- Checking for the total deflection:
  i. Classify the structural system into a ***floor system*** supports ***nonstructural elements likely to be damaged by large deflections elements*** or ***not***.
  ii. Compute the ***total deflection*** occurring ***after attachment of nonstructural elements***, which is the sum of the ***time-dependent deflection due to all sustained loads*** and ***the immediate deflection due to any additional live load***.

$$\Delta_{\text{total}} = \left(\left(\Delta_d + \Delta_\ell \left(\frac{Live\ sustained}{Live\ total} \right) \right) \times \left(\frac{I_g}{I_e} \right)_{modification\ for\ crack} \times (\lambda_\Delta)_{modification\ for\ long-term} + (\Delta_\ell) \times \left(\frac{I_g}{I_e} \right)_{modification\ for\ crack} \right) \leqslant \begin{Bmatrix} if \text{ nonstructural elements likely to be damaged} & \dfrac{\ell}{480} \\ if \text{ nonstructural elements not likely to be damaged} & \dfrac{\ell}{240} \end{Bmatrix}$$

7.7 Examples for Deflection Control

Example 7.7-1

Check adequacy for the simply supported beam indicated in ***Figure 7.7-1*** for the deflection control requirements of the code. In your checking assume that:

- The selfweight of the beam is already included in the indicated dead loads,
- Sixty percent of the live load is sustained,
- The beam is part from a flooring system, and it supports non-structural element likely to be damaged by the deflection,
- Material strengths are $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.

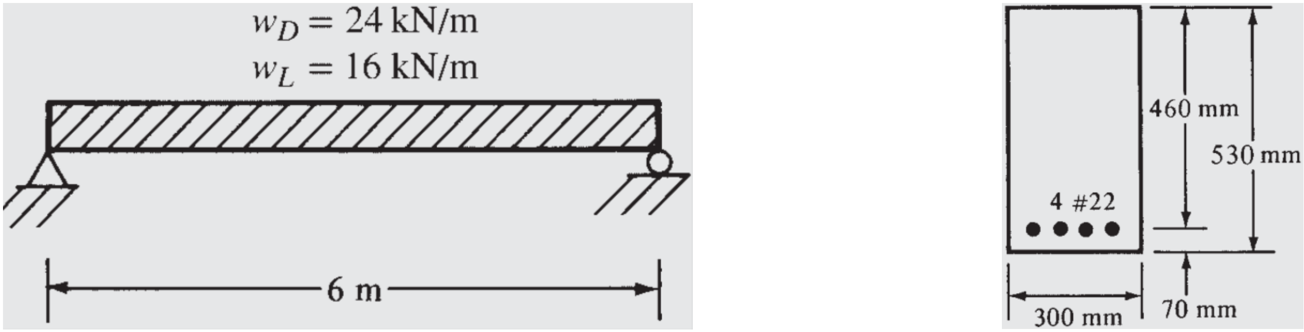


Figure 7.7-1: Simply supported beam for Example 7.7-1.

Solution

1. Determination of the deflections due to dead and live loads:
   Based on the mechanics of materials, see ***Table 7.3-1***, the immediate deflection in terms of I_g would be:

$$\Delta = \frac{5}{384}\left(\frac{w\ell^4}{E_c I_g}\right)$$

$$E_c = 4700\sqrt{f_c'} = 4700 \times \sqrt{28} = 24870\ MPa, E_s = 200000\ MPa \Rightarrow n = \frac{E_s}{E_c} = \frac{200000}{24870} \approx 8$$

$$I_g = \frac{bh^3}{12} = \frac{300 \times 530^3}{12} = 3.72 \times 10^9\ mm^4$$

Before substitution in the above relation, it is useful to note that:

$$\frac{kN}{m} = \frac{N}{mm}$$

Therefore, no unit transformation is required for the distributed loads.

$$\Delta_\text{d} = \frac{5}{384} \times \left(\frac{24 \times 6000^4}{24870 \times 3.72 \times 10^9}\right) = 4.38\ mm$$

$$\Delta_\ell = \frac{5}{384} \times \left(\frac{16 \times 6000^4}{24870 \times 3.72 \times 10^9}\right) = 2.92\ mm$$

2. Modification for the crack effect if necessary:

$$M_a = \frac{w_{d+\ell}\ell^2}{8} = \frac{(24+16) \times 6^2}{8} = 180\ kN.m$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \left(\frac{\left((0.62 \times 1.0 \times \sqrt{28}) \times 3.72 \times 10^9\right)}{\left(\frac{530}{2}\right)}\right) \times \left(\frac{1}{10^6}\right) = 46.1\ kN.m < M_a$$

Therefore, the section is a ***partially*** or ***full cracked*** one.
The centroid for the cracked section measured from the top face is:

$$(\bar{y} \times b) \times \frac{\bar{y}}{2} = nA_s \times (d - \bar{y})$$

$$nA_s = 8 \times \left(4 \times \frac{\pi \times 22^2}{4}\right) = 12164\ mm^4$$

$$(\bar{y} \times 300) \times \frac{\bar{y}}{2} = (12164) \times (460 - \bar{y}) \Rightarrow \bar{y} = 157\ mm$$

$$I_{cr} = \frac{300 \times 157^3}{3} + 12164 \times (460 - 157)^2 = 1.5 \times 10^9\ mm^4$$

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) I_{cr}$$

$$I_e = \left(\left(\frac{46.1}{180}\right)^3 \times 3.72 + \left(1 - \left(\frac{46.1}{180}\right)^3\right) \times 1.50\right) \times 10^9 \Rightarrow I_e = 1.54 \times 10^9 mm^4$$

Therefore, the modification factor for crack would be:

$$\frac{I_g}{I_e} = \frac{3.72}{1.54} = 2.42 \blacksquare$$

3. Modification for the log-term effect:
   It is next necessary to find the sustained-load deflection multiplier, λ_Δ given by ***Eq. 7.4-1***:

$$\lambda_\Delta = \frac{\xi}{1 + 50\rho'}$$

   The time-dependent coefficient, ξ, can be taken as 2.0 based on ***Figure 7.4-1*** or ***Table 7.4-1***. As the beam is singly reinforced, therefore $\rho' = 0$, then λ_Δ would be:

$$\lambda_\Delta = \frac{2}{1 + 50\rho'} = 2 \blacksquare$$

4. Determine the final deflections and compare with the code permissible values:
- Checking for immediate live load deflection:
  i. Classify the structural system into a flat roof system or into a floor system:
     In the examples statement, the structural system is a floor system.
  ii. Determine the immediate live load deflection with modification for the crack effect if necessary and compare with the permissible value of the code:

$$\Delta_{\text{immediate}\,\ell} = \Delta_\ell \times \left(\frac{I_g}{I_e}\right)_{modification\ for\ crack} \leqslant \frac{\ell}{360}$$

$$\Delta_{\text{immediate}\,\ell} = 2.92 \times 2.42 = 7.05\ mm < \frac{\ell}{360} = \frac{6000}{360} = 16.7\ mm \therefore Ok.$$

- Checking for the total deflection:
  i. Classify the structural system into a ***floor system*** supports ***nonstructural elements likely to be damaged by large deflections elements*** or ***not***.
     Example statements mentions that ***the beam supports nonstructural partitions that would be damaged if large deflections were to occur***.
  ii. Compute the ***total deflection*** occurring ***after attachment of nonstructural elements***, which is the sum of the ***time-dependent deflection due to all sustained loads*** and ***the immediate deflection due to any additional live load***.

$$\Delta_{\text{total}} = \left(\left(\Delta_d + \Delta_\ell\left(\frac{Live\ sustained}{Live\ total}\right)\right) \times \left(\frac{I_g}{I_e}\right)_{modification\ for\ crack} \times (\lambda_\Delta)_{modification\ for\ long-term} + (\Delta_\ell) \times \left(\frac{I_g}{I_e}\right)_{modification\ for\ crack}\right) \leqslant \frac{\ell}{480}$$

$$\Delta_{\text{total}} = \left(\left(4.38 + 2.92 \times \left(\frac{60}{100}\right)\right) \times (2.42) \times (2) + (2.92) \times (2.42)\right) = 36.7\ mm < \frac{\ell}{480} = \frac{6000}{480} = 12.5\ mm \therefore Not\ Ok.$$

Aforementioned computations and comparisons indicating that ***the stiffness of the proposed member is insufficient***.

Example 7.7-2

The beam shown in ***Figure 7.7-2*** is a part of the floor system of an apartment house and is designed to carry calculated ***dead load*** w_d ***of 24 kN/m*** and a ***service live load*** w_ℓ ***of 48 kN/m***. Of the total live load, ***20 percent is sustained in nature***, while ***80 percent will be applied only intermittently over the life of the structure***. Under ***full dead and live load***, the moment diagram is as shown in ***Figure 7.7-2c*** and the ***total deflection is*** $\Delta_{d+\ell} = 2.82$ mm.

The ***beam will support nonstructural partitions that would be damaged if large deflections were to occur***. They will be installed shortly after construction shoring is removed and dead loads take effect, but before significant creep occurs.

Check beam adequacy for deflection requirements of the ACI code. Material strengths are $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.

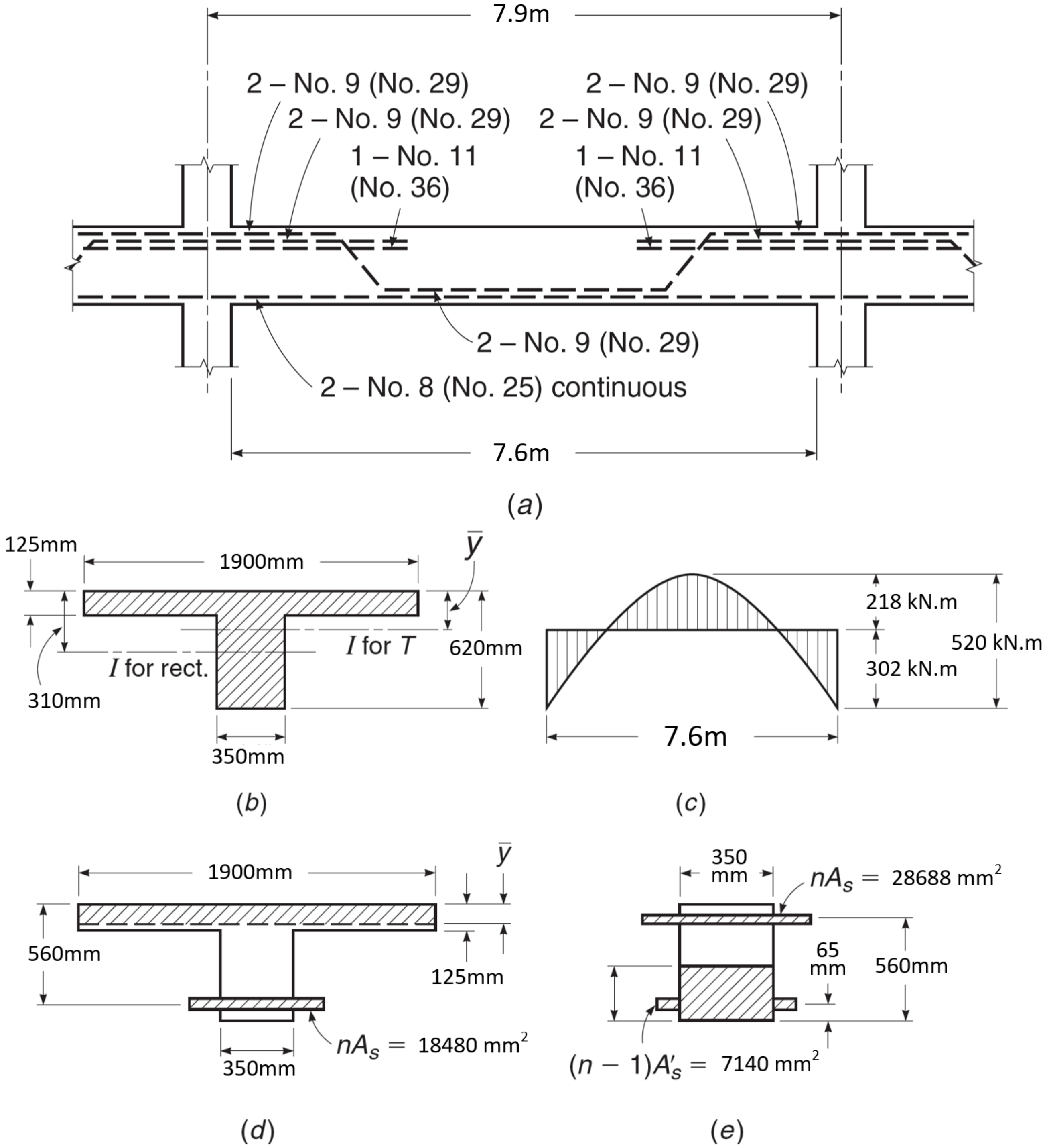


Figure 7.7-2: Continuous T beam for deflection calculations in Example 7.7-2. The uncracked section is shown in (b), the cracked transformed section in the positive moment region is shown in (d), and the cracked transformed section in the negative moment region is shown in (e).

Solution

1. Determination of the deflections due to dead and live loads:
   As the deflection due to dead and live loads, $\Delta_{d+\ell}$, is already given in the example statement, therefore what is necessary at this step is to determine the deflection due to dead load alone, Δ_d , and the live load alone, Δ_ℓ based on linear proportionalities of ***Eq. 7.6-1*** and ***Eq. 7.6-2***:
   $$\Delta_d = \frac{W_d}{W_d + W_\ell} \times \Delta_{d+\ell} = \frac{24}{24+48} \times 2.82 = 0.94\,mm$$
   $$\Delta_\ell = \frac{W_\ell}{W_d + W_\ell} \times \Delta_{d+\ell} = \frac{48}{24+48} \times 2.82 = 1.88\,mm$$
2. Modification for the crack effect if necessary:
   For the specified materials:
   $$E_c = 4700\sqrt{f_c'} = 4700 \times \sqrt{28} = 24870\,MPa, E_s = 200000\,MPa \Rightarrow n = \frac{E_s}{E_c} = \frac{200000}{24870} \approx 8$$
   The modulus of rupture, f_r, is:
   $$f_r = 0.62\lambda\sqrt{f_c'} = 0.62 \times 1 \times \sqrt{28} = 3.28\,MPa$$
   The effective moment of inertia will be calculated for the moment diagram shown in ***Figure 7.7-2c*** corresponding to ***the full-service load***, on the basis that ***the extent of cracking will be governed by the full-service load***, ***even though that load is intermittent***.
   Determine the instantaneous deflection due to dead and live loads:
   As the structure is assumed linear in traditional structural analysis, therefore the instantaneous deflection due to deal load, Δ_d, and due to live load, Δ_ℓ, can be determined from $\Delta_{d+\ell}$ based on the following linear proportionalty
   <u>The positive region:</u>
   In the positive-moment region, the centroidal axis of the uncracked T section of ***Figure 7.7-2b*** is found by taking moments about the top surface, to be;
   $$\bar{y}_{for\ the\ gross\ positve\ section} = \frac{\Sigma A_i y_i}{\Sigma A_i}$$
   $$= \frac{\left(1900 \times 125 \times \frac{125}{2} + 350 \times (620 - 125) \times \left(\frac{(620-125)}{2} + 125\right)\right)}{\left(1900 \times 125 + 350 \times (620 - 125)\right)}$$
   $$= 193\,mm < 310mm \therefore Ok.$$
   The moment of inertia, I_g, for the gross section is:
   $$I_g = \left(\frac{350 \times 620^3}{12} + 350 \times 620 \times (310 - 193)^2\right) + \frac{(1900 - 350) \times 125^3}{12}$$
   $$+ (1900 - 350) \times 125 \times \left(193 - \frac{125}{2}\right)^2 = 1.347 \times 10^{10} mm^4$$
   The cracking moment, M_{cr}, is then found by means of Eq. 7.3-5:
   $$M_{cr} = \frac{f_r I_g}{y_t} = \left(\frac{3.28 \times 1.347 \times 10^{10}}{620 - 193}\right) \times \frac{1}{1000000} = 104\,kN.m$$
   With
   $$\frac{M_{cr}}{M_a} = \frac{104}{218} = 0.477 < 1.0$$
   Therefore, the section is cracked and I_{cr} and I_e should be determined and the deflection should be modified accordingly.
   The centroidal axis of the cracked transformed T section shown in ***Figure 7.7-2d*** is determined as follows, assume that $\bar{y} \leq 125mm$ to be checked later:
   $$(\bar{y} \times b) \times \frac{\bar{y}}{2} = nA_s \times (d - \bar{y}) \Rightarrow (\bar{y} \times 1900) \times \frac{\bar{y}}{2} = 18480 \times (560 - \bar{y})$$
   $$\Rightarrow \bar{y} = 95.7\,mm < 125mm \therefore Ok.$$
   below the top of the slab and I_{cr} would be:
   $$I_{cr} = \frac{1900 \times 95.7^3}{3} + 18480 \times (560 - 95.7)^2 = 0.4539 \times 10^{10}\,mm^4$$
   The effective moment of inertia in the positive bending region is found from Eq. 7.3-7 to be:

$$I_e=\left(\frac{M_{cr}}{M_a}\right)^3 I_g+\left(1-\left(\frac{M_{cr}}{M_a}\right)^3\right)I_{cr}\Rightarrow I_e$$
$$=\left(\frac{104}{218}\right)^3\times 1.347\times 10^{10}+\left(1-\left(\frac{104}{218}\right)^3\right)\times 0.4539\times 10^{10}$$
$$I_e=0.551\times 10^{10}\ mm^4$$

<u>The negative region:</u>

In the negative bending region, the gross moment of inertia will be based on the rectangular section shown in ***Figure 7.7-2b***. For this area, the centroid is:

$$\bar{y}=\frac{620}{2}=310mm$$

from the top surface and I_g would be:

$$I_g=\frac{bh^3}{12}=\frac{350\times 620^3}{12}=0.695\times 10^{10}mm^4$$

Therefore, the crack moment, M_{cr}, would be:

$$M_{cr}=\frac{f_rI_g}{y_t}=\left(\frac{3.28\times 0.695\times 10^{10}}{310}\right)\times\frac{1}{1000000}=73.5\ kN.m$$
$$\frac{M_{cr}}{M_a}=\frac{73.5}{302}=0.243<1.0$$

Therefore, the section is cracked and I_{cr} and I_e should be determined and the deflection should be modified accordingly.

For the cracked transformed section shown in ***Figure 7.7-2e***, the centroidal axis is found, taking moments about the bottom surface, to be:

$$(\bar{y}\times 350)\times\frac{\bar{y}}{2}+7140\times(\bar{y}-65)=28688\times(560-\bar{y})\Rightarrow\bar{y}=222mm$$

from that level, and I_{cr} would be:

$$I_{cr}=\frac{350\times 222^3}{3}+7140\times(222-65)^2+28688\times(560-222)^2=0.473\times 10^{10}mm^4$$

Thus, for the negative-moment regions,

$$I_e=\left(\frac{M_{cr}}{M_a}\right)^3 I_g+\left(1-\left(\frac{M_{cr}}{M_a}\right)^3\right)I_{cr}\Rightarrow I_e$$
$$=\left(\frac{73.5}{302}\right)^3\times 0.695\times 10^{10}+\left(1-\left(\frac{73.5}{302}\right)^3\right)\times 0.473\times 10^{10}$$
$$I_e=0.476\times 10^{10}mm^4$$

The average effective moment of inertia:

The average value of I_e to be used in calculation of deflection is:

$$I_{e\ avg.}=\frac{1}{2}(0.551+0.476)\times 10^{10}=0.514\times 10^{10}\ mm^4$$

<u>The modification factor for the crack:</u>

Based on Eq. 7.6-2, the deflection should be modified for the crack based on the following relation:

$$\Delta_{with\ crack\ effects}=\Delta_{without\ crack\ effect}\times\frac{I_g}{I_e}\Rightarrow$$
$$\Delta_{with\ crack\ effects}=\frac{1.347\times 10^{10}}{0.514\times 10^{10}}\Delta_{without\ crack\ effect}=2.62\Delta_{without\ crack\ effect}\blacksquare$$

3. Modification for the log-term effect:

   It is next necessary to find the sustained-load deflection multiplier, λ_Δ given by Eq. 7.4-1:

$$\lambda_\Delta=\frac{\xi}{1+50\rho'}$$

The time-dependent coefficient, ξ, can be taken as 2.0 based on ***Figure 7.4-1*** or ***Table 7.4-1***. For the positive bending zone, with no compression reinforcement, $\rho'=0$, then λ_Δ would be:

$$\lambda_\Delta=\frac{2}{1+50\rho'}=2\blacksquare$$

4. Determine the final deflections and compare with the code permissible values:
   - Checking for immediate live load deflection:
     i. Classify the structural system into a flat roof system or into a floor system: In the examples statement, the structural system is a floor system.
     ii. Determine the immediate live load deflection with modification for the crack effect if necessary and compare with the permissible value of the code:

$$\Delta_{\text{immediate}\,\ell} = \Delta_\ell \times \left(\frac{I_g}{I_e}\right)_{modification\ for\ crack} \leqslant \frac{\ell}{360}$$

$$\Delta_{\text{immediate}\,\ell} = 1.88 \times 2.62 = 4.92\ mm < \frac{\ell}{360} = \frac{7900}{360} = 21.9\ mm \therefore Ok.$$

   - Checking for the total deflection:
     i. Classify the structural system into a ***floor system*** supports ***nonstructural elements likely to be damaged by large deflections elements*** or ***not***.
     Example statements mentions that ***the beam will support nonstructural partitions that would be damaged if large deflections were to occur***.
     ii. Compute the ***total deflection*** occurring ***after attachment of nonstructural elements***, which is the sum of the ***time-dependent deflection due to all sustained loads*** and ***the immediate deflection due to any additional live load***.

$$\Delta_{\text{total}} = \left(\left(\Delta_d + \Delta_\ell\left(\frac{Live\ sustained}{Live\ total}\right)\right) \times \left(\frac{I_g}{I_e}\right)_{modification\ for\ crack} \times (\lambda_\Delta)_{modification\ for\ long-term} + (\Delta_\ell) \times \left(\frac{I_g}{I_e}\right)_{modification\ for\ crack}\right) \leqslant \frac{\ell}{480}$$

$$\Delta_{\text{total}} = \left(\left(0.94 + 1.88 \times \left(\frac{20}{100}\right)\right) \times 2.62 \times (2) + (1.88) \times 2.62\right) = 11.8\ mm < \frac{\ell}{480} = \frac{7900}{480} = 16.5mm$$

$\therefore Ok.$

Aforementioned computations and comparisons indicating that ***the stiffness of the proposed member is sufficient***.

Contents

CHAPTER 8
ANALYSIS AND DESIGN FOR TORSION

8.1 INTRODUCTION

- Torsional forces may act, tending to twist a member about its longitudinal axis.
- Torsional forces seldom act alone and are ***usually concurrent*** with ***bending moment*** and ***transverse shear***, and sometimes with axial force.

8.1.1 Torsion in Old Design Philosophy

For many years, ***torsion was regarded as a secondary effect*** and ***was not considered explicitly in design***, its influence being absorbed in the overall factor of safety of rather conservatively designed structures.

8.1.2 Torsion in Current Design Philosophy

- Current methods of analysis and design have ***resulted in less conservatism***, ***leading to somewhat smaller members that***, in many cases, ***must be reinforced to increase torsional strength***.
- Torsion should be included explicitly especially with the increasing use of structural members for which ***torsion is a central feature of behavior***; examples include ***curved beam***, ***curved bridge girders***, and ***helical stairway slabs***.

