- Compression steel having area A'_s and tension steel with area A_s, are located at distances d and d', respectively, from the compression face (See Figure 9.5-5 "b" above).

9.5.2.2 Constitutive Relationships

The corresponding stresses and forces are shown in Figure 9.5-5 "c", just as for simple bending, the actual concrete compressive stress distribution is replaced by an equivalent rectangular distribution having depth $a = \beta_1 c$.

9.5.2.3 Equilibrium Equations

- Equilibrium between external and internal axial forces shown in Figure 9.5-5 "c"; requires that:

  $$\sum \boldsymbol{F_y} = \mathbf{0.0}$$

  $$P_n = 0.85 f'_c ab + A'_s f'_s - A_s f_s$$

- Also, the moment about the centerline of the section of the internal stresses and forces must be equal and opposite to the moment of the external force. P_n, so that:

  $$\sum M = 0.0$$

  $$M_n = P_n e = 0.85 f'_c ab\left(\frac{h}{2} - \frac{a}{2}\right) + A'_s f'_s\left(\frac{h}{2} - d'\right) + A_s f_s(d - \frac{h}{2})$$

- These are the two basic equilibrium relations for rectangular eccentrically compressed members.
- For a given eccentricity determined from the frame analysis (i.e., $e = \frac{M}{P}$) it is possible to solve above equations for the load and moment M_n that would result in failure as follows:
  - In both equations, f'_s, f_s, and a can be expressed in terms of a single unknown c, the distance to the neutral axis. This is easily done based on the geometry of the strain diagram, with ϵ_u taken equal to 0.003 as usual, and using the stress-strain curve of the reinforcement.
  - The result is that the two equations contain only two unknowns, P_n and c, and can be solved for those values simultaneously. However, to do so in practice would be complicated algebraically particularly because of the need to incorporate the limit f_y on both f'_s, and f_s.

9.5.3 Concept of Interaction Diagram

9.5.3.1 Basic Concepts

- A better approach, providing the basis for practical design, is to construct a *Strength Interaction Diagram* defining the failure load and failure moment for a given column for the full range of eccentricities from zero *to* infinity (see Fig. below):

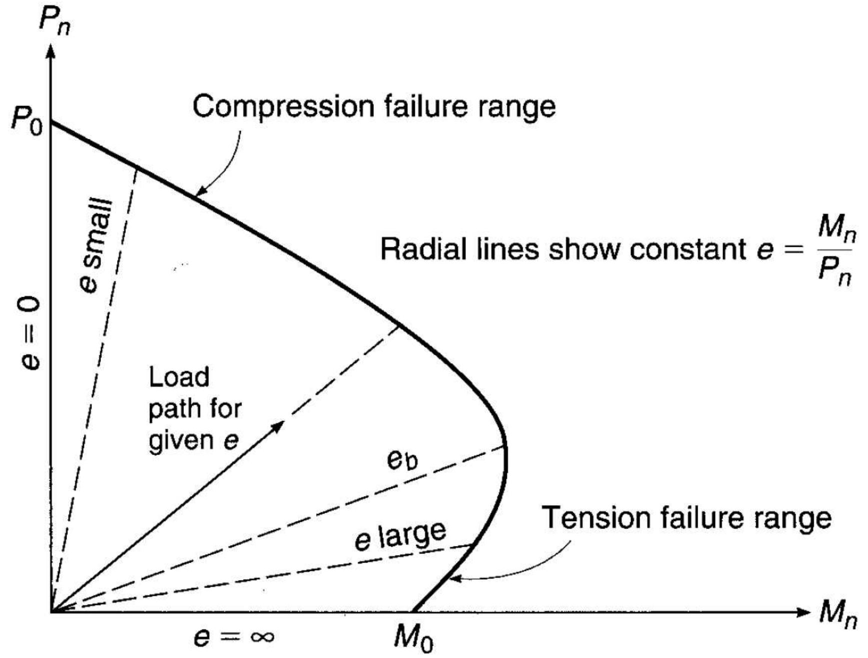


Figure 9.5-6: Interaction diagram for nominal column strength in combined bending and axial load.

