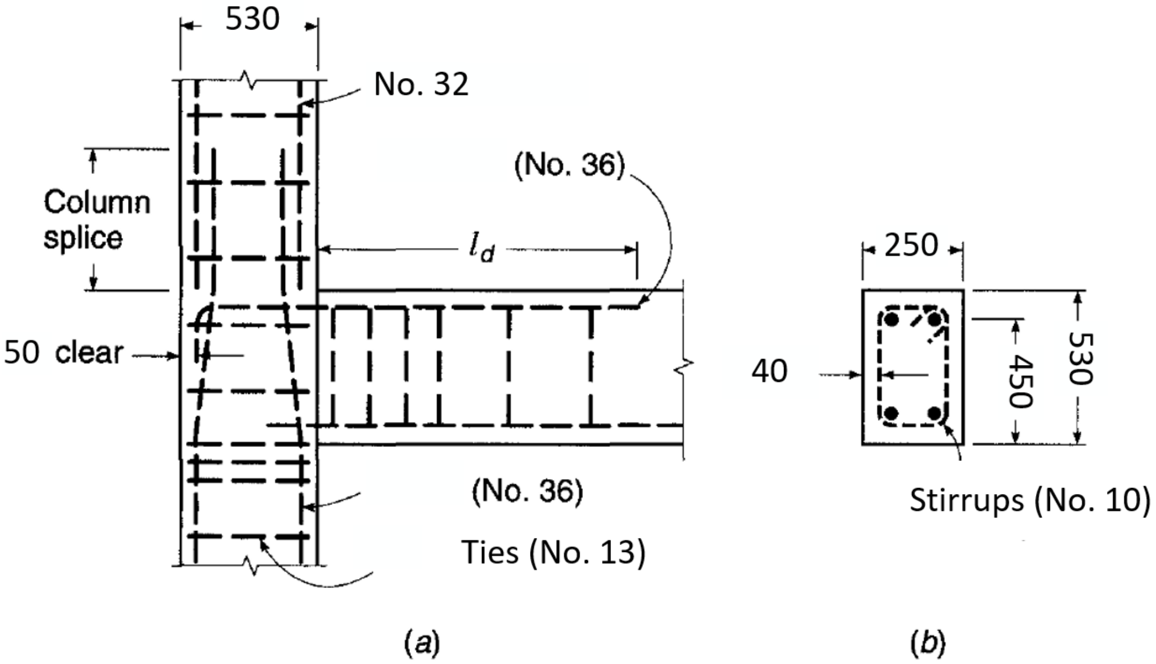


Figure 6.3-2: Bar details at beam-column joint for bar development examples.

Solution

- The simplified equations:

Checking for lateral spacing in the No. 36 bars determines that the clear distance between the bars is:

$$\frac{Clear\ distance}{d_b} = \frac{250 - 40 \times 2 - 2 \times 10 - 2 \times 36}{36} = 2.17 > 2d_b$$

$$\frac{Clear\ side\ cover}{d_b} = \frac{40 + 10}{36} = 1.39 > d_b$$

$$\frac{Clear\ top\ cover}{d_b} = \frac{530 - 450 - \frac{36}{2}}{36} = 1.72$$

These dimensions meet the restrictions stated in the second row of Table 6.3-1, and as $d_b > No. 22$ then:

Spacing and cover	No. 19 and smaller bars and deformed wires	No. 22 and larger bars
Clear spacing of bars or wires being developed or spliced not less than d_b, clear cover not less than d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum or Clear spacing of bars or wires being developed or spliced not less than $2d_b$ and clear cover not less than d_b	$\left(\frac{f_y \Psi_t \Psi_e}{2.1\lambda\sqrt{f_c'}}\right) d_b$	$\left(\frac{f_y \Psi_t \Psi_e}{1.7\lambda\sqrt{f_c'}}\right) d_b$
Other cases	$\left(\frac{f_y \Psi_t \Psi_e}{1.4\lambda\sqrt{f_c'}}\right) d_b$	$\left(\frac{f_y \Psi_t \Psi_e}{1.1\lambda\sqrt{f_c'}}\right) d_b$

$$l_d = \left(\frac{f_y \psi_t \psi_e}{1.7\lambda\sqrt{f_c'}}\right) d_d$$

For uncoated top bars with normal-density concrete:

$\psi_t = 1.3, \psi_e = 1.0, \lambda = 1.0$

$$l_d = \left(\frac{420 \times 1.3 \times 1.0}{1.7 \times 1.0 \times \sqrt{28}}\right) d_d = 60.7 d_b = 2185\ mm$$

This can be reduced by the ratio of steel required to that provided, so that the final development length is

$$l_d = 2185 \times \frac{1870}{2012} = 2031\ mm > 300\text{mm} \therefore \text{Ok.} \blacksquare$$

- Further simplified tabulated values:

		No. 6 (No. 19) and Smaller[a]			No. 7 (No. 22) and Larger		
		f'_c, MPa			f'_c, MPa		
	f_y, MPa	28	35	42	28	35	42
(1) Bottom bars							
Spacing, cover and ties as per Case *a* or *b*	280	25	23	21	32	28	26
	350	32	28	26	40	35	32
	420	38	34	31	47	42	39
Other cases	280	38	34	31	47	42	39
	350	47	42	39	59	53	48
	420	57	51	46	71	64	58
(2) Top bars							
Spacing, cover and ties as per Case *a* or *b*	280	33	29	27	41	37	34
	350	41	37	34	51	46	42
	420	49	44	40	62	55	50
Other cases	280	49	44	40	62	55	50
	350	62	55	50	77	69	63
	420	74	66	60	92	83	76

Case *a*: Clear spacing of bars being developed or spliced $\geq d_b$, clear cover $\geq d_b$, and stirrups or ties throughout l_d not less than the Code minimum.

Case *b*: Clear spacing of bars being developed or spliced $\geq 2d_b$, and clear cover not less than d_b.

[a] ACI Committee 408 recommends that the values indicated for bar sizes No. 7 (No. 22) and larger be used for all bar sizes.

$$\frac{l_d}{d_b} = 62 \Longrightarrow l_d = 62 \times 36 \times \frac{1870}{2012} = 2074mm > 300mm \therefore Ok. \blacksquare$$

- The basic equation:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}} \frac{\psi_t\psi_e\psi_s}{\frac{c_b + K_{tr}}{d_b}} \right) d_b$$

$$c_b = minimum\left(side\ cover\ to\ center\ of\ bar, top\ cover\ to\ center\ of\ bar, \frac{1}{2}S_{\frac{c}{c}}\right)$$

$$c_b = minimum\left(40 + 10 + \frac{36}{2}, 530 - 450, \frac{1}{2} \times (250 - 40 \times 2 - 10 \times 2 - \frac{36}{2} \times 2)\right)$$

$$c_b = minimum(68, 80, 57) = 57\ mm$$

The smallest of these three distances controls and $c_b = 57mm$. Potential splitting would be in the horizontal plane of the bars.

$$K_{tr} = \frac{40A_{tr}}{sn} = \frac{40 \times 2 \times \frac{(\pi \times 10^2)}{4}}{125 \times 2} = 25$$

$$Confinement\ Term\ = \frac{c_b + K_{tr}}{d_b} = \frac{57 + 25}{36} = 2.27 < 2.5 \therefore Ok.$$

$$\because d_b > 19 \therefore \psi_s = 1.0$$

$$l_d = \left(\frac{420}{1.1 \times 1.0 \times \sqrt{28}} \times \frac{1.3 \times 1.0 \times 1.0}{2.27}\right) d_b = 41.3d_d = 41.3 \times 36 = 1487\ mm$$

This can be reduced by the ratio of steel required to that provided, so that the final development length is

$$l_d = 1487 \times \frac{1870}{2012} = 1382\ mm > 300mm \therefore Ok\blacksquare$$

- Main conclusions about different approaches to compute l_d:
  - Clearly, the use of the more accurate equation permits a considerable reduction in development length.
  - Even though its use requires much more time and effort, it is justified if the design is to be repeated many times in a structure.

Example 6.3-2

Figure 6.3-3 below shows cross-section of a simply supported beam reinforced with four No. 25 bars that are confined with No. 10 stirrups spaced at 150mm. Determine the development length of the bars if the beam is made of normal-weight concrete, bars are not coated, $f_c' = 21\ MPa$, and $f_y = f_{yt} = 420\ MPa$.

In your solution, use:

- The simplified equations,
- Simplified tabulated values,
- The basic equation.

In your solution, assume that $S_{maximum}$ could be computed based on:

$$S_{maximum} = minimum\left(\frac{d}{2}, 600\right)$$

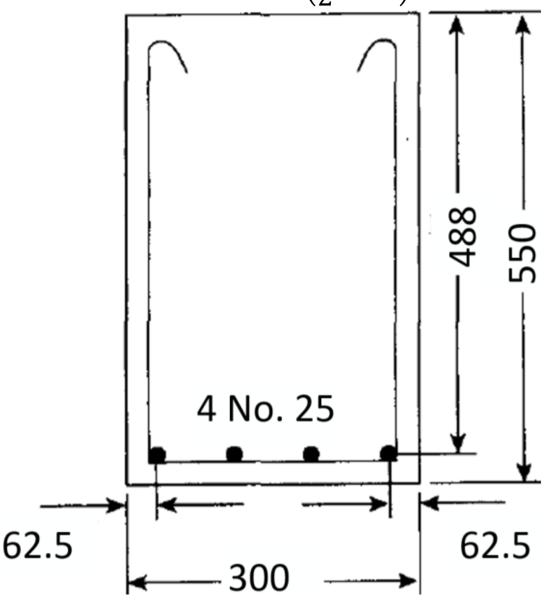
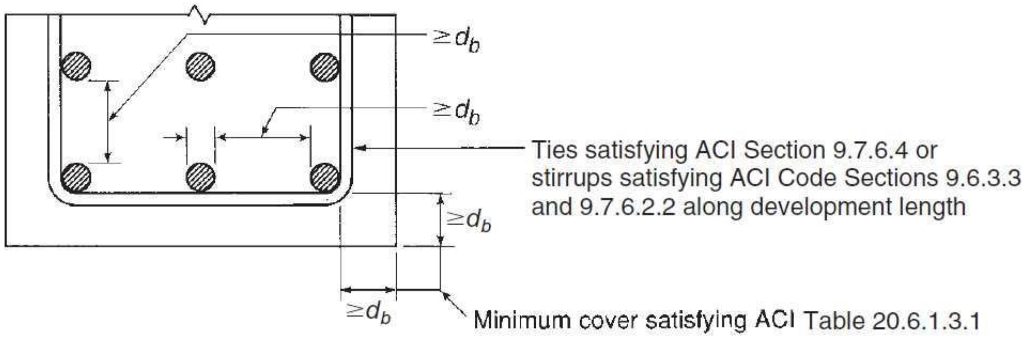


Figure 6.3-3: Beam Cross Section for Example 6.3-2.

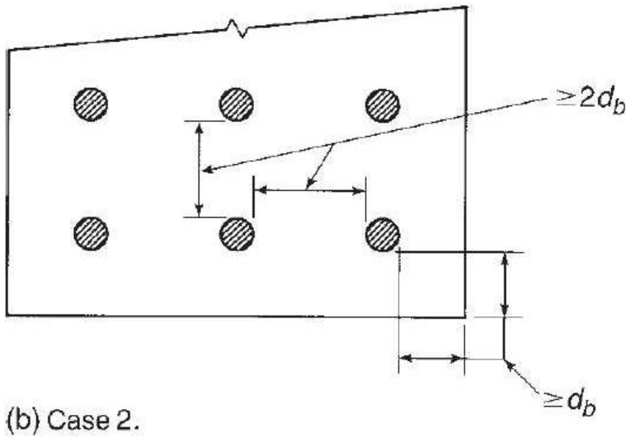
Solution

- The Simplified Relation:
  - Case for Spacing and Concrete Cover:

    Check conditions to see if spacing and concrete cover are to be classified as case 1, case 2, or other cases.



(a) Case 1.



(b) Case 2.

- For No. 25 bars, $d_b = 25mm$.

  $$Clear\ cover = 62.5 - \frac{25}{2} = 50mm > d_b$$

  $$Clear\ spacing\ between\ bars = \frac{300 - 2 \times 62.5 - 3 \times 25}{3} = 33.3 > d_b$$

  $$\because Clear\ Spacing = 33.3 < 2d_b$$

- Therefore, the provided stirrups should be compared with minimum limitations required by ACI Code:

  $$S_{maximum} = minimum\left(\frac{488}{2}, 600\right) = 244 > 150mm \therefore Ok.$$

  $$\mathrm{S_{for\ Av\ minimum}} = \mathrm{minimum}\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'} b_w} \mathrm{or} \frac{A_v f_{yt}}{0.35 b_w})$$

  $$\mathrm{S_{for\ Av\ minimum}} = \mathrm{minimum}\ (\frac{\frac{\pi \times 10^2}{4} \times 2 \times 420}{0.062\sqrt{21} \times 300} \mathrm{or} \frac{\frac{\pi \times 10^2}{4} \times 2 \times 420}{0.35 \times 300})$$

  $$\mathrm{S_{for\ Av\ minimum}} = \mathrm{minimum}\ (774\ \mathrm{or}\ 628) = 628\ mm > 150mm \therefore Ok.$$

  Then, rebars confinement could be classified as case 1, and as bar diameter is greater 22mm, then development could be computed based on following relation:

  $$l_d = \left(\frac{f_y \psi_t \psi_e}{1.7\lambda\sqrt{f_c'}}\right) d_b$$

  For bottom rebars:

  $\psi_t = 1.0$

  For uncoated rebars:

  $\psi_e = 1.0$

  For normal weight concrete:

  $\lambda = 1.0$

  $$l_d = \left(\frac{1.0 \times 1.0 \times 420}{1.7 \times 1.0 \times \sqrt{21}}\right) d_d = 53.9 d_b = 53.9 \times 25 \Longrightarrow l_d = 1348\ mm > 300mm\ \blacksquare$$

- Simplified Tabulated Values:

  As $f_c' = 21\ MPa$ has not been tabulated within the Table, then tabulated values cannot be used in this example.

- The basic equation:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s}{\frac{c_b + K_{tr}}{d_b}}\right) d_b$$

$$K_{tr} = \frac{40A_{tr}}{sn} = \frac{40 \times 2 \times \frac{(\pi \times 10^2)}{4}}{150 \times 4} = 10.5$$

$$c_b = minimum\left(side\ cover\ to\ center\ of\ bar, bottom\ cover\ to\ center\ of\ bar, \frac{1}{2} S_{\frac{c}{c}}\right)$$

$$c_b = minimum\left(62.5, 550 - 488, \left((300 - 2 \times 62.5) \times \frac{1}{3}\right) \times \frac{1}{2}\right)$$

$$c_b = minimum(62.5, 62, 29.2) = 29.2$$

$$\frac{c_b + K_{tr}}{d_b} = \frac{29.2 + 10.5}{25} = 1.59 < 2.5 \therefore Ok.$$

$$l_d = \left(\frac{420}{1.1 \times 1.0 \times \sqrt{21}} \frac{1.0 \times 1.0 \times 1.0}{1.59}\right) d_b = 52.4\ d_b$$

$$l_d = 52.4 \times 25 = 1310\ mm > 300mm \therefore Ok. \blacksquare$$

Example 6.3-3

Repeat ***Example 6.3-2*** if the beam is made of lightweight aggregate concrete, the bars are epoxy coated, and A_s required from analysis is 1800 mm^2. In your solution, use the simplified equations.

Solution

As confinement is same as Example 6.3-2, then confinement case could be classified as case 1, and development length could be computed based on following relation:

$$l_d = \left(\frac{f_y \psi_t \psi_e}{1.7\lambda\sqrt{f_c'}}\right) d_d$$

For bottom rebars:

$\psi_t = 1.0$

For coated rebars, and with cover less than 3d, then:

$\psi_e = 1.5$

According to ACI:

$\psi_t \psi_e = 1.0 \times 1.5 = 1.5 < 1.7 \therefore Ok.$

For a lightweight aggregate concrete:

$\lambda = 0.75$

$$l_d = \left(\frac{1.0 \times 1.5 \times 420}{1.7 \times 0.75 \times \sqrt{21}}\right) d_b \Rightarrow l_d = 108 d_d$$

Development could be reduced by ratio of steel required to that provided,

$$l_d = 108 \times \frac{1800}{4 \times \left(\pi \times \frac{25^2}{4}\right)} d_b$$

$$l_d = 108 \times \frac{1800}{4 \times \left(\pi \times \frac{25^2}{4}\right)} d_b = 99 \times 25 = 2475\, mm > 300mm \therefore Ok. \blacksquare$$

Example 6.3-4

Check adequacy of the embedded length of $1000mm$ indicated in ***Figure 6.3-4*** for development requirement of tension member. In your checking assume the following:

- $f_c' = 28\, MPa$ and $f_y = 420\, MPa$,
- Use ACI basic equation,
- Uncoated rebar,
- $A_{s\, required}/A_{s\, provided} \approx 1.0$.

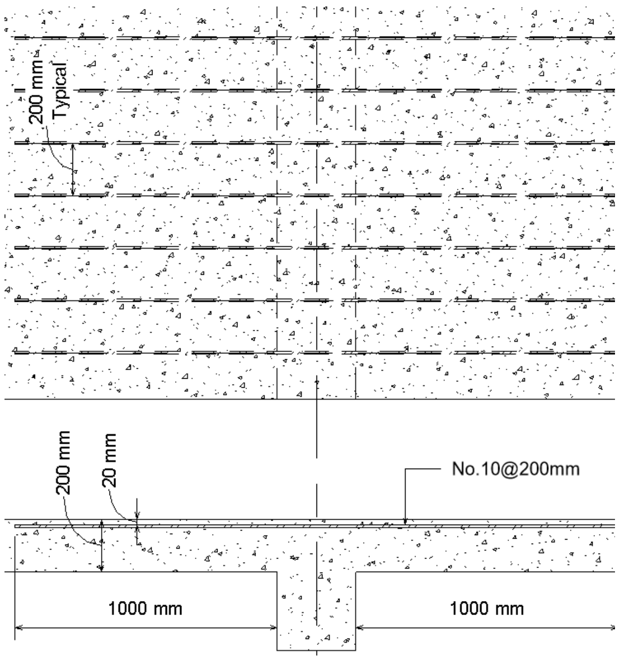


Figure 6.3-4: Slab rebars for Example 6.3-4.