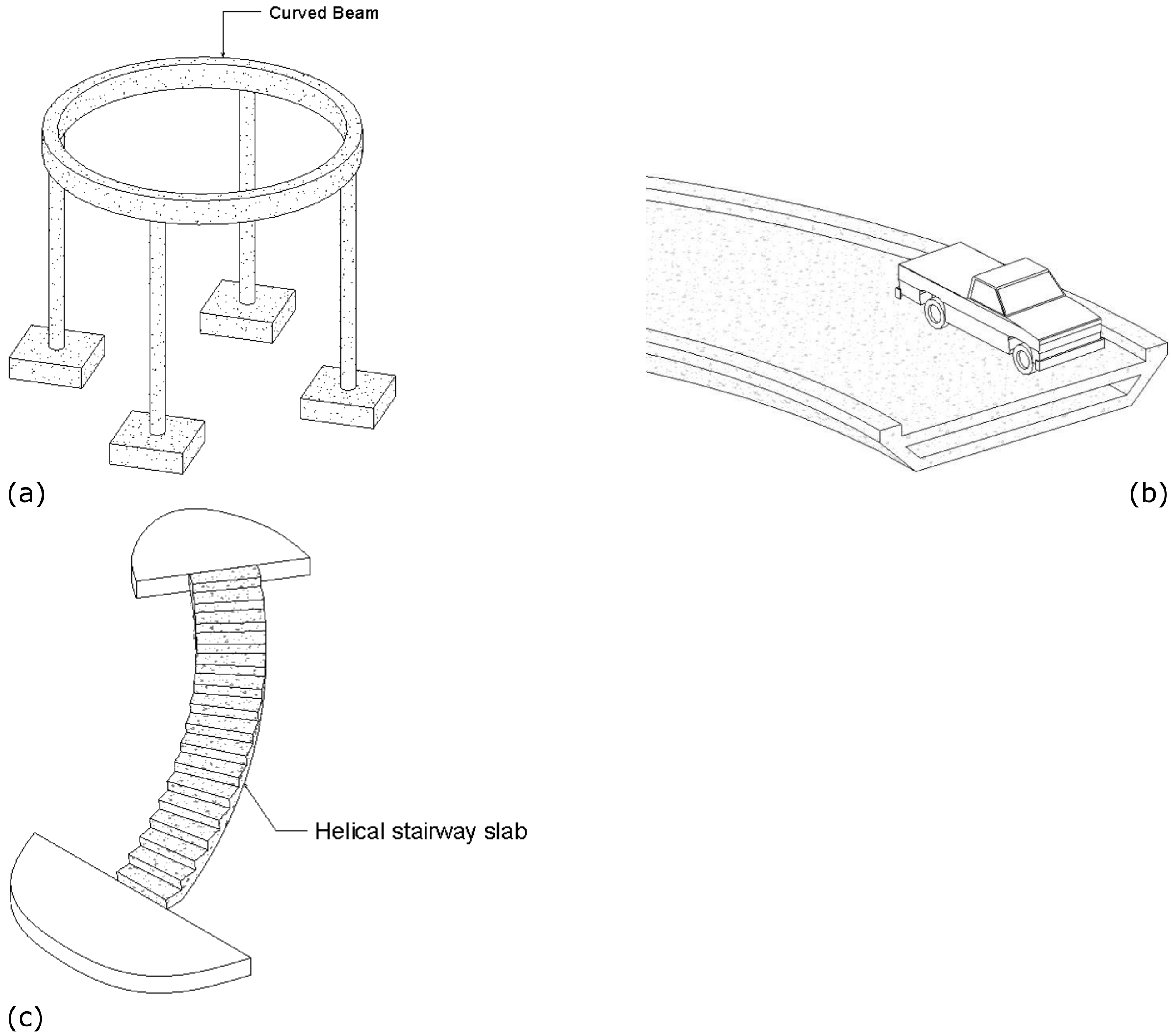


Figure 8.1-1: Members subjected to significant torsion: (a) curved beams; (b) bridge girders; (c) helical stairway slabs.

8.1.3 Primary versus Secondary Torsions

It is useful in considering torsion to distinguish between ***primary*** and ***secondary*** torsion in reinforced concrete structures.

8.1.3.1 Primary Torsion

- Sometimes called ***equilibrium torsion*** or ***statically determinate torsion***, exists when the external load has no alternative load path but must be supported by torsion.
- For such cases, the torsion required to maintain static equilibrium could be uniquely determined.
- An example is the cantilevered slab of ***Figure 8.1-2*** below. Loads applied to the slab surface cause twisting moments m_t to act along the length of the supporting beam. These are equilibrated by the resisting torque T provided at the columns. Without the torsional moments, the structure will collapse.

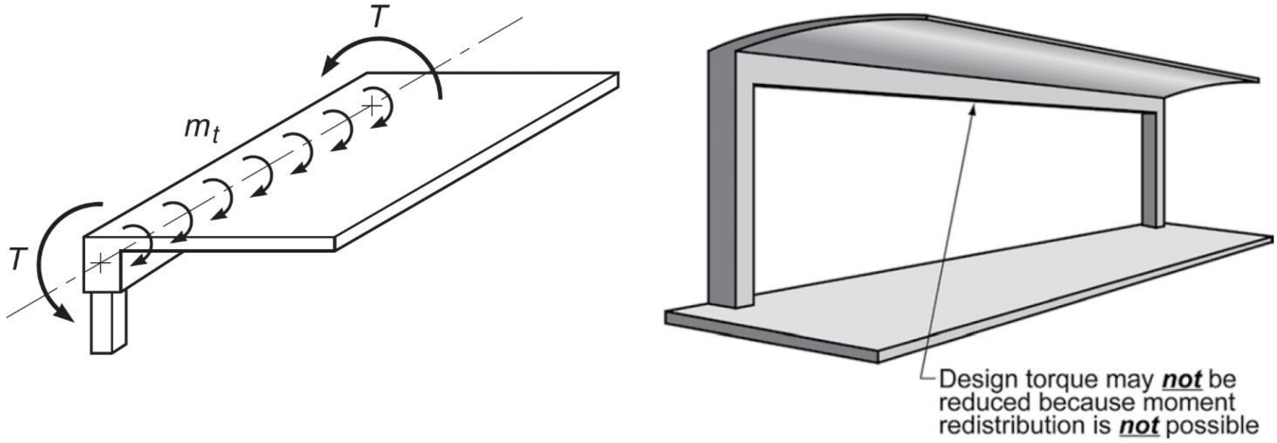
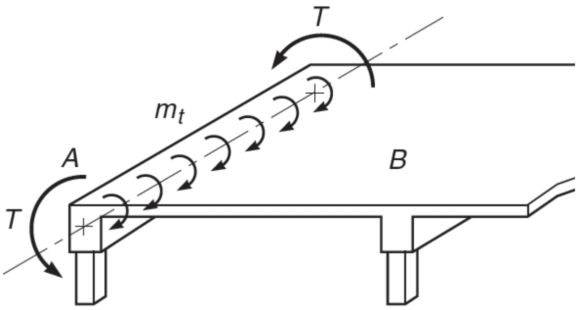


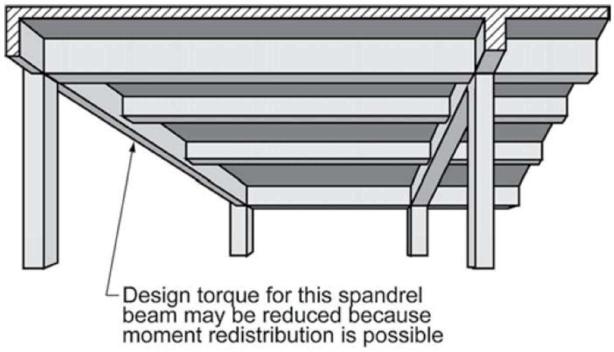
Figure 8.1-2: Primary or equilibrium torsion at a cantilevered slab.

8.1.3.2 Secondary Torsion,

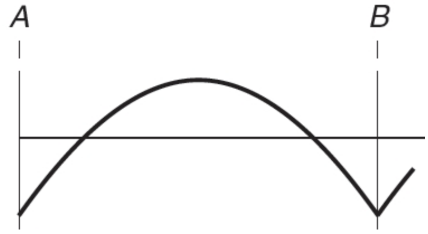
- Also called ***compatibility torsion*** or ***statically indeterminate torsion***, arises from the requirements of continuity, that is, compatibility of deformation between adjacent parts of a structure.
- For this case, the torsional moments cannot be found based on static equilibrium alone. ***Disregard of continuity in the design will often lead to extensive cracking, but generally will not cause collapse***. ***An internal readjustment of forces is usually possible and an alternative equilibrium of forces found***.
- An example of secondary torsion is found in the spandrel or edge beam supporting a monolithic concrete slab, shown in ***Figure 8.1-3a***.
  - **First Load Path:**
    If the spandrel beam is torsionally stiff and suitably reinforced, and if the columns can provide the necessary resisting torque T, then the slab moments will approximate those for a rigid exterior support as shown in ***Figure 8.1-3b***.
  - **Second Load Path:**
    However, if the beam has little torsional stiffness and inadequate torsional reinforcement, cracking will occur to further reduce its torsional stiffness, and the slab moments will approximate those for a hinged edge, as shown in ***Figure 8.1-3c***.

  If the slab is designed to resist the altered moment diagram, collapse will not occur.

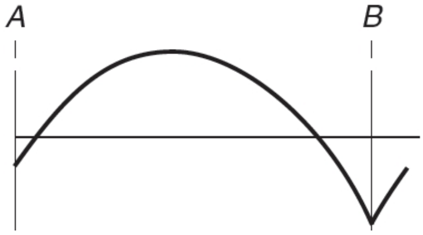




(a) Secondary or compatibility torsion at an edge beam.



(b) slab moments if edge beam is stiff torsionally



(c) Slab moments if edge beam is flexible torsionally.

Figure 8.1-3: Secondary or compatibility torsion.

8.1.4 Torsion in Uncracked Plain Concrete Members

If the material is elastic, St. Venant's torsion theory indicates that torsional shear stresses are distributed over the cross section, as shown in ***Figure 8.1-4*** below.

- Stress Distribution in Elastic Martial:
  The largest shear stresses occur at the middle of the wide faces.
- Stress Distribution in Inelastic Martial:
  If the material deforms inelastically, as expected for concrete, the stress distribution is closer to that shown by the dashed line.
- Diagonal Stresses Associated with Torsional Shear Stresses:
  - Shear stresses in pairs act on an element at or near the wide surface, as shown in ***Figure 8.1-4a***.
  - As explained in strength of materials texts, this state of stress corresponds to equal tension and compression stresses on the faces of an element at 45° to the direction of shear.
  - These inclined tension stresses are of the same kind as those caused by transverse shear, discussed in ***Chapter 5***.
  - However, in the case of torsion, since the torsional shear stresses are of opposite sign on opposing sides of the member (***Figure 8.1-4b***), the corresponding diagonal tension stresses are at right angles to each other (***Figure 8.1-4a***).

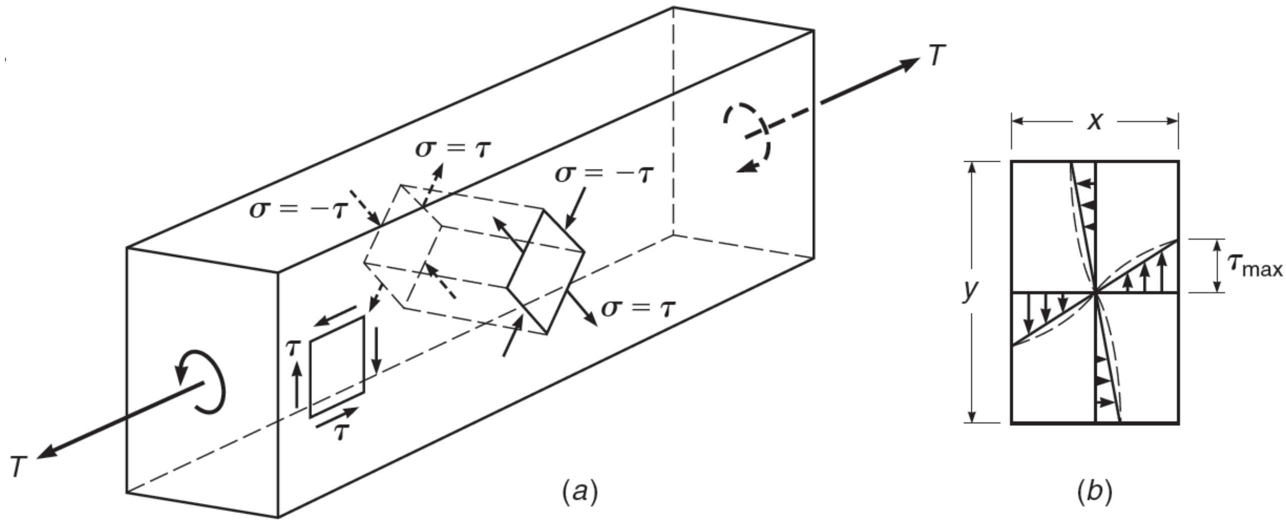


Figure 8.1-4: Stresses caused by torsion.

8.1.5 Cracking Torque T_{cr}

- Definition of Cracking Torque T_{cr}
  - When the diagonal tension stresses exceed the tensile resistance of the concrete, a crack forms at some accidentally weaker location and spreads immediately across the beam.
  - The value of torque corresponding to the formation of this diagonal crack is known as the cracking torque T_{cr}.
- Using ***Thin-walled Tube***, ***Space Truss Analogy*** to Compute T_{cr}:
  - The nonlinear stress distribution shown by the dotted lines in ***Figure 8.1-4b*** lends itself to the use of the ***thin-walled tube***, ***space truss analogy***.
  - Using this analogy, ***the shear stresses are treated as constant over a finite thickness t around the periphery of the member***, allowing the beam to be represented by an equivalent tube, as shown in ***Figure 8.1-5*** below.
- Shear Flow According to Thin-walled Tube Model:
  In the analogy, shear flow q is treated as a constant around the perimeter of the tube and related to applied torque, T, as follows:
  $T = q(x_0 t)_{Area} \times y_{0\,Arm} + q(y_0 t)_{Area} \times x_{0\,Arm} \Longrightarrow T = 2q\, x_0 y_0 t$
  The product $x_0 y_0$ represents the area enclosed by the shear flow path A_0 , giving
  $\because x_0 y_0 = A_0 \Longrightarrow \therefore T = 2qA_0 \Longrightarrow q = \dfrac{T}{2A_0}$ ■
- Shear Stress τ According to Thin-walled Tube Model:
  $\because \tau = \dfrac{q}{t} \Longrightarrow \therefore \tau = \dfrac{T}{2A_0 t}$
- Corresponding Diagonal Tension:
  From ***Figure 8.1-4a*** above
  $\sigma = \tau$
  Let tensile strength of concrete approximated with
  $\sigma = 0.33\lambda\sqrt{f_c'}$
  Therefore, the cracking torque would be:
  $T_{cr} = 0.33\lambda\sqrt{f_c'}(2A_0 t)$
  Let
  $A_0 \approx \dfrac{2}{3}A_{cp}, t = \dfrac{3}{4}\dfrac{A_{cp}}{p_{cp}}$
  The cracking moment would be:
  $T_{cr} = 0.33\lambda\sqrt{f_c'}\left(2 \times \dfrac{2}{3}A_{cp} \times \dfrac{3}{4}\dfrac{A_{cp}}{p_{cp}}\right)$
  $T_{cr} = 0.33\lambda\sqrt{f_c'}\left(\dfrac{A_{cp}^2}{p_{cp}}\right)$ ■
  where
  A_{cp} is area enclosed by outside perimeter of concrete cross section, in mm^2,
  p_{cp} is outside perimeter of concrete cross section, in mm.

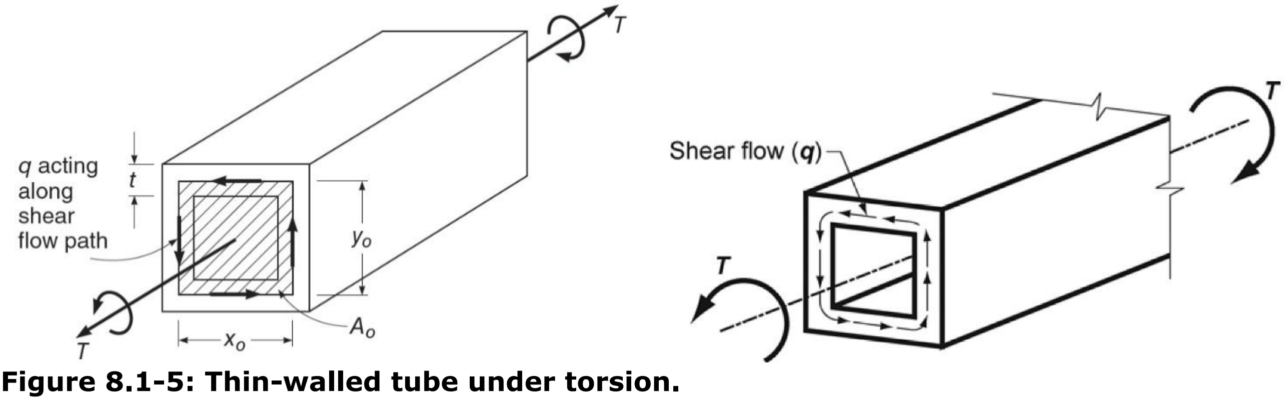


Figure 8.1-5: Thin-walled tube under torsion.

8.1.6 Torsion in Reinforced Concrete Members

8.1.6.1 Reinforcement for Torsion

- To resist torsion for values of T above T_{cr} , reinforcement must consist of ***closely spaced stirrups*** and ***longitudinal bars***,
- Reinforcement for torsion with cracking pattern are presented in ***Figure 8.1-6*** below.

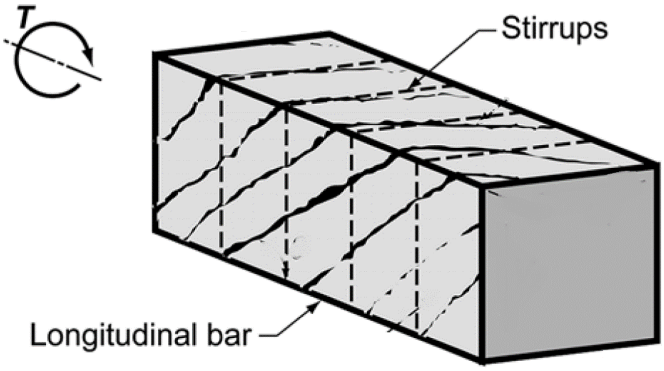


Figure 8.1-6: Reinforcement for torsion.

8.1.6.2 Shear Path after Cracking

- Tests show that, after cracking, the area enclosed by the shear path is defined by the dimensions x_0 and y_0 ***measured to the centerline of the outermost closed transverse reinforcement***.
- These dimensions define the gross area
  $A_{oh} = x_o\, y_o$
  and the shear perimeter
  $p_h = 2(x_o + y_o)$
  measured at the steel centerline, see ***Figure 8.1-7*** above

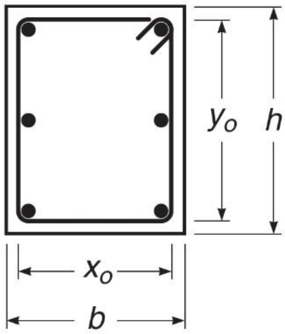


Figure 8.1-7: Notations for shear flow path after cracking of a reinforced concrete beam.

8.1.6.3 Basic Relation for Stirrups Torsional Reinforcement

- With referring to ***Figure 8.1-8*** below, the relation for stirrups torsional reinforcement can be formulated based on basic principles of equilibrium as presented in below:
  $$T_4 = \frac{V_4 x_0}{2}$$
- With referring to ***Figure 8.1-9*** below, the vertical shear force, V_4, can be related to the provided stirrups as follows:
  $$V_4 = A_t f_{yt} n$$
  where
  A_t is area of one leg of a closed stirrup,
  f_{yt} is yield strength of transverse reinforcement,
  n is number of stirrups intercepted by torsional crack.
  $$\because n = y_0 \frac{\cot\theta}{s} \Longrightarrow \therefore V_4 = \frac{A_t f_{yt} y_0}{s}\cot\theta$$
  and the pertained torsion, T_4, would be:
  $$T_4 = \frac{A_t f_{yt} y_0 x_0}{2s}\cot\theta$$
- The contributions of the horizontal walls T_1, T_2, and T 3 can be determined in the same way. Summing over all four sides, the nominal capacity of the section is:
  $$T_{Resisted\ by\ stirrups} = T_n = \sum_{i=1}^{4} \mathrm{T_i} = \frac{2A_t f_{yt} y_0 x_0}{s}\cot\theta \quad \blacksquare$$
- Noting that $y_o x_o = A_{oh}$ and rearranging slightly give
  $$T_n = \frac{2A_{0h} A_t f_{yt}}{s}\cot\theta$$

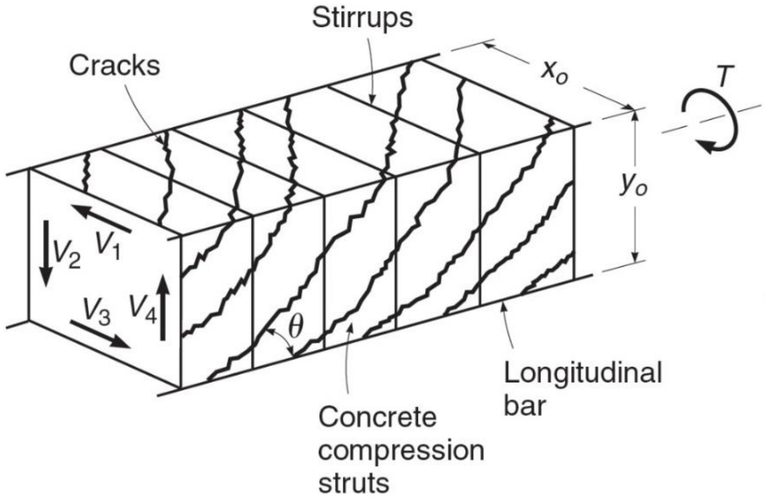


Figure 8.1-8: Space truss analogy.

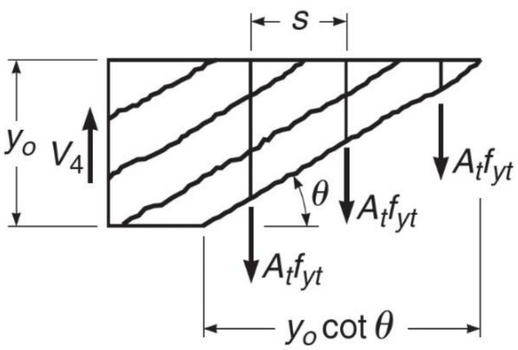


Figure 8.1-9: Vertical tension in stirrups.

8.1.6.4 Basic Relation for Longitudinal Reinforcement

- As shown in ***Figure 8.1-10 a*** and ***b***, the horizontal component of compression in the struts in the vertical wall must be equilibrated by an axial tensile force ΔN_4.

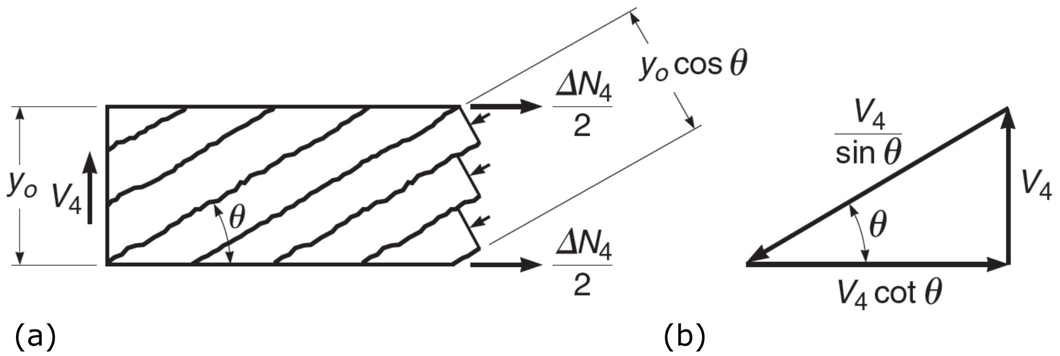


Figure 8.1-10: Basis for contribution of longitudinal reinforcement for torsional strength: (a) diagonal compression in vertical wall of beam; and (b) equilibrium diagram of forces due to shear in vertical wall.