- On such a diagram, any radial line represents a particular eccentricity $e = \frac{M}{P}$. For that eccentricity, gradually increasing the load will define a load path as shown, and when that load path reaches the limit curve, failure will result.
- The vertical axis corresponds to $e = 0$, and P_0 is the capacity of the column if concentrically loaded, as given by equations of articles 9.2 and 9.3.
- The horizontal axis corresponds to an infinite value of e, i.e., pure bending at moment capacity M_0.
- Failure Regions on Interaction Diagram:
  - Small eccentricities will produce failure governed by concrete compression.
  - Large eccentricities give a failure triggered by yielding of the tension steel.

9.5.3.2 Construction of A nominal Interaction Diagram

For a given column, the interaction diagram is most easily constructed by following procedure:

- Selecting successive choices of neutral axis distance "c", from infinity (axial load with eccentricity 0) to a very small value found by trial to give $P_n = 0$ pure bending).
- For each selected value of "c", the steel strains and stresses and the concrete force are easily calculated as follows:
  - For the tension steel:

$$\epsilon_s = \epsilon_u \frac{d-c}{c}$$

$$f_s = E_s \epsilon_u \frac{d-c}{c} \leq f_y$$

  - While for the compression steel:

$$\epsilon_s' = \epsilon_u \frac{c-d'}{c}$$

$$f_s' = E_s \epsilon_u \frac{c-d'}{c} \leq f_y$$

  - The concrete stress block has depth:

$$a = \beta_1 c \ \leq h$$

- Substitute the values of f_s, f_s', and a into the following relations to compute the values of P_n and M_n that corresponding to assume "c" value.

$$\sum F_y = 0.0 \Rightarrow P_n = 0.85 f_c' ab + A_s' f_s' - A_s f_s$$

$$\sum M = 0.0 \Rightarrow M_n = P_n e = 0.85 f_c' ab \left(\frac{h}{2} - \frac{a}{2}\right) + A_s' f_s' \left(\frac{h}{2} - d'\right) + A_s f_s (d - \frac{h}{2})$$

- These steps (starting from assuming of "c" to obtain the corresponding P_n and M_n) represent a point on the interaction diagram. Then these will be repeated until enough number of points on interaction is obtained to draw the required diagram.
- Construct interaction diagram through connecting between points drawn.

9.5.3.3 Design Interaction Diagram

- As was discussed in Chapter 3, the strength reduction factor "∅" is a function of steel strain and as shown in ***Figure 9.5-7*** below.

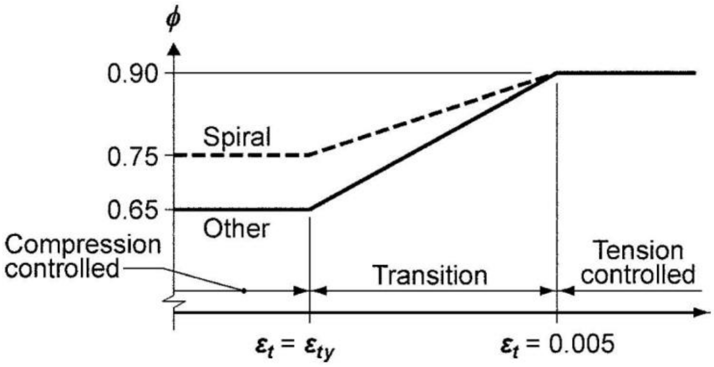


Figure 9.5-7: Variation of ϕ with net tensile strain in extreme tension reinforcement, ε_t.

- Each point on the interaction diagram has its strain, see ***Figure 9.5-8*** below, and in turn has own factor of safety, see ***Figure 9.5-9*** below.