Solution

According to the ACI basic relation, the development length for tension is:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s}{\frac{c_b + K_{tr}}{d_b}}\right) d_b$$

As no more 300mm of fresh concrete is cast below the rebar, therefore $\psi_t = 1.0$. For the uncoated rebars, $\psi_e = 1.0$. As the rebars have a size smaller than No.22, $\psi_s = 0.8$.

Regarding the confinement term, as there is no shear reinforcement, therefore $K_{tr} = 0$. The c_b is:

$$c_b = minimum\left(\left(\frac{10}{2} + 20\right) or \frac{200}{2}\right) = 25mm \Rightarrow \frac{c_b + K_{tr}}{d_b} = \frac{(25 + 0)}{10} = 2.5 \ngtr Code\ limit\ of\ 2.5 \therefore Ok.$$

The development length would be:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s}{\frac{c_b + K_{tr}}{d_b}}\right) d_b = \left(\left(\frac{420}{1.1 \times 1.0 \times \sqrt{28}}\right) \times \left(\frac{1.0 \times 1.0 \times 0.8}{2.5}\right)\right) d_b = 23 d_b = 23 \times 10$$
$$= 230\, mm < 300mm \therefore No\ Ok.$$

$\therefore l_d = minimum\ value\ according\ to\ ACI\ code = 300mm < l_{\text{embedded}} = 1000\, mm \therefore Ok$

Therefore, the proposed embedment is adequate from bond point of view but for the final decision, it should be checked for the requirements of the cutoff points.

Example 6.3-5

Use ACI basic relation to determine the development length of the bottom tensile rebars indicated in ***Figure 6.3-5***. In your solution assumes that $A_{s\,required} \approx 750\ mm^2$

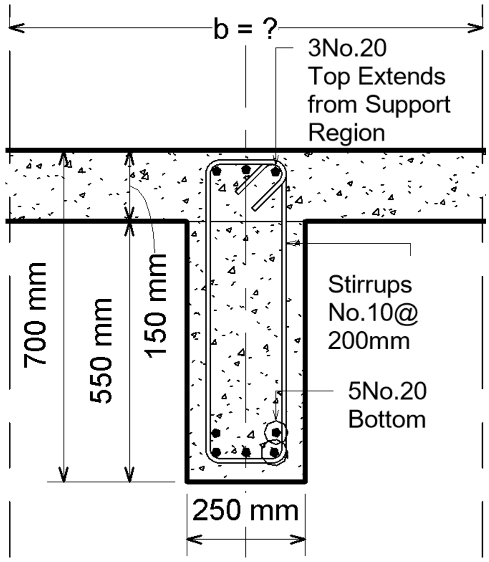


Figure 6.3-5: Beam section for Example 6.3-5.

Solution

The basic equation:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}}\frac{\psi_t\psi_e\psi_s}{\frac{c_b+K_{tr}}{d_b}}\right)d_b$$

$$c_b = minimum\left(\begin{matrix} side\ cover\ to\ center\ of\ bar, \\ top\ cover\ to\ center\ of\ bar, \frac{1}{2}S_{\frac{c}{c}}\end{matrix}\right)$$

$$c_b = minimum\left(40+10+\frac{20}{2}, 40+10+\frac{20}{2}, \frac{1}{2}\times(250 - 40\times 2 - 10\times 2 - \frac{20}{2}\times 2)\right)$$

$$c_b = minimum(60, 60, 65) = 60\ mm$$

The smallest of these three distances controls and $c_b = 60mm$.

$$K_{tr} = \frac{40A_{tr}}{sn} = \frac{40\times 2\times\frac{(\pi\times 10^2)}{4}}{150\times 5} = 8.38$$

$$Confinement\ Term = \frac{c_b+K_{tr}}{d_b} = \frac{60+8.38}{20} = 3.42 > 2.5 \therefore \text{Not } Ok.$$

Therefore, use confinement term of 2.5.

For uncoated bottom bars with normal-density concrete:

$\psi_t = 1.0, \psi_e = 1.0, \lambda = 1.0$

$\because d_b > 19 \therefore \psi_s = 1.0$

$$l_d = \left(\frac{420}{1.1\times 1.0\times\sqrt{28}}\times\frac{1.0\times 1.0\times 1.0}{2.5}\right)d_b = 28.9d_d = 28.9\times 20 = 578\ mm$$

This can be reduced by the ratio of steel required to that provided, so that the final development length is

$$l_d = 578\times\frac{750}{5\times 314} = 276\ mm < 300mm \therefore \text{Not } Ok$$

$\therefore \ell_{\text{d}} = 300\text{mm}$ ■

Example 6.3-6

Based on a structural analysis and design, a designer has adopted reinforcement of $No.12@200mm$ for the cantilever slab indicated in ***Figure 6.3-6*** above. Using ACI basic equation, determine the development length, l_d, for the adopted negative reinforcements and then check to see if the available overhang part is adequate for their anchorage.

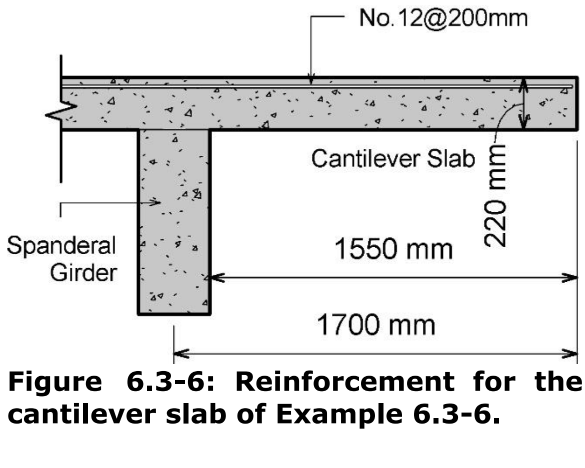


Figure 6.3-6: Reinforcement for the cantilever slab of Example 6.3-6.

Solution

According to basic equation of the ACI code, the development length for tension rebars, l_d, would be:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}}\frac{\psi_t\psi_e\psi_s}{\frac{c_b+K_{tr}}{d_b}}\right)d_b$$

$$c_b = minimum\left(\begin{matrix} side\ cover\ to\ center\ of\ bar, \\ top\ cover\ to\ center\ of\ bar, \frac{1}{2}S_{\frac{c}{c}}\end{matrix}\right)$$

$$c_b = \min\begin{pmatrix} \left(20+\frac{12}{2}\right), \\ \left(20+\frac{12}{2}\right), \left(\frac{1}{2}\times 200\right) \end{pmatrix} \Rightarrow c_b = \min(26,26,100) = 26\,mm$$

As there is no shear reinforcement in the cantilever slab, therefore:

$K_{tr} = 0$

As the concrete is normal weight concrete, $\lambda = 1.0$. For uncoated rebars, $\psi_e = 1.0$. As slab has thickness less than $300mm$, therefore, the rebars are considered bottom rebars from bond point of view. Finally, for rebars with size less than 19mm, $\psi_s = 0.8$.

$$l_d = \left(\left(\frac{420}{1.1\times 1.0\times\sqrt{28}}\right)\times\left(\frac{1.0\times 1.0\times 0.8}{\frac{26+0}{12}}\right)\right)d_b = 26.6d_b$$

As nothing is mentioned about $A_{s\,required}/A_{s\,provided}$, therefore it can be conservatively assumed 1.0.

$l_d = 26.6d_b = 26.6\times 12 = 319\,mm > 300\,mm \therefore Ok.$

Check adequacy of the available overhang part for anchorage of negative slab reinforcement:

$\because l_d = 319mm \ll 1550mm \therefore Ok.$

Example 6.3-7

Referring to beam section of ***Figure 6.3-7*** and based on ACI basic equation, what is the development length, l_d, for bottom rebars with 25mm diameter? In your solution, assume that the coefficient of $A_{s\,required}/A_{s\,provided}$ can conservatively be neglected. Based on your calculations, is a standard hook should adopt for the bottom rebars?

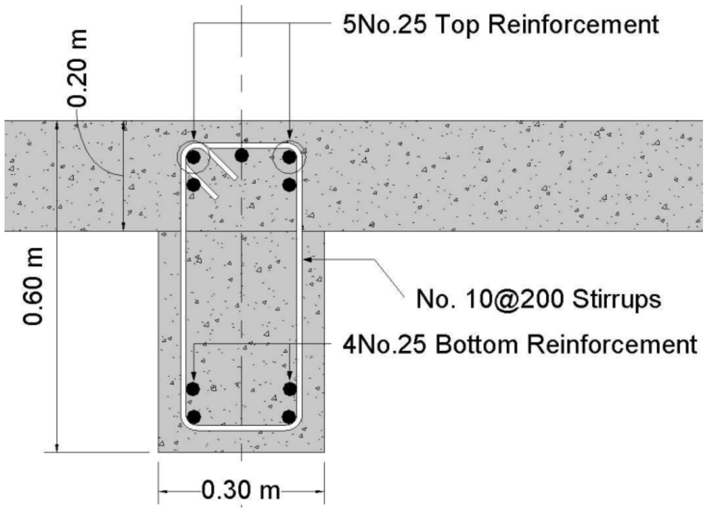


Figure 6.3-7: Beam for Example 6.3-7.

Solution

According to basic relation, development length for rebars in tension is:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}}\,\frac{\psi_t\psi_e\psi_s}{\frac{c_b+K_{tr}}{d_b}}\right)d_b$$

$$c_b = minimum\begin{pmatrix} side\ cover\ to\ center\ of\ bar, \\ \text{bottom}\ cover\ to\ center\ of\ bar, \frac{1}{2}S_{\frac{c}{c}} \end{pmatrix}$$

$$c_b = minimum\left(40+10+\frac{25}{2}, 40+10+\frac{25}{2}, \frac{1}{2}\times(300-40\times 2-10\times 2-\frac{25}{2}\times 2)\right)$$

$$c_b = minimum(62.5, 62.5, 87.5) = 62.5\,mm \Rightarrow K_{tr} = \frac{40A_{tr}}{sn} = \frac{40\times 2\times\frac{(\pi\times 10^2)}{4}}{200\times 4} = 7.9$$

$$Confinement\ Term = \frac{c_b+K_{tr}}{d_b} = \frac{62.5\ +7.9}{25} = 2.82 > 2.5 \therefore Not\ Ok.$$

To avoid overemphasis of confinement role, use:

$$\frac{c_b+K_{tr}}{d_b} = 2.5$$

For bottom bars, uncoated, and with normal-density concrete, we have the values of:

$\psi_t = 1.0, \psi_e = 1.0, \lambda = 1.0$

For 25mm rebars, greater than 19mm, we have:

$\psi_s = 1.0$

$$l_d = \left(\frac{420}{1.1\times 1.0\times\sqrt{28}}\,\frac{1.0\times 1.0\times 1.0}{2.5}\right)d_b = 28.7\,d_b = 28.7\times 25 = 717.5\,mm < \frac{6000}{2}\,mm \therefore Ok$$

Therefore, bottom rebars can be developed with available room and no hook should be adopted.

Example 6.3-8

Referring to beam section of ***Figure 6.3-7*** and based on ACI basic equation, what is the development length, l_d, for bottom rebars with 20mm diameter? In your solution, assume that the coefficient of $A_{s\,required}/A_{s\,provided}$ can conservatively be neglected. Based on your calculations, is a standard hook should adopt for the bottom rebars?

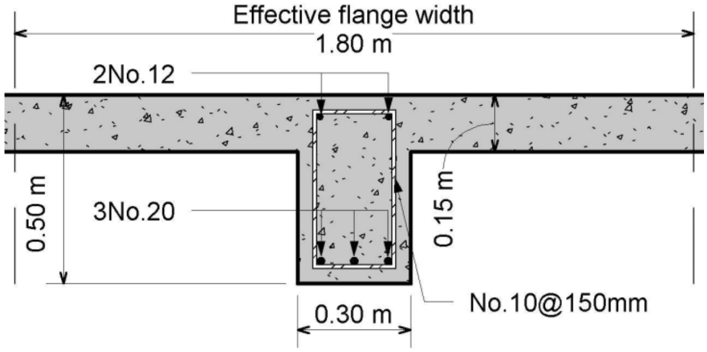


Figure 6.3-8: Beam for Example 6.3-8

Solution

According to basic relation, development length for rebars in tension is:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}} \frac{\psi_t\psi_e\psi_s}{\frac{c_b + K_{tr}}{d_b}} \right) d_b$$

$$c_b = minimum \left(\begin{array}{c} side\ cover\ to\ center\ of\ bar, \\ \text{bottom}\ cover\ to\ center\ of\ bar, \frac{1}{2} S_{\frac{c}{c}} \end{array} \right)$$

$$c_b = minimum \left(40 + 10 + \frac{20}{2}, 40 + 10 + \frac{20}{2}, \frac{1}{2} \times (300 - 40 \times 2 - 10 \times 2 - \frac{20}{2} \times 2) \right)$$

$$c_b = minimum(60, 60, 90) = 60\ mm \implies K_{tr} = \frac{40A_{tr}}{sn} = \frac{40 \times 2 \times \frac{(\pi \times 10^2)}{4}}{150 \times 3} \approx 14$$

$$Confinement\ Term = \frac{c_b + K_{tr}}{d_b} = \frac{60\ + 14}{20} = 3.7 > 2.5 \therefore Not\ Ok.$$

To avoid overemphasis of confinement role, use:

$$\frac{c_b + K_{tr}}{d_b} = 2.5$$

For bottom uncoated bars and with normal-density concrete, values of ψs would be:

$\psi_t = 1.0, \psi_e = 1.0, \lambda = 1.0$

For 20mm rebars, i.e. greater than 19mm, ψ_s is:

$\psi_s = 1.0$

$$l_d = \left(\frac{420}{1.1 \times 1.0 \times \sqrt{28}} \frac{1.0 \times 1.0 \times 1.0}{2.5} \right) d_b = 28.7\ d_b = 28.7 \times 20 = 574\ mm < \frac{6300 + 300 \times 2}{2}$$
$$= 3450mm \therefore Ok$$

Therefore, bottom rebars can be developed with available room and no hook should be adopted.

6.4 ANCHORAGE OF TENSION BARS BY HOOKS

In the event that the desired tensile stress in a bar cannot be developed by bond alone, it is necessary to provide special anchorage at the ends of the bar, usually by means of:

- a 90° hook,
- a 180° hook,
- a headed bar (out the scope of this course).

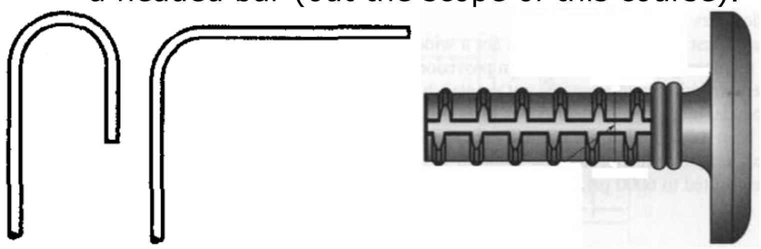


Figure 6.4-1: 90° Hook, 180° Hook, and Headed Bar.

6.4.1 BASIC CONCEPTS

- Forces Acting on Hooks:
  - A 90° hook loaded in tension develops forces in the manner shown in ***Figure 6.4-2*** below.
  - The stress in the bar is resisted by the bond on the surface of the bar and by the bearing on the concrete inside the hook.

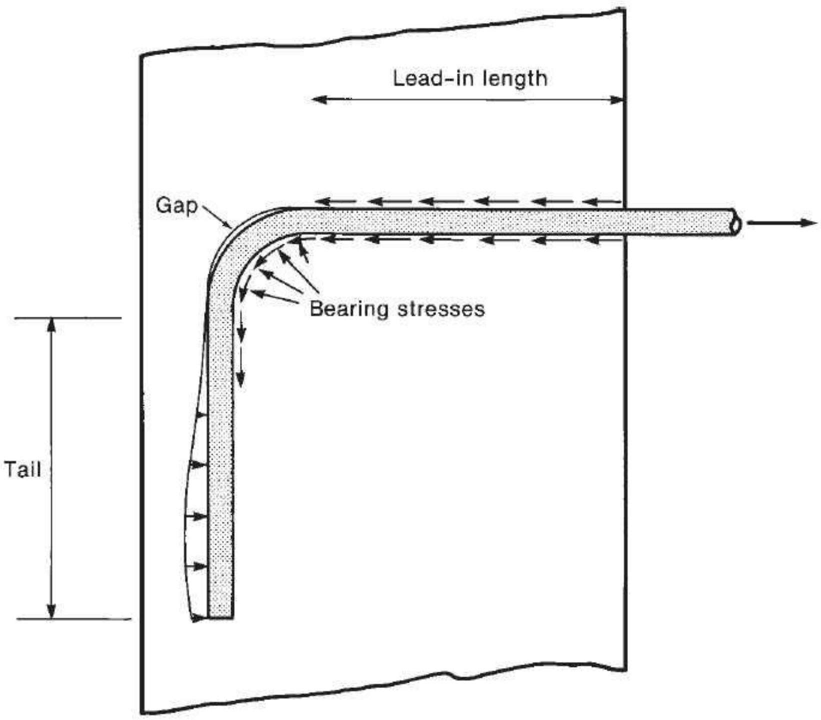


Figure 6.4-2: Forces Acting on a 90-degree Hooked Bars.

- Mode of Failure:
  - Hooked bars resist pullout by the combined actions of bond along the straight length of bar leading to the hook and anchorage provided by the hook. Then ***Pullout strength of hook is okay.***
  - Tests indicate that the ***main cause of failure of hooked bars in tension is splitting of the concrete in the plane of the hook***.
- Splitting Stresses:
  - This splitting is due to the very high stresses in the concrete inside of the hook; these stresses are influenced mainly by:
    - the bar diameter d_b for a given tensile force,
    - the radius of bar bend.
  - Resistance to splitting:

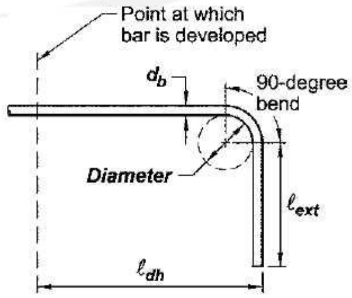
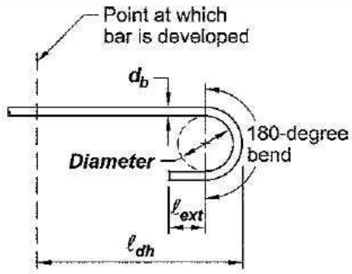
    Resistance to splitting has been found to depend on ***the concrete cover for the hooked bar***, measured laterally from the edge of the member to the bar perpendicular to the plane of the hook, and measured to the top (or bottom) of the member from the point where the hook starts, parallel to the plane of the hook.