- Based on the assumed ***uniform distribution of shear flow around the perimeter of the member***, the ***diagonal stresses in the struts must be uniformly distributed***, resulting in a line of action of the resultant axial force that ***coincides with the mid-height of the wall***.
- Referring to ***Figure 8.1-10***b , the total contribution of the right-hand vertical wall to the change in axial force of the member due to the presence of torsion is:

$$\Delta N_4 = V_4 \cot\theta = \frac{A_t f_{yt} y_0}{s} \cot^2\theta$$

- Summing over all four sides, the total increase in axial force for the member is:

$$\Delta N = \sum_{i=1}^{4} \Delta \mathrm{N_i} = \frac{A_t f_{yt}}{s} 2(x_0 + y_0) \cot^2\theta \Longrightarrow \Delta N = \frac{A_t f_{yt} p_h}{s} \cot^2\theta$$

where p_h is the perimeter of the centerline of the closed stirrups.

- Longitudinal reinforcement must be provided to carry the added axial force Δ N. If that steel is designed to yield, then:

$$A_l f_y = \frac{A_t f_{yt} p_h}{s} \cot^2\theta$$

Solve for A_l to obtain:

$$A_l = \frac{A_t}{s} p_h \frac{f_{yt}}{f_y} \cot^2\theta$$

where A_ℓ is total area of longitudinal reinforcement to resist torsion,

- Finally, as

$$T_n = \frac{2A_{0h} A_t f_{yt}}{s} \cot\theta$$

therefore,

$$A_t f_{yt} = \frac{T_n s}{2A_{0h} A_t f_{yt} \cot\theta}$$

Substitute $A_t f_{yt}$ into equation above for A_l

$$A_l = \left(\frac{1}{s} p_h \frac{1}{f_y} \cot^2\theta\right)\left(\frac{T_n s}{2A_{0h} A_t f_{yt} \cot\theta}\right)$$

and for T_n to obtain:

$$T_n = \frac{2A_{oh} A_l f_y}{p_h} \tan\theta$$

8.1.7 Torsion plus Shear

- Members are rarely subjected to torsion alone. The prevalent situation is that of a beam subject to the usual flexural moments and shear forces, which, in addition, must resist torsional moments.
- Basic Shear and Torsion Stresses in Reinforced Concrete Members:
  Using the usual representation for reinforced concrete, the nominal shear stress caused by an applied shear force V is:
  $$\tau_v = \frac{V}{b_w d}$$
  While using the concept of thin-walled tube, the shear stress caused by torsion would be:
  $$\tau_t = \frac{T}{2A_0 t}$$
- Superposition of Shear and Torsion Stresses in a Hollow Section:
- As shown in ***Figure 8.1-11*** for hollow sections, these stresses are directly additive on one side of the member. Thus, for a cracked concrete cross section with
  $A_o = 0.85\,A_{oh}$ and $t = \frac{A_{oh}}{p_h}$
  the maximum shear stress can be expressed as:
  $$\tau = \tau_v + \tau_t = \frac{V}{b_w d} + \frac{T p_h}{1.7A_{oh}^2}$$

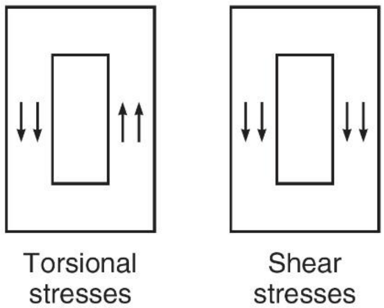


Figure 8.1-11: Addition of torsional and shear stresses in a hollow section.

- For a member with a solid section, ***Figure 8.1-12***, τ_t is predominately distributed around the perimeter, as represented by the hollow tube analogy, but the full cross section contributes to carrying τ_v.
- ***Comparisons with experimental results show that equation above for a hollow section is somewhat overconservative for solid sections*** and that a better representation for maximum shear stress is provided by the square root of the sum of the squares, SRSS[1], of the nominal shear stresses:
  $$\tau = \sqrt{\left(\frac{V}{b_w d}\right)^2 + \left(\frac{T p_h}{1.7A_{oh}^2}\right)^2}$$

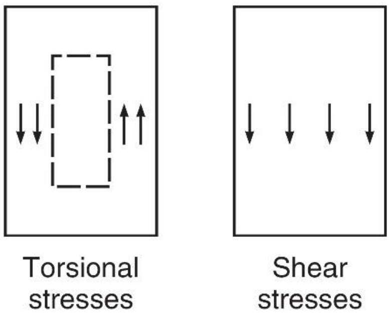


Figure 8.1-12: Addition of torsional and shear stresses in a solid section.

[1] SRSS summation is usually adopted to superimpose two quantities that their maximum values occurs at different positions or at different times.

8.2 ACI CODE PROVISIONS FOR TORSION DESIGN

8.2.1 Basic Design Principle

The basic principles upon which ACI Code design provisions are based have been presented in the preceding chapters for flexure and shear. ACI Code ***9.5.1.1*** safety provisions require that:

$T_u \le \phi T_n$

where

T_n = nominal torsional strength of member,

T_u = required torsional strength at factored loads. The strength reduction factor $\phi = 0.75$ applies for torsion.

8.2.2 Computing of T_u

- In accordance with ***ACI Code 9.4.4.3***, sections located less than a distance d from the face of a support may be designed for the same torsional moment T_u as that computed at a distance d, recognizing the beneficial effects of support compression.
- However, if a concentrated torque is applied within this distance, the critical section must be taken at the face of the support.
- ***These provisions parallel those used in shear design***.

8.2.3 Effective Section

8.2.3.1 Before Cracking

- For T beams, a portion of the overhanging flange contributes to the cracking torsional capacity and, ***if reinforced with closed stirrups***, to the torsional strength.
- According to ***ACI Code 9.2.4.4***, the contributing width of the overhanging flange on either side of the web would be as indicated in ***Figure 8.2-1*** below.

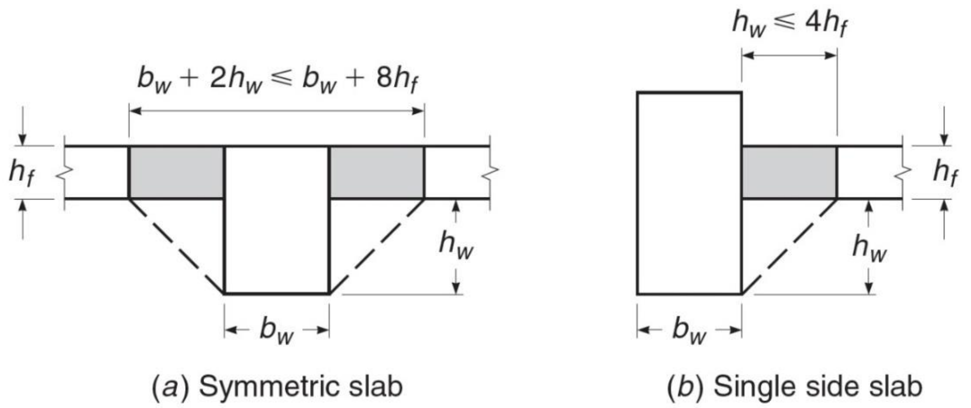


Figure 8.2-1: Portion of slab to be included with beam for torsional design.

- The overhanging flanges ***shall be neglected in cases where the parameter*** A_{cp}^2/p_{cp} ***for solid sections or*** A_g^2/p_{cp} ***for hollow sections calculated for a beam with flanges is less than that calculated for the same beam ignoring the flanges***.

8.2.3.2 After Cracking

After torsional cracking, the applied torque is resisted by the portion of the section represented by A_{oh}, the area enclosed by the centerline of the outermost closed transverse torsional reinforcement. For ***rectangular***, ***box***, and ***T sections***, A_{oh} is illustrated in ***Figure 8.2-2*** below.

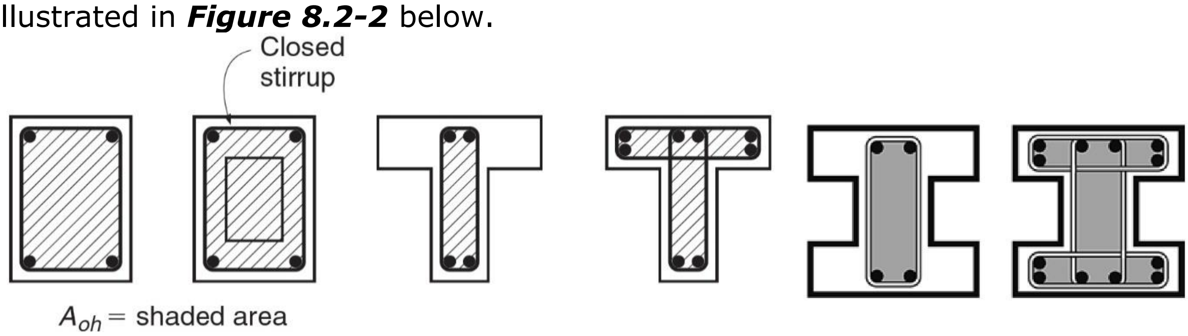


Figure 8.2-2: Definition of A_{oh}.

8.2.3.3 Sections before and after Cracking

For sections with flanges, the Code does not require that the section used to establish A_{cp} coincide with that used to establish A_{oh}.

8.2.4 Threshold Torsion

- If the value of factored torsional moment T_u is low enough, the effects of torsion may be neglected, according to ***ACI Code 22.7.1.1***.
- This lower limit is ϕ times the threshold torsion T_{th}, which equals 25 percent of the ***cracking torque***, given by:

  $$T_{th} = \frac{1}{4} T_{cr}$$

  $$\because T_{cr} = 0.33\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

  $$\therefore T_{th\,for\,solid\,section} = 0.083\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

- For hollow cross sections, the threshold torsion is:

  $$T_{th\,for\,hollow\,section} = 0.083\lambda\sqrt{f_c'}\left(\frac{A_g^2}{p_{cp}}\right)$$

- The value of λ is as specified in ***ACI Code 19.2.4.2*** and previously described with λ = 0.85, 0.75, and 1.0 for ***sand-lightweight***, ***all-lightweight***, and ***normalweight*** concrete, respectively.

8.2.5 Equilibrium vs. Compatibility Torsion

- As discussed in Article 8.1.3, a distinction is made in the ACI Code between ***equilibrium*** (***primary***) torsion and ***compatibility*** (***secondary***) torsion.
- For the ***equilibrium*** (***primary***) torsion***, the supporting member must be designed to provide the torsional resistance required by static equilibrium***.
- For ***secondary torsion*** resulting from compatibility requirements, it is assumed that cracking will result in a redistribution of internal forces; and according to ***ACI Code 22.7.3.2***, the maximum torsional moment T_u may be reduced to:

  $$T_u = \phi T_{cr}$$

  or

  $$T_u = \phi\left(0.33\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\right)$$

8.2.6 Limitations on Shear Stress

- Based largely on ***empirical observations***, the ***width of diagonal cracks*** caused by combined shear and torsion under service loads can be limited by limiting the calculated shear stress under factored shear and torsion.
- In accordance with ACI Code 22.7.7.1, shear stresses should be limited to the following values:
  - For hollow sections:

    $$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right) \le \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right)$$

  - For solid sections:

    $$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right)$$

8.2.7 Reinforcement for Torsion

8.2.7.1 Stirrups for Torsion

- As discussed in Article 8.1.6 above, stirrups for torsion can be determined from following relation:

  $$A_t = \frac{T_u s}{2\phi A_o f_{yt} \cot\theta}$$

- According to ACI Code ***22.7.6.1***, the angle θ may assume any value between ***30***

and 60°, with a value of ***θ = 45°*** suggested.

- The Code limits f_{yt} ***to a maximum of 420 MPa*** for reasons of crack control.
- The reinforcement provided for torsion must be combined with that required for shear. Based on the typical two-leg stirrup, this may be expressed as
$$\frac{A_{v+t}}{s} = \frac{A_v}{s} + 2\frac{A_t}{s}$$
- Anchorage of torsional stirrups is presented in ***Figure 8.2-3*** below.

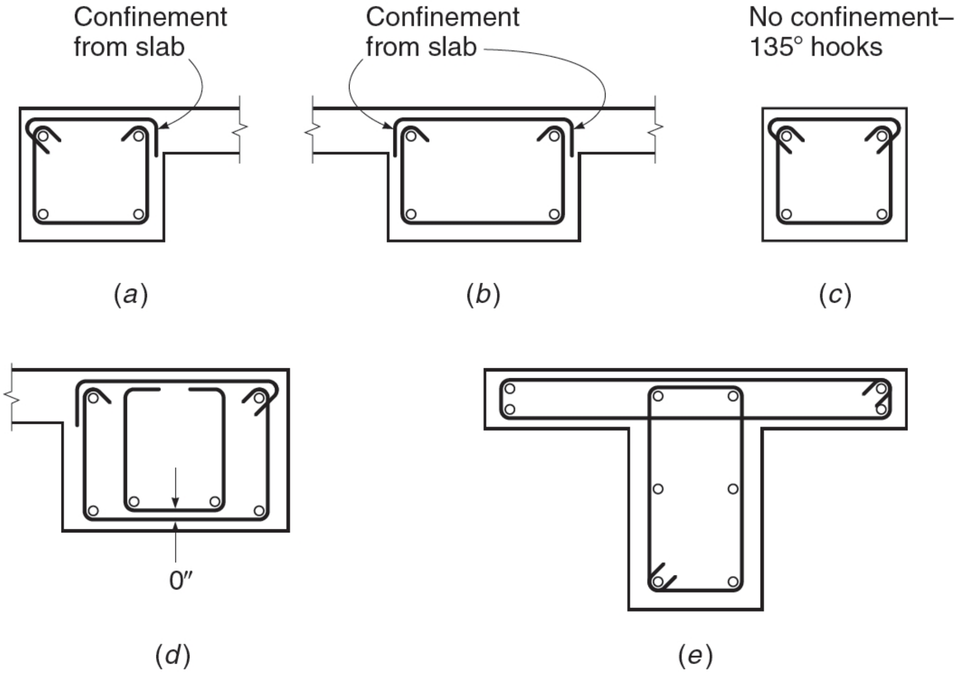


Figure 8.2-3: Stirrup-ties and longitudinal reinforcement for torsion: (a) spandrel beam with flanges on one side; (b) interior beam; (c) isolated rectangular beam; (d) wide spandrel beam; and (e) T beam with torsional reinforcement in flanges.

- Maximum Spacing for Torsional Stirrups
According to ACI Code ***9.6.4.2***, to ***control spiral cracking***, the maximum spacing of torsional stirrups should be:
$$s_{Maximum} = Minimum\left(\frac{p_h}{8} \ or \ 300mm\right)$$
- Minimum Area of Closed Stirrups
In addition, for members requiring both shear and torsion reinforcement, the minimum area of closed stirrups is equal to:
$$A_v + 2A_t = 0.062\sqrt{f_c'}\frac{b_w s}{f_{yt}} \geq 0.35\frac{b_w s}{f_{yt}}$$

8.2.7.2 Longitudinal Reinforcement

- Based on discussion of Article ***8.1.6*** above, the area of longitudinal bar reinforcement A_ℓ required to resist T_n is given by:
$$A_\ell = \left(\frac{A_t}{s}\right)p_h\left(\frac{f_{yt}}{f_y}\right)\cot^2\theta$$
where θ must have the same value used to calculate A_t.
- The term A_t/s should be taken as the value calculated, not modified based on minimum transverse steel requirements.
- Based on an evaluation of the performance of reinforced concrete beam torsional test specimens, ACI Code ***9.6.4.3*** requires a minimum value of A_ℓ equal to the lesser:
$$a.\ \ 0.42\sqrt{f_c'}\left(\frac{A_{cp}}{f_{yt}}\right) - \left(\frac{A_t}{s}\right)p_h\left(\frac{f_{yt}}{f_y}\right)$$
$$b.\ \ 0.42\sqrt{f_c'}\left(\frac{A_{cp}}{f_{yt}}\right) - \left(\frac{0.175b_w}{f_{yt}}\right)p_h\left(\frac{f_{yt}}{f_y}\right)$$

8.3 DESIGN PROCEDURES AND EXAMPLES

Designing a reinforced concrete flexural member for torsion involves a series of steps. The following sequence ensures that each is covered:

- Compute V_u and T_u. When pertinent conditions are satisfied, V_u and T_u can be determined at distance d from face of support.
- Determine if the factored torque is less than:

  $$\phi T_{Th} = \phi 0.083\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

  If so, torsion may be neglected. If not, proceed with the design. Note that in this step, portions of over-hanging flanges, as defined in ***Figure 8.2-1*** above, must be included in the calculation of A_{cp} and p_{cp}.

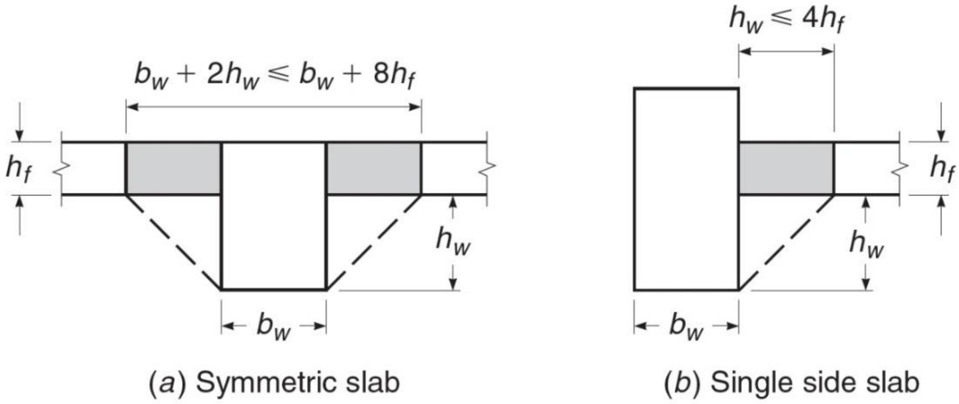


Figure 8.2-1: Portion of slab to be included with beam for torsional design. Reproduce for convenience.

- If the ***torsion is compatibility torsion***, rather than equilibrium torsion, as described in ***Section 8.1.3*** above, the maximum factored torque may be reduced to:

  $$\phi 0.33\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

  Equilibrium torsion cannot be adjusted.
- Check the shear stresses in the section under combined torsion and shear, using the following criteria:
  - For hollow sections:

    $$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right) \leq \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right)$$

  - For solid sections:

    $$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \leq \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right)$$

- Calculate the required transverse reinforcement for torsion using following relation:

  $$A_t = \frac{T_u s}{2\phi A_o f_{yt} \cot\theta}$$

  Combine A_t and A_v using following relation:

  $$\frac{A_{v+t}}{s} = \frac{A_v}{s} + 2\frac{A_t}{S}$$

- Check that the minimum transverse reinforcement requirements are met for both torsion and shear. These include:
  - The maximum spacing:

    $$s_{Maximum} = Minimum\left(\frac{p_h}{8} \ or \ 300mm\right)$$

  - The minimum area:

    $$A_v + 2A_t = Maximum\left(0.062\sqrt{f_c'}\frac{b_w s}{f_{yt}}, 0.35\frac{b_w s}{f_{yt}}\right)$$

    As in ***Chapter 5***, solve for $s_{\text{For minimum value of } A_v + 2A_t}$

$$s_{\text{For minimum value of } A_v+2A_t} = \text{minimum}\left(\frac{(A_v+2A_t)f_{yt}}{0.062\sqrt{f_c{}'}b_w}, \frac{(A_v+2A_t)f_{yt}}{0.35b_w}\right)$$

- Calculate the required longitudinal torsional reinforcement A_ℓ, using the following relation:

  $$A_l = \frac{A_t}{s} p_h \frac{f_{yt}}{f_y} \cot^2 \theta$$

  then comparing with $A_{l\,minimum}$ given by:

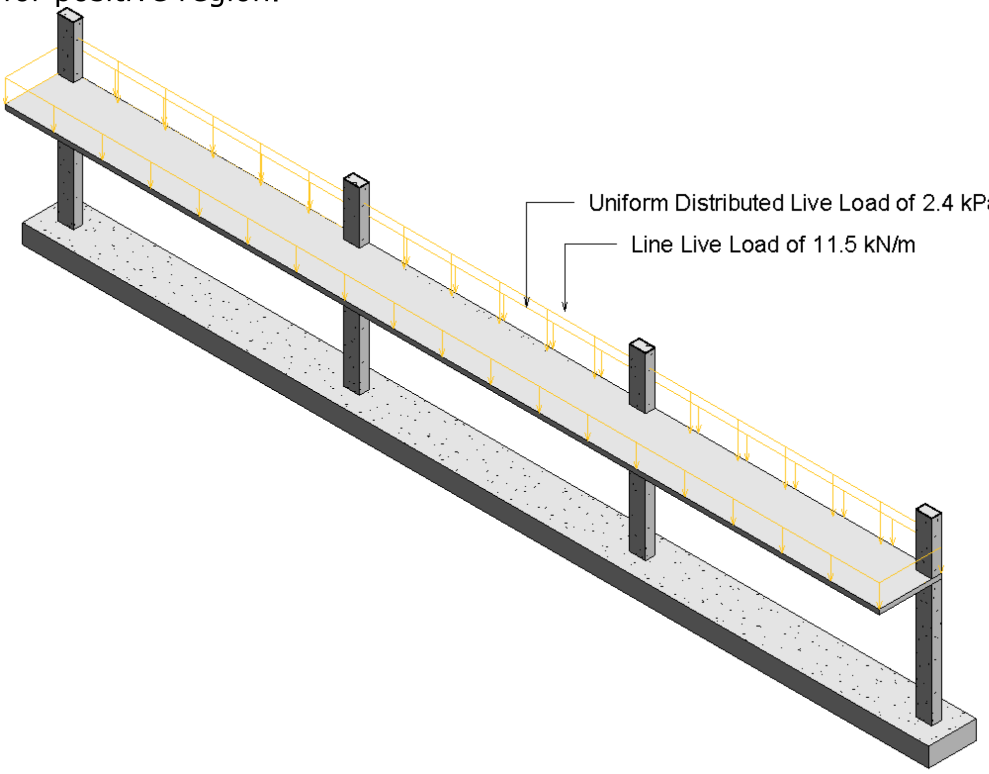
  $$A_{l\,minimum} = minimum\left(0.42\sqrt{f_c{}'}\frac{A_{cp}}{f_{yt}} - \left(\frac{A_t}{s}\right)p_h\frac{f_{yt}}{f_y}, 0.42\sqrt{f_c{}'}\frac{A_{cp}}{f_{yt}} - \left(\frac{0.175b_w}{f_{yt}}\right)p_h\frac{f_{yt}}{f_y}\right)$$

- Details for Torsional Longitudinal Bars:
  According to ***ACI Code 9.7.5***,
  - The spacing of the longitudinal bars should not exceed 300mm,
  - They should be distributed around the perimeter of the cross section to control cracking and to ensure that the centroid of the additional longitudinal reinforcement for torsion should approximately coincide with the centroid of the section.
  - The bars shall have a diameter at least 0.042 times the transverse reinforcement spacing, but not less than 10 mm.
  - At least one longitudinal bar must be placed at each corner of the stirrups.
  - Careful attention must be paid to the anchorage of longitudinal torsional reinforcement so that it is able to develop its yield strength at the face of the supporting columns, where torsional moments are often maximum.