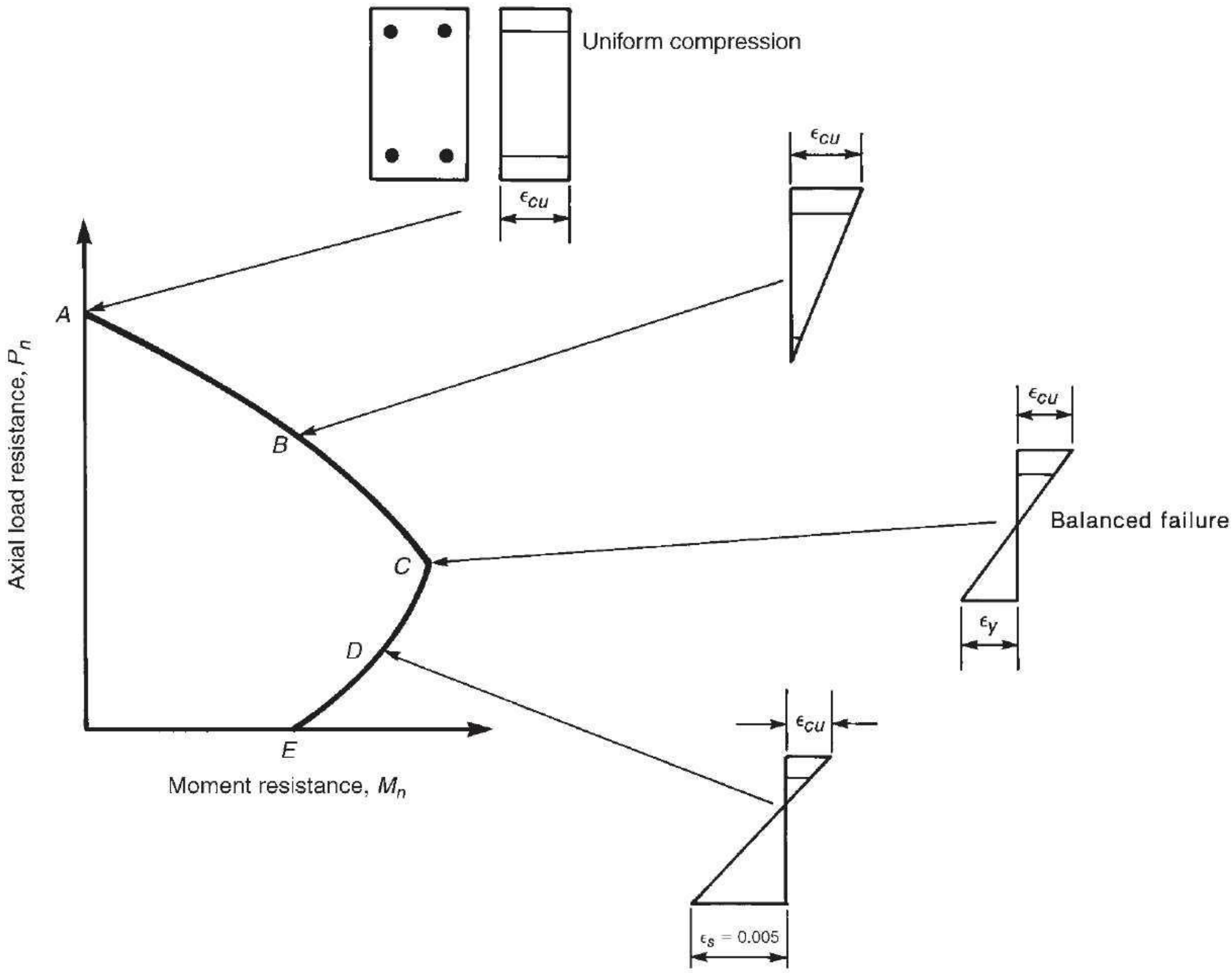


Figure 9.5-8: Strain distributions corresponding to points on the interaction diagram.

- Column design strengths ($\emptyset P_n$, $\emptyset M_n$) can be obtained by multiplied the nominal strengths (P_n, M_n) by the corresponding factor of safety "$\emptyset$" to obtained the *Design Interaction Diagram* and as shown ***Figure 9.5-9*** below.