- Increase resistance to splitting:
  The strength of the anchorage can be substantially increased by providing confinement steel in the form of closed stirrups or ties.

6.4.2 Standard Hook Dimensions

According to (ACI318M, 2014), ***25.3.1***, Standard hooks for the development of deformed bars in tension shall conform to ***Table 6.4-1*** below, ***Table 25.3.1*** of the (ACI318M, 2014).

Table 6.4-1: Standard hook geometry for development of deformed bars in tension, Table 25.3.1 of the (ACI318M, 2014).

Type of standard hook	Bar size	Minimum inside bend diameter, mm	Straight extension[1] ℓ_{ext}, mm	Type of standard hook
90-degree hook	No. 10 through No. 25	$6d_b$	$12d_b$	
	No. 29 through No. 36	$8d_b$		
	No. 43 and No. 57	$10d_b$		
180-degree hook	No. 10 through No. 25	$6d_b$	Greater of $4d_b$ and 65 mm	
	No. 29 through No. 36	$8d_b$		
	No. 43 and No. 57	$10d_b$		

[1]A standard hook for deformed bars in tension includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

6.4.3 Design of Hooked Anchorages

- The design process described in ACI Code does not distinguish between 90° and 180° hooks or between top and bottom bar hooks.
- ACI design procedure for hooked anchorage can be summarized as follows:
  - Compute l_{dh} based on a basic relation of ***ACI 25.4.3.1***,
  - Reduce l_{dh} by multiplier of ***ACI 25.4.3.2*** when applicable,
  - Check ACI provisions related to discontinuous end (***ACI 25.4.3.3***),
  - Check l_{dh} with minimum code limitations (***ACI 25.4.3.1***).

6.4.3.1 Basic Relation for l_{dh}

According to ***ACI 25.4.3.1***, a total development length, l_{dh}, defined as shown in ***Table 6.4-1*** above, for deformed bars in tension terminating in a standard hook shall be:

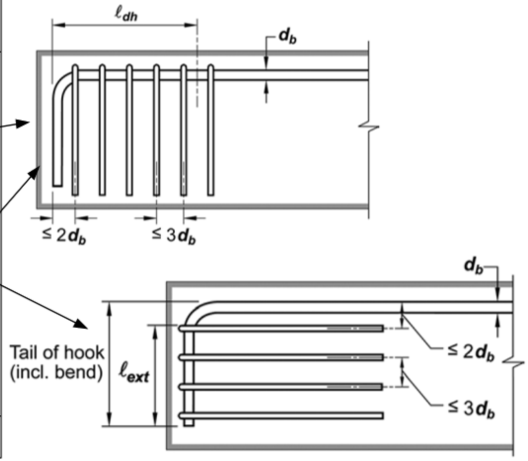
$$l_{dh} = \left(\frac{0.24\, f_y \psi_e \psi_c \psi_r}{\lambda \sqrt{f_c'}}\right) d_b \qquad \text{Eq. 6.4-1}$$

6.4.3.2 Multiplier Factors of ACI 12.5.3

- According to ***ACI 25.4.3.2***, for the calculation of l_{dh}, modification factors shall be in accordance with ***Table 6.4-2*** below, ***Table 25.4.3.2*** of (ACI318M, 2014).
- Factors ψ_c and ψ_r shall be permitted to be taken as 1.0.
- According to ***25.4.10.1*** of (ACI318M, 2014), reduction of development lengths defined by provision above shall be permitted by use of the ratio:
  $(As_{required})/(As_{provided})$

Table 6.4-2: Modification factors for development of hooked bars in tension, Table 25.4.3.2 of (ACI318M, 2014).

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Epoxy ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Cover ψ_c	For No. 36 bar and smaller hooks with side cover (normal to plane of hook) $\geq$ 65 mm and for 90-degree hook with cover on bar extension beyond hook $\geq$ 50 mm	0.7
	Other	1.0
Confining reinforcement ψ_r[2]	For 90-degree hooks of No. 36 and smaller bars (1) enclosed along ℓ_{dh} within ties or stirrups[1] perpendicular to ℓ_{dh} at $s \leq 3d_b$, or (2) enclosed along the bar extension beyond hook including the bend within ties or stirrups[1] perpendicular to ℓ_{ext} at $s \leq 3d_b$; For 180-degree hooks of No. 36 and smaller bars enclosed along ℓ_{dh} within ties or stirrups[1] perpendicular to ℓ_{dh} at $s \leq 3d_b$	0.8
	Other	1.0



[1]The first tie or stirrup shall enclose the bent portion of the hook within $2d_b$ of the outside of the bend.

[2]d_b is the nominal diameter of the hooked bar.

6.4.3.3 Transverse Confinement Steel at Discontinuous Ends

6.4.3.3.1 Provisions for Discontinues Ends

According to (ACI318M, 2014), ***25.4.3.3***, for bars being developed by a standard hook:

- At discontinuous ends of members,
- With **both** side cover and top (or bottom) cover to hook less than 65 mm

provisions (a) through (c) shall be satisfied, see ***Figure 6.4-3*** below:

(a) The hook shall be enclosed along l_{dh} within ties or stirrups perpendicular to ℓ_{dh} at $s \leq 3d_b$,

(b) The first tie or stirrup shall enclose the bent portion of the hook within $2d_b$ of the outside of the bend,

(c) ψ_r shall be taken as 1.0 in calculating ℓdh in accordance with Table 6.4-2 above.

where d_b is the nominal diameter of the hooked bar.

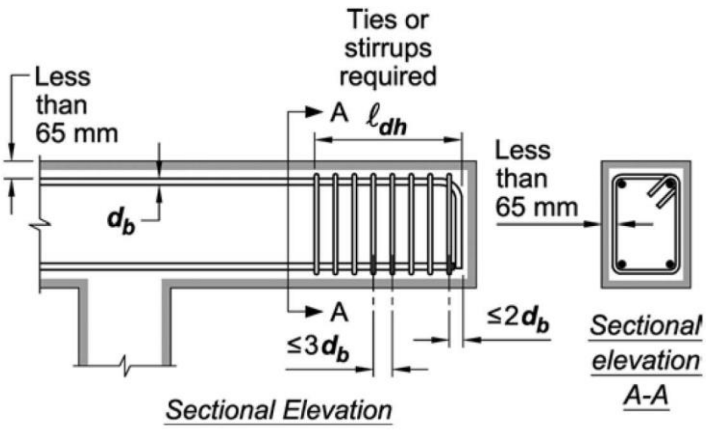


Figure 6.4-3: Transverse confinement steel at discontinuous ends.

6.4.3.3.2 Discontinuous Ends

Cases where hooks may require ties or stirrups for confinement are, adopted from ***R25.4.3.3*** of (ACI318M, 2014)

- At ends of simply-supported beams,
- At the free end of cantilevers,
- At ends of members framing into a joint where members do not extend beyond the joint.

6.4.3.3.3 Discontinuous Ends of Slabs

Above provisions do not apply for hooked bars at discontinuous ends of slabs where confinement is provided by the slab on both sides and perpendicular to the plane of the hook.

6.4.3.4 ACI Minimum Limitations on Hook Development Length

According to ***ACI 25.4.3.1***,
$l_{dh} \geq maximum\ (8d_b, 150mm)$

6.4.3.5 Hooks Effectiveness in Compression Rebars

According to ***ACI 25.4.1.2***, hooks shall not be considered effective in developing bars in compression.

6.4.4 DESIGN EXAMPLES FOR TENSION ANCHORAGE WITH HOOK

Example 6.4-1

Referring to the beam-column joint of ***Example 6.3-1*** that is represented below for convenience, the No. 36 negative bars are to be extended into the column and terminated in a standard 90° hook, keeping 50mm clear to the outside face of the column. The column width in the direction of beam width is 400mm. Find the minimum length of embedment of the hook past the column face, and specify the hook details. As in ***Example 6.3-1***, assume that normal weight concrete is to be used, with $f_c' = 28\ MPa$, and reinforcing bars have $f_y = 420\ MPa$.

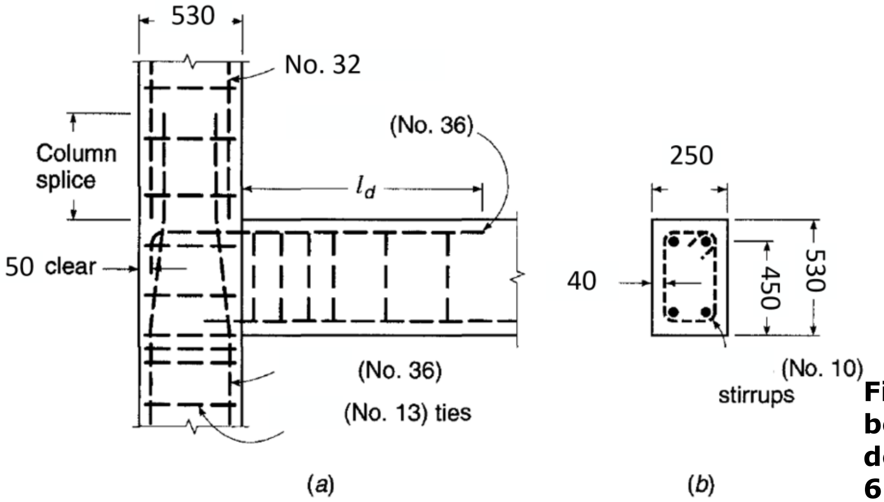


Figure 6.4-4: Bar details at beam-column joint for bar development of Example 6.3-1 (Re-presenting).

Figure 6.4-5: Bar details at beam-column joint for bar development examples (3D Views).

Solution

Basic Relation

The development length for hooked bars, measured from the critical section along the bar to the far side of the vertical hook, is given by:

$$l_{dh} = \left(\frac{0.24 f_y \psi_e \psi_c \psi_r}{\lambda \sqrt{f_c'}}\right) d_b$$

For uncoated or zinc-coated (galvanized):

$\psi_e = 1.0$

Confinement Provided by Increased Cove:

$$Side\ cover = 40 + \frac{400 - 250}{2} = 115\ mm > 65mm$$

In this case, side cover for the (No. 36) bars exceeds 65mm and cover beyond the bent bar is adequate, so:

$\psi_c = 0.7$

As there is no confinement reinforcement, therefore:

$\psi_r = 1.0$

$$l_{dh} = \left(\frac{0.24 \times 420 \times 1.0 \times 0.7 \times 1.0}{1.0 \times \sqrt{28}}\right) d_b = 13.3\ d_b$$

Multiplication Factor of $A_{S\ Required}$/ $A_{S\ Provided}$

As

$$\frac{A_{S\ Required}}{A_{S\ Provided}} = \frac{1870}{2012} = 0.929$$

Then:

$$l_{dh} = 0.929 \times 13.3\ d_b = 12.4\ d_b = 12.4 \times 36 = 446\ mm$$

Code Minimum Limitations

Check with code minimum limitations:

$$l_{dh} = \ 446mm\ ?\ maximum\ (8 \times 36, 150mm) \Rightarrow l_{dh} = \ 446mm\ ?\ maximum\ (288, 150mm)$$

$$l_{dh} = \ 446mm > \ 288 \therefore Ok.$$

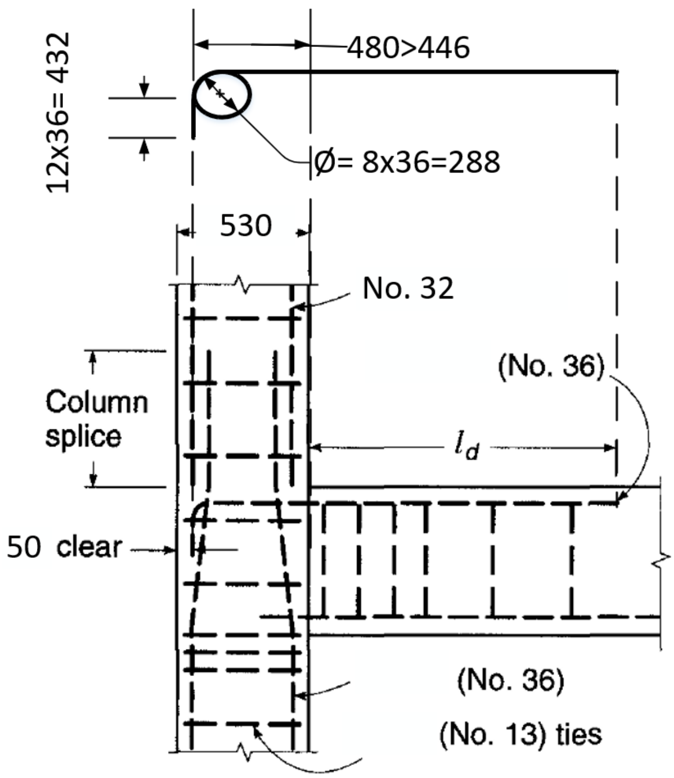
$$l_{dh} = \ 446\ mm < 530 - 50 = 480 \therefore Ok.$$

$$l_{dh} = \ 445\ mm\ \blacksquare$$

Transverse Confinement Steel at Discontinuous Ends:

As side, bottom, and top coves are greater than 65mm, then no need for transverse confinement reinforcement.

Final Details for Hooked Bar:



Example 6.4-2

Design standard hook details to anchor exterior negative slab reinforcement to the supporting beam, see ***Figure 6.4-6*** below. In your solution assume $f_c' = 28\,MPa$, $f_y = 420\,MPa$, and $A_{s\,required}/A_{s\,provided}$ is $\frac{216}{524}$.

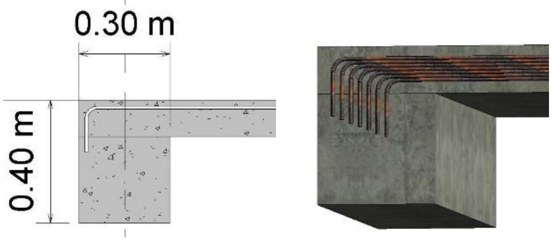


Figure 6.4-6: Details for exterior negative reinforcement of a slab to the supporting beam.

Solution

Basic Relation

The development length for hooked bars, measured from the critical section along the bar to the far side of the vertical hook, is given by:

$$l_{dh} = \left(\frac{0.24 f_y \psi_e \psi_c \psi_r}{\lambda\sqrt{f_c'}}\right) d_b$$

For normal weight concrete:

$\lambda = 1.0$

For uncoated or zinc-coated (galvanized):

$\psi_e = 1.0$

As cover extends beyond hook is less than 50mm, then:

$\psi_c = 1.0$

As there are no transverse confinement reinforcement, then

$\psi_r = 1.0$

$$l_{dh} = \left(\frac{0.24 \times 420 \times 1.0 \times 1.0 \times 1.0}{1.0 \times \sqrt{28}}\right) d_b = 19 d_b = 19 \times 10 = 190mm$$

Multiplication Factor of As Required/ As Provided

Finally, reduction factor of $A_{s\,Required}/A_{s\,Provided}$ is applicable.

$$l_{dh} = 190 \times \frac{216}{524} = 78.3\,mm$$

Transverse Confinement Steel at Discontinuous Ends:

Provisions of discontinuous edges do not apply for hooked bars at discontinuous ends of slabs where confinement is provided by the slab on both sides and perpendicular to the plane of the hook.

ACI Minimum Limitations on Hook Development Length:

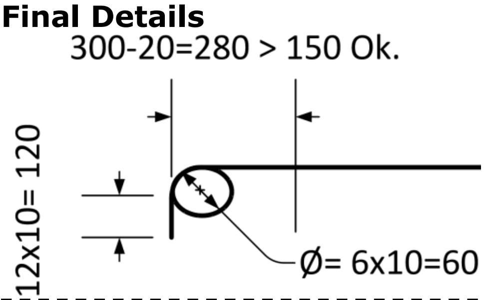
$l_{dh} > maximum\ (8d_b, 150mm)$

$l_{dh} = 78.3\,mm\ > maximum\ (8 \times 10, 150mm) = 150mm \therefore Not\ Ok.$

Then, use

$l_{dh} = 150mm$

Final Details



Example 6.4-3

The short beam shown in ***Figure 6.4-7*** cantilevers from a supporting column at the left. It must carry a calculated dead load of $29\,kN/m$ including its own weight and a service live load of $38\,kN/m$. Based on these loads, required reinforcement of $A_{s\,Required} = 1593\ mm^2$ have been determined. Tensile flexural reinforcement consists of ***two No. 36 bars*** have been provided. ***Transverse No. 10 U stirrups with 40mm*** cover are provided at the following spacings from the face of the column: $\mathbf{100mm}$, $\mathbf{3\ at\ 200mm}$, and $\mathbf{5\ at\ 260mm}$.

a) If the flexural and shear steel use $f_y = 420\ MPa$ and if the beam uses ***lightweight concrete*** having $f_c' = 28\ MPa$, check to see if proper development length can be provided for the No. 36 bars. Use the ***simplified development length equations***.

b) If the column material strengths are $f_y = 420\ MPa$ and $f_c' = 35\ MPa$ (***normalweight concrete***), check to see if adequate embedment can be provided within the column for the No. 36 bars. In your checking, use ***the basic equation***.

c) If hooks are required, specify detailed dimensions.