Example 8.3-1

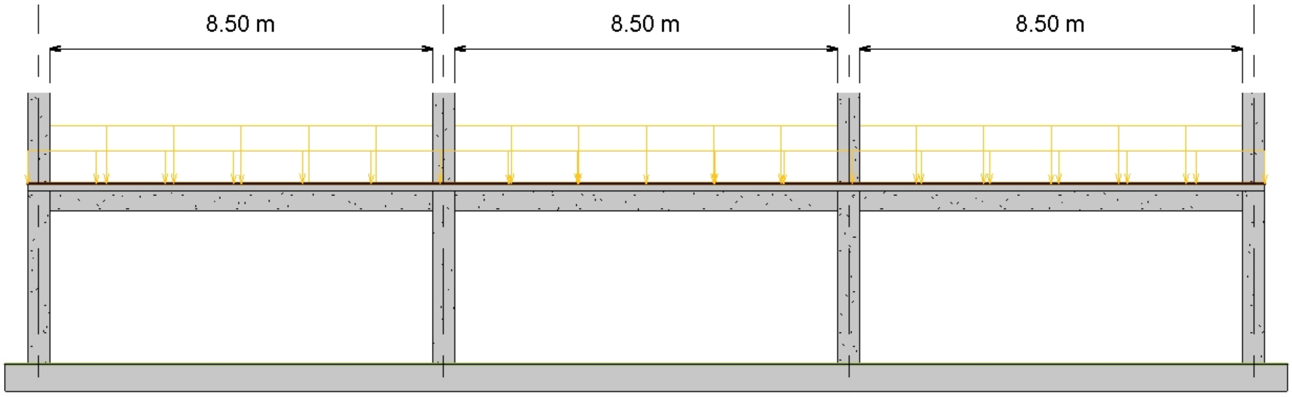
The ***8.5m*** span beam shown in ***Figure 8.3-1*** below carries a monolithic slab cantilevering ***1.8m*** past the beam centerline. The resulting L beam supports a live load of ***11.5 kN/m*** along the beam centerline plus ***2.4 kPa*** uniformly distributed over the upper slab surface. The effective depth to the flexural steel centroid is ***546mm***, and the distance from the beam surfaces to the centroid of stirrup steel is ***45mm***. Material strengths are $f_c' = 35\ MPa$ and $f_y = f_{yt} = 420\ MPa$. Using same stirrup spacing along beam span, design the torsional and shear reinforcement for the beam.

It is useful to note that based on flexure requirement a longitudinal reinforcement of $1191\ mm^2$ should be provided for negative region and about $900 mm^2$ should be provided for positive region.

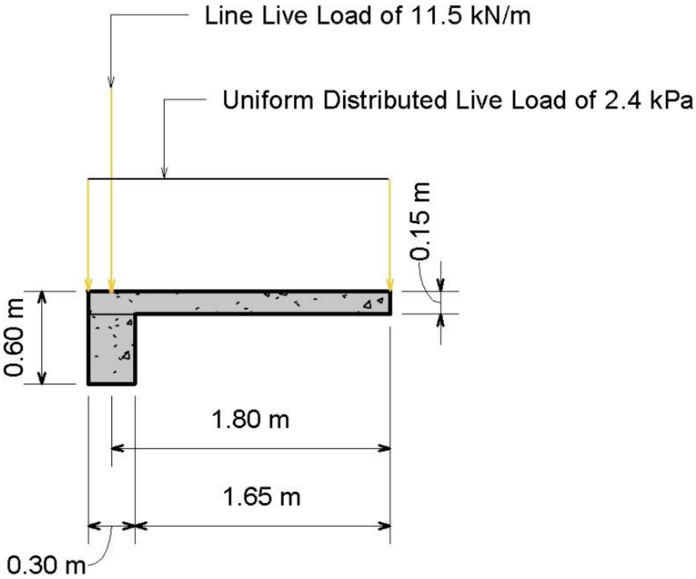


3D view

Figure 8.3-1: Structure for Example 8.3-1.



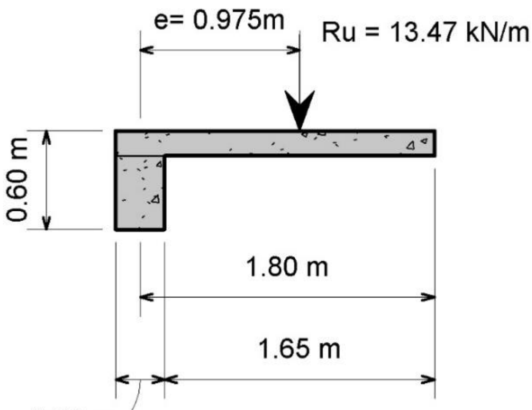
Elevation View.



Sectional View.

Figure 8.3-1: Structure for Example 8.3-1. Continue.

Solution



<u>Factored Loads</u>

Factored uniformly distributed load:

$$W_u = 1.2 \times (0.15 \times 24) + 1.6 \times 2.4 = 8.16\ kPa$$

The resultant for this UDL would be:

$$R_{u\ of\ UDL} = 8.16 \times 1.65 = 13.47 \frac{kN}{m}$$

Located at eccentricity of:

$$e = \frac{1.65}{2} + \frac{0.30}{2} = 0.975\ m$$

Factored live load:

$$q_u = 1.2 \times (0.3 \times 0.6 \times 24) + 1.6 \times 11.5 \approx 24\ kN/m$$

<u>Factored Shear Force and Torsion</u>

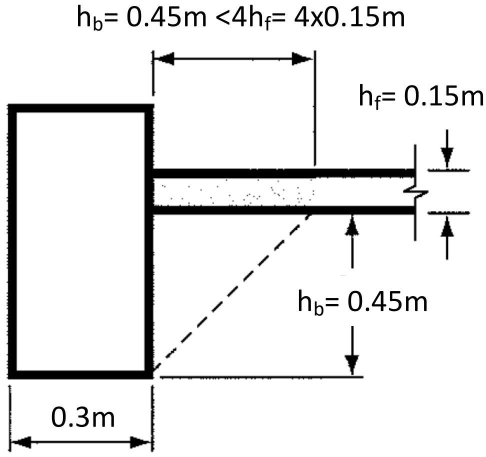
As all related conditions are satisfied, therefore shear force and torsion can be determined at distance

$$V_{u\ @\ distance\ d\ from\ face\ of\ support} = \frac{1}{2}\left((24 + 13.47) \times (8.50 - 0.546 \times 2)\right) \approx 139\ kN$$

$$T_{u\ @\ distance\ d\ from\ face\ of\ support} = \frac{1}{2}\left((13.47 \times 0.975) \times (8.50 - 0.546 \times 2)\right) = 48.6\ kN.m$$

<u>Comparing with ϕT_{th}</u>

$$\phi T_{Th} = \phi 0.083 \lambda \sqrt{f_c'} \left(\frac{A_{cp}^2}{p_{cp}}\right)$$



The effective section would be as indicated in below:

$$A_{cp} = (300 \times 600 + 150 \times 450) = 247500\ mm^2$$

$p_{ch} = (300 + 600) \times 2 + 450 \times 2 = 2700\ mm$

$$\phi T_{Th} = \phi 0.083\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right) = \frac{0.75 \times 0.083 \times 1.0 \times \sqrt{35}\left(\frac{247500^2}{2700}\right)}{10^6} = 8.36\ kN.m < T_u$$

Clearly, torsion must be considered in the present case.

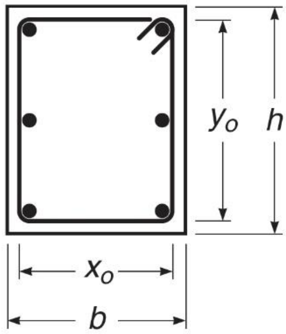
Primary versus compatibility torsion:

Since the torsional resistance of the beam is required for equilibrium, no reduction in T_u may be made.

Checking for shear stresses:

Check the shear stresses in the section under combined torsion and shear. As the section is a solid one, therefore shear stresses would be checked using the following criteria:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right)$$



Proposed the stirrups indicated in below, with $45mm$ cover to the center of the stirrup bars from all faces,

$x_0 = 300 - 90 = 210\ mm, y_0 = 600 - 90 = 510\ mm$

$A_{oh} = 210 \times 510 = 107100\ mm^2, p_h = 2 \times (210 + 510) = 1440\ mm$

Substitute in the criterion to obtain

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} = \sqrt{\left(\frac{139 \times 10^3}{300 \times 546}\right)^2 + \left(\frac{48.6 \times 10^6 \times 1440}{1.7 \times 107100^2}\right)^2}$$

$$\le 0.75 \times \left(0.17\sqrt{35} + 0.66\sqrt{35}\right)$$

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} = \sqrt{\left(\frac{139 \times 10^3}{300 \times 546}\right)^2 + \left(\frac{48.6 \times 10^6 \times 1440}{1.7 \times 107100^2}\right)^2} = 0.75 \times \left(0.17\sqrt{35} + 0.66\sqrt{35}\right)$$

$$= 3.68MPa = 3.68\ MPa \therefore Ok.$$

Therefore, the cross section is of adequate size for the given concrete strength.

Design of Transverse Reinforcement

The values of A_t and A_v will now be calculated at the distance d from column face. With choosing $\theta = 45°$,

$$A_t = \frac{T_u s}{2\phi A_o f_{yt}\cot\theta}$$

$A_o = 0.85A_{oh} = 0.85 \times 210 \times 510 = 91035\ mm^2$

$$A_t = \frac{48.6 \times 10^6}{2 \times 0.75 \times 91035 \times 420 \times 1.0}s = 0.847s$$

$$\phi V_c = \frac{0.75 \times 0.17 \times \sqrt{35} \times 546 \times 300}{1000} = 124\ kN$$

$$V_s = \frac{V_u - \phi V_c}{\phi} = \frac{139 - 124}{0.75} = 20\ kN$$

From Chapter 5,

$$V_s = \left(A_v f_{yt}\right) \times \frac{d}{s} \Rightarrow A_v = \frac{V_s}{f_{yt}d}s = \frac{20000}{420 \times 546}s = 0.0872s$$

Combine A_t and A_v using following relation:

$2A_t + A_v = 2 \times 0.847s + 0.0872s = 1.78s$

Try stirrups of $No.13$

$$2A_t + A_v = 2 \times \frac{\pi \times 13^2}{4} = 1.78s$$

Solve for spacing s:

$s = 149\ mm$

Try $No.13$ @ $125\ mm$

Check with maximum spacing for shear and torsion:

$\because V_s < 0.33\lambda\sqrt{f_c'}b_w d$

Therefore,

$$s_{Maximum\ for\ shear} = Minimum\left(\frac{d}{2}, 600\right)$$

While the maximum spacing for torsion is:

$s_{Maximum\ for\ torsion} = Minimum\left(\frac{p_h}{8}\ or\ 300mm\right)$

Therefore, $s_{Maximum}$ for both aspects would be:

$$s_{Maximum} = Minimum\left(\frac{d}{2}, \frac{p_h}{8}, 300\right) = Minimum\left(\frac{546}{2}, \frac{1440}{8}, 300\right) = Minimum(273, 180, 300)$$
$$= 180mm > s_{Provided} \therefore Ok.$$

Finally, checking the limitation on minimum area of transverse reinforcement:

$$s_{\text{For minimum value of } A_v+2A_t} = \text{minimum}\left(\frac{(A_v + 2A_t)f_{yt}}{0.062\sqrt{f_c'}b_w}, \frac{(A_v + 2A_t)f_{yt}}{0.35b_w}\right)$$

$$s_{\text{For minimum value of } A_v+2A_t} = \text{minimum}\left(\frac{2 \times \frac{\pi \times 13^2}{4} \times 420}{0.062 \times \sqrt{35} \times 300}, \frac{2 \times \frac{\pi \times 13^2}{4} \times 420}{0.35 \times 300}\right)$$

$$= \text{minimum}\ (1013, 1061) = 1013\ mm \gg s_{Provided} \therefore Ok.$$

Therefore use $No.13$ @ $125mm$ along whole span of the beam.

<u>Design for Longitudinal Reinforcement for Torsion:</u>

Calculate the required longitudinal torsional reinforcement A_l, using the following relation:

$$A_l = \frac{A_t}{s} p_h \frac{f_{yt}}{f_y} \cot^2\theta$$

The longitudinal steel required for torsion at a distance d from the column face is:

$$\because A_t = 0.847s \Rightarrow \frac{A_t}{s} = 0.847$$

Then

$$A_l = \frac{A_t}{s} p_h \frac{f_{yt}}{f_y} \cot^2\theta = 0.847 \times 1440 \times 1.0 \times 1.0^2 = 1219\ mm^2$$

Comparing with $A_{l\ minimum}$ given by:

$$A_{l\ minimum} = minimum\left(0.42\sqrt{f_c'}\frac{A_{cp}}{f_{yt}} - \left(\frac{A_t}{s}\right)p_h\frac{f_{yt}}{f_y}, 0.42\sqrt{f_c'}\frac{A_{cp}}{f_{yt}} - \left(\frac{0.175b_w}{f_{yt}}\right)p_h\frac{f_{yt}}{f_y}\right)$$

$$A_{l\ minimum} = minimum\left(0.42 \times \sqrt{35} \times \frac{247500}{420} - (0.847) \times 1440 \times 1.0,\right.$$
$$\left. 0.42 \times \sqrt{35} \times \frac{247500}{420} - \left(\frac{0.175 \times 300}{420}\right) \times 1440 \times 1.0\right) = minimum(225, 1284)$$
$$= 225\ mm^2 < A_l \therefore Ok.$$

Reinforcement will be placed at the top, mid-depth, and bottom of the member each level to provide not less than $1219/3 = 406$. Try rebar with No.20:

$$No.of\ rebars\ at\ mid\ depth = \frac{406}{\frac{\pi \times 20^2}{4}} = 1.29$$

Use $2No.20$ @ $mid\ depth$

$$No.of\ top\ rebars = \frac{1191 + 406}{\frac{\pi \times 20^2}{4}} \approx 5.0$$

Use $5No.20$ @ top

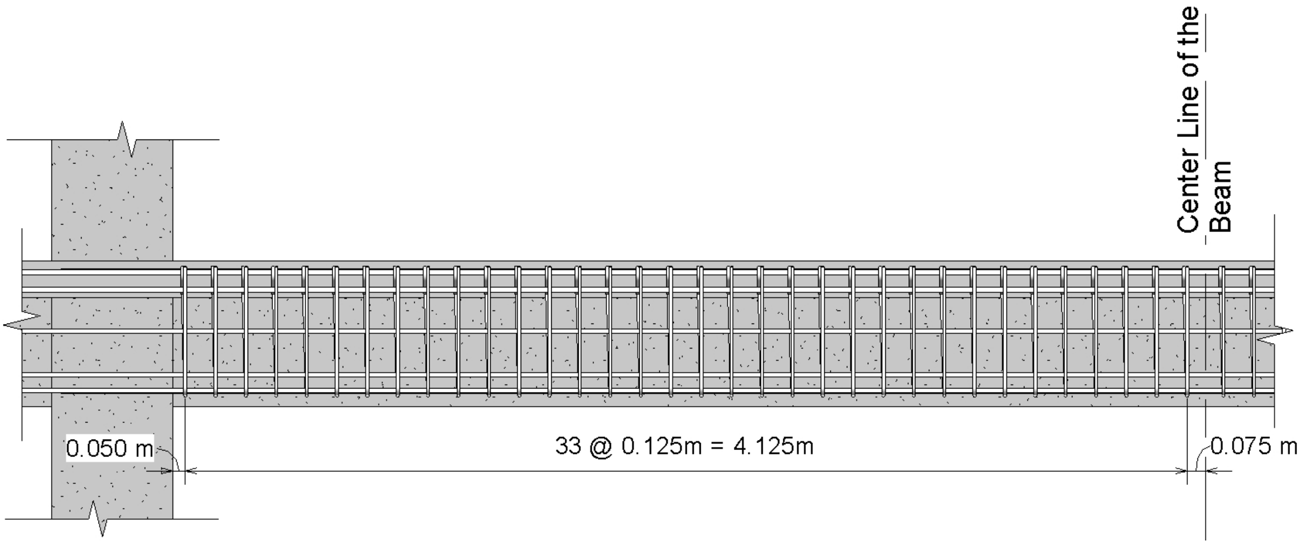
$$No.of\ bottom\ rebars = \frac{900 + 406}{\frac{\pi \times 20^2}{4}} = 4.15$$

Use $5No.20$ @ $Bottom$.

Proposed beam reinforcement are presented in ***Figure 8.3-2*** below. To be a final decision, proposed reinforcement should be checked for ACI requirements for details of torsional longitudinal bars, ACI Code 9.7.5,

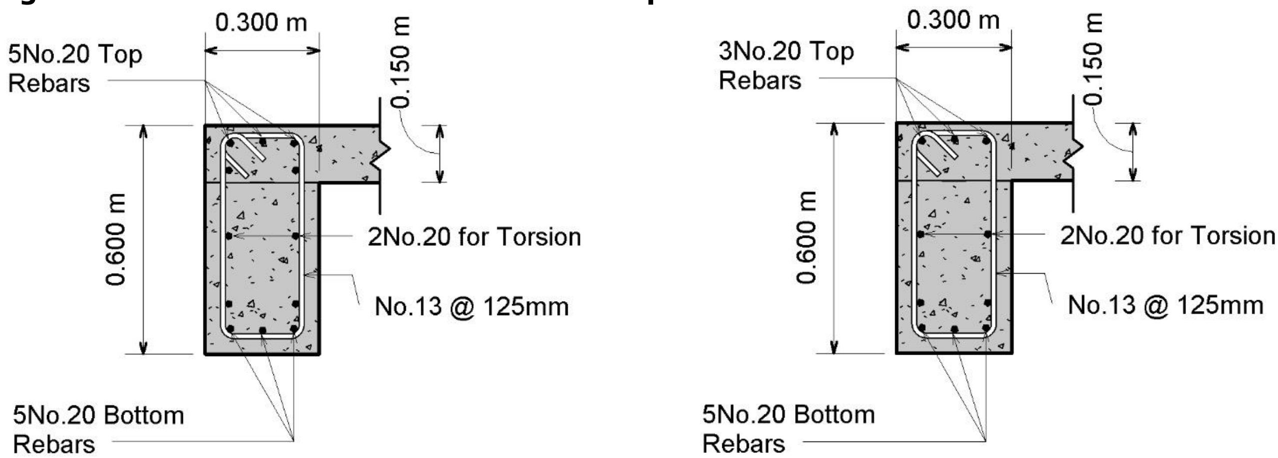
- The spacing of the longitudinal bars should not exceed 300mm, Ok.
- They should be distributed around the perimeter of the cross section to control cracking, Ok.
- The bars shall have a diameter at least 0.042 times the transverse reinforcement spacing, but not less than 10 mm., Ok.
- At least one longitudinal bar must be placed at each corner of the stirrups, Ok.

- Careful attention must be paid to the anchorage of longitudinal torsional reinforcement so that it is able to develop its yield strength at the face of the supporting columns, where torsional moments are often maximum. ***This should be as discussed in Chapter 7***.



Longitudinal section view.

Figure 8.3-2: Beam reinforcement for Example 8.3-1.



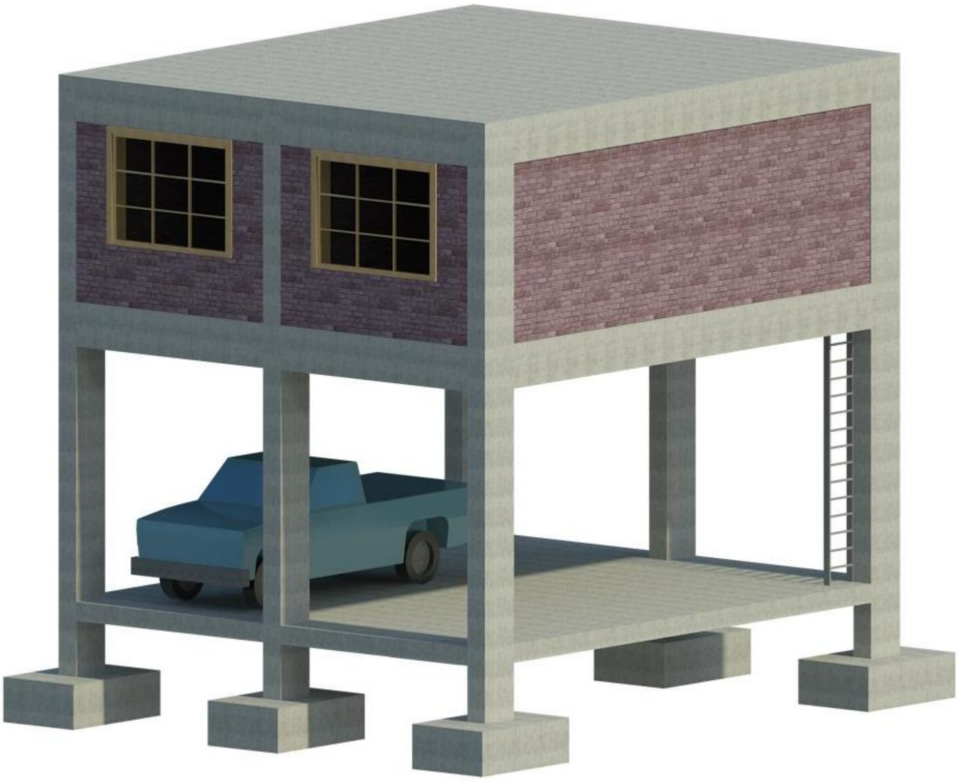
Section @ supports **Section @ mid-span**

Figure 8.3-2: Beam reinforcement for Example 8.3-1. Continue.

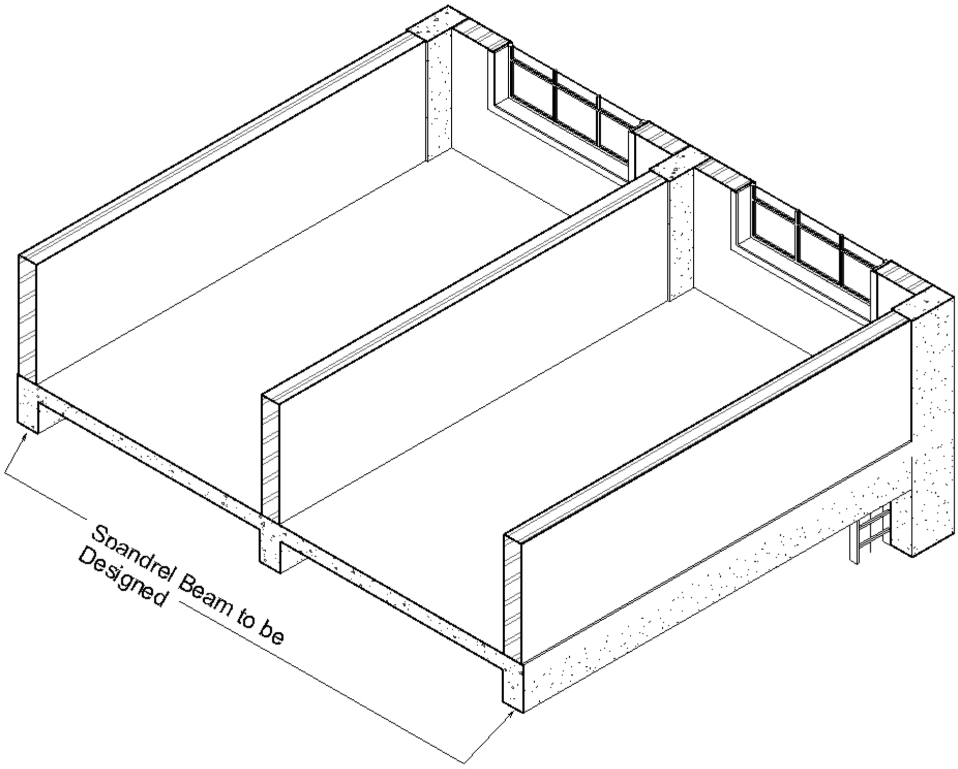
Example 8.3-2

For a maintenance shop indicated in ***Figure 8.3-3*** below, design a floor supporting spandrel beam for torsion and shear. The floor slab is subjected to a ***live load of 2.5 kPa*** and a ***superimposed dead load of 2.0 kPa*** in addition of its own weight. In your design, assume that:

- $f_c' = 28\ MPa$, and $f_y = f_{yt} = 420\ MPa$,
- Based on flexural design, $A_{top\ required} \approx 700\ mm^2$ and $A_{bottom\ required} \approx 610\ mm^2$,
- Try two layers with $No.20$ for longitudinal reinforcement and $No.10$ for stirrups.