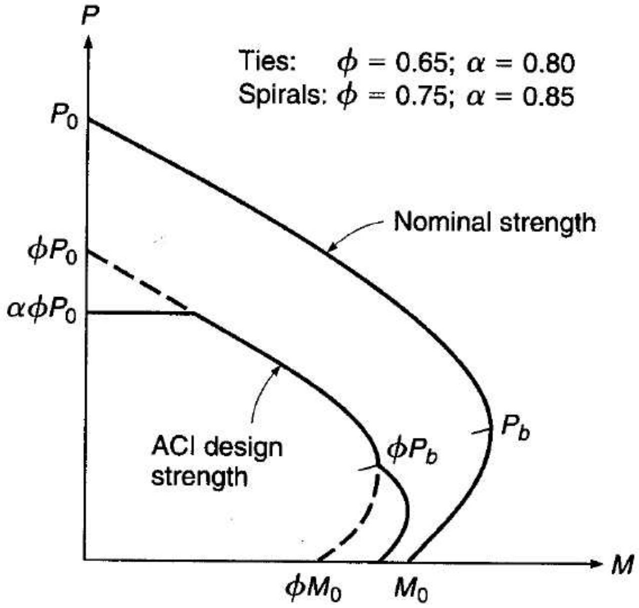


Figure 9.5-9: ACI safety provisions superimposed on column interaction diagram.

9.5.3.4 Notes on Design Interaction Diagram

- For high eccentricities, as the eccentricity increases to infinity (pure: bending), the ACI Code recognizes that the member behaves progressively more like a flexural member and less like a column. This is acknowledged in ACI Code by providing a linear transition in $\emptyset$ from values of 0.65 (for tied column) and 0.75 (for spiral column) to 0.90 (for beam) as the net tensile strain in the extreme tensile steel ϵ_s increases from 0.002 for Grade 60 reinforcement to 0.005.
- At the other extreme, for columns with very small or zero calculated eccentricities, the. ACI Code recognizes that accidental construction misalignments and other unforeseen factors may produce actual eccentricities in excess of these small design values. Therefore, regardless of the magnitude of the calculated eccentricity, ACI Code limits the maximum design strength to $0.80\ \emptyset P_{nmax}$, for tied columns and to $0.85\emptyset\ P_{nmax}$ for spirally reinforced.

9.5.3.5 A Set of Design Interaction Curves

Our textbook (Design of Concrete Structures, 15th Edition, by David Darwin, Charles W. Dolan, and A. H. Nilson) includes the group of useful ***Design Interaction Diagrams***.

Example 9.5-1

Check the adequacy of column shown below for general ACI requirement then use an appropriate interaction diagram to find its design axial load and design bending moment.

Use $f_c^{'} = 28\,MPa$ and $f_y = 420\,MPa$. $A_{Bar\ of\ 29mm} = 645mm$.

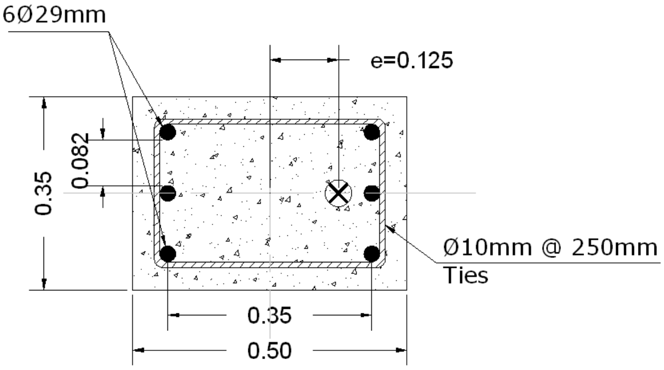


Figure 9.5-10: Proposed column section for Example 9.5-1.