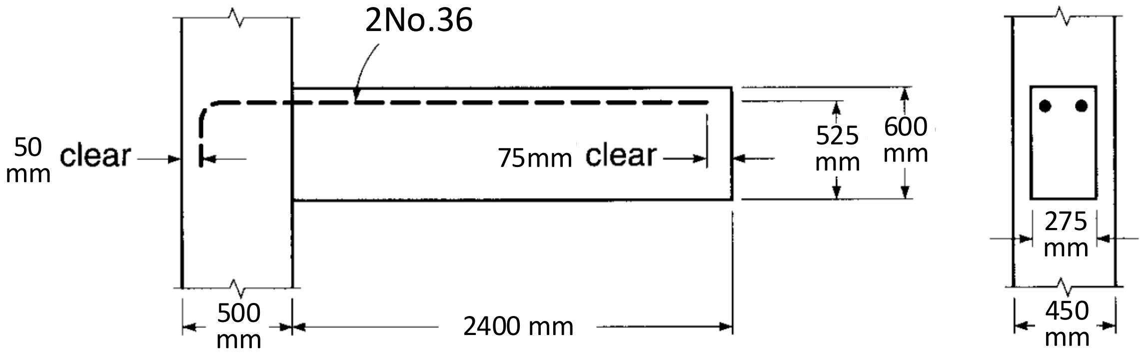
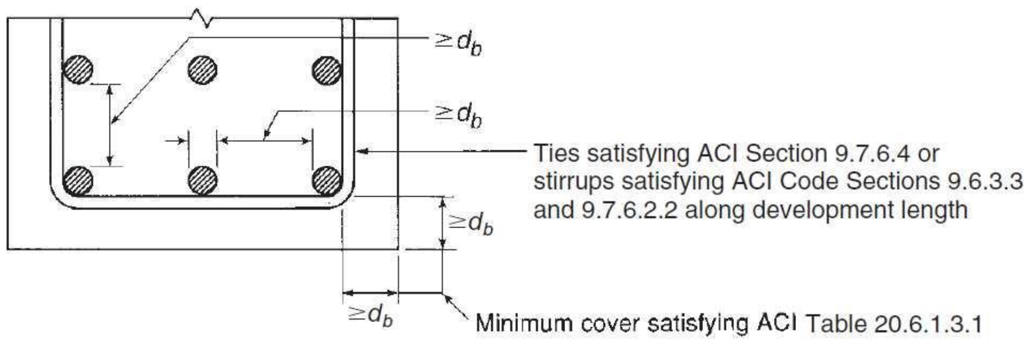


Figure 6.4-7: Cantilever beam for Example 6.4-3.

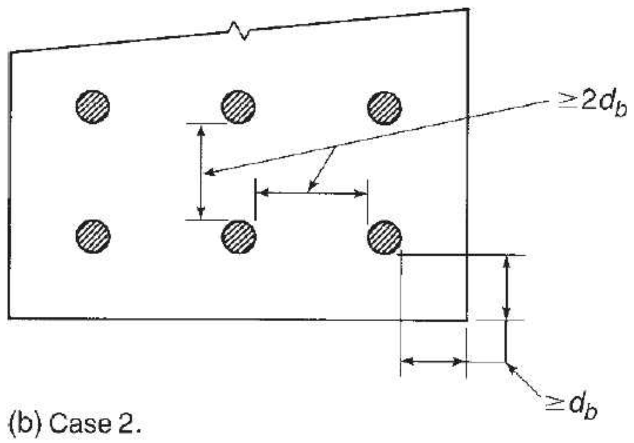
Solution

a) Anchorage to beam based on the simplified equations:

Checking for lateral spacing to determine if rebar is confined according to Case 1 or Case 2 below:



(a) Case 1.



(b) Case 2.

Start with checking for Case 2 where confinement depends on concrete mass only:

$$\frac{Clear\ distance}{d_b} = \frac{275 - 40 \times 2 - 2 \times 10 - 2 \times 36}{36} = 2.86 > 2d_b \therefore \text{Ok.}$$

$$\frac{Clear\ side\ cover}{d_b} = \frac{40 + 10}{36} = 1.39 > d_b \therefore Ok.$$

$$\frac{Clear\ top\ cover}{d_b} = \frac{600 - 525 - \frac{36}{2}}{36} = 1.58 > \text{d}_\text{b} \therefore Ok.$$

Therefore, the rebars can be considered confined depends on concrete mass, Case 2. With bar diameter of $36mm$, i.e. greater than No. 22, the required development length would be:

Spacing and cover	No. 19 and smaller bars and deformed wires	No. 22 and larger bars
Clear spacing of bars or wires being developed or spliced not less than d_b, clear cover not less than d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum or Clear spacing of bars or wires being developed or spliced not less than $2d_b$ and clear cover not less than d_b	$\left(\frac{f_y\psi_t\psi_e}{2.1\lambda\sqrt{f_c'}}\right)d_b$	$\left(\frac{f_y\psi_t\psi_e}{1.7\lambda\sqrt{f_c'}}\right)d_b$
Other cases	$\left(\frac{f_y\psi_t\psi_e}{1.4\lambda\sqrt{f_c'}}\right)d_b$	$\left(\frac{f_y\psi_t\psi_e}{1.1\lambda\sqrt{f_c'}}\right)d_b$

$$l_d = \left(\frac{f_y\psi_t\psi_e}{1.7\lambda\sqrt{f_c'}}\right)d_d$$

Then for top bars, uncoated, and with lightweight concrete, we have the values of:

$\psi_t = 1.3, \psi_e = 1.0, \lambda = 0.75$

$$l_d = \left(\frac{420 \times 1.3 \times 1.0}{1.7 \times 0.75 \times \sqrt{28}}\right)d_d = 80.9d_b = 80.9 \times 36 = 2912\ mm$$

This can be reduced by the ratio of steel required to that provided, so that the final development length is

$$l_d = 2912 \times \frac{\mathrm{A_{s\,Required}}}{A_{s\,Provided}} = 2912 \times \frac{1593}{2 \times \frac{\pi \times 36^2}{4}} = 2279 > 300\mathrm{mm} \therefore \mathrm{Ok.}$$

$l_d = 2279mm$ ■

$\because l_d = 2279mm < 2400mm \therefore Ok.$

b) Anchorage to the column using straight rebar with l_d determined based on the basic relation:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}}\frac{\psi_t\psi_e\psi_s}{\frac{c_b + K_{tr}}{d_b}}\right)d_b$$

$$c_b = minimum\left(side\ cover\ to\ center\ of\ bar, top\ cover\ to\ center\ of\ bar, \frac{1}{2}S_{\frac{c}{c}}\right)$$

$$c_b = \min\left(\left(\frac{450-275}{2} + 40 + 10 + \frac{36}{2}\right), \infty, \left(\frac{1}{2} \times (275 - 40 \times 2 - 10 \times 2 - \frac{36}{2} \times 2)\right)\right)$$

$$= \min(156, \infty, 69.5) = 69.5mm$$

As no stirrups have been adopted in column joint region, therefore the parameter of k_{tr} which simulate stirrup confinement would be:

$$K_{tr} = \lim_{s\to\infty}\frac{40A_{tr}}{sn} = 0$$

Finally, the confinement term would be

$$Confinement\ Term = \frac{c_b + K_{tr}}{d_b} = \frac{69.5 + 0}{36} = 1.93 < 2.5 \therefore Ok.$$

$\because d_b > 19 \therefore \psi_s = 1.0$

$$l_d = \left(\frac{420}{1.1 \times 1.0 \times \sqrt{35}} \times \frac{1.3 \times 1.0 \times 1.0}{1.93}\right)d_b = 43.5d_d = 43.5 \times 36 = 1566\ mm$$

This can be reduced by the ratio of steel required to that provided, so that the final development length is:

$$l_d = 1566 \times \frac{\mathrm{A_{s\,Required}}}{A_{s\,Provided}} = 1566 \times \frac{1593}{2 \times \frac{\pi \times 36^2}{4}} = 1225mm > 300\mathrm{mm} \therefore \mathrm{Ok.}$$

$l_d = 1225\ mm$ ■ $\Rightarrow \because l_d = 1225mm > (500 - 50) = 450mm \therefore Not\ Ok.$

Therefore, hook should be adopted to anchor the rebar to the column.

c) Anchorage to the column using standard hook:

The development length for hooked bars, measured from the critical section along the bar to the far side of the vertical hook, is given by:

$$l_{dh} = \left(\frac{0.24 f_y \psi_e \psi_c \psi_r}{\lambda\sqrt{f_c'}}\right) d_b$$

For uncoated or zinc-coated (galvanized):

$\psi_e = 1.0$

Confinement provided by increased cove:

$$Side\ cover = 40 + \frac{450 - 275}{2} = 127.5\ mm > 65mm$$

In this case, side cover for the (No. 36) bars exceeds 65mm and cover beyond the bent bar is adequate, so:

$\psi_c = 0.7$

As there is no confinement reinforcement, therefore:

$\psi_r = 1.0$

With above parameters, the relation for l_{dh} would be:

$$l_{dh} = \left(\frac{0.24 \times 420 \times 1.0 \times 0.7 \times 1.0}{1.0 \times \sqrt{35}}\right) d_b = 11.9\ d_b$$

Multiplication factor of $A_{s\ Required}/A_{s\ Provided}$:

As

$$\frac{A_{S\ Required}}{A_{S\ Provided}} = \frac{1593}{2 \times \frac{\pi \times 36^2}{4}}$$

Then:

$$l_{dh} = \frac{1593}{2 \times \frac{\pi \times 36^2}{4}} \times 11.9\ d_b = 9.31\ d_b = 9.31 \times 36 = 335\ mm$$

Code Minimum Limitations

Check with code minimum limitations:

$l_{dh} =\ 335mm\ ?\ maximum\ (8 \times 36, 150mm) \Rightarrow l_{dh} =\ 335mm\ ?\ maximum\ (288, 150mm)$

$l_{dh} =\ 335mm >\ 288 \therefore Ok.$

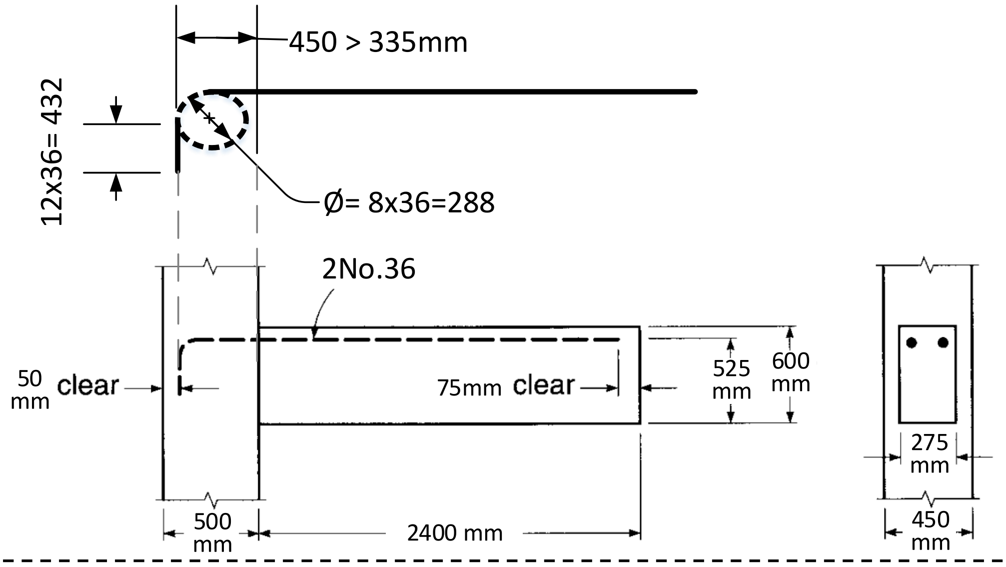
Check with available room:

$l_{dh} =\ 335\ mm < 500 - 50 = 450 \therefore Ok.$

Transverse Confinement Steel at Discontinuous Ends:

As side, or bottom or top coves are greater than 65mm, then no need for transverse confinement reinforcement.

Final details for hooked bar:



6.5 Anchorage Requirements for Web Reinforcement

This issue has been discussed thoroughly in ***Chapter 5***.

6.6 Development of Bars in Compression

6.6.1 Basic Concepts

- Reinforcement may be required to develop its compressive strength by embedment under various circumstances, e.g., where bars transfer their share of column loads to a supporting footing or basement walls or where lap splices are made of compression bars in column.
- In the case of bars in compression,
  - A part of the total force is transferred by bond along the embedded length,
  - And a part is transferred by end bearing of the bars on the concrete.
- Main difference between development length in tension and in compression:
  - Because the surrounding concrete is relatively free of cracks
  - And because of the beneficial effect of end bearing,

  shorter basic development lengths are permissible for compression bars than for tension bars.
- Transverse confinement steel:

  If transverse confinement steel is present, such as spiral column reinforcement or special spiral steel around an individual bar, the ***required development length is further reduced***.

6.6.2 ACI Relations

- Basic Relation

  According to ACI ***25.4.9.2***, development length for rebars in compression (l_{dc}) shall be computed based on following relation:

$$l_{dc} = maximum\left(\frac{0.24 f_y \psi_r}{\lambda\sqrt{f_c{}'}} d_b \; or \; 0.043 f_y \psi_r d_b\right) \quad \text{Eq. 6.6-1}$$

- Modifications Factors
  - According to ***25.4.9.3***, for the calculation of ℓ_{dc}, modification factors shall be in accordance with ***Table 6.6-1*** above, except ψ_r shall be permitted to be taken as 1.0.
  - According to ACI ***25.4.10.1***, length l_{dc} shall be permitted to reduce by ratio of $A_{s\,required}/A_{s\,provided}$.
- $\boldsymbol{l_{dc}}$ Lower Bound

  According to ACI **25.4.9.1**, $l_{dc} \geq 200\ mm$

Table 6.6-1: Modification factors for deformed bars and wires in compression, Table 25.4.9.3 of (ACI318M, 2014).

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Lightweight concrete, if f_{ct} is specified	In accordance with 19.2.4.3
	Normalweight concrete	1.0
Confining reinforcement ψ_r	Reinforcement enclosed within (1), (2), (3), or (4): (1) a spiral (2) a circular continuously wound tie with $d_b \geq 6$ mm and pitch 100 mm (3) No. 13 bar or MD130 wire ties in accordance with 25.7.2 spaced ≤ 100 mm on center (4) hoops in accordance with 25.7.4 spaced ≤ 100 mm on center	0.75
	Other	1.0

6.6.3 EXAMPLES

Example 6.6-1

The forces in the column bars of ***Figure 6.6-1*** below are to be transferred into the footing with No. 29 dowels.

Determine the development lengths needed for the dowels:

- Down into the footing.
- Up into the column.

In your solution assume that, $f_y = 420\ MPa$ and that column is under a compressive force.

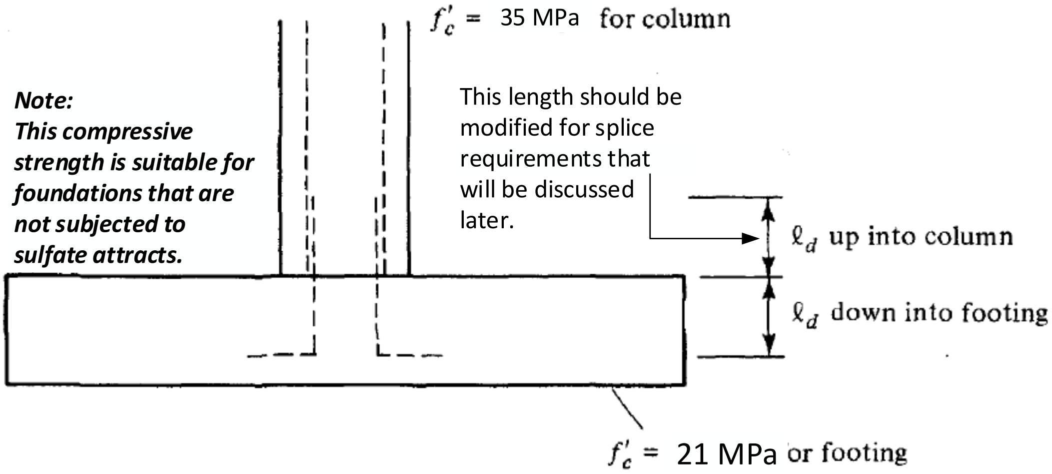


Figure 6.6-1: Foundation and column dowels for Example 6.6-1.

Solution

<u>Down into the Footing</u>

$$l_{dc} = maximum\left(\frac{0.24f_y\psi_r}{\lambda\sqrt{f_c'}}d_b\ or\ 0.043f_y\psi_r d_b\right)$$

As no confining reinforcement are included,

$\psi_r = 1.0$

For normal weight concrete,

$\lambda = 1.0$

$$l_{dc} = maximum\left(\frac{0.24 \times 420 \times 1.0}{1.0 \times \sqrt{21}}d_b\ or\ 0.043 \times 420 \times 1.0d_b\right)$$

$$l_{dc} = maximum(22d_b\ or\ 18d_b) = 22\ d_b = 22 \times 29 = 638mm > 200\ mm\ \therefore Ok.$$

<u>Up into the Column</u>

$$l_{dc} = maximum\left(\frac{0.24f_y\psi_r}{\lambda\sqrt{f_c'}}d_b\ or\ 0.043f_y\psi_r d_b\right)$$

As no confining reinforcement are included,

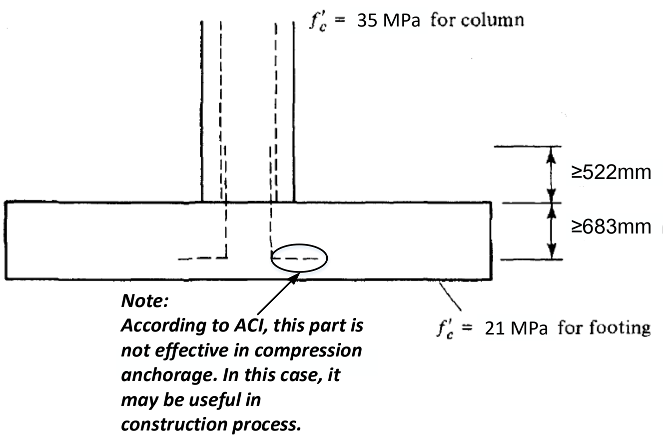
$\psi_r = 1.0$

For normal weight concrete,

$\lambda = 1.0$

$$l_{dc} = maximum\left(\frac{0.24 \times 420 \times 1.0}{1.0 \times \sqrt{35}}d_b\ or\ 0.043 \times 420 \times 1.0d_b\right)$$

$$l_{dc} = maximum(17d_b\ or\ 18d_b) = 18\ d_b = 18 \times 29 = 522mm > 200mm\ \therefore Ok.$$



Example 6.6-2

To anchor an axially compressed column, that reinforced with $4\phi25$, to its foundation, a designer has proposed the detail shown in below.

Assuming that $A_{s\,provided} \approx A_{s\,Required}$, $f_c' = 28\ MPa$, and $f_y = 420\ MPa$.