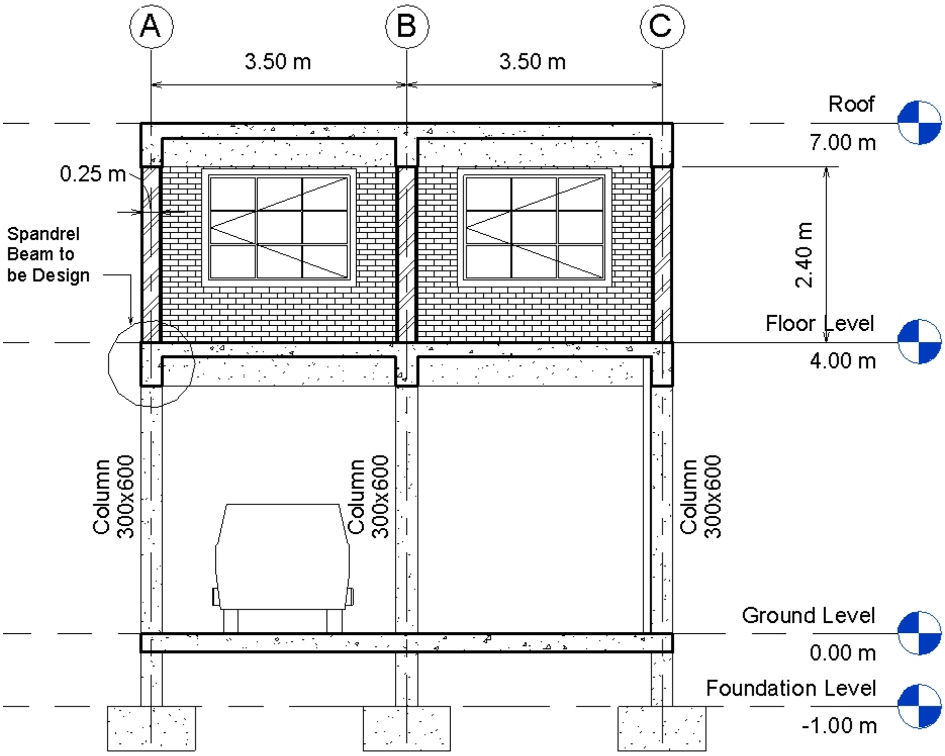


3D View.

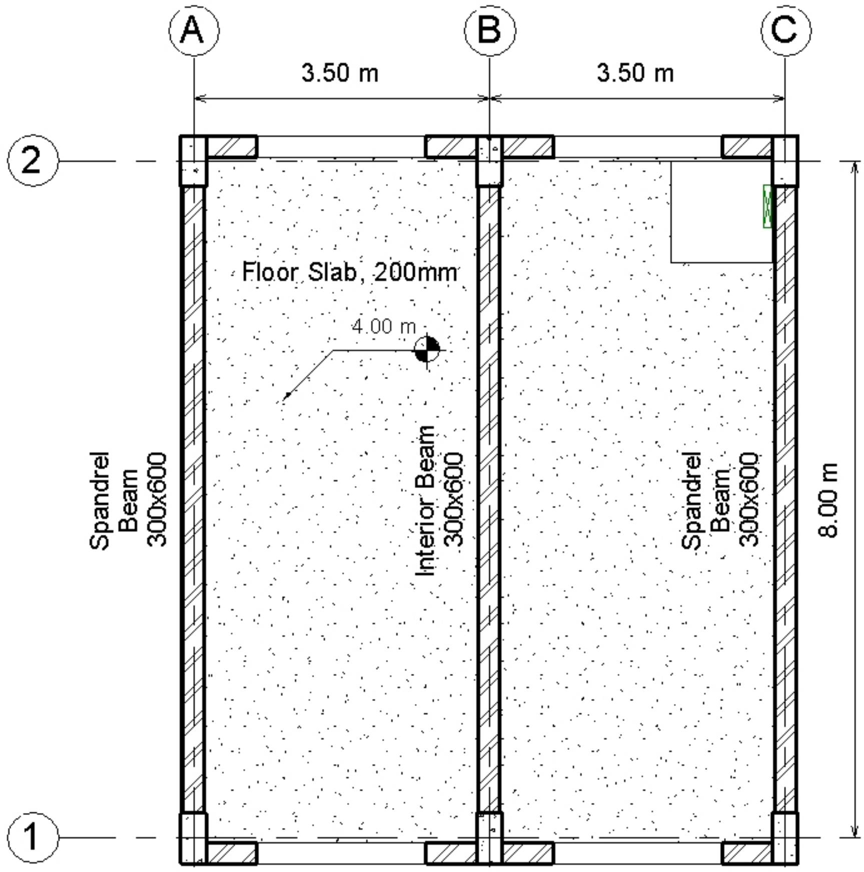


3D Sectional View.

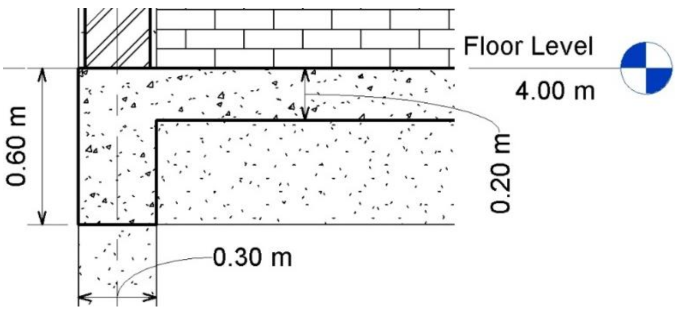
Figure 8.3-3: Maintenance shop for Example 8.3-2.



Sectional View.



Plan View.



Callout View for the Spandrel Beam.

Figure 8.3-3: Maintenance shop for Example 8.3-2. Continue.

Solution

Factored Loads

Factored uniformly distributed, W_u, that acting on the slab would be:

$W_D = 0.2 \times 24 + 2.0 = 6.8\ kPa$

$W_u = \max(1.4 \times 6.8, 1.2 \times 6.8 + 1.6 \times 2.5) = 12.2\ kPa$

With considering brick cladding as a dead load and with assuming $\gamma_{Brick} = 19\ kN/m^3$, the factored line load, q_u, would be:

$$q_u = 1.2 \times \left((0.25 \times 2.40 \times 19)_{Weight\ of\ brick\ wall} + (0.3 \times 0.6 \times 24)_{selfweight\ of\ the\ beam}\right) = 18.9 \frac{kN}{m}$$

Factored Shear Force, V_u, and Torsion, T_u, Acting on Beam

As would be discussed in ***Chapter 12***, ***Analysis and Design of One-way Slabs***, an edge supported slab is classified as one-way slab when its length to width ratio is more than 2.

$$\frac{l}{s} = \frac{8.00}{3.50} = 2.28 > 2$$

Therefore, floor system is classified as one-way slab system.

In ***Chapter 12***, it is shown that shear force and torsion transferred from the slab to the supporting beam can be estimated from following relations, see Figure 8.3-4 below:

$$M_{u\ exterior-ve\ of\ slab} = T_{u\ torsional\ moment\ acting\ on\ beam} = \frac{W_u l_n^2}{24}$$

$$V_{shear\ force\ acting\ on\ slab} = Load\ acting\ on\ beam = \frac{W_u l_n}{2}$$

where:

$W_u = Factored\ UDL\ acting\ on\ slab = 12.2\ kPa$

$$l_n = clear\ span\ of\ slab = clear\ spacing\ between\ supporting\ beam = 3.5 - \frac{0.3}{2} \times 2 = 3.2\ m$$

$$M_{u\ exterior-ve\ of\ slab} = T_{u\ torsional\ moment\ acting\ on\ beam} = \frac{12.2 \times 3.2^2}{24} = 5.21\ kN.m\ per\ m\ ■$$

$$V_{shear\ force\ acting\ on\ slab} = Load\ acting\ on\ beam = \frac{12.2 \times 3.2}{2} = 19.5 \frac{kN}{m}$$

Including the factored loads that acting directly on beam, the total factored line load acting on the beam would be:

$$q_{u\ total\ line\ load\ acting\ on\ the\ beam} = 19.5 + 18.9 = 38.4 \frac{kN}{m}\ ■$$

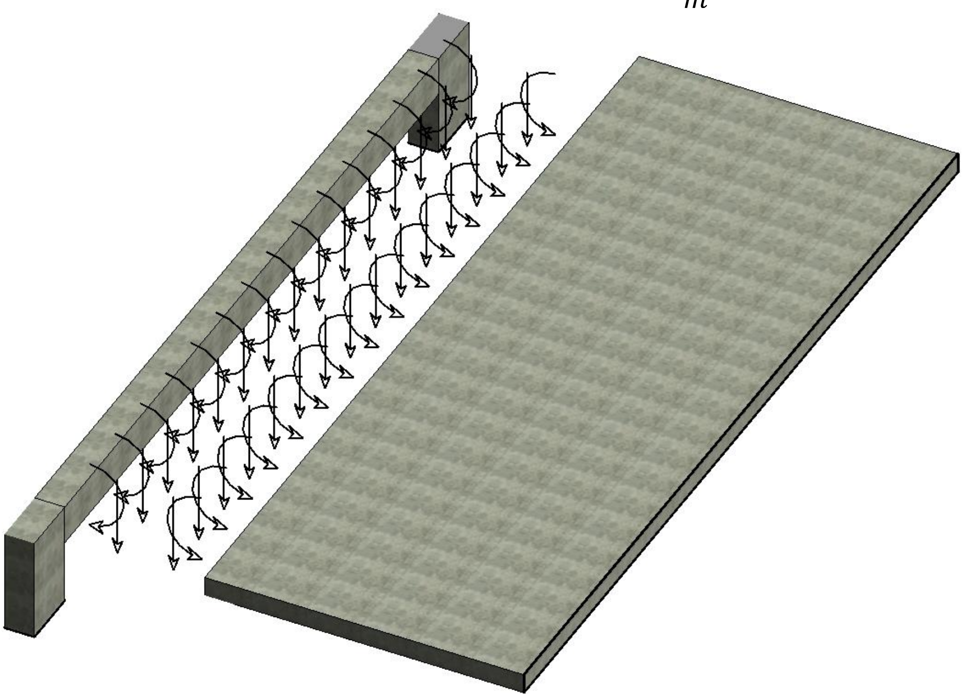


Figure 8.3-4: Forces transformed from supported slab to the supporting beam.

As all pertinent conditions are satisfied, therefore, design force can be determined at distance d from face of support. With two layers of reinforcement and with adopting of $No.20$ for longitudinal reinforcement and $No.10$ for stirrups, the effective depth would be:

$$d = 600 - 40 - 10 - 20 - \frac{25}{2} = 517\ mm$$

$$V_{u\,@\,distance\,d\,from\,face\,of\,support}=\frac{38.4\times\left(8.0-\frac{0.6}{2}\times2-0.517\times2\right)}{2}=122\,kN$$

$$T_{u\,@\,distance\,d\,from\,face\,of\,support}=\frac{5.21\times\left(8.0-\frac{0.6}{2}\times2-0.517\times2\right)}{2}=16.6\,kN.m\,per\,m$$

A more accurate torque can be determined with considering of offset between transferred shear force, V_u, and the center line of the spandrel beam:

$$T_{u\,@\,distance\,d\,from\,face\,of\,support}=\frac{\left(5.21+19.5\times\frac{0.3}{2}\right)\times\left(8.0-\frac{0.6}{2}\times2-0.517\times2\right)}{2}$$
$$=25.9\,kN.m\,per\,m$$

Comparing with ϕT_{th}

$$\phi T_{Th}=\phi 0.083\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

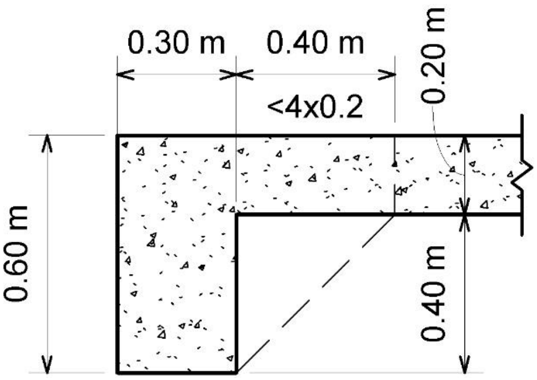
The effective section would be as indicated in below:

$A_{cp}=(300\times600+200\times400)=260000\,mm^2$

$p_{ch}=(300+600)\times2+400\times2=2600\,mm$

$$\phi T_{Th}=\phi 0.083\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)=\frac{\left(0.75\times0.083\times1.0\times\sqrt{28}\times\left(\frac{260000^2}{2600}\right)\right)}{10^6}=8.56\,kN.m<T_u$$

Clearly, torsion must be considered in the present case.



Primary versus compatibility torsion:

Since the torsional resistance of the beam is required for computability, therefore T_u .can be reduced to the value indicated in below:

$$T_{u\,reduced}=\phi 0.33\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$
$$=\frac{0.75\times0.33\times1.0\times\sqrt{28}\times\left(\frac{260000^2}{2600}\right)}{10^6}$$
$$=34.1\,kN.m>T_{u\,applied}$$

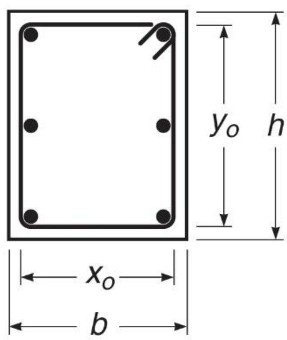
Therefore, no benefit can be obtained for torque reduction and the design should be based on T_u of $16.6\ kN.m$.

Checking for shear stresses:

Check the shear stresses in the section under combined torsion and shear. As the section is a solid one, therefore shear stresses would be checked using the following criteria:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2+\left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2}\le\phi\left(\frac{V_c}{b_w d}+0.66\sqrt{f_c'}\right)$$

Proposed the stirrups indicated in below, with $45mm$ cover to the center of the stirrup bars from all faces,



$$x_0=300-40\times2-\frac{10}{2}\times2=210\,mm$$

$$y_0=600-40\times2-\frac{10}{2}\times2=510\,mm$$

$A_{oh}=210\times510=\ 107100\,mm^2$

$p_h=2\times(210+510)=\ 1440\,mm$

Substitute in the criterion to obtain

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2+\left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2}=\sqrt{\left(\frac{122\times10^3}{300\times517}\right)^2+\left(\frac{25.9\times10^6\times1440}{1.7\times107100^2}\right)^2}$$
$$\le0.75\times\left(0.17\sqrt{28}+0.66\sqrt{28}\right)$$

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2+\left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2}=\ 2.06\,MPa\le3.29\,MPa\therefore Ok.$$

Therefore, the cross section is of adequate size for the given concrete strength.

<u>Design of Transverse Reinforcement</u>

The values of A_t and A_v will now be calculated at the distance d from column face. With choosing $\theta = 45°$,

$$A_t = \frac{T_u s}{2\phi A_o f_{yt} \cot\theta}$$

$$A_o = 0.85A_{oh} = 0.85 \times 210 \times 510 = 91035\ mm^2$$

$$A_t = \frac{25.9 \times 10^6}{2 \times 0.75 \times 91035 \times 420 \times 1.0} s = 0.452s$$

$$\phi V_c = \frac{0.75 \times 0.17 \times \sqrt{28} \times 517 \times 300}{1000} = 105\ kN$$

$$V_s = \frac{V_u - \phi V_c}{\phi} = \frac{122 - 105}{0.75} = 22.6\ kN$$

From Chapter 5,

$$V_s = (A_v f_{yt}) \times \frac{d}{s} \Rightarrow A_v = \frac{V_s}{f_{yt} d} s = \frac{22.6 \times 10^3}{420 \times 517} s = 0.104s$$

Combine A_t and A_v using following relation:

$$2A_t + A_v = 2 \times 0.452s + 0.104s = 1.00s$$

Try stirrups of $No.10$

$$2A_t + A_v = 2 \times \frac{\pi \times 10^2}{4} = 1.00s$$

Solve for spacing s:

$$s = 157\ mm$$

Try $No.10\ @\ 150\ mm$

Check with maximum spacing for shear and torsion:

$$\because V_s < 0.33\lambda\sqrt{f_c'} b_w d$$

Therefore,

$$s_{Maximum\ for\ shear} = Minimum\left(\frac{d}{2}, 600\right)$$

While the maximum spacing for torsion is:

$$s_{Maximum\ for\ torsion} = Minimum\left(\frac{p_h}{8}\ or\ 300mm\right)$$

Therefore, $s_{Maximum}$ for both aspects would be:

$$s_{Maximum} = \min\left(\frac{d}{2}, \frac{p_h}{8}, 300\right) = \min\left(\frac{517}{2}, \frac{1440}{8}, 300\right) = 180mm > s_{Provided} \therefore Ok.$$

Try $No.10\ @\ 150\ mm$

Finally, checking the limitation on minimum area of transverse reinforcement:

$$\mathrm{s_{For\ minimum\ value\ of\ A_v+2A_t}} = \min\left(\frac{(\mathrm{A_v} + 2\mathrm{A_t})\mathrm{f_{yt}}}{0.062\sqrt{\mathrm{f_c}'}\mathrm{b_w}}, \frac{(\mathrm{A_v} + 2\mathrm{A_t})\mathrm{f_{yt}}}{0.35\mathrm{b_w}}\right)$$

$$\mathrm{s_{For\ minimum\ value\ of\ A_v+2A_t}} = \min\left(\frac{2 \times \frac{\pi \times 10^2}{4} \times 420}{0.062 \times \sqrt{28} \times 300}, \frac{2 \times \frac{\pi \times 10^2}{4} \times 420}{0.35 \times 300}\right) = 628\ \mathrm{mm} > s_{Provided}$$

$$\therefore Ok.$$

Therefore ***use*** $\mathrm{No.10\ @\ 150mm}$ ***along whole span of the beam***.

<u>Design for Longitudinal Reinforcement for Torsion:</u>

Calculate the required longitudinal torsional reinforcement A_l, using the following relation:

$$A_l = \frac{A_t}{s} p_h \frac{f_{yt}}{f_y} \cot^2\theta$$

The longitudinal steel required for torsion at a distance d from the column face is:

$$\because A_t = 0.452s \Rightarrow \frac{A_t}{s} = 0.452$$

Then

$$A_l = \frac{A_t}{s} p_h \frac{f_{yt}}{f_y} \cot^2\theta = 0.452 \times 1440 \times 1.0 \times 1.0^2 = 650\ mm^2$$

Comparing with $A_{l\ minimum}$ given by:

$$A_{l\,minimum} = \min\left(0.42\sqrt{f_c'}\frac{A_{cp}}{f_{yt}} - \left(\frac{A_t}{s}\right)p_h\frac{f_{yt}}{f_y}, 0.42\sqrt{f_c'}\frac{A_{cp}}{f_{yt}} - \left(\frac{0.175 b_w}{f_{yt}}\right)p_h\frac{f_{yt}}{f_y}\right)$$

$$A_{l\,minimum} = \min\left(0.42 \times \sqrt{28} \times \frac{260000}{420} - (0.452) \times 1440 \times 1.0,\right.$$
$$\left. 0.42 \times \sqrt{28} \times \frac{260000}{420} - \left(\frac{0.175 \times 300}{420}\right) \times 1440 \times 1.0\right) = 724\,mm^2 > A_l \therefore Not\ Ok.$$

$\therefore A_l = 724\,mm^2$

Reinforcement will be placed at the top, mid-depth, and bottom of the member. Each level to provide not less than $724/3 = 241$. Try No.20 rebar:

$$No.of\ rebars\ at\ mid\ depth\ = \frac{241}{\frac{\pi \times 20^2}{4}} \approx 0.767$$

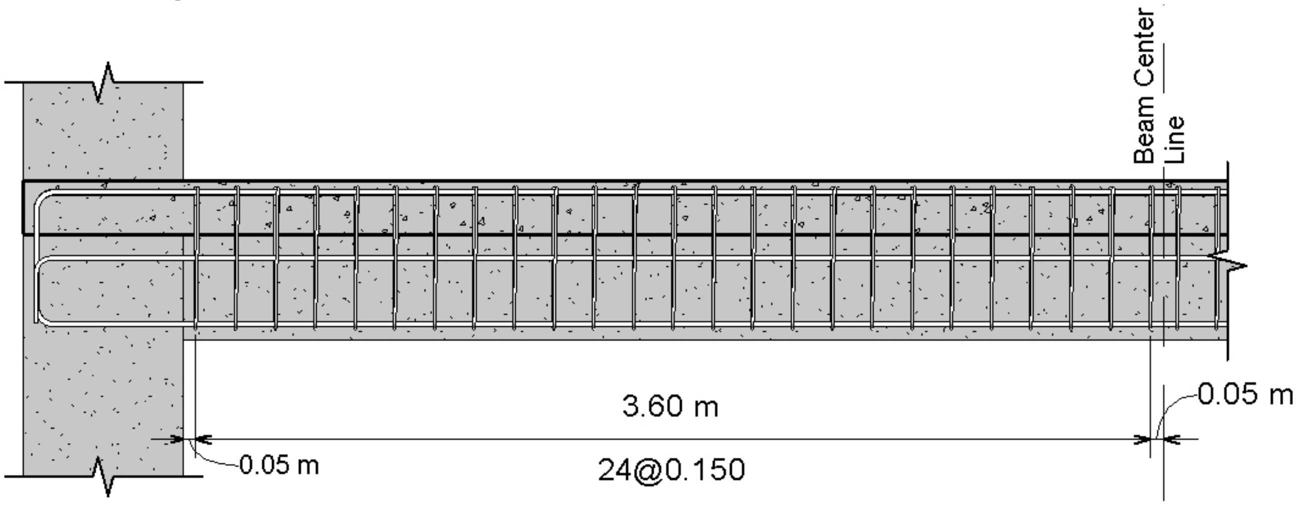
Therefore, using $2No.16 @ mid - depth$ seems more suitable and economical.

$$No.of\ top\ rebars = \frac{700 + 241}{\frac{\pi \times 20^2}{4}} = 2.99$$

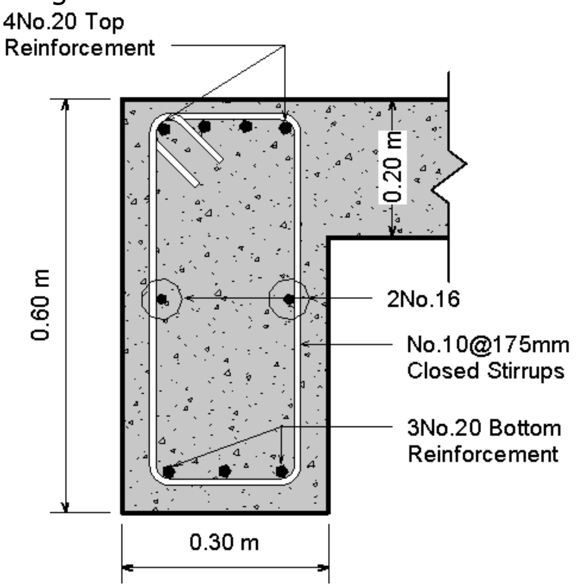
Use $3No.20 @ top$

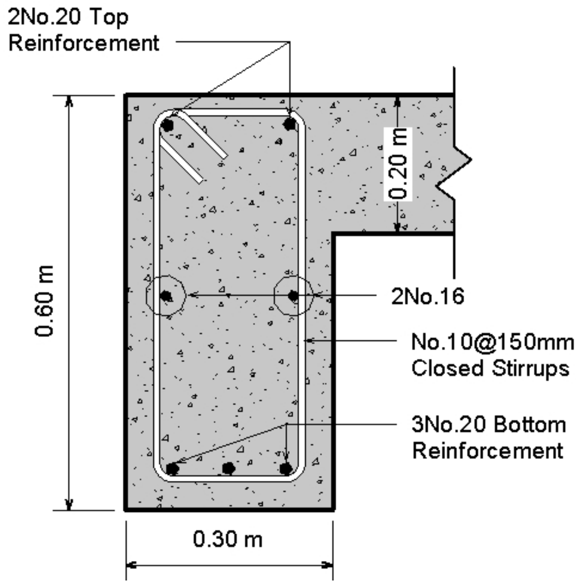
$$No.of\ bottom\ rebars = \frac{610 + 241}{\frac{\pi \times 20^2}{4}} = 2.7$$

Use $3No.20 @ Bottom$.



Longitudinal section.