Solution

- The procedure for analysis of an eccentrically loaded column is exactly similar to the procedure of a concentrically loaded column in all steps except in the computing of design axial force and bending moment ($\emptyset P_n, \emptyset M_n$).
- Longitudinal reinforcement:
  Check ρ_g within acceptable limits:
  $A_g = 500 \times 350 = 175\,000\,mm^2$
  $A_{st} = 645 \times 6 = 3\,870\,mm^2$
  $0.01 < \rho_g = \frac{3\,870}{175\,000} = 2.2\% < 0.08$
  Check minimum number of longitudinal bars:
  $6 > 4 \quad \therefore Ok.$
  Check minimum distance between longitudinal bars:
  $S_{Minimum} = Maximum[1.5 \times 29^{mm}, 40^{mm}]$
  $S_{Minimum} = 43.5^{mm} < 82^{mm} \quad \therefore Ok.$
- Calculate the design axial load strength and bending moment for given eccentricity *($\emptyset P_n, \emptyset M_n$)*:
  $\gamma = \frac{350}{500} = 0.7$
  Based on γ value and as the reinforcements are distributed on two faces of the rectangular column, then the interaction diagram that will be used is as shown in Figure below.
  For
  $\frac{e}{h} = \frac{125}{500} = 0.25$
  the R_n for the interaction diagram will be:
  $R_n = \frac{P_n.e}{f_c' A_g h} = 0.17$
  $M_n = P_n.e = 0.17 \times 28 \times (500 \times 350) \times 500 = 417\ \mathrm{kN.m}$
  As we working with compression controlled section (i.e. with a section has $\epsilon_t < 0.002$) then the strength reduction factor is $\emptyset = 0.65$
  $\emptyset M_n = 0.65 \times 417\ \mathrm{kN.m} = 271\ \mathrm{kN.m}$ ■
  and the K_n for the interaction diagram will be:
  $K_n = 0.69 = \frac{P_n}{f_c' A_g}$
  $P_n = 0.69 \times 28 \times 500 \times 350 = 3\,381\ \mathrm{kN}$
  $\emptyset P_n = 0.65 \times 3\,381\ \mathrm{kN} = 2\,198\ \mathrm{kN}$ ■

- Lateral reinforcement (Ties):
  Ties diameter:
  $\because\ \emptyset = 29^{\mathrm{mm}} < 32^{\mathrm{mm}}, \therefore$ we can use $\emptyset = 10^{\mathrm{mm}}$ for ties
  Ties spacing:
  $S_{\mathrm{Maximum}} = \min[16 \times 29^{\mathrm{mm}}, 48 \times 10^{\mathrm{mm}}, 350^{\mathrm{mm}}] = 350^{\mathrm{mm}} > S_{\mathrm{Provided}}\ \therefore \mathrm{Ok}.$
  Ties arrangement:
  $\because S_{\mathrm{Spacing\ between\ longitudinal\ bars}} < 150^{\mathrm{mm}}$
  Then, alternate longitudinal bars will be supported by corner bars.