- Is the proposed down in to foundation anchorage adequate according to ACI code?
- If proposed anchorage is inadequate, propose two different alternatives to solve the problem.

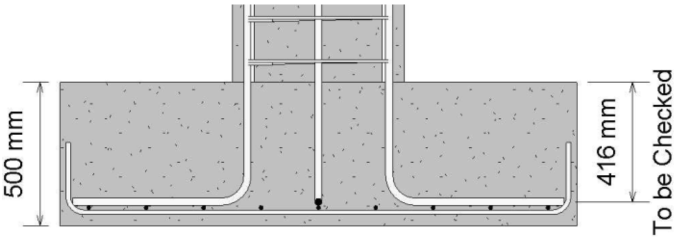


Figure 6.6-2: Column to foundation anchor of Example 6.6-2.

Solution

Adequacy Checking

Down into the footing:

$$l_{dc} = maximum\left(\frac{0.24 f_y \psi_r}{\lambda\sqrt{f_c'}} d_b \text{ or } 0.043 f_y \psi_r d_b\right)$$

As no confining reinforcement are included,

$\psi_r = 1.0$

For normal weight concrete,

$\lambda = 1.0$

$$l_{dc} = maximum\left(\frac{0.24 \times 420 \times 1.0}{1.0 \times \sqrt{28}} d_b \text{ or } 0.043 \times 420 \times 1.0 d_b\right)$$

$l_{dc} = maximum(19d_b \text{ or } 18d_b) = 19\, d_b = 19 \times 25 = 475mm > 416 \text{ mm} \therefore \text{Not ok.}$

Proposed Alternatives

1st Alternative:

First alternative is to use $\phi 16mm$ instead of $\phi 25mm$ for column longitudinal reinforcement and recalculate required number accordingly:

$l_{dc\ for\ \phi 16} = 19 \times 16 = 304\ mm < 416\ mm \therefore Ok.$

$$No.of\ \phi 16 = \frac{\left(4 \times \frac{\pi \times 25^2}{4}\right)}{\pi \times \frac{16^2}{4}} = 9.76$$

Used $10\phi 16mm$.

2nd Alternative:

This alternative is based on using an area for longitudinal reinforcement greater than the required one to activate the reduction factor of $A_{s\ Required}/A_{s\ Provided}$.

$$\frac{A_{s\ Required}}{A_{s\ Provided}} \times 475 = 416 \Rightarrow \frac{A_{s\ Required}}{A_{s\ Provided}} = 0.876$$

$$A_{s\ Provided} = \frac{\left(\frac{\pi \times 25^2}{4} \times 4\right)}{0.876} = 2241\ mm^2$$

$$Modified\ No.of\ \phi 25 = \frac{2241}{\pi \times \frac{25^2}{4}} = 4.56$$

Then use $6\phi 25$ instead of $4\phi 25$.

6.7 Development of Bundled Bars

6.7.1 General Requirements

- According to (ACI318M, 2014), ***25.6.1.1***, groups of parallel reinforcing bars bundled in contact to act as a unit ***shall be limited to <u>four</u> in any one bundle***.
- According to (ACI318M, 2014), ***25.6.1.2***, bundled bars in compression members shall be enclosed by ***transverse reinforcement at least No. 13 in size***. This aspect has been discussed thoroughly ***in analysis and design of doubly reinforced beams in Chapter 3***.
- According to (ACI318M, 2014), ***25.6.1.3***, ***bars larger than a No. 36 shall not be bundled in beams***.

6.7.2 Development Length for Bundled Bars

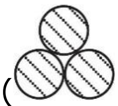
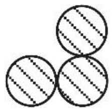
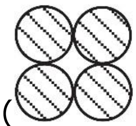
- According to (ACI318M, 2014), ***25.6.1.5***, development length of individual bars within a bundle, ***in tension or compression***, shall be that for the individual bar,
  - Increased 20 percent for three-bar bundle, ( or )
  - Increased 33 percent for four-bar bundle ().
- The extra extension is needed because the grouping makes it more difficult to mobilize bond resistance from the core between the bars.
- According to (ACI318M, 2014), ***25.6.1.6***, a unit of bundled bars shall be treated as a single bar of a diameter derived from the equivalent total area and having a centroid that coincides with that of the bundled bars for determining the following:
  - Spacing limitations based on d_b,
  - Cover requirements based on d_b,
  - Spacing and cover values in ***Article 25.4.2.2***, i.e. cover and spacing related to Table 6.3-1. ***For bundled bars, bar diameter*** $\mathrm{d_b}$ ***outside the brackets is that of a single bar***.

Table 6.3-1: Simplified ACI Relations for Development Length (Table 25.4.2.2 of (ACI318M, 2014)). Represented for convenience.

Spacing and cover	No. 19 and smaller bars and deformed wires	No. 22 and larger bars
Clear spacing of bars or wires being developed or lap spliced not less than d_b, clear cover at least d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum or Clear spacing of bars or wires being developed or lap spliced at least $2d_b$ and clear cover at least d_b	$\left(\frac{f_y\Psi_t\Psi_e}{2.1\lambda\sqrt{f_c'}}\right)d_b$	$\left(\frac{f_y\Psi_t\Psi_e}{1.7\lambda\sqrt{f_c'}}\right)d_b$
Other cases	$\left(\frac{f_y\Psi_t\Psi_e}{1.4\lambda\sqrt{f_c'}}\right)d_b$	$\left(\frac{f_y\Psi_t\Psi_e}{1.1\lambda\sqrt{f_c'}}\right)d_b$

  - Confinement term in 25.4.2.3, i.e., the term $(c_b + K_{tr})/d_b$ in the basic equation below. ***For bundled bars, bar diameter*** $\mathrm{d_b}$ ***outside the brackets is that of a single bar***.

$$\ell_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s}{\left(\frac{c_b + K_{tr}}{d_b}\right)} \right) d_b$$

- ψ_e factor in ***Article 25.4.2.4***, i.e., ψ_e in the Table 6.3-2, represented in below for convenience in below.

Table 6.3-2: Modification factors for development of deformed bars and deformed wires in tension, Table 25.4.2.4 of (ACI318M, 2014). Represented for convenience.

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Lightweight concrete, where f_{ct} is specified	In accordance with 19.2.4.3
	Normalweight concrete	1.0
Epoxy[1] ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement with clear cover less than $3d_b$ or clear spacing less than $6d_b$	1.5
	Epoxy-coated or zinc and epoxy dual-coated reinforcement for all other conditions	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Size ψ_s	No. 22 and larger bars	1.0
	No. 19 and smaller bars and deformed wires	0.8
Casting position[1] ψ_t	More than 300 mm of fresh concrete placed below horizontal reinforcement	1.3
	Other	1.0

[1]The product $\psi_t\psi_e$ need not exceed 1.7.

6.7.3 EXAMPLES

Example 6.7-1

Compute the development length required for the uncoated bundled bars shown in ***Figure 6.7-1*** below if $f_y = 420\,MPa$ and $f_c' = 28\,MPa$ with normal weight concrete. Use ACI basic relation and assume that $K_{rt} = 0$.

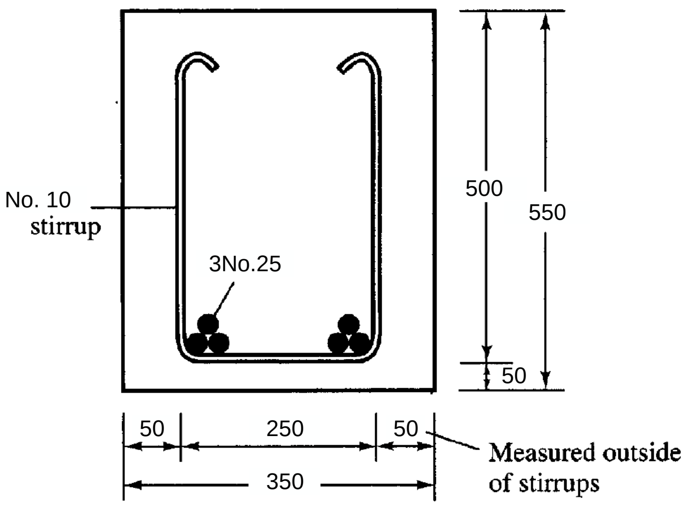


Figure 6.7-1: Cross sectional area for a beam reinforced with bundled bars.

Solution

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s}{\frac{c_b + K_{tr}}{d_b}} \right) d_b$$

$\psi_t = \psi_e = \psi_s = \lambda = 1.0$

In confinement term, c_b and d_b will be computed based on an equivalent single rebar:

$$\frac{\pi d_{b\ equivalent}^2}{4} = 3 \times \frac{\pi \times 25^2}{4}$$

$$d_{b\ equivalent} = \sqrt{3 \times 25^2} = 43.3\ mm$$

$$c_b = minimum\left(side\ cover\ to\ center\ of\ bar, bottom\ cover\ to\ center\ of\ bar, \frac{1}{2} S_{\frac{c}{c}}\right)$$

$$c_b = minimum\left(50 + 10 + \frac{43.3}{2}, \quad 50 + 10 + \frac{43.3}{2}, \quad \frac{1}{2}\left(250 - 2 \times 10 - \frac{43.3}{2} \times 2\right)\right)$$

$$c_b = minimum(81.6, \quad 81.6, \quad 93.4) = 81.6\ mm$$

$$\frac{c_b + K_{tr}}{d_{b\ equivalent}} = \frac{81.6 + 0.0}{43.3} = 1.88 < 2.5 \therefore Ok$$

$$l_d = \left(\frac{420}{1.1 \times 1.0 \times \sqrt{28}} \frac{1.0 \times 1.0 \times 1.0}{1.88}\right) d_b$$

$$l_d = 38.4 d_b$$

This value should be increased 20% for a 3-bar bundle according to ACI Section ***25.6.1.5***.

$l_d = 1.2 \times 38.4 \times 25 = 1152\ mm$■

6.8 Lap Splices

6.8.1 Basic Concepts

- Need for Splices:

  In general, reinforcing bars are stocked by suppliers in lengths of 12m. For this reason, and because it is often more convenient to work with shorter bar lengths, it is frequently necessary to splice bars in the field.

- Splice Types:

  Rebars are spliced to each other by:

  - Lap Splices:

    In this type, rebars are usually made simply by lapping the bars a sufficient distance to transfer stress by bond from one bar to the other. The lapped bars are usually placed in contact and lightly wired so that they stay in position as the concrete is placed.

  - Mechanical Splices:

    Sample of mechanical splice is presented in ***Figure 6.8-2***.

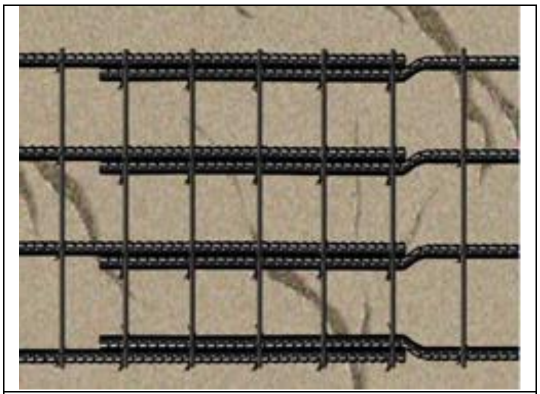


Figure 6.8-1: Lap Splices.

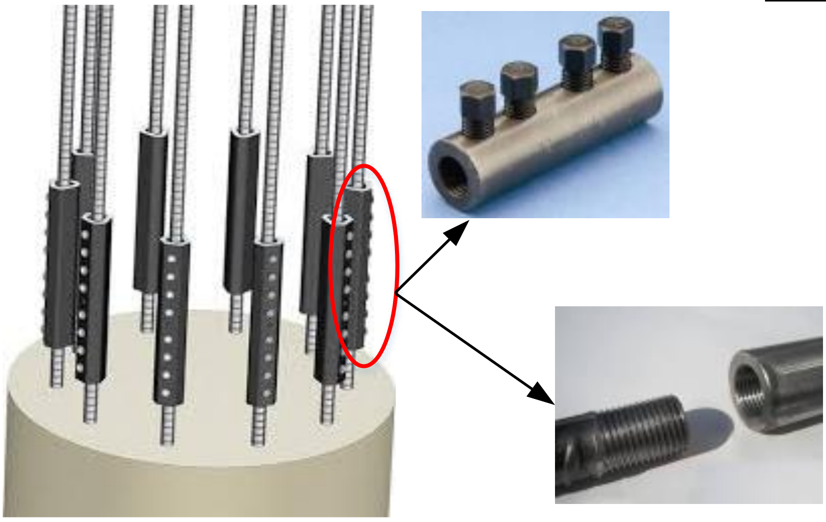


Figure 6.8-2: Mechanical Splice for Rebars.

  - Welding Splice:

    Splice with welding splice, with fillet weld, is presented in ***Figure 6.8-3***.

Figure 6.8-3: Welding Splice for Rebars.

- Only lap splice is considered in this article.

6.8.2 General Notes on Lap Splices

- According to (ACI318M, 2014), ***Article 25.5.1.1***, ***Lap splices shall not be used for bars larger than No. 36 except as provided in 25.5.5.3 (***compression lap splices of No. 43 and No. 57 bars with smaller bars***)***. This because of lack of adequate experimental data on lap splices for larger diameters.
- According to (ACI318M, 2014), ***Article 25.5.1.4***, ***Lap splices of bars in a bundle shall be based on the lap splice length required for individual bars within the bundle, increased in accordance with Article 25.6.1.7 (***increased by 20 percent and 33 percent for 3- and 4-bar bundles, respectively***).***
- According to (ACI318M, 2014), ***Article 25.5.1.4***, reduction of development length in accordance with $A_{s\,required}/A_{s\,provided}$ is not permitted in calculating lap splice lengths because the splice classifications already reflect any excess reinforcement at the splice location.

6.8.3 LAP SPLICES IN TENSION

- According to (ACI318M, 2014), ***Article 25.5.2.1***, tension lap splice length l_{st} for deformed bars and deformed wires in tension shall be in accordance with ***Table 6.8-1*** below, Table 25.5.2.1 of (ACI318M, 2014):

Table 6.8-1: Lap splice lengths of deformed bars and deformed wires in tension, Table 25.5.2.1 of (ACI318M, 2014).

$A_{s,provided}/A_{s,required}$[1] over length of splice	Maximum percent of A_s spliced within required lap length	Splice type	ℓ_{st}	
≥ 2.0	50	Class A	Greater of:	$1.0\ell_d$ and 300 mm
	100	Class B	Greater of:	$1.3\ell_d$ and 300 mm
< 2.0	All cases	Class B		

[1]Ratio of area of reinforcement provided to area of reinforcement required by analysis at splice location.

- The two-level lap splice requirements encourage splicing bars at points of minimum stress and staggering splices to improve behavior of critical details.
- For calculating ℓ_d for staggered splices, the clear spacing is taken as the minimum distance between adjacent splices, as illustrated in ***Figure 6.8-4*** below.
- According to (ACI318M, 2014), ***Article 25.5.2.2***, if bars of different size are lap spliced in tension, ℓ_{st} shall be the greater of ℓ_d of the larger bar and ℓ_{st} of the smaller bar.

6.8.4 TENSION LAP SPLICE FOR COLUMNS

- For tension lap splice in columns, see (ACI318M, 2014) Article ***10.7.5.2***.
- Tension lap splice in columns is out of our scope in this article.

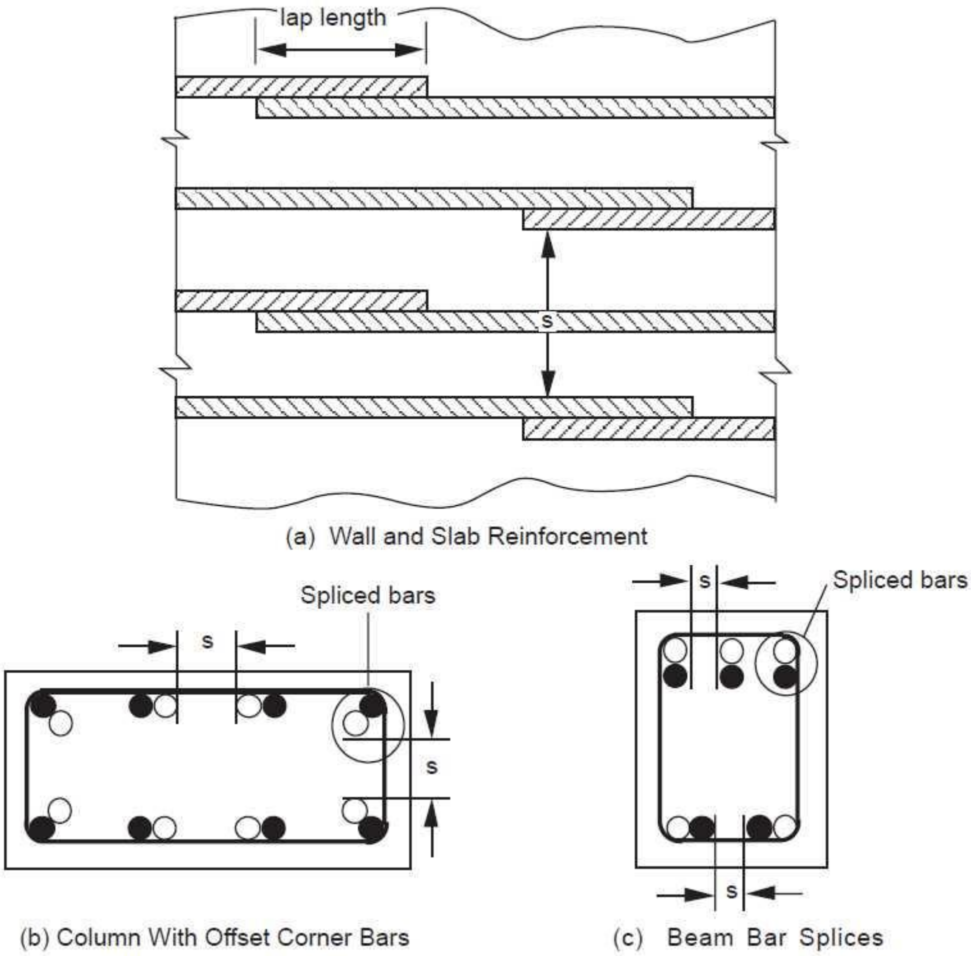


Figure 6.8-4: Effective Clear Spacing of Spliced Bars for determination of ℓ_d for staggered splices, adopted from (Kamara, 2005).