Beam Section at Supports **Section at Mid-span**

Figure 8.3-5: Reinforcement for Example 8.3-2.

Proposed beam reinforcement are presented in ***Figure 8.3-5*** above. To be a final decision, proposed reinforcement should be checked for ACI requirements for details of torsional longitudinal bars, ACI Code 9.7.5,

- The spacing of the longitudinal bars should not exceed 300mm, Ok.

- They should be distributed around the perimeter of the cross section to control cracking, Ok.
- The bars shall have a diameter at least 0.042 times the transverse reinforcement spacing, but not less than 10 mm, Ok.
- At least one longitudinal bar must be placed at each corner of the stirrups, Ok.
- Careful attention must be paid to the anchorage of longitudinal torsional reinforcement so that it is able to develop its yield strength at the face of the supporting columns, where torsional moments are often maximum. ***This should be as discussed in Chapter 6***.

Contents

CHAPTER 9
SHORT COLUMNS

9.1 INTRODUCTION

9.1.1 Definition of Column

- Columns are defined as ***members that carry loads chiefly in compression with a ratio of height to least lateral dimension exceeding 3***
- According to ***Article 14.3.3*** of the code, vertical member with ratio of ***unsupported*** height to average least lateral dimension ***not exceed 3***, is classified as ***pedestal*** and can be designed as a plain concrete member.
- Pedestals are usually used in steel structures to protect steel against corrosion due to soil contact, see ***Figure 9.1-1*** and ***Figure 9.1-2*** shown below.
- Usually columns ***carry bending moments as well, about one or both axes of the cross section***, and ***the bending action may produce tensile forces over a part of the cross section***. Even in such cases, columns are generally referred to as compression members, because the compression forces dominate their behavior.

9.1.2 Other Compression Members

- In addition to the most common type of compression member, i.e., vertical elements in structures, other member can be classified as compression members and design on same basis adopted for column.
- These compression members include ***arches***, ***inclined members in gable frame***, and ***compression elements in trusses***. It is interested to know that truss can be constructed with reinforced concrete, for more information ***Advanced Reinforced Concrete Design by N. K. Raju Page 281***.

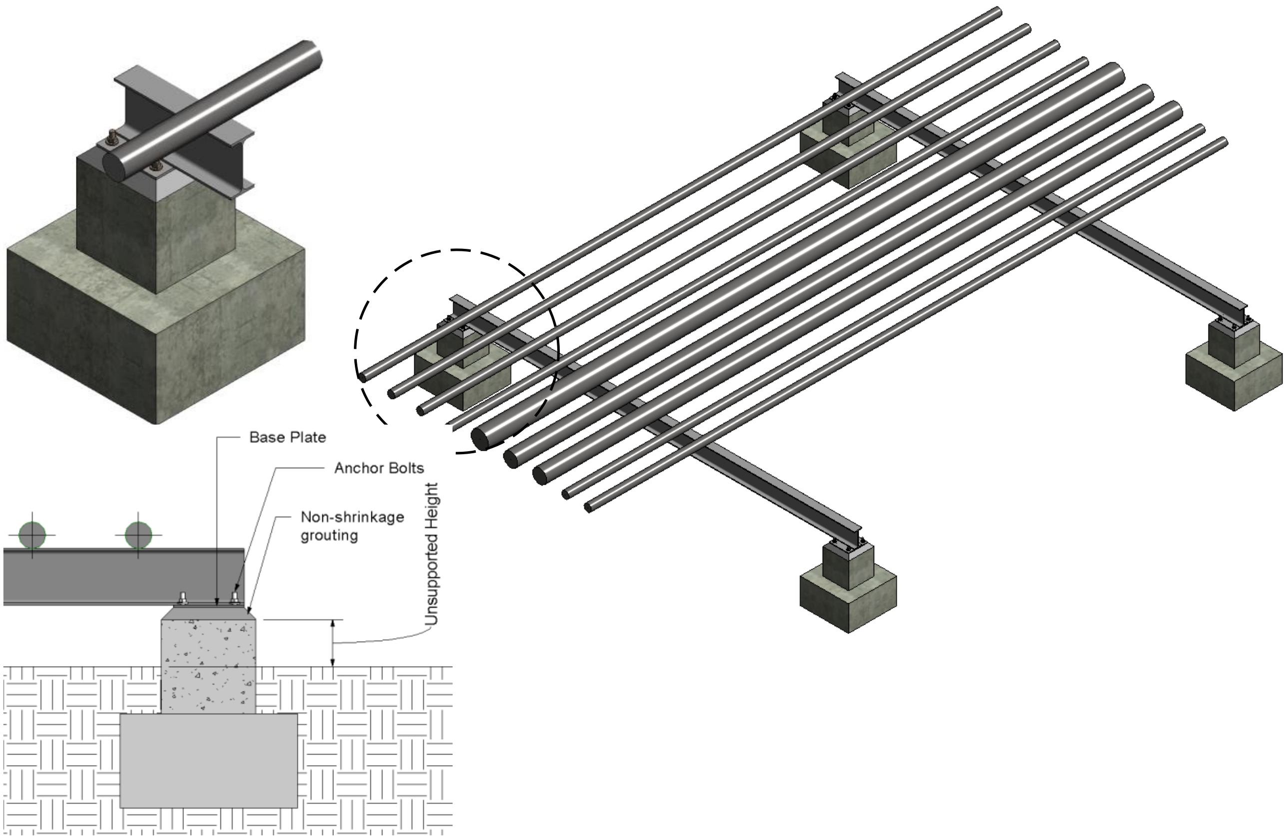


Figure 9.1-1: Pedestal used in a pipe supporting structure.

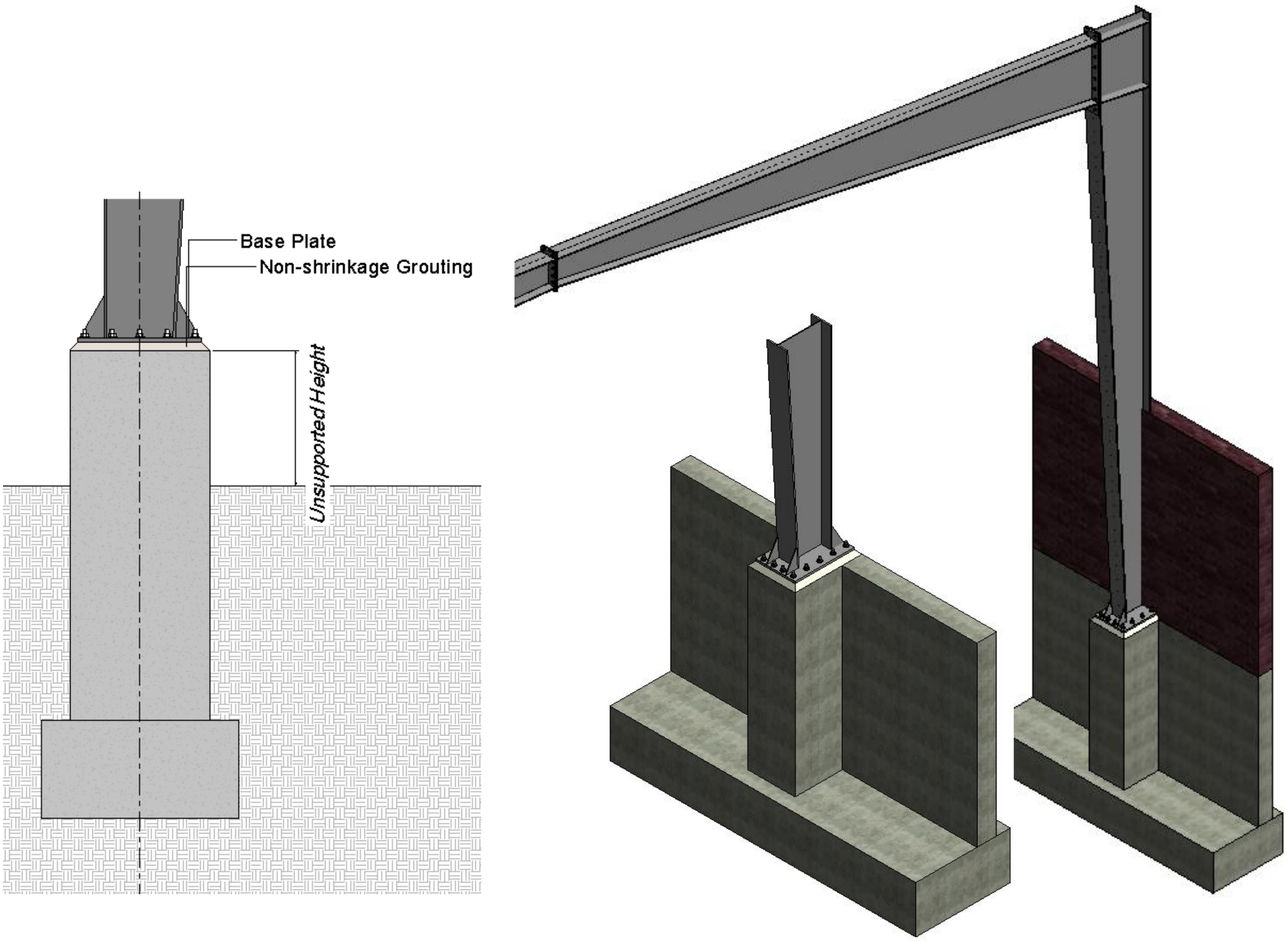


Figure 9.1-2: Pedestal used in a gable steel structure.

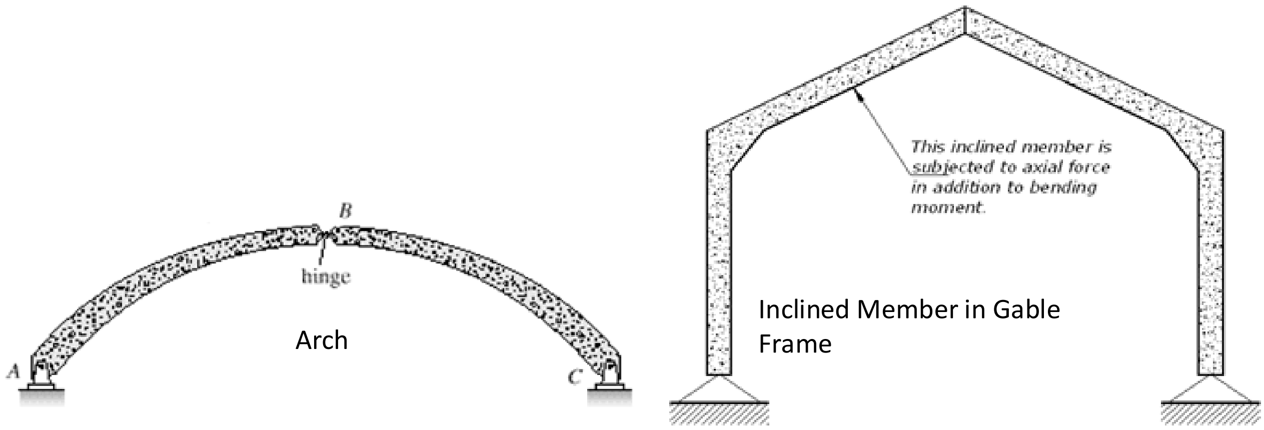


Figure 9.1-3: Other compression members.

9.1.3 Columns Classification According to Their Reinforcement

According to details of their reinforcement, reinforced concrete columns can be classified into:

- Tied Columns:
  Members reinforced with longitudinal bars and lateral ties, see ***Figure 9.1-4*** below.
- Spiral Columns
  Members reinforced with longitudinal bars and continuous spirals, ***Figure 9.1-5*** below.

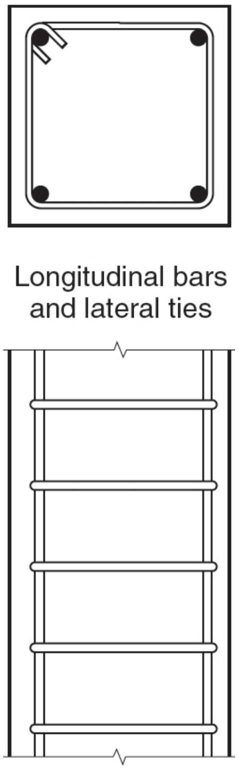


Figure 9.1-4: Tied columns.

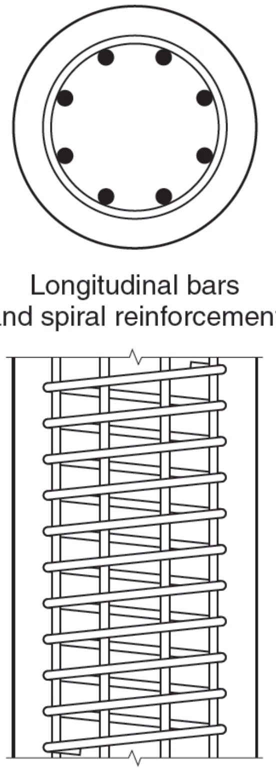


Figure 9.1-5: Spiral columns.

- Composite columns:
  Composite compression members reinforced longitudinally with structural steel shapes, pipe, or tubing, with or without additional longitudinal bars, and various types of lateral reinforcement, ***Figure 9.1-6*** above.
- Types 1 and 2 are by far the most common, and the discussion of this chapter will refer to them.

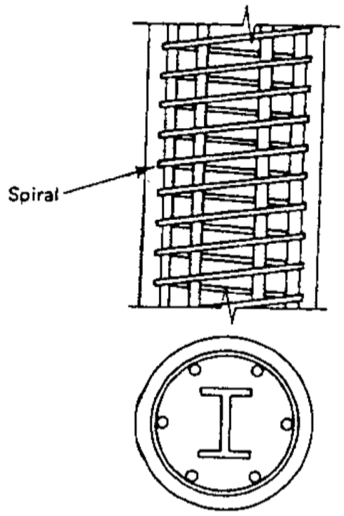


Figure 9.1-6: Composite columns.

9.1.4 Columns Classification According to Their Slenderness

- According to their length or slenderness, columns may be divided into two broad categories:
  - Short columns, for which the strength is governed by the strength of the materials and the geometry of the cross section.
  - Slender columns, for which the strength may be significantly reduced by lateral deflections.
- Only short columns will be discussed in this Chapter; the effects of the slenderness in reducing column strength will be covered in ***Chapter 10***.

9.1.5 Columns Classification According to Nature of Applied Forces

According to the nature of applied loads, columns can be calcified into following types.

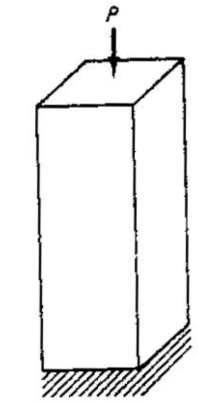


Figure 9.1-7: Axially loaded columns.

9.1.5.1 Axially Loads Columns

- Sometimes columns are almost subjected to concentric forces with negligible moments, ***Figure 9.1-7*** above.
- Interior columns in building with equal spans are with in this category when subjected to gravity loads, see columns B2, C2 of ***Figure 9.1-10*** below.
- Analysis of columns under axial loads, i.e., checking the adequacy of proposed longitudinal and lateral reinforcements for given axial loads has been presented in ***Article 9.2***. While design of columns under axial loads, i.e., select the required longitudinal and lateral reinforcements for the axial loads has been presented in ***Article 9.3*** below.

9.1.5.2 Columns Subjected to Axial Force and Uniaxial Moment

- In buildings with equal spans, edge columns are mainly subjected to axial force and uniaxial moment, see ***Figure 9.1-8*** and see ***columns B1, C1, A2, D2, B2, and D2*** of ***Figure 9.1-10*** below.
- Analysis and design of columns that subjected to axial force and uniaxial moment are presented in ***Article 0*** and ***Article 9.6*** respectively.

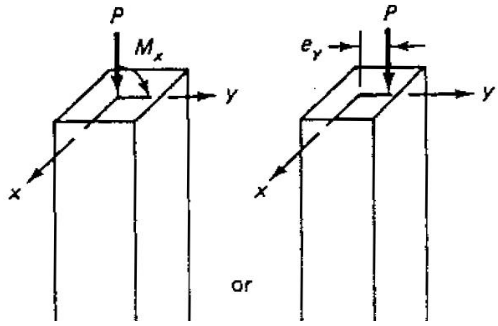


Figure 9.1-8: Columns subjected to axial force and uniaxial moment.

9.1.5.3 Columns Subjected to Axial Force and Biaxial Moments

- Columns at corner of buildings, columns A1, A2, D1, and D2 of ***Figure 9.1-10***, are usually subjected to axial force and biaxial moments as indicated in ***Figure 9.1-9*** below.
- As the design of these columns is iterative in nature, only their analysis is presented in ***Article 9.8***.

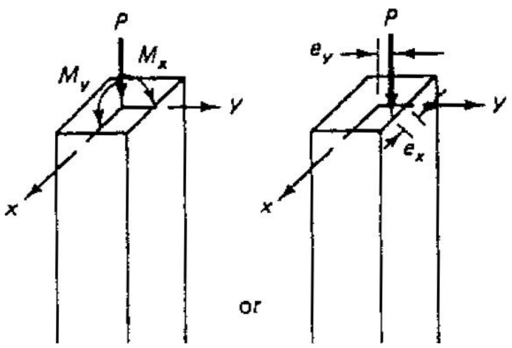


Figure 9.1-9: Columns subjected to axial force and biaxial moments.

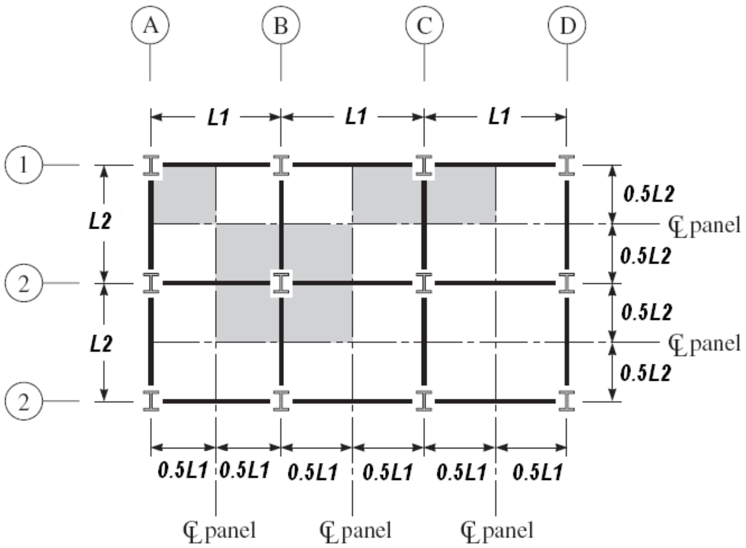


Figure 9.1-10: Columns layout for a building with equal spans.

9.2 ACI Analysis Procedure for a Short Column under an Axial Load (Small Eccentricity)

- Earlier ACI versions have defined small eccentricity as follows:
  - For spirally reinforced columns: $e/h \leq 0.05$.
  - For tied reinforced columns: $e/h \leq 0.10$.
- For short columns, definition of minimum eccentricity is implicitly included as will be discussed in ***Articles 9.5*** and ***9.6***.
- ACI procedures for the analysis of short columns under axial loads can be summarized as follows:

9.2.1 Checking of Longitudinal Reinforcement for Nominal Requirements

Reinforcement Limits

- Check ρ_g within acceptable limits.

$$0.01 \leq \rho_g = \frac{A_{st}}{A_g} \leq 0.08$$

- According to AC1 Code (10.6.1.1), the ratio of longitudinal steel area A_{st}, to gross concrete cross section A_g should be in the range from 0.01 to 0.08.
- The lower limit is necessary:
  - To ensure resistance to bending moments not accounted for in the analysis.
  - To reduce the effects of creep and shrinkage of the concrete under sustained compression.
- Ratios higher than 0.08 not only are uneconomical, but also would cause difficulty owing to congestion of the reinforcement, particularly where the steel must be spliced.
- Most columns are designed with ratios below 0.04. Larger-diameter bars are used to reduce placement costs and to avoid unnecessary congestion.
- The special large-diameter, No. 43 and No. 37 bars are produced mainly for use in columns.

Number of Rebars

- Check the rebar number with the minimum number of longitudinal bars:
  - Four bars for tied columns.
  - Six bars for spiral columns.
- According to ACI Code 10.7.3.1, a minimum of four longitudinal bars is required when the bars are enclosed by spaced rectangular or circular ties, and a minimum of six bars must be used when the longitudinal bars are enclosed by a continuous spiral.

Minimum Spacing between Longitudinal Bras

- According to (ACI318M, 2014), article ***25.2.3***, for longitudinal reinforcement in columns, pedestals, struts, and boundary elements in walls, clear spacing between bars shall be

$$\text{S}_{\text{Minimum}} = \text{Maximum}\left(1.5\text{d}_{\text{Bar}}, 40^{\text{mm}}, \frac{4}{3} \times \text{maximum size of aggregate}\right)$$

- As the student in his course on concrete technology how to select the maximum size of aggregate as a function of rebar spacing, the third condition related to maximum size of aggregate is assumed satisfied in this course.

9.2.2 Design Strength of Axially Loaded Columns

- According to (ACI318M, 2014), article 22.4, design strength of axially loaded column, can be determined as follows:
  - For spiral column the design strength is:
    $\emptyset P_{nMaximum} = 0.85\emptyset\left[0.85 f_c'\left(A_g - A_{st}\right) + A_{st} f_y\right]$
    with $\emptyset = 0.75$.
  - For tied columns:
    $\emptyset P_{nMaximum} = 0.80\emptyset\left[0.85 f_c'\left(A_g - A_{st}\right) + A_{st} f_y\right]$
    with $\emptyset = 0.65$.

- In articles 0 and 9.6, it will be shown that a column has its maximum strength when it is subjected to concentrically loaded with a compression force.
- Nominal strength of axially loaded column can be derived as follows:
  Nominal Strength of an Axially Loaded Column can be found recognizing the nonlinear response of both materials (steel and concrete) by:
  $P_n = 0.85 f'_c A_c + A_{st} f_y$
  or
  $P_n = 0.85 f'_c (A_g - A_{st}) + A_{st} f_y$
  i.e., by summing the strength contributions of the two components of the column.
- Strength Reduction Factor for Columns:
  The ACI strength reduction factors, ϕ, are lower for columns than for beams, see article 21.2.1. of the (ACI318M, 2014),
  - ***Reflecting their greater importance in a structure***,
  - A beam failure would normally affect only a local region whereas a column failure could result in the collapse of the entire structure,

  In addition, these factors reflect differences in the behavior of tied columns and spirally reinforced columns that shown in Figure 9.2-1 below.
  $\emptyset_{Tied\ Column} = 0.65$
  $\emptyset_{Spiral\ Column} = 0.75$

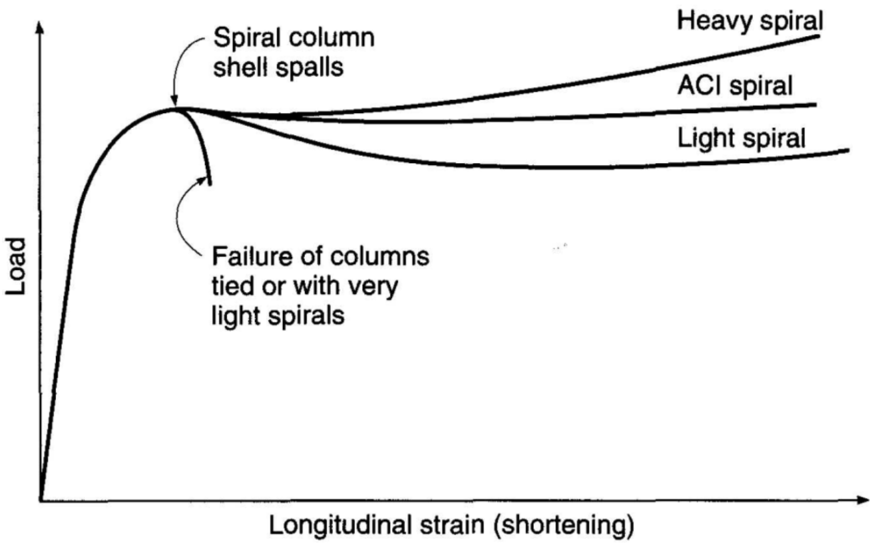


Figure 9.2-1: Behavior of spirally reinforced and tied columns.