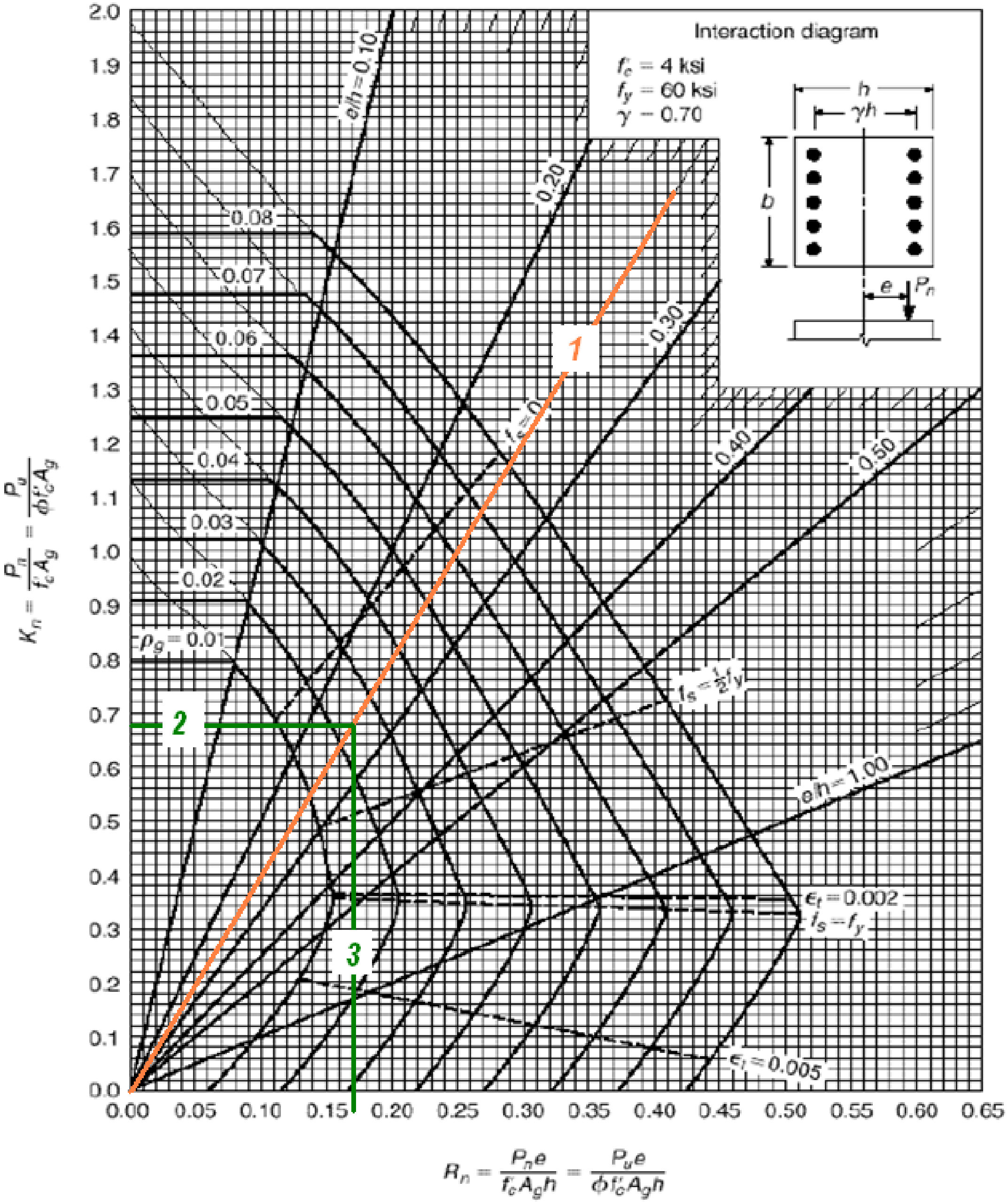


Figure 9.5-11: Adopting interaction diagram to computed design strength for the column of Example 9.5-1.

Example 9.5-2

In ***Example 9.2-1*** above, it was required to check the column shown in Figure below to general requirements of ACI code and to compute its design strength. Material properties where $f_c' = 27.5\,\text{MPa}$, and $f_y = 420\text{MPa}$. Resolve this example based on interaction diagram instead of equations for axially loaded columns.

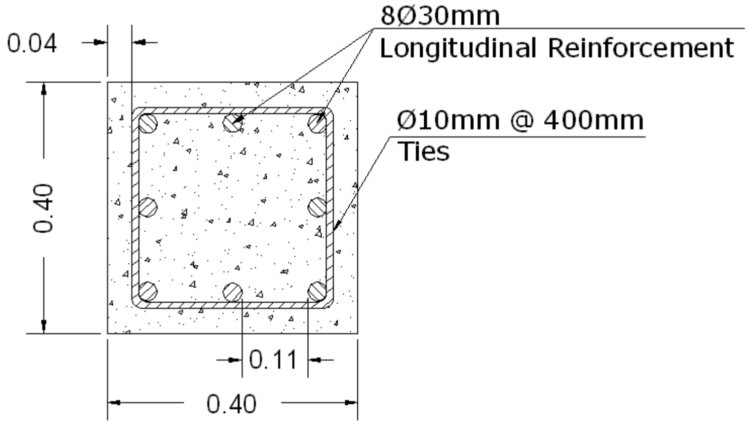


Figure 9.2-5: Proposed tied column for Example 9.2-1. Reproduce for convenience.

Solution

<u>Checking for General Requirements</u>

General requirements of ACI code are nominal in nature and do not related to use of interaction diagram or use equation in