Example 6.8-1

Calculate the lap-splice length for six No. 25 tension bottom bars (in two rows) with clear spacing of 63.5 mm, clear cover of 40 mm and stirrups of 10mm for the following cases:

- When three bars are spliced and (As provided)/(As required) > 2.
- When four bars are spliced and (As provided)/(As required) < 2.
- When all bars are spliced at the same location.

In your solution assume $f_c' = 35\ MPa$ and $f_y = 420\ MPa$ and use basic equation to compute l_d

Solution

Compute the development length, l_d:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}} \frac{\psi_t\psi_e\psi_s}{\frac{c_b + K_{tr}}{d_b}} \right) d_b$$

$$c_b = minimum\left(side\ cover\ to\ center\ of\ bar, top\ cover\ to\ center\ of\ bar, \frac{1}{2}S_{\frac{c}{c}}\right)$$

$$c_b = minimum\left(40 + 10 + \frac{25}{2}, 40 + 10 + \frac{25}{2}, \frac{1}{2} \times (63.5 + 25)\right)$$

$$c_b = minimum(62.5, 62.5, 44.3) = 44.3\ mm$$

For bottom rebars,

$$\psi_t = 1.0$$

For epoxy uncoated rebars,

$$\psi_e = 1.0$$

For bar with diameter of 25mm > 19mm,

$$\psi_s = 1.0$$

For normal weight concrete,

$$\lambda = 1.0$$

As nothing has been mentioned about stirrups spacing,

$$K_{tr} = 0$$

and the confinement factor would be:

$$\frac{c_b + K_{tr}}{d_b} = \frac{44.3 + 0}{25} = 1.77 < 2.5 \therefore Ok.$$

$$l_d = \left(\frac{420}{1.1 \times 1.0 \times \sqrt{35}} \frac{1.0 \times 1.0 \times 1.0}{\frac{44.3 + 0}{25}} \right) d_b = 36.4\ d_b = 910\ mm$$

<u>Splice</u>

When three bars are spliced and (As provided)/(As required) > 2:

As

$$\frac{A_{s\ provided}}{A_{s\ required}} > 2.0$$

and only 50% of reinforcement to be spliced, therefore splice can be classified as Class A.

$$l_{splice} = 1.0\ l_d = 910\ mm > 300mm \therefore Ok.$$

When four bars are spliced and (As provided)/(As required) < 2:

Class B splice should be adopted.

$$l_{splice} = 1.3 l_d = 1.3 \times 910 = 1183\ mm > 300\ mm \therefore Ok.$$

When all bars are spliced at the same location:

Class B splice should be adopted.

$$l_{splice} = 1.3 l_d = 1.3 \times 910 = 1183\ mm > 300\ mm \therefore Ok.$$

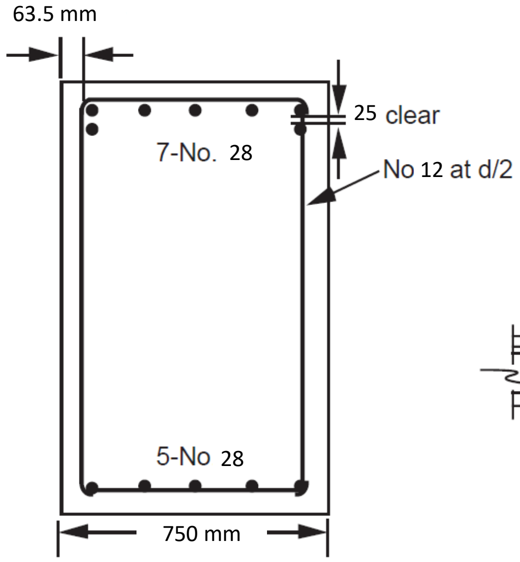
Example 6.8-2

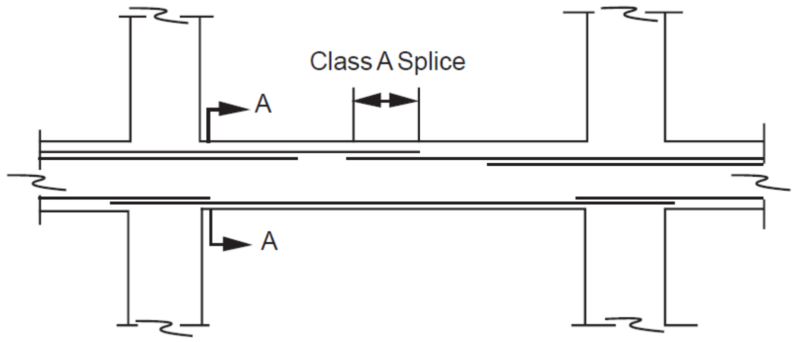
A beam at the perimeter of the structure has 7-No. 28 top bars over the support. Structural integrity provisions require that at least one-sixth of the tension reinforcement be made continuous, but not less than 2 bars (9.7.7.1).

Bars are to be spliced with a Class A splice at mid-span. Determine required length of Class A lap splice for the following two cases:

Case I - Development length computed based on simplified equation.
Case II - Development length computed based basic equation.
In your solution assume:

- Lightweight concrete
- Epoxy-coated bars
- $f_c' = 28\ MPa$
- $f_y = 420\ MPa$





Solution

<u>Case I - Development computed from simplified equation:</u>

Check confinement: try Case 2:

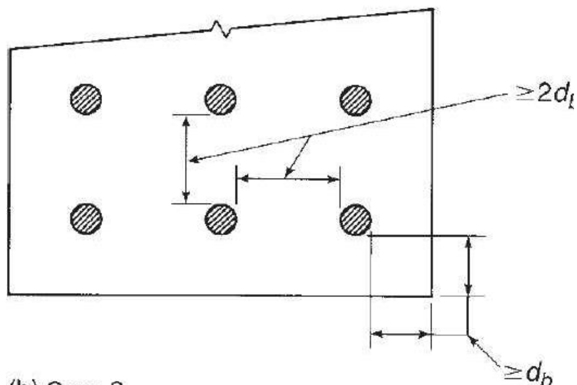
$$Clear\ Spacing = \frac{(750 - 2 \times 63.5 - 2 \times 12 - 2 \times 28)}{28}$$

$$= 19.4 \gg 2$$

$$Side\ Cover = \frac{63.5 + 12}{28} = 2.7 > 2$$

Then rebar is confined according to requirement of Case 2. As rebar diameter is greater than No. 19, therefore development length would be:

$$l_d = \left(\frac{f_y \psi_t \psi_e}{1.7\lambda\sqrt{f_c'}}\right) d_b$$



(b) Case 2.

	Spacing and cover	No. 19 and smaller bars and deformed wires	No. 22 and larger bars
Case 1 Case 2	Clear spacing of bars or wires being developed or spliced not less than d_b, clear cover not less than d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum or Clear spacing of bars or wires being developed or spliced not less than $2d_b$ and clear cover not less than d_b	$\left(\frac{f_y \Psi_t \Psi_e}{2.1\lambda\sqrt{f_c'}}\right) d_b$	$\left(\frac{f_y \Psi_t \Psi_e}{1.7\lambda\sqrt{f_c'}}\right) d_b$
	Other cases	$\left(\frac{f_y \Psi_t \Psi_e}{1.4\lambda\sqrt{f_c'}}\right) d_b$	$\left(\frac{f_y \Psi_t \Psi_e}{1.1\lambda\sqrt{f_c'}}\right) d_b$

For top rebars, assuming concrete below rebar is greater than 300 as it is clear from Figure above assuming it has been drawn to scale.

$\psi_t = 1.3$

For epoxy coated with clear cover less than 3d,

$\psi_e = 1.5$

$\psi_t \psi e = 1.3 \times 1.5 = 1.95 > 1.7 \therefore Not\ ok$

Let

$\psi_t \psi_e = 1.7$

For lightweight concrete:

$\lambda = 0.75$

$$l_d = \left(\frac{f_y \psi_t \psi_e}{1.7\lambda\sqrt{f_c'}}\right) d_b = \left(\frac{420 \times 1.7}{1.7 \times 0.75 \times \sqrt{28}}\right) d_b = 106\ d_b = 106 \times 28 = 2968\ mm$$

For Case A splice:

$l_{splice} = 1.0\ l_d = 2968\ mm > 300\ mm$ ■

<u>Case II - Development computed from basic equation.</u>

According to basic equation below:

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s}{\frac{c_b + K_{tr}}{d_b}}\right) d_b$$

As discussed above,

$\psi_t \psi_e = 1.7$

$\lambda = 0.75$

$\because d_b = 28mm > 19mm \therefore \psi_s = 1.0$

For rebars in the second layer, only side cover and center to center rebar spacing to be considered:

$$c_b = minimum\left(63.5 + 12 + \frac{28}{2}, \frac{1}{2} \times (750 - 2 \times 63.5 - 2 \times 12 - 28)\right) = minimum\ (89.5\ or\ 286)$$
$$= 89.5\ mm$$

Without computing K_{tr}, one can conclude that:

$$\frac{c_b + K_{tr}}{d_b} = \frac{89.5}{28} = 3.2 > 2.5$$

Therefore,

$$\frac{c_b + K_{tr}}{d_b} = 2.5$$

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s}{\frac{c_b + K_{tr}}{d_b}}\right) = \left(\frac{420}{1.1 \times 0.75 \times \sqrt{28}} \frac{1.7 \times 1.0}{2.5}\right) d_b = 65.4\ d_b$$

$l_d = 65.4 \times 28 = 1831\ mm$

For Case A splice:

$l_{splice} = 1.0\ l_d = 1831\ mm > 300mm$ ■

Example 6.8-3

Based on structural analysis and design, a designer has adopted bottom reinforcement of No. 12@200mm for the slab indicated in ***Figure 6.8-5***. Using ACI basic equation, determine the development length, l_d, for the adopted positive slab reinforcement and then compute the corresponding lap splice length.

Solution

Computing of the development length, $\boldsymbol{l_d}$:

According to basic equation of the ACI code, the development length for tension rebars, l_d, would be:

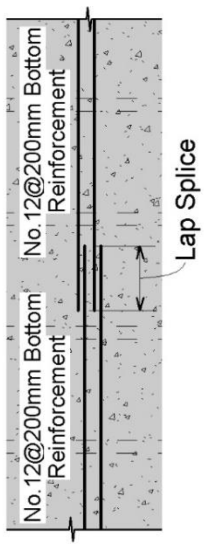


Figure 6.8-5: Slab reinforcement for Example 6.8-3

$$l_d = \left(\frac{f_y}{1.1\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s}{\frac{c_b + K_{tr}}{d_b}} \right) d_b$$

$$c_b = \text{minimum} \begin{pmatrix} side\ cover\ to\ center\ of\ bar, \\ top\ cover\ to\ center\ of\ bar, \frac{1}{2} S_{\frac{c}{c}} \end{pmatrix} \Rightarrow c_b = \min \begin{pmatrix} \left(20 + \frac{12}{2}\right), \\ \left(20 + \frac{12}{2}\right), \left(\frac{1}{2} \times 200\right) \end{pmatrix}$$

$c_b = \min(26,26,100) = 26\ \text{mm}$

As there is no shear reinforcement in the cantilever slab, therefore:

$K_{tr} = 0$

As the concrete is normal weight concrete, $\lambda = 1.0$. For uncoated rebars, $\psi_e = 1.0$.

As the rebars are bottom rebars from bond point of view.

$\psi_t = 1.0$

Finally, for rebars with size less than 19mm, $\psi_s = 0.8$.

$$l_d = \left(\left(\frac{420}{1.1 \times 1.0 \times \sqrt{28}} \right) \times \left(\frac{1.0 \times 1.0 \times 0.8}{\frac{26+0}{12}} \right) \right) d_b = 26.6 d_b$$

As nothing is mentioned about $A_{s\,required}/A_{s\,provided}$, therefore it can be conservatively assumed 1.0.

$l_d = 26.6 d_b = 26.6 \times 12 = 319\ \text{mm} > 300\ \text{mm} \therefore \text{Ok.} \Rightarrow l_d = 319\ mm$ ■

Splice Length:

As nothing has been mentioned about $A_{s\,provided}/A_{s\,required}$, therefore the splice would conservatively be classified as ***Class B***.

$l_{st} = maximum(1.3 l_d, 300) = 1.3 \times 319 = 415\ mm$ ■

6.8.5 LAP SPLICE LENGTHS OF DEFORMED BARS IN COMPRESSION

6.8.5.1 General Requirements

According to (ACI318M, 2014), ***25.5.5.2***, compression lap splices shall not be used for bars larger than No. 36, except to No. 36 or smaller bars.

6.8.5.2 Compression Lap Splice Length ℓ_{sc}

- According to (ACI318M, 2014), ***25.5.5.1***, compression lap splice length ℓ_{sc} of No. 36 or smaller deformed bars in compression shall be calculated in accordance with (a) or (b):

  (a) For $f_y \leq 420\ MPa$:

  $$\ell_{sc} = maximum\left(0.071 f_y d_b and\ 300\ mm\right)$$

  (b) For $f_y > 420\ MPa$:

  $$\ell_{sc} = maximum\left(\left(0.13 f_y - 24\right) d_b and\ 300\ mm\right)$$

- For $f_c' < 21\ MPa$, the length of lap shall be increased by one-third.

6.8.5.3 Reducing in Compression Lap Splice Length ℓ_{sc}

According to (ACI318M, 2014), ***10.7.5.2.1***, it shall be permitted to decrease the compression lap splice length in accordance with (a) or (b), but the lap splice length shall be at least 300 mm.

(a) For tied columns, where ties throughout the lap splice length have an effective area not less than $\mathbf{0.0015}\boldsymbol{hs}$ in ***both directions***, lap splice length shall be permitted to be multiplied by ***0.83***. Tie legs perpendicular to dimension h shall be considered in calculating effective area.

(b) For spiral columns, where spirals throughout the lap splice length satisfy ***25.7.3***, ***this will be discussed thoroughly in column design***, lap splice length shall be permitted to be multiplied by ***0.75***.

6.8.5.4 Lap Splice for Bars with Different Size

- According to (ACI318M, 2014), ***25.5.5.3***, compression lap splices of No. 43 or No. 57 bars to No. 36 or smaller bars shall be permitted and shall be in accordance with (ACI318M, 2014), ***25.5.5.4***, presented in below.
- According to (ACI318M, 2014), ***25.5.5.4***, where bars of different size are lap spliced in compression, ℓ_{sc} shall be:

  $$l_{sc} = maximum\ (l_{dc}\ for\ larger\ bar\ or\ \boldsymbol{l_{sc}}\ \text{of smaller bar})$$

6.8.5.5 Common Details for Columns Splices

- The ***most common method of splicing column steel*** is ***the simple lapped bar splice***, with the bars in contact throughout the lapped length.
- It is standard practice to offset the lower bars, as shown in ***Figure 6.8-6*** below

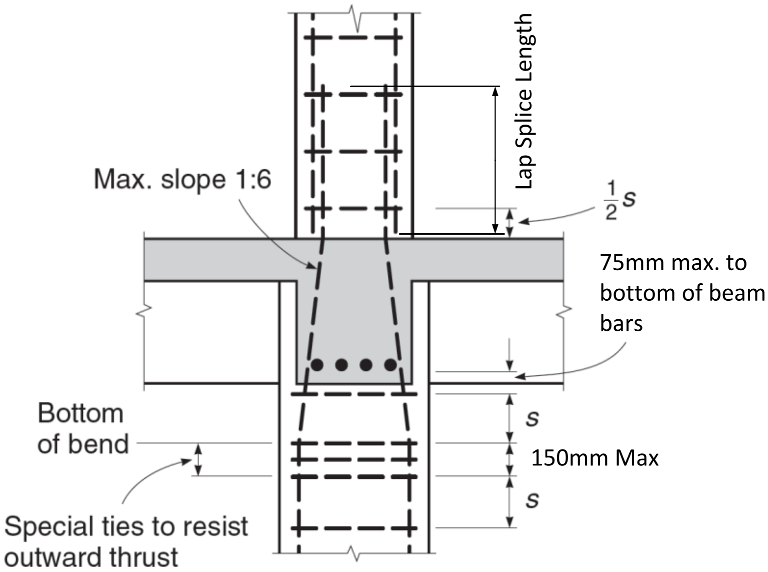
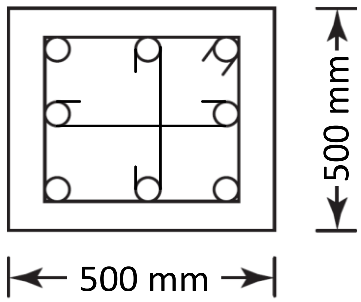


Figure 6.8-6: Splice details at typical interior column. Beams frame into joint from four directions.

6.8.5.6 Examples

Example 6.8-4

Calculate the lap-splice length for a tied column. The column has eight No. 32 longitudinal bars and No. 10 ties with spacing of 450mm. Given $f_c' = 35\ MPa$, $f_y = 420\ MPa$, $f_y = 560\ MPa$ and all rebars under compression.



Solution

For $f_y \leq\ 420\ MPa$:

$\ell_{sc} =\ maximum\ \left(0.071 f_y d_b and\ 300\ mm\right)$

$\ell_{sc} =\ maximum\ (0.071 \times 420 \times 32\ and\ 300\ mm)$

$\ell_{sc} =\ maximum\ (954\ \ and\ 300\ mm) = 954\ mm$

Reduction in l_{sc}:

Determine column tie requirements to allow 0.83 reduce lap-splice length according to ACI Code, Section 10.7.5.2.1.

$Effective\ area\ of\ ties\ \geq 0.0015\ hs$

$$3 \times \frac{\pi \times 10^2}{4} = 236\ mm^2 < 0.0015 \times 500 \times 450 = 338\ mm^2$$

Modifier 0.83 will not apply. Lap-splice length is $954\ mm$.

For $f_y >\ 420\ MPa$:

$\ell_{sc} =\ maximum\ \left(\left(0.13 f_y - \ 24\right) d_b and\ 300\ mm\right)$

$\ell_{sc} =\ maximum\ \left((0.13 \times 560 -\ 24) \times 32\ and\ 300\ mm\right)$

$\ell_{sc} =\ maximum\ (1562\ and\ 300\ mm) = 1562\ mm$

Modifier 0.83 will not apply as previously calculated.