- Provisions for Small Eccentricity:
  - A farther limitation on column strength is imposed by ACI Code 22.4 to allow for accidental eccentricities of loading not considered in the analysis.
  - This is done by imposing an upper limit on the axial load that is less than the calculated design strength:
    $Reduction\ Factor\ for\ Accidental\ Eccentricities_{Tied\ Column} = 0.8$
    $Reduction\ Factor\ for\ Accidental\ Eccentricities_{Spiral\ Column} = 0.85$
- Based on above discussion, design strength of axially loaded columns would be:
  - For spiral column the design strength is:
    $\emptyset P_{nMaximum} = 0.85\emptyset\left[0.85 f'_c \left(A_g - A_{st}\right) + A_{st} f_y\right]$
    with $\emptyset = 0.75$.
  - For tied columns:
    $\emptyset P_{nMaximum} = 0.80\emptyset\left[0.85 f'_c \left(A_g - A_{st}\right) + A_{st} f_y\right]$
    with $\emptyset = 0.65$.

9.2.3 Checking of Lateral Reinforcement (Ties), (ACI318M, 2014), Article 25.7.2

- All bars of tied columns shall be enclosed by lateral ties at least No 10 in size for longitudinal bars up to No. 32 and at least No. 13 in size for Nos. 36, 43, and 57 and bundled longitudinal bars.
- The spacing of the ties shall not exceed:
  $S_{Maximum} = \min[16 d_{bar}, 48 d_{ties}, Least\ dimension\ of\ column]$

- Arrangement of Rectilinear Ties
  - The ties shall be so arranged that every corner and alternate longitudinal bar shall have lateral support provided by the corner of a tie having an included angle of not more than 135, and no bar shall be farther than 150mm clear on either side from such a laterally supported bar.
  - Rectilinear ties arrangement according to ACI Code requirements can be summarized as follows, ***Figure 9.2-2*** below.

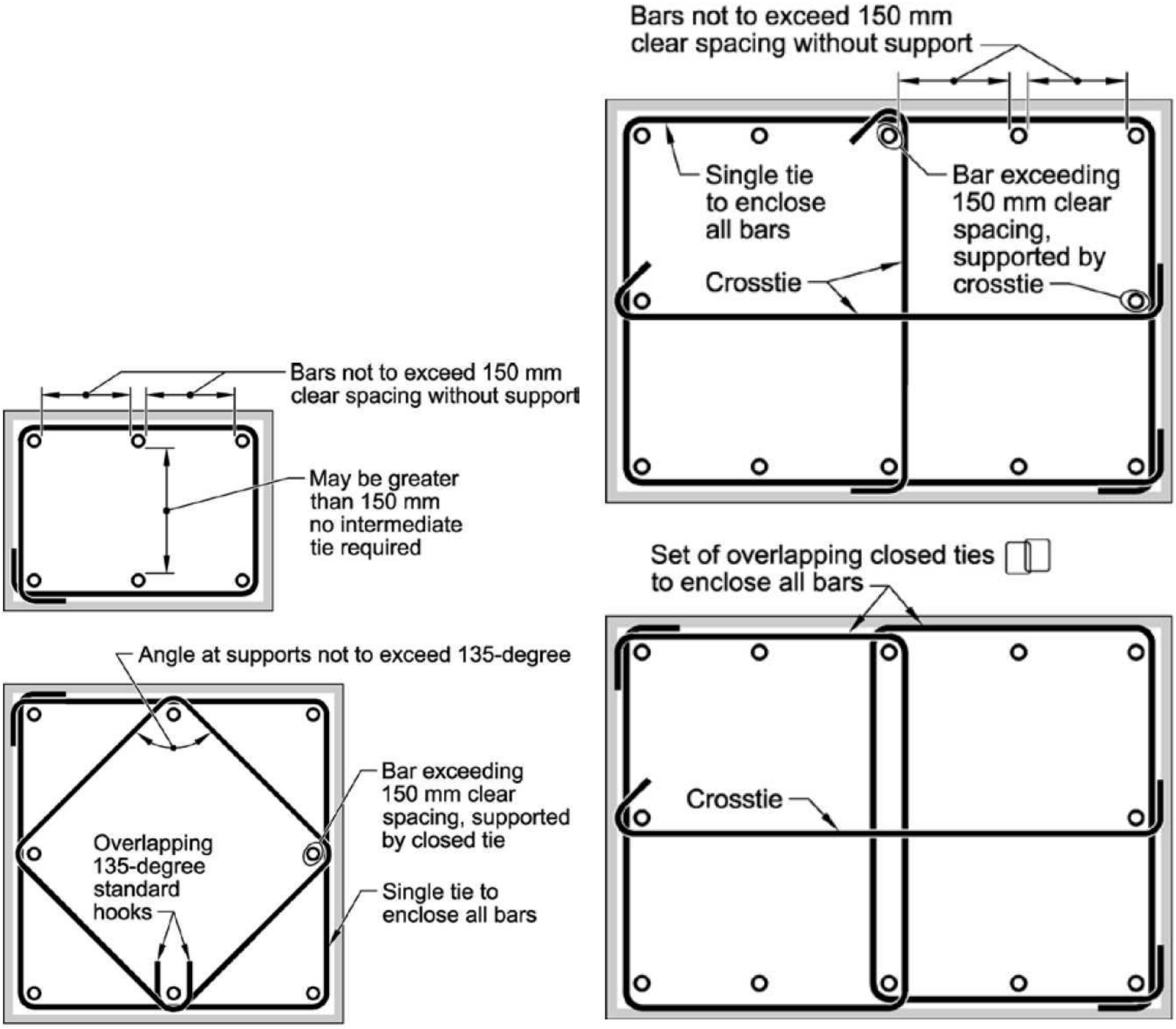


Figure 9.2-2: Tie arrangements for square and rectangular columns.

- Anchorage of Circular Ties
  - Circular ties shall be permitted where longitudinal bars are located around the perimeter of a circle.
  - Anchorage of individual circular ties shall be in accordance with:
    1. Ends shall overlap by at least 150 mm
    2. Ends shall terminate with standard hooks,
    3. Overlaps at ends of adjacent circular ties shall be staggered around the perimeter enclosing the longitudinal bars.
  - Above anchorage requirements have been summarized in ***Figure 9.2-3*** above.

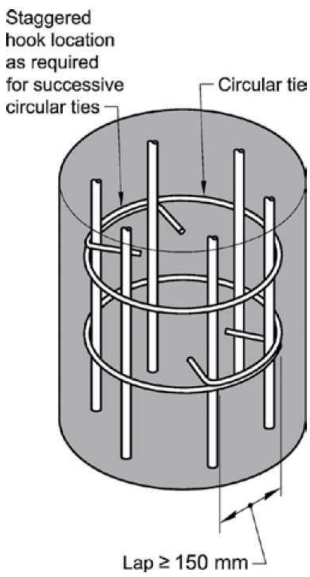


Figure 9.2-3: Circular tie anchorage.

9.2.4 Checking of Lateral Reinforcement (Spiral)

- For spirally reinforced columns, ACI Code requirements (25.7.3) for lateral reinforcement may be summarized as follows:
- Spirals shall consist of a continuous bar or wire not less than 10mm. in diameter.
- Compare the spiral ratio provided by the designer ($\rho_{s\ Provided}$) with the minimum recommended spiral ratio by the ACI Code ($\rho_{s\ Minimum}$):

$$\rho_{s\ Provided} = \frac{Volume\ of\ the\ spiral\ steel\ in\ one\ revolution}{volume\ of\ concrete\ core\ contained\ in\ one\ revoltion}$$

$$= \frac{4A_{sp}}{D_c\ S}$$

$$\rho_{sMinimum} = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f'_c}{f_{sy}}$$

- The provided clear spacing ($S_{Provided\ CLear}$) between turns of the spiral must be:

$S_{Provided\ CLear} \leq 80^{mm}$ and $S_{Provided\ Clear} \geq 25^{mm}$

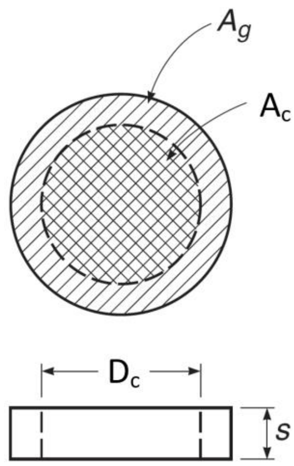


Figure 9.2-4: Notations adopted in analysis and design of spiral reinforcements.

Example 9.2-1

For a column that has the cross section area shown in ***Figure 9.2-5***, check the column adequacy with ACI Code requirements and compute the design axial load. Use $f_c' = 27.5\ \mathrm{MPa}$, and $f_y = 420\mathrm{MPa}$.

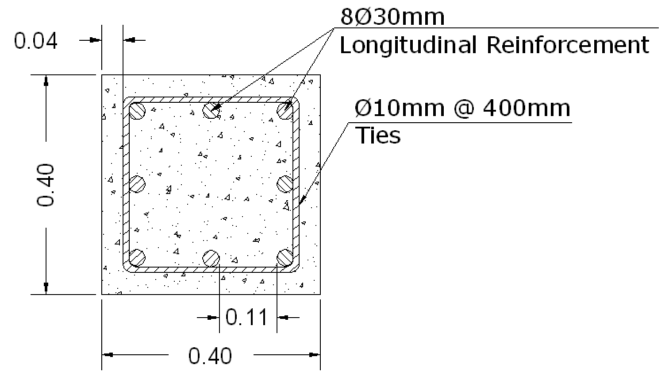


Figure 9.2-5: Proposed tied column for Example 9.2-1.

Solution

Longitudinal reinforcement

Check ρ_g within acceptable limits:

$A_g = 400^2 = 160\ \ 000\ mm^2$

$$A_{st} = \frac{\pi \times 30^2}{4} \times 8 = 5\ \ 652\ mm^2$$

$$0.01 < \rho_g = \frac{5\ 652}{160\ 000} = 3.53\% < 0.08$$

Check minimum number of longitudinal bars:

$8 > 4 \quad \therefore Ok.$

Check minimum distance between longitudinal bars:

$S_{Minimum} = Maximum[1.5 \times 30^{mm}, 40^{mm}] = 45^{mm} < 110^{mm} \quad \therefore Ok.$

Design Axial Strength, ϕP_n

Calculate the maximum design axial load strength $\emptyset P_{n(max)}$:

$\emptyset P_{nMaximum} = 0.80 \times 0.65[0.85 \times 27.5(160\ 000 - 5\ 652) + 5\ 652 \times 420] =$

$\emptyset P_{nMaximum} = 3\ \ 110\ kN$

Lateral reinforcement (Ties)

Checking of Lateral Reinforcement (Ties):

Ties diameter:

$\because\ \emptyset = 30^{\mathrm{mm}} < 32^{\mathrm{mm}}, \therefore$ we can use $\emptyset = 10^{\mathrm{mm}}$ for ties

Ties spacing:

$S_{\mathrm{Maximum}} = \min[16 \times 30^{\mathrm{mm}}, 48 \times 10^{\mathrm{mm}}, 400^{\mathrm{mm}}] = 400^{\mathrm{mm}} = S_{\mathrm{Provided}} \quad \therefore \mathrm{Ok}.$

Ties arrangement:

$\because S_{\text{Spacing between longitudinal bars}} < 150^{\mathrm{mm}}$

Then, alternate longitudinal bars will be supported by corner bars.

Example 9.2-2

Check the column shown in ***Figure 9.2-6*** with general requirements of the ACI Code, then determine whether this column is adequate to carry a factored load of P_u= 2250 kN or not.

In your analysis:

- Assume small eccentricity.
- Use $f_c' = 27.5\ \mathrm{MPa}$, and $f_y = 420\mathrm{MPa}$.

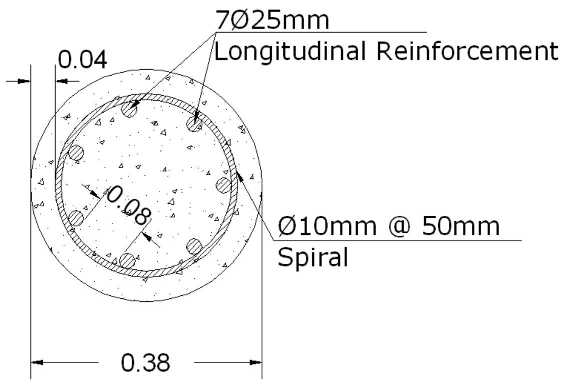


Figure 9.2-6: Spiral column of Example 9.2-2.

Solution

Longitudinal reinforcement

Check ρ_g within acceptable limits:

$$A_g = \frac{\pi \times 380^2}{4} = 113\ \ 354\ mm^2$$

$$A_{st} = \frac{\pi \times 25^2}{4} \times 7 = 3\ 434\ mm^2 \Rightarrow \rho_g = \frac{3\ \ 434}{113\ \ 354} = 3.0\% \Rightarrow 0.01 < \rho_g < 0.08 \ \therefore Ok.$$

Check minimum number of longitudinal bars

$7 > 6 \quad \therefore Ok.$

Check minimum distance between longitudinal bars

$S_{Minimum} = Maximum[1.5 \times 25^{mm}, 40^{mm}] = 40.0^{mm} < 80^{mm} \quad \therefore Ok.$

Design Axial Strength

Calculate the maximum design axial load strength $\emptyset P_{n(max)}$:

$\emptyset P_{nMaximum} = 0.85 \times 0.75[0.85 \times 27.5(113\ 354 - 3\ 434) + 3\ 434 \times 420] = 2\ \ 557\ kN > P_u \quad \therefore Ok.$

Lateral reinforcement (Ties)

Check the lateral reinforcement (Spiral):

Check Spiral Diameter:

$\emptyset_{Spiral} = 10mm\ Ok.$

Check Spiral Steel Ratio:

$$A_{sp} = \frac{\pi \times 10^2}{4} = 78.5^{mm^2} \Rightarrow \rho_{sProvided} = \frac{4 \times 78.5^{mm^2}}{(380 - 2 \times 40)^{mm} \times 50^{mm}} = 0.0209$$

$$\rho_{sMinimum} = 0.45 \times \left(\frac{113\ 354}{\frac{\pi \times 300^2}{4}} - 1 \right) \times \frac{27.5}{420} = 0.0178 < 0.0209 \ \therefore Ok.$$

Check the Clear Spacing:

$25^{mm} < [S_{Clear\ Provided} = 50^{mm} - 10^{mm} = 40^{mm}] < 80^{mm}\ \therefore Ok.$

9.3 ACI DESIGN PROCEDURE FOR A SHORT COLUMN UNDER AN AXIAL LOAD (SMALL ECCENTRICITY)

ACI Code procedure for design of a short column under an axial compression force can be summarized as follows:

- Determine the applied factored axial load P_u:
- Establish a desired ρ_g.
- Determine the required gross column area A_g:
  For tied column:
  $$A_{gRequired} = \frac{P_U}{0.80 \times \emptyset[0.85 f_c'(1-\rho_g) + f_y \rho_g]}$$
  For spiral column:
  $$A_{gRequired} = \frac{P_U}{0.85 \times \emptyset[0.85 f_c'(1-\rho_g) + f_y \rho_g]}$$
- Select the column dimensions. Round the answer to the nearest 25^{mm}.
- Find the load that carried by the concrete:
  For tied column:
  $$\emptyset \mathrm{P}_{\mathrm{n\ Carried\ by\ Concrete}} = 0.80 \times \emptyset[0.85 \mathrm{f}_c' \mathrm{A_g}(1-\rho_g)]$$
  For spiral column:
  $$\emptyset \mathrm{P}_{\mathrm{n\ Carried\ by\ Concrete}} = 0.85 \times \emptyset[0.85 \mathrm{f}_c' \mathrm{A_g}(1-\rho_g)]$$
- Determine the load required to be carried by the longitudinal steel:
  $$\emptyset \mathrm{P}_{\mathrm{nCarried\ by\ Steel}} = \mathrm{P_u} - \emptyset \mathrm{P}_{\mathrm{n\ Carried\ by\ Concrete}}$$
- Determine the required steel area of longitudinal bars:
  For tied column:
  $$\emptyset \mathrm{P}_{\mathrm{nCarried\ by\ Steel}} = 0.80\emptyset[\mathrm{A_{stRequired}} \mathrm{f_y}]$$
  For spiral column:
  $$\emptyset \mathrm{P}_{\mathrm{nCarried\ by\ Steel}} = 0.85\emptyset[\mathrm{A_{stRequired}} \mathrm{f_y}]$$
- Determine the required number of bars:
  $$No.of\ Bar_{Required} = \frac{A_{stRequired}}{A_{Bar}}$$
  Round required number to the nearest integer and check with requirement of the ACI for the minimum number of longitudinal bars:
  $$\mathrm{No.\ of\ Bars_{Provided}} \geq 4_{\mathrm{for\ tied\ columns}}$$
  $$\mathrm{No.\ of\ Bars_{Provided}} \geq 6_{\mathrm{for\ spiral\ columns}}$$
- Check the spacing between the longitudinal bars:
  $$S_{Provided} \geq Maximum\ of\ [1.5 d_{Bar}, 40^{mm}]$$
- Design the lateral reinforcement:
  Ties:
  Select ties diameter:
  If $\varphi_{Longitudinal} \leq 32^{mm}$ then:
  $$\varphi_{Ties} = 10^{mm}$$
  Else
  $$\varphi_{Ties} = 13^{mm}$$
  Select ties spacing:
  $$S_{Required} \leq Minimum[16\varphi_{Bar}, 48\varphi_{ties}, Least\ Column\ Dimensions]$$
  Arrange the ties according to requirements of the ACI for maximum spacing between longitudinal bars (use the standard arrangements of Figure 9.2-2 above).
  Spiral:
  $$\varphi_{Spiral} \geq 10^{mm}$$
  Compute $\rho_{sMinimum}$
  $$\rho_{sMinimum} = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f_c'}{f_{sy}}$$
  Let $\rho_s = \rho_{sMinimum}$ to compute the required $S_{Required}$:

$S_{Required} = \frac{4A_{sp}}{D_c\ \rho_{sMinimum}}$

The clear spacing $S_{Required\ Clear}$ between turns of the spiral must be:

$25 \leq S_{Clear} \leq 80^{mm}$

Example 9.3-1

Design a tied column to carry a factored axial load of P_u=3 184 kN.

- Assume that there is no identified applied moment.
- Assume that the column is short.
- Assume $\rho_{Preferable} = 0.03$.
- Assume $f_c' = 27.5MPa$, $f_y = 420MPa$.
- Try square section.
- Try $\varphi_{Longitudinal\ Bar} = 29^{mm}$, $A_{Bar} = 645mm^2$
- Try $\varphi_{Lateral\ Reiforcement} = 10^{mm}$.

Solution

Compute $A_{gRequired}$:

$$A_{gRequired} = \frac{3184 \times 10^3 N}{0.80 \times 0.65[0.85 \times 27.5(1 - 0.03) + 420 \times 0.03]} = 173\ 587mm^2$$

Try square section:

$B = \sqrt{173\ 587mm^2} = 416.6^{mm}$

Try $B = 425^{mm}, \therefore A_g = 180\ \ 625mm^2$.

Compute $\emptyset P_{n\ Carried\ by\ Concrete}$:

$\emptyset P_{n\ Carried\ by\ Concrete} = 0.8 \times 0.65[0.85 \times 27.5 \times 180\ 625(1 - 0.03)] = 2\ \ 130kN$

Compute $\emptyset P_{n\ Carried\ by\ Steel}$:

$\emptyset P_{n\ Carried\ by\ Steel} = 3\ 184 - 2\ \ 130 = 1\ 054\ kN$

Compute $A_{stReqired}$:

$1\ 054 \times 10^3 = 0.8 \times 0.65 \times [420 \times A_{stReqired}] \Longrightarrow A_{stReqired} = 4\ 826\ mm^2$

Compute Number of longitudinal bars:

Try $\varphi_{Longitudinal} = 29^{mm}$:

$No. = \frac{4\ 826}{645} = 7.48$

Try $8\varphi 29^{mm}$:

$\because 8 \geq 4\ \therefore Ok.$

Check spacing between longitudinal bars:

$$S_{Provided} = [425^{mm} - 2 \times 40^{mm} - 2 \times 10^{mm} - 3 \times 29^{mm}]\frac{1}{2} = 119^{mm}$$

$$S_{Minimum} = Maximum[1.5d_{bar}, 40^{mm}] = 43.5^{mm} < 119^{mm}$$

$\therefore Ok.$

Design of Ties:

Ties diameter:

$\because \varphi_{Longitudinal\ Bars} < 32^{mm}$, $\therefore Use\ \varphi_{Ties} = 10^{mm}$

Tie spacing:

$$S_{Required} = Minimum[16 \times 29^{mm}, 48 \times 10^{mm}, 425^{mm}] = 425^{mm}$$

Try $\varphi 10^{mm}@425^{mm}$

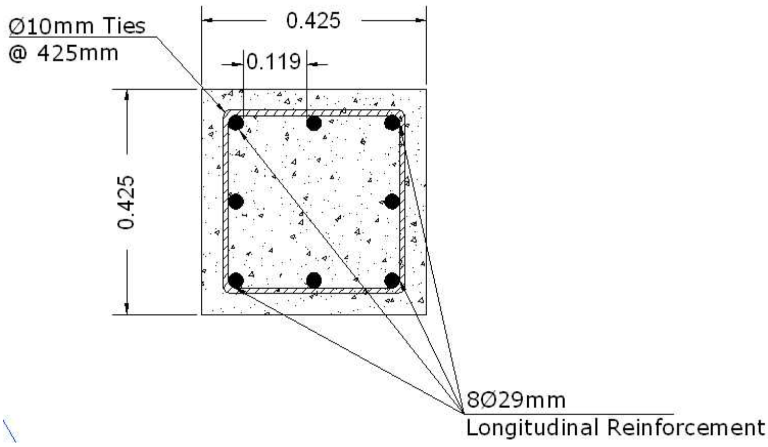


Figure 9.3-1: Final design section for the column of Example 9.3-1.

Ties arrangement:

As we intend to use eight rebars and spacing between rebars is less than 150mm, then the ties reinforcement is presented in ***Figure 9.3-1***.

Example 9.3-2

Redesign the column of Example 9.3-1 as a circular spirally reinforced column with P_u = 3 429 kN.

Solution

Compute $A_{gRequired}$:

$$A_{gRequired} = \frac{3429 \times 10^3 N}{0.85 \times 0.75[0.85 \times 27.5(1-0.03) + 420 \times 0.03]} = 152\,488\ mm^2$$

$$\frac{\pi D^2}{4} = 152\,488\ mm^2, \therefore D_{Required} = 441^{mm}, Try\ D_{Provided} = 450^{mm}$$

Compute $\emptyset P_{n\ \text{Carried by Concrete}}$:

$$\emptyset P_{n\ \text{Carried by Concrete}} = 0.85 \times 0.75 \times \left[0.85 \times 27.5 \times \frac{\pi \times 450^2}{4}(1-0.03)\right] = 2\,298 kN$$

Compute $\emptyset P_{n\ \text{Carried by Steel}}$

$$\emptyset P_{n\ \text{Carried by Steel}} = 3\,429 kN - 2\,298 kN = 1\,131\ kN$$

Compute $A_{stRequired}$

$$0.85 \times 0.75 \times [A_{stRequired} \times 420] = 1\,131\,000\ N$$

$$A_{stRequired} = 4\,224 mm^2$$

Compute number of longitudinal bars:

Try $\varphi_{Longitudinal} = 29^{mm}$, $A_{Bar} = 645 mm^2$.

$$No. = \frac{4\,224}{645} = 6.55$$

Try $7\varphi 29^{mm}$.