6.11 Contents

CHAPTER 7
SERVICEABILITY

7.1 INTRODUCTION

- What have been achieved in Chapters 4, 5, and 6?
  - Chapters 4, 5, and 6 have dealt mainly with the ***strength design*** of reinforced concrete beams.
  - Methods have been developed to ensure that beams will have ***a proper safety margin against failure*** in ***flexure*** or ***shear***, or due to ***inadequate bond and anchorage*** of the reinforcement.
  - The member has been ***assumed*** to be at a ***hypothetical*** ***overload state*** for this purpose.
- Performance in normal service:
  - It is also important that ***member performance*** in ***normal service*** be ***satisfactory***.
  - ***Normal service conditions*** are ***when*** loads are ***those actually expected to act***, that is, when ***load factors are 1.0***.
- Member adequacy in strength is not necessarily adequate in service conditions.
  - ***Normal service conditions are not guaranteed*** simply by ***providing adequate strength***.
- Aspects that such be checked under normal conditions:
  - ***Deflection:***
    It may be:
    - Excessively large under full-service,
    - Long-term due to sustained loads,

    such that may ***cause damage***.
  - ***Tension cracks***:
    They in beams may be wide enough to:
    - Be visually disturbing,
    - reduce the durability of the structure.
  - ***Vibration or Fatigue***:
    Vibration or Fatigue are other questions that require consideration under service conditions. These aspects are out the scope of this course.
- The theory adopted to study the elastic conditions:
  - ***Serviceability studies*** are ***carried out based on elastic theory***.
  - Assumptions of the elastic theory:
    The elastic theory for analysis assumes that:
    - ***Stresses*** in ***both concrete*** and ***steel*** are ***proportional to strain***.
    - The ***concrete on the tension side*** of the neutral axis may be ***uncracked***, ***partially cracked***, or ***fully cracked***, depending on the loads and material strengths.
- Past versus current design philosophies:
  - ***In early*** reinforced concrete designs, ***questions of serviceability*** were dealt with ***indirectly***, by ***limiting the stresses in concrete and steel at service loads*** to the rather ***conservative values*** that had resulted in satisfactory performance.
  - The current design methods:
    - It ***permits more slender members*** through:
      1. More accurate assessment of capacity,
      2. Higher-strength materials.
    - It contributs to the trend toward ***smaller member sizes***, such that the old indirect methods no longer work.
    - The ***current approach*** is to ***investigate service load cracking*** and ***deflections*** specifically, ***after*** ***proportioning members based on strength requirements***.
- Scope of this Chapter:
  According to the text book:
  - ***Tension cracks***
    - This chapter develops methods to ensure that the cracks associated with flexure of reinforced concrete beams are narrow and well distributed.
    - For structures other than ***liquid retaining structures***, the concept of $s_{maximum}$ that has been discussed in Chapter 4 is adequate to ensure narrow and well distributed cracks.

    - Therefore, due to limited time, the explicit checking of cracks may be skipped in buildings-oriented design courses.
- Deflection control:
  After reviewing the deflection determinations from the mechanics of material and theory of structures, this chapter aims to:
    - Modify deflections determined based on assumptions of the uncracked section and short-term effect to be more accurate and representative for actual structures where the sections are fully or partially cracked, and the loads are sustained in nature.
    - Give permissible limits for the deflections.

7.2 CONTROL OF DEFLECTIONS

7.2.1 BASIC CONCEPTS

- Main Concerns of Excessive Deflection:
  Excessive deflections can lead to:
  - Cracking of supported walls and partitions,
  - Ill-fitting doors and windows,
  - Poor roof drainage,
  - Misalignment of sensitive machinery and equipment,
  - Visually offensive sag.

  It is important, therefore, to maintain control of deflections, in one way or another. ***So that members designed mainly for strength at prescribed overloads will also checked in normal service.***
- Approaches for Deflection Control:
  There are presently two approaches to deflection control:
  - Indirect Approach.
  - Direct Approach.

  These approaches are discussed in some details in following articles, and different illustrated examples are presented.

7.2.2 INDIRECT APPROACH

- The approach consists of setting proper upper limits on the span-depth ratio. These limits are as follows:
  - For one-way slabs:
    According to the code ***7.3.1.1***, for ***solid nonprestressed slabs not supporting or attached to partitions or other construction likely to be damaged by large deflections***, overall slab thickness h shall not be less than the limits in ***Table 7.2-1***, ***unless the calculated deflection limits are satisfied***.
  - For beams:
    According to the code ***9.3.1.1***, ***for nonprestressed beams not supporting or attached to partitions or other construction likely to be damaged by large deflections***, overall beam depth h shall satisfy the limits in ***Table 7.2-2***, ***unless the calculated deflection limits are satisfied***.

Table 7.2-1: Minimum thickness of solid nonprestressed one-way slabs, Table 7.3.1.1 of the code.

Support condition	Minimum h[1]
Simply supported	$\ell/20$
One end continuous	$\ell/24$
Both ends continuous	$\ell/28$
Cantilever	$\ell/10$

[1]Expression applicable for normalweight concrete and f_y = 420 MPa. For other cases, minimum h shall be modified in accordance with 7.3.1.1.1 through 7.3.1.1.3, as appropriate.

Table 7.2-2: Minimum depth of nonprestressed beams, Table 9.3.1.1 of the code.

Support condition	Minimum h[1]
Simply supported	$\ell/16$
One end continuous	$\ell/18.5$
Both ends continuous	$\ell/21$
Cantilever	$\ell/8$

[1]Expressions applicable for normalweight concrete and Grade 420 reinforcement. For other cases, minimum h shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

- This approach is simple, and it is ***satisfactory*** in many cases ***where spans***, ***loads*** and ***load distributions***, and ***member sizes*** and ***proportions fall in the usual ranges***.
- The condition of "***Members not supporting or attached to partitions or other construction likely to be damaged by large deflections***" has been left for designer judgment.

7.2.3 Direct Approach

- In this approach, deflection has to be calculated and to be compared with specific limitations that may be imposed by codes or by special requirements.
- Methods for predicating deflection in RC beams and ACI limitations on deflection are discussed in articles below.

7.2.4 Deflection Types in RC Beams

Two types of deflections are usually noted in RC beams:

- Immediate Deflection:
  As its name implies, this type of deflection occurs immediately when the load is applied.
- Long-term Deflection (Time Dependent Deflections):
  - These time-dependent deformations take place gradually over an extended time.
  - They are chiefly due to concrete creep and shrinkage.
  - Because of these influences, reinforced concrete members continue to deflect with the passage of time. Long-term deflections continue over a period of several years.
  - They may eventually be 2 or more times the initial elastic deflections.

7.3 IMMEDIATE DEFLECTION

- Safety provisions of the ACI Code and similar design specifications ensure that, under loads up to the full-service load, ***stresses in both steel and concrete remain within the elastic ranges***.
- Consequently, ***deflections that occur at once upon application of load, can be calculated based on the properties of the uncracked elastic member, the cracked elastic member, or some combination of these***.
- From the mechanics of materials, it is well known that ***elastic deflections*** can be expressed in the general form:

$$\Delta_{Immediate\ or\ Elastic} = \frac{f(Loads, Spans, Supports)}{EI} \qquad \text{Eq. 7.3-1}$$

- Deflection relations can easily be computed and tabulated for many loadings and spans arrangements as shown in ***Table 7.3-1***, ***Table 7.3-2***, and ***Table 7.3-3***.
- With the tabulated relations, the deflection computing in reinforced concrete structures are reduced into:
  - What should load values be in deflection computing?
  - What is the appropriate flexural rigidity EI for the member?

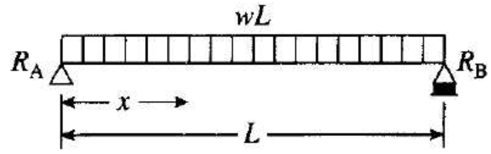
  These two issues are discussed below.

7.3.1 LOADS USED IN DEFLECTION CALCULATIONS

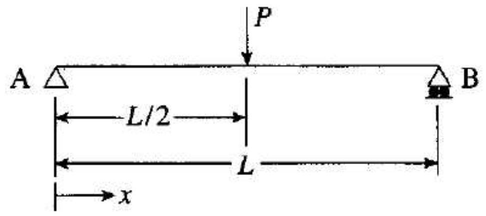
- The deflections of concern are generally those that occur during the ***normal service life of the member***.
- In service, a member sustains ***the full dead load***, plus ***some fraction*** or ***all*** of the specified ***service live load***.

Table 7.3-1: Deflection of Simply Supported Beams.

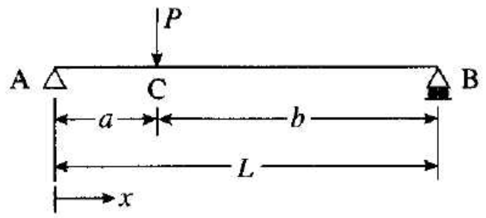
$$\Delta_{max} = \frac{5}{384} \times \frac{WL^3}{EI} \quad \text{(at center)}$$
$$W = \text{total load} = wL$$



$$\Delta_{max} = \frac{PL^3}{48EI} \quad \text{(at midspan)}$$



$$\Delta_C = \frac{Pa^2b^2}{3EIL} \quad \text{(at point load)}$$
$$\Delta_{max} = \frac{PL^3}{48EI}\left[\frac{3a}{L} - 4\left(\frac{a}{L}\right)^3\right] \quad \text{(when } a \ge b\text{)}$$
$$\text{at } x = \sqrt{a(b+L)/3}$$



$$\Delta_{max} = \frac{PL^3}{6EI}\left[\frac{3a}{4L} - \left(\frac{a}{L}\right)^3\right] \quad \text{(at midspan)}$$
$$\Delta_{max} = \frac{23PL^3}{648EI} \quad \text{(at midspan) when } a = L/3$$

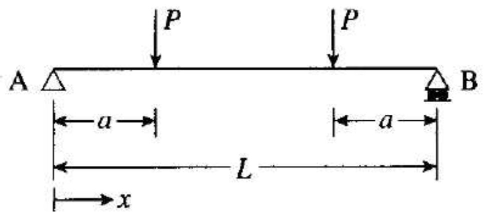
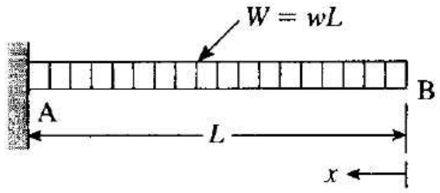


Table 7.3-2: Deflection for Cantilever Beams.

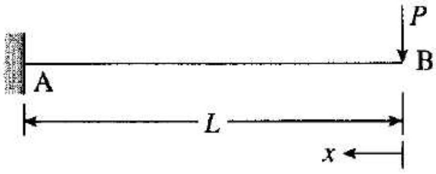
$$\Delta_B \max = \frac{WL^3}{8EI} \quad (W = wL)$$

$$\Delta_x = \frac{w}{24EIL}(x^4 - 4L^3x + 3L^4)$$



$$\Delta_B \max = \frac{PL^3}{3EI}$$

$$\Delta_x = \frac{P}{6EI}(2L^3 - 3L^2x + x^3)$$



$$\Delta_C = Pa^3/3EI$$

$$\Delta_B \max = \frac{Pa^3}{3EI}\left(1 + \frac{3b}{2a}\right) \quad \text{(at free end)}$$

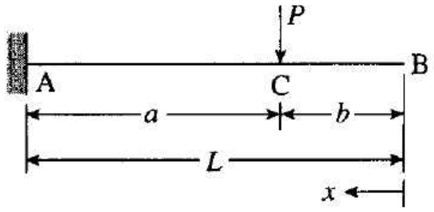
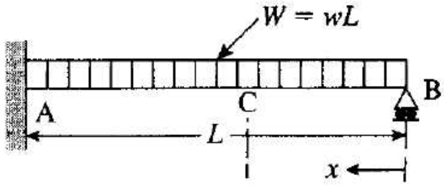


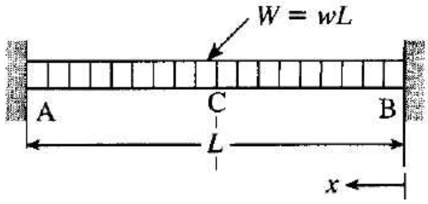
Table 7.3-3: Deflection of Statically Indeterminate Single Span Beams.

$$\Delta_{max} = \frac{WL^3}{185EI}$$

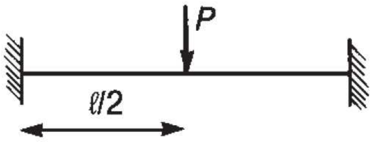
at a distance $x = 0.4215L$ (from support B)



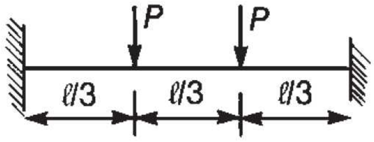
$$\Delta_{max} = \frac{WL^3}{384EI} \quad \text{(at midspan)}$$



$$\Delta_{mid} = \frac{1}{192} \cdot \frac{P\ell^3}{EI}$$



$$\Delta_{mid} = \frac{5}{684} \cdot \frac{P\ell^3}{EI}$$



7.3.2 MEMBER FLEXURAL RIGIDITY EI

7.3.2.1 MODULUS OF ELASTICITY E

- According to ACI 24.2.3.4, ***immediate deflection shall be computed with the modulus of elasticity for concrete,*** E_c.

  $E_{for\ immddiate\ deflection\ calculations} = E_c$ Eq. 7.3-2

- According to ACI 19.2.2, ***modulus of elasticity,*** E_c ***for normal weight concrete*** could be computed based on following relation.

  $E_c = 4700\sqrt{f_c'}$ Eq. 7.3-3

7.3.2.2 EFFECTIVE MOMENT OF INERTIA I_e

7.3.2.2.1 Uncracked Elastic Range

- If the maximum moment in a flexural member is so small that the tensile stress in the concrete does not exceed the modulus of rupture f_r, no flexural tension cracks will occur. The full, uncracked section is then available for resisting stress and providing rigidity, see ***Figure 7.3-1***.

- Then for applied moment M_a less than cracking moment M_{cr}, effective moment of inertia I_e that could be used for composite RC beams is:
  $\because M_a \leq M_{cr} \Rightarrow \therefore I_e \approx I_g$ Eq. 7.3-4
  where:
  I_e is effective moment of inertia for computation of deflection, mm4.
  I_g is moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement, mm4.
  M_{cr} is cracking moment, according to ACI ***24.2.3.5***, it could be computed as follows:
  $$M_{cr} = \frac{f_r I_g}{y_t}$$ Eq. 7.3-5
  and
  f_r is modulus of rupture of concrete, MPa:
  $$f_r = 0.62\lambda\sqrt{f_c'}$$ Eq. 7.3-6
  y_t is distance from centroidal axis of gross section, neglecting reinforcement, to tension face, mm. Above terms are more clarified with referring to Figure 7.3-2.

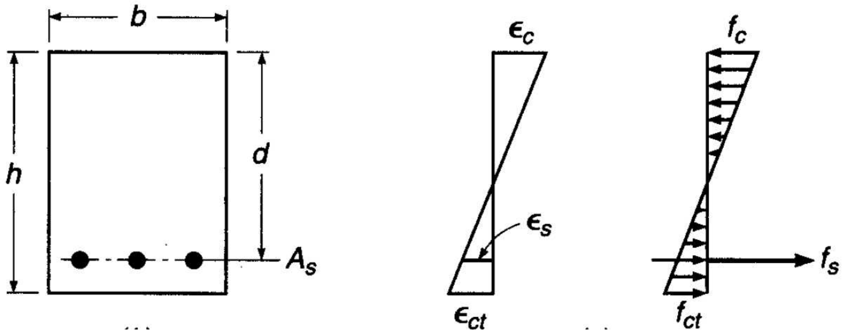


Figure 7.3-1: Uncracked elastic section.

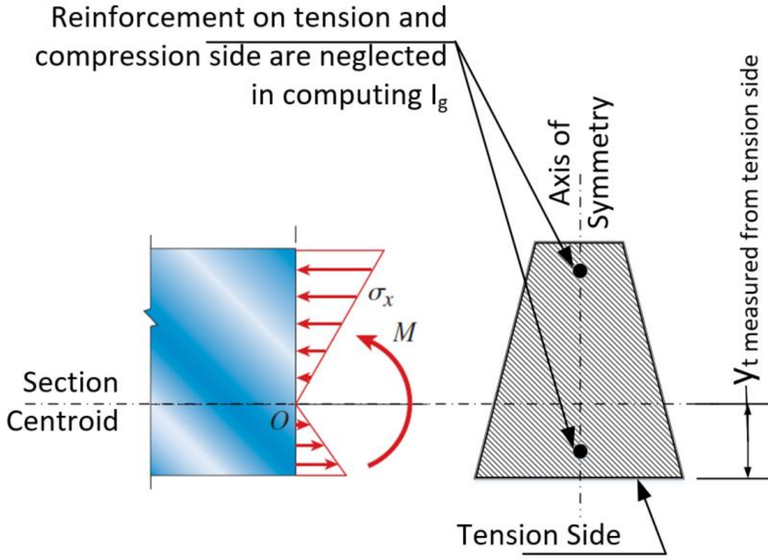


Figure 7.3-2: Gross moment of inertia for a RC beam with general symmetrical shape.

Example 7.3-1

For the rectangular concrete section that shown in ***Figure 7.3-3***, calculate

- Modulus of rupture, f_r,
- Gross moment of inertia, I_g,
- Cracking moment, M_{cr}.

Use $f_c' = 28\,MPa$ and $f_y = 420\,MPa$.

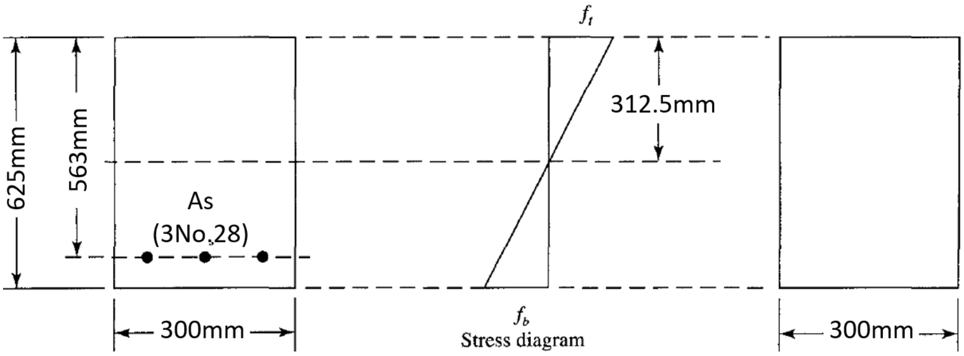


Figure 7.3-3: Beam for Example 7.3-1.