$\because 7 \geq 6\ \therefore Ok.$

Check spacing between longitudinal bars:

$$D_{Center\ of\ Longitudinal\ Bars} = 450^{mm} - 2 \times 40^{mm} - 2 \times 10^{mm} - 29^{mm} = 321^{mm}$$

$$S_{Provided} = \frac{[\pi \times 321^{mm} - 7 \times 29^{mm}]}{7} = 115^{mm}$$

$$S_{Minimum} = Maximum\ [1.5 d_{Bar}, 40^{mm}] = 43.5^{mm} < 115^{mm}\ \therefore Ok.$$

Spiral Design:

Spiral diameter:

$\because \varphi_{Spiral} = 10^{mm}\ \therefore Ok.$

Compute $\rho_{sMinimum}$:

$$D_c = 450^{mm} - 2 \times 40^{mm} = 370^{mm}$$

$$A_c = \frac{\pi \times 370^2}{4} = 107\,467\ mm^2$$

$$A_g = \frac{\pi \times 450^2}{4} = 158\,962\ mm^2$$

$$\rho_{sMinimum} = 0.45\left(\frac{158\,962}{107\,467} - 1\right) \times \frac{27.5}{420} = 14.2 \times 10^{-3}$$

$$A_{sp} = \frac{\pi \times 10^2}{4} = 78.5^{mm^2}$$

$$\therefore S_{Required} = \frac{4 \times 78.5^{mm^2}}{370^{mm} \times 14.2 \times 10^{-3}} = 59.8^{mm}$$

Try $\varphi 10^{mm}$ @ 60^{mm}

$\because S_{Clear} = 50^{mm} < 80^{mm}\ \therefore Ok.$

$\because S_{Clear} = 50^{mm} > 25^{mm}\ \therefore Ok.$

Use $\varphi 10^{mm}$ @ 60^{mm}

The final section of the column is shown in Figure 9.3-2.

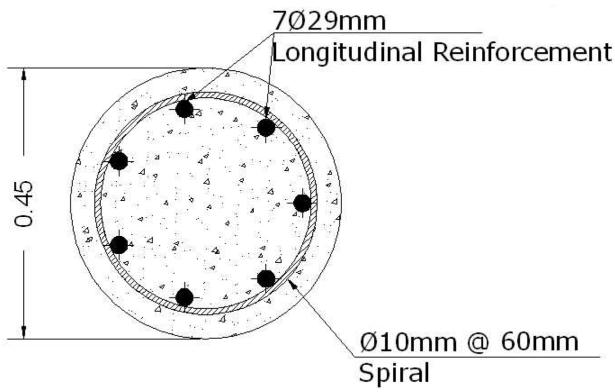


Figure 9.3-2: Final design section for the column of Example 9.3-2.

9.4 HOMEWORK: ANALYSIS AND DESIGN OF AXIALLY LOADED COLUMNS

Problem 9.4-1

Check the adequacy of the column that shown below according to the requirement of the ACI Code and compute its design strength.

Assume:

- Short column
- $f_c' = 27.5$ MPa
- $f_y = 420$ MPa
- $A_{Bar} = 637.5\ mm^2$

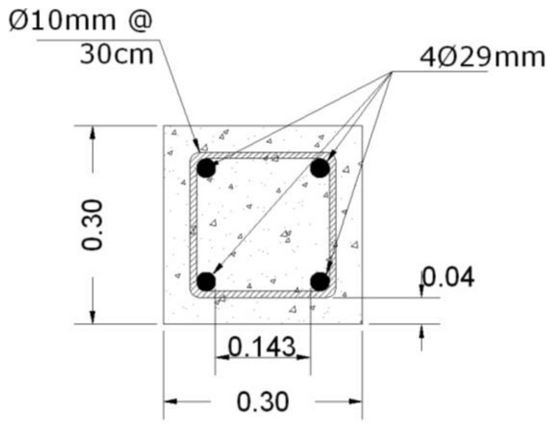


Figure 9.4-1: Proposed tied column for Problem 9.4-1.

Answers

Longitudinal reinforcement:

Check ρ_g within acceptable limits:

$A_g = 90\ 000\ mm^2, A_{st} = 2\ 550\ mm^2$

$0.01 < \rho_g = 2.83\% < 0.08$

Check minimum number of longitudinal bars:

$No. of\ Bars = 4\quad \therefore Ok.$

Check minimum distance between longitudinal bars:

$S_{Minimum} = 43^{mm} < 143^{mm}\ \therefore Ok.$

Calculate the maximum design axial load strength $\emptyset P_{n(max)}$:

$\emptyset P_{nMaximum} = 1\ 620\ kN$

Lateral reinforcement (Ties):

Ties diameter:

$\because\ \emptyset = 29^{mm} < 32^{mm}, \therefore we\ can\ use\ \emptyset = 10^{mm}\ for\ ties$

Ties spacing:

$S_{Maximum} = 300^{mm} = S_{Provided}\ \therefore Ok.$

Ties arrangement:

For a column with four rebars only, no interior ties are required.

Problem 9.4-2

Design a square tied column to support an axial load of $P_u = 4\ 078\ kN$. Design the necessary ties also.

Assume:

- Short column
- $f_c' = 34.5$ MPa
- $f_y = 420$ MPa
- $\rho_g = 0.05$
- $\varphi_{Longitudinal\ Bars} = 32^{mm}$
- $\varphi_{Ties} = 10^{mm}$

Answers

Compute $A_{gRequired}$:

$A_{gRequired} = 160\ 510 mm^2$

Try square section:

$B \approx 400^{mm}$

Try $B = 400^{mm}, \therefore A_g = 160\ 000 mm^2$.

Compute $\emptyset P_{n\ Carried\ by\ Concrete}$:

$\emptyset P_{n\ Carried\ by\ Concrete} = 2\ 318 kN$

Compute $\emptyset P_{n\ Carried\ by\ Steel}$:

$\emptyset P_{n\ Carried\ by\ Steel} = 1\ 760 kN$

Compute $A_{stReqired}$:

$A_{stReqired} = 8\ 059\ mm^2$

Compute Number of longitudinal bars:

$No. \approx 10$

$\because 10 > 4\ \therefore Ok.$

Check spacing between longitudinal bars:

$S_{Provided} = 57.3^{mm}, S_{Minimum} = Maximum[1.5d_{bar}, 40^{mm}] = 48^{mm} < 57.3^{mm} \therefore Ok.$

Design of Ties:

Ties diameter:

$\because \varphi_{Longitudinal\ Bars} = 32^{mm}, \therefore Use\ \varphi_{Ties} = 10^{mm}$

Tie spacing

$S_{Required} = 400^{mm}$

Try $\varphi 10^{mm}@400^{mm}$

Ties arrangement:

Sketch for details of longitudinal and lateral reinforcements are shown in ***Figure 9.4-2***.

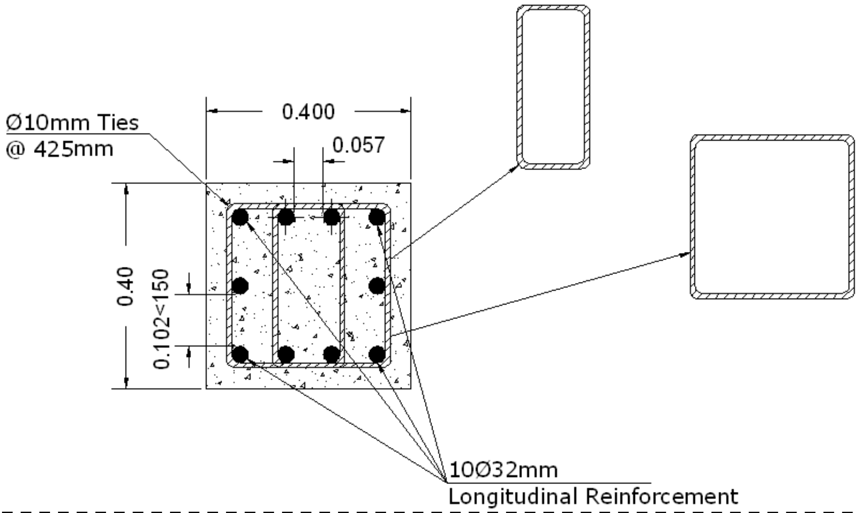


Figure 9.4-2: Final design section Problem 9.4-2.

Problem 9.4-3

Repeat Problem 9.4-2, using a rectangular section that has width $b = 350^{mm}$.

Answers

Compute $A_{gRequired}$:

$A_{gRequired} = 160\,510 mm^2$

Try rectangular section with $b = 350^{mm}$, therefore $h = 459\ mm$

Try $b = 350^{mm}, h = 460^{mm} \ \therefore A_g = 161\,000\ mm^2$.

Compute $\emptyset P_{n\ Carried\ by\ Concrete}$:

$\emptyset P_{n\ Carried\ by\ Concrete} = 2\ \ 332 kN$

Compute $\emptyset P_{n\ Carried\ by\ Steel}$:

$\emptyset P_{n\ Carried\ by\ Steel} = 1\,746 kN$

Compute $A_{stReqired}$:

$A_{stReqired} = 7\,994\ mm^2$

Compute Number of longitudinal bars:

$No. = \frac{7\,994}{804} = 9.94$, Try $10\varphi 32^{mm}$. $\because 10 > 4 \ \therefore Ok.$

Check spacing between longitudinal bars:

$S_{Provided} = 77.3^{mm}, S_{Minimum} = Maximum[1.5d_{bar}, 40^{mm}] = 48^{mm} < 74^{mm} \ \therefore Ok.$

Design of Ties:

Ties diameter:

$\because \varphi_{Longitudinal\ Bars} = 32^{mm}, \therefore Use\ \varphi_{Ties} = 10^{mm}$

Tie spacing

$S_{Required} = 350^{mm}$

Try $\varphi 10^{mm}@350^{mm}$

Ties arrangement:

Sketch for details of longitudinal and lateral reinforcements are shown in Fig. below.

$S_{Provided} = 77^{mm} < 150^{mm}$

No additional interior ties are required.

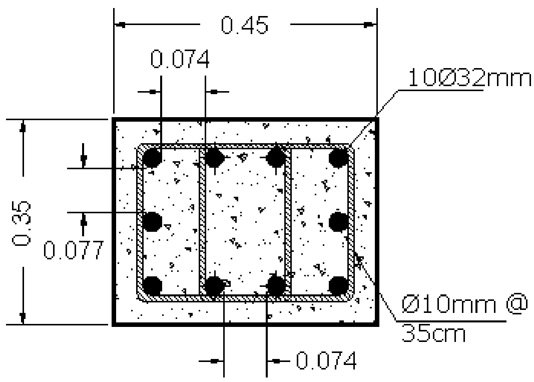


Figure 9.4-3: Final design section for Problem 9.4-3.

Problem 9.4-4

Design the spiral column that supports four girders of bridge shown in Figure 9.4-4 below. In your design, assume that.

- Each girder has a dead load reaction of 150 kN and has a live load reaction of 100 kN.
- f_c' = 28 MPa and f_y = 420 MPa.
- Rebar No. 25 for longitudinal reinforcement and No. 10 for spiral reinforcement.
- Column has a height of 4m, and it is assumed short.
- Column and cap selfweight should be included in your solution.

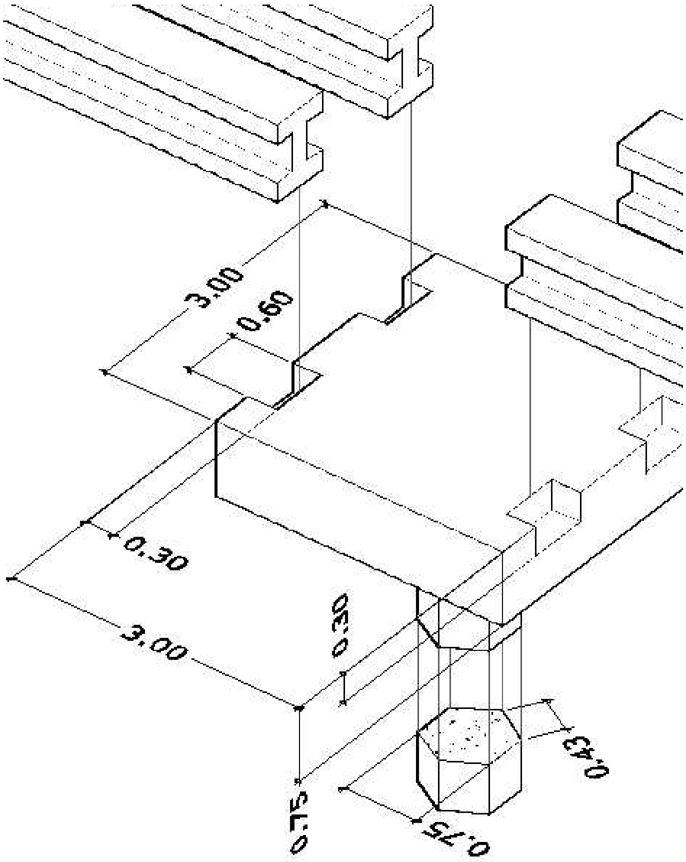


Figure 9.4-4: Four girders that supported on the column of Problem 9.4-4.

Hint for Solution: According to ACI Code, ***version 2011***, ***Article (10.8.3)*** "As an alternative to using the full gross area for design of a compression member with a square, octagonal, or other shaped cross section, it shall be permitted to use a circular section with a diameter equal to the least lateral dimension of the actual shape. Gross area considered, required percentage of reinforcement, and design strength shall be based on that circular section". Then this column can be transformed from hexagonal shape to the following circular shape.

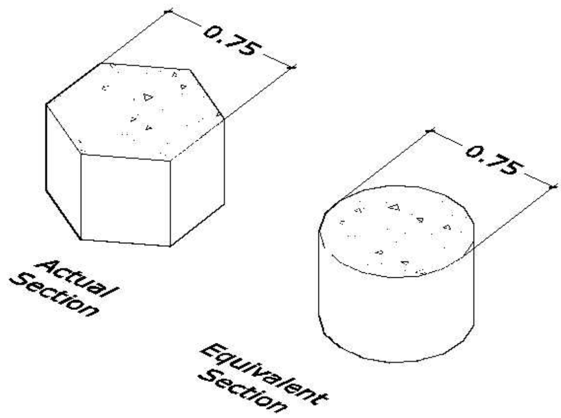


Figure 9.4-5: Transformation of an octagonal column into the equivalent circular section.

9.5 Analysis of a Column with Compression Load Plus Uniaxial Moment

9.5.1 Introduction

- Members that are axially loaded, i.e., concentrically compressed, occur rarely, if ever in buildings and other structures. Components such as columns chiefly carry loads in compression but simultaneous bending is usually present.
- Bending moments are caused by:
  - Continuity, i.e., by the fact that building columns are parts of monolithic frames in which the support moments of the girders are partly resisted by the abutting columns.

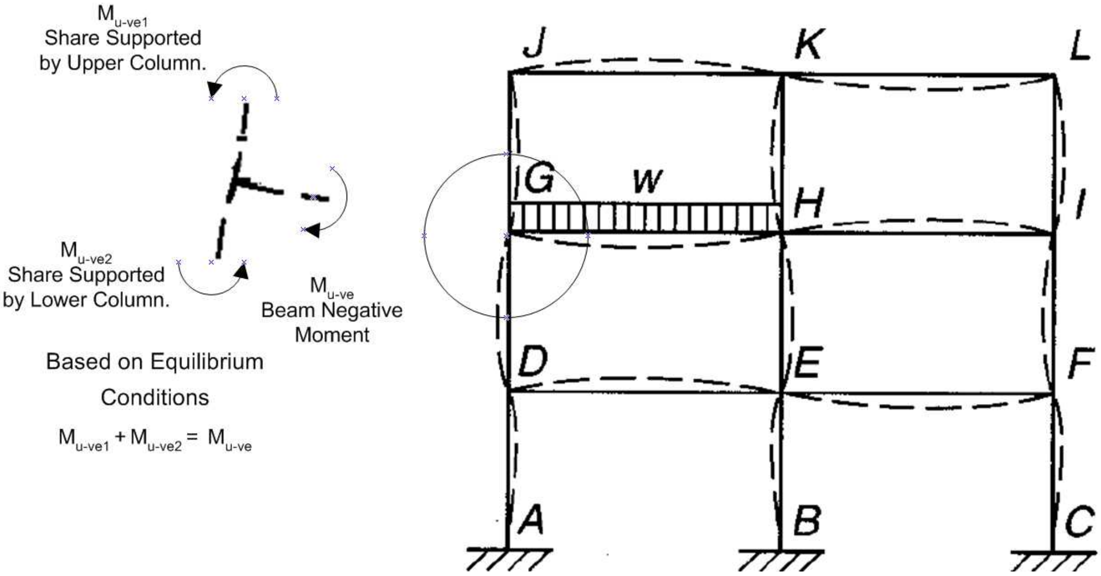


Figure 9.5-1: Moments in columns due to frame continuity.

  - Transverse loads such as wind forces.

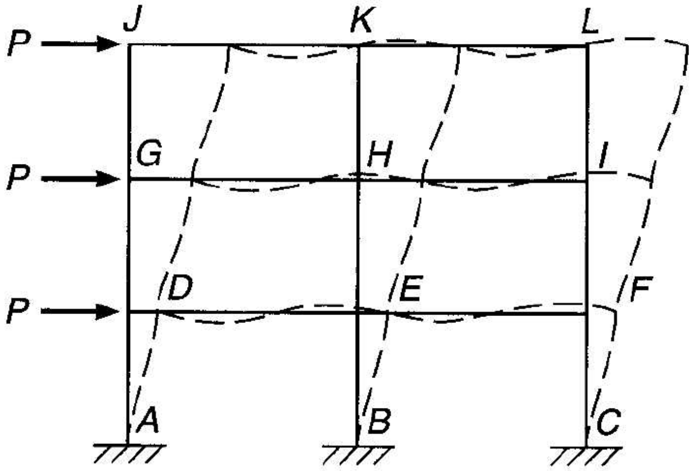


Figure 9.5-2: Moments in columns due to lateral forces.

  - Loads carried eccentrically on column brackets when the column axis does not coincide with the pressure line.
  - Imperfections of construction.

    For these reasons, members that should be designed for simultaneous compression and bending are very frequent in almost all types of concrete structures.
- When a member is subjected to combined axial compression P and moment M, it is usually convenient to replace the axial load and moment with an equal load P applied at eccentricity $e = M/P$. The two loadings are statically equivalent.

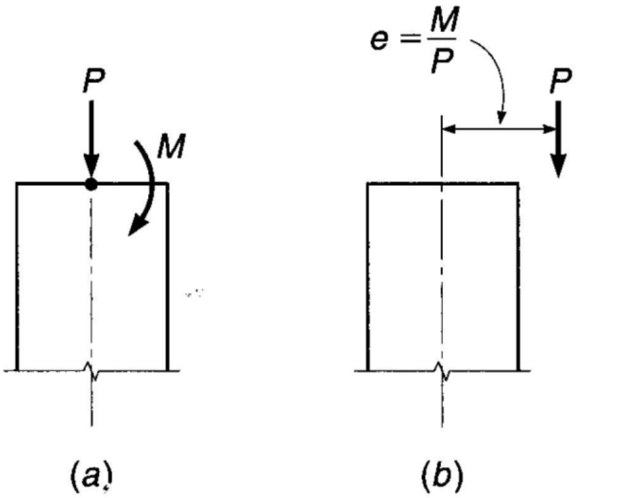


Figure 9.5-3: Equivalent eccentricity of column load.

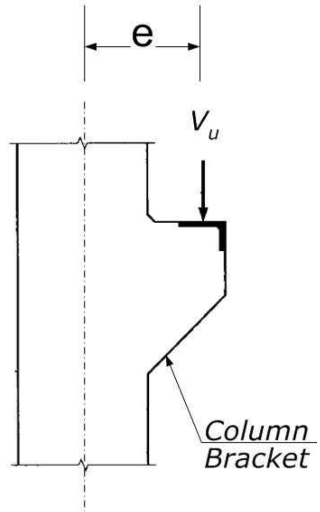


Figure 9.5-4: Moments in columns in precast frames.

- Two approaches for analysis of a column with axial force and uniaxial moment will be discussed in ***Articles 9.5.2*** and ***9.5.3*** below.

9.5.2 Column Analysis by Direct Application of Basic Principles

- Figure 9.5-5 "a" shows a member loaded parallel to its axis by a compressive force P, at an eccentricity e measured from the centerline.

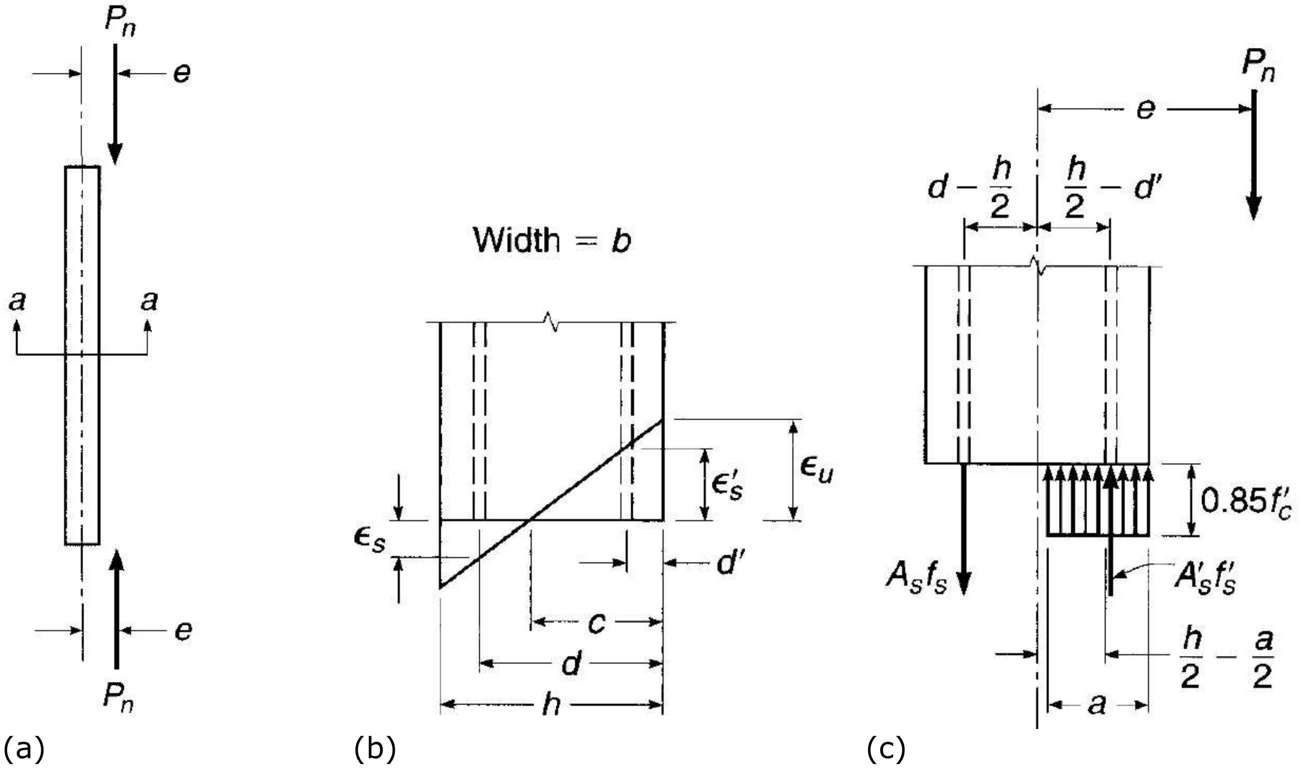


Figure 9.5-5: Column subject to eccentric compression: (a) loaded column; (b) strain distribution at section a-a; (c) stresses and forces at nominal strength.

- Above column can be analyzed based on direct application of basic principles of applied mechanics and as follows:

9.5.2.1 Compatibility

- With plane sections assumed to remain plane, concrete strains vary linearly with distance from the neutral axis which is located a distance "c" from the more heavily loaded side of the member.
- With full compatibility of deformations, the steel strains at any location are the same as the strains in the adjacent concrete; thus if the ultimate concrete strain is ϵ_u, the strain in the bars nearest the load is ϵ'_s while that in the tension bars at the far side is ϵ_s.