Solution

Modulus of rupture:

$f_r = 0.62\lambda\sqrt{f_c'}$

With normal weight concrete,

$\lambda = 1.0$

$f_r = 0.62 \times 1.0 \times \sqrt{28} = 3.28\ MPa$ ■

Gross Moment of Inertia:

With neglecting of reinforcement, gross section is a rectangular one:

$$I_g = \frac{bh^3}{12} = \frac{300 \times 625^3}{12} = 6104 \times 10^6\ mm^4 \blacksquare$$

$$y_t = \frac{625}{2} = 313\ mm$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{3.28 \times 6104 \times 10^6}{313} = 64.0\ kN.m \blacksquare$$

7.3.2.2.2 Partially to Fully Cracked Range

- According to ACI ***24.2.3.5***, when applied bending moment, M_a, greater than section cracking moment, M_{cr}, section would be in ***partially*** to ***fully cracked stage*** and its effective moment of inertia could be estimated based on following relation (***Eq. 24.2.3.5a of ACI Code***).

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) I_{cr} \qquad \text{Eq. 7.3-7}$$

- Trends of ***Eq. 7.3-7*** are presented in ***Figure 7.3-4***. Graphically it is presented in ***Figure 7.3-5***.

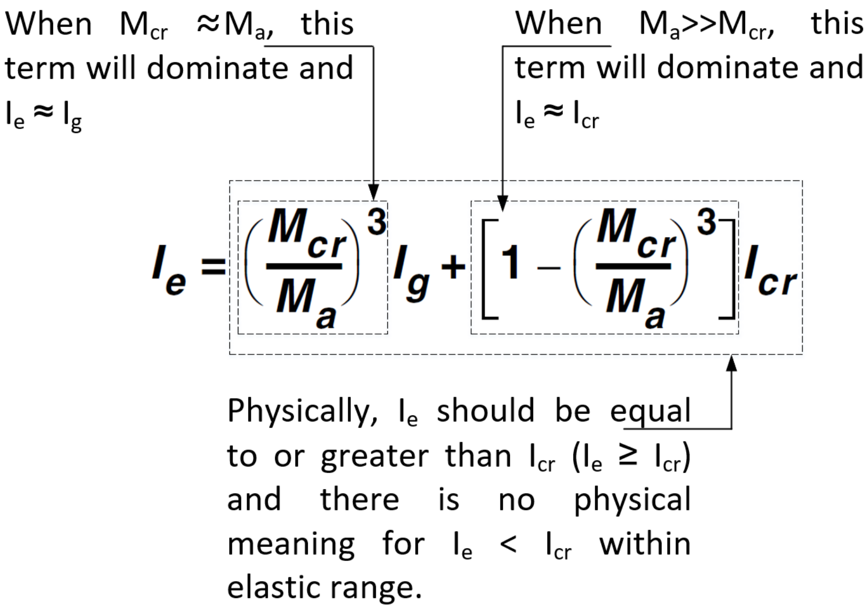


Figure 7.3-4: Trends of Eq. 7.3-7.

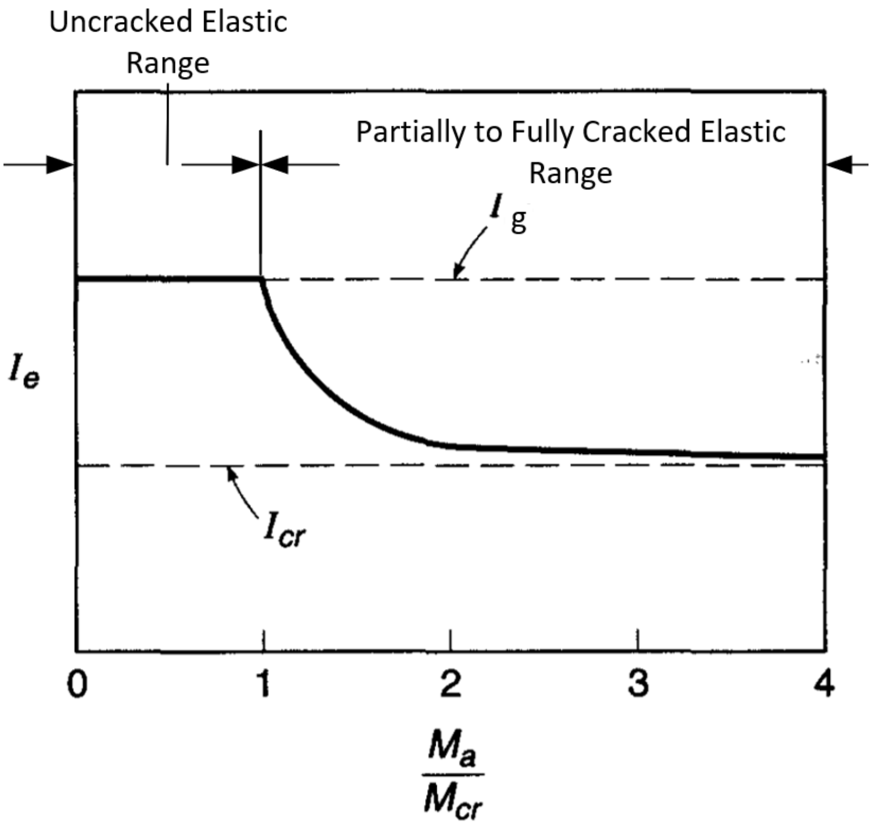


Figure 7.3-5: Variation of I_e with Moment Ratio.

- Variation of I_e along Beam Span:
  As I_e depends on $\frac{M_{cr}}{M_a}$, then it inversely varies with M_a along beam span as indicated in ***Figure 7.3-6***.

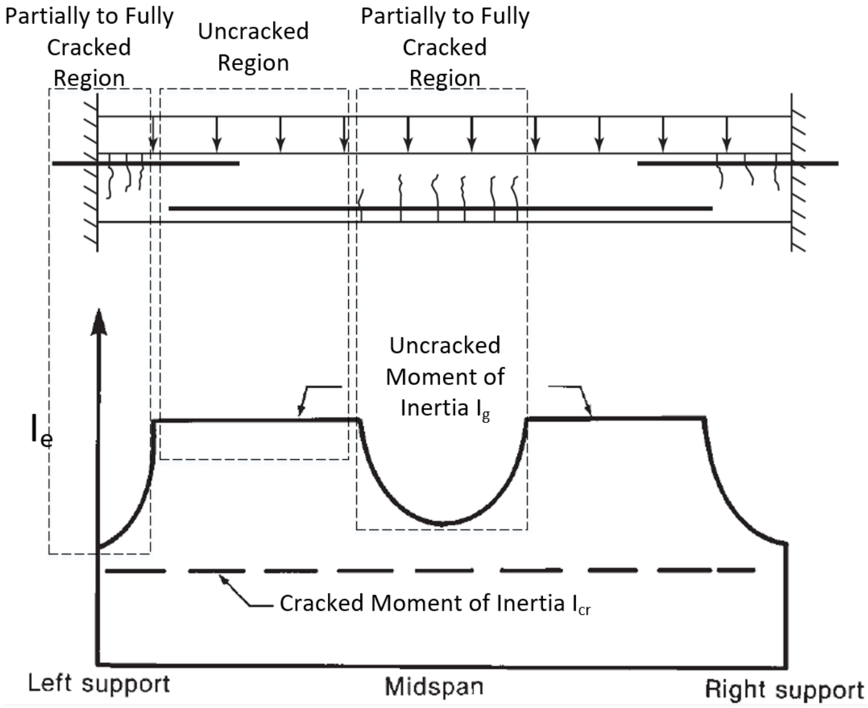


Figure 7.3-6: Variation of I_e along the Length of a Continuous Beam.

- According to ***ACI 24.2.3***, above variation of I_e along beam span could be approximated as follows for different support conditions:
  - Simply Supported Beam:

$I_{e\ for\ simply\ supported\ beam} = I_{e\ @\ mid-span}$

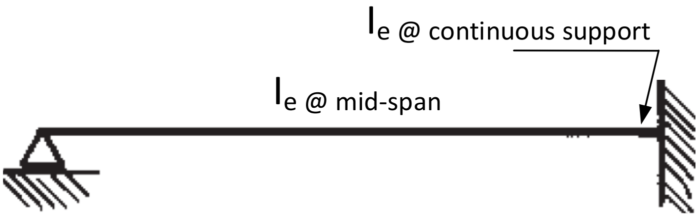
  - Both-end Continuous Beam:

$I_{e\ for\ Both\ end\ continuous\ beam} = 0.5 I_{e\ @\ mid-span} + 0.25(I_{e\ @\ left\ support} + I_{e\ @\ right\ support})$

or:

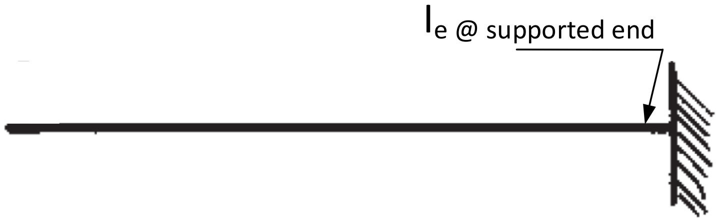
$I_{e\ for\ Both\ end\ continuous\ beam} \approx I_{e\ @\ mid-span}$

  - One-end Continuous Beam (Nilson, Design of Concrete Structures, 14th Edition, 2010):



$I_{e\ for\ one\ end\ continuous\ beam} = 0.85 I_{e\ @\ mid-span} + 0.15 I_{e\ @\ continuous\ support}$

  - Cantilever Beam:



$I_{e\ for\ cantilever\ beam} = I_{e\ @\ supported\ end}$

7.4 Deflections Due to Long-term Loads

- ***Initial deflections*** are ***increased significantly*** if ***loads are sustained over a long period of time***, due to the ***effects of shrinkage and creep***.
- Creep or shrinkage, which one is more dominant?
  - These two effects are usually combined in deflection calculations.
  - ***Creep generally dominates***,
  - But for ***some types of members***, ***shrinkage deflections are large*** and ***should be considered separately***.
- On the basis of ***empirical studies***, ACI Code ***24.2.4.1*** specifies that ***additional long-term deflections*** Δ_t due to the ***combined effects of creep and shrinkage*** be calculated by multiplying the immediate deflection Δ_i by a factor λ_Δ:

$$\lambda_\Delta = \frac{\xi}{1 + 50\rho'} \qquad \text{Eq. 7.4-1}$$

where

$$\rho' = \frac{A'_s}{bd}$$

and ξ is a time-dependent coefficient. It is a ***material property*** depending on ***creep*** and ***shrinkage*** characteristics and it can be estimated from ***Figure 7.4-1*** or from ***Table 7.4-1***.

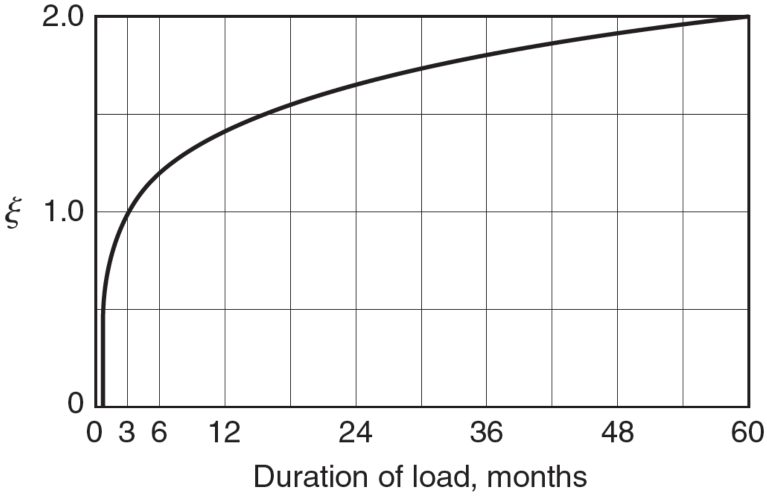


Figure 7.4-1: Time variation of ξ for long-term deflections.

Table 7.4-1: Time-dependent factor, ξ, for sustained loads, Table 24.2.4.1.3 of the code.

Sustained load duration, months	Time-dependent factor ξ
3	1.0
6	1.2
12	1.4
60 or more	2.0

- In Eq. 7.4-1, the quantity

$$\frac{1}{1 + 50\rho'}$$

is a ***reduction factor*** that is essentially a ***section property***, reflecting the ***beneficial effect of compression reinforcement*** A_s' in ***reducing long-term deflections***.
- When should ρ' be determined along the beam span:
  According to the ACI Code the value of ρ' used in Eq. 7.4-1 should be:
  - For simple and continuous spans that at the midspan section,
  - For cantilevers that at the support.

7.5 Permissible Deflections

- To ensure satisfactory performance in service, ACI Code ***24.2.2*** imposes ***certain limits*** on ***deflections calculated according to the procedures just described***.
- These limits are given in Table 7.5-1.
- Limits depend on:
  - Whether or not the member supports or is attached to other nonstructural elements,
  - Whether or not those nonstructural elements are likely to be damaged by large deflections.
- Span length ℓ:
  - According notations and terminology in Chapter 2 of the code, the length ℓ has been defined as ***span length of beam or one-way slab; clear projection of cantilever***.
  - According the ***textbook***, this statement has been understood as that ***center to center span should be used for ℓ for spans other than cantilever where clear span should be used***.

Table 7.5-1: Maximum permissible calculated deflections, Table 24.2.2 of the code.

Member	Condition		Deflection to be considered	Deflection limitation
Flat roofs	Not supporting or attached to nonstructural elements likely to be damaged by large deflections		Immediate deflection due to maximum of L_r, S, and R	$\ell/180$[1]
Floors			Immediate deflection due to L	$\ell/360$
Roof or floors	Supporting or attached to non-structural elements	Likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements, which is the sum of the time-dependent deflection due to all sustained loads and the immediate deflection due to any additional live load[2]	$\ell/480$[3]
		Not likely to be damaged by large deflections		$\ell/240$[4]

[1]Limit not intended to safeguard against ponding. Ponding shall be checked by calculations of deflection, including added deflections due to ponded water, and considering time-dependent effects of sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

[2]Time-dependent deflection shall be calculated in accordance with 24.2.4, but shall be permitted to be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be calculated on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

[3]Limit shall be permitted to be exceeded if measures are taken to prevent damage to supported or attached elements.

[4]Limit shall not exceed tolerance provided for nonstructural elements.

- When ***long-term deflections are computed***, ***that part of the deflection that occurs before attachment of the nonstructural elements may be deducted***; information from ***Figure 7.4-1*** or from ***Table 7.4-1*** may be useful for this purpose.
- As indicated in footnotes (3) and (4), the last two limits of ***Table 7.5-1 may be exceeded under certain conditions***, according to the ACI Code.

7.6 A STEP BY STEP PROCEDURE TO CHECK THE DEFLECTION

The aforementioned discussions of Sections 7.2 through 7.5 have been put in a step by step procedure for to be used in the practical checking of deflection problem.

1. Determination of the deflections due to dead and live loads:
   - Use the mechanics of materials relations presented in ***Table 7.3-1***, ***Table 7.3-2***, and ***Table 7.3-3*** to determine the deflections due to the dead load, Δ_d, and the live load, Δ_ℓ.
   - Other analytical methods such as ***moment-area method*** can be used to determine these deflections.
   - ***Almost in all of current practical problems***, these deflections are ***determined by the software***.
   - If the computations give the total defection due to dead and live load together, $\Delta_{d+\ell}$, the deflection due to each part can be determined based on the following linear interpolations:

     $$\Delta_d = \frac{W_d}{W_d + W_\ell} \times \Delta_{d+\ell} \quad \text{Eq. 7.6-1}$$

     $$\Delta_\ell = \frac{W_\ell}{W_d + W_\ell} \times \Delta_{d+\ell} \quad \text{Eq. 7.6-2}$$

     As traditional structural analysis in civil engineering applications are based on ***linear behavior assumptions***[1], the above ***linear proportionalities seem justifiable***.
   - Irrespective of the computation approach, these deflections are usually ***instantaneous in nature*** and ***determined based on gross moment of inertia***, I_g. Therefore, they ***should be modified for the cracking effect and the long-term effect***.
2. Modification for the crack effect if necessary:
   - Determine the service moment, M_a, due to the dead and live loads.
   - Determine the cracking moment, M_{cr}, based on ***Eq. 7.3-5***.
   - If $M_a < M_{cr}$ then ***the section is uncracked***, and the deflections determined based on I_g are correct and ***no modification is required***.
   - If $M_a > M_{cr}$, the section is ***partially*** to ***fully cracked stage***, and the deflections should be modified as follows:
     i. Determine the effective moment of inertia, I_e, based on ***Eq. 7.3-7***.
     ii. Modified the deflection based on the following relation:

     $$\Delta_{with\ crack\ effects} = \Delta_{without\ crack\ effect} \times \frac{I_g}{I_e} \quad \text{Eq. 7.6-3}$$
3. Modification for the log-term effect:
   The deflection due to sustained loads including ***selfweight***, ***superimposed dead load***, and a ***permanent*** part of the live load should be modified with the factor λ_Δ of ***Eq. 7.4-1***.
4. Determine the final deflections and compare with code permissible values:
   As indicated in ***Table 7.5-1***, the code offers two deflection checking, one for the immediate live loads and the second for the total loads.
   - Checking for immediate live load deflection:
     i. Classify the structural system into a flat roof system or into a floor system.
     ii. Determine the immediate live load deflection with modification for the crack effect if necessary and compare with the permissible value of the code:

     $$\Delta_{\text{immediate}\,\ell} = \Delta_\ell \times \left(\frac{I_g}{I_e}\right)_{modification\ for\ crack} \leqslant \begin{Bmatrix} if\ flat\ roof & \dfrac{\ell}{180} \\ if\ floor & \dfrac{\ell}{360} \end{Bmatrix}$$

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[1] Analytically, this assumption is valid only when the ***materials are linearly elastic***, and the ***deformations are small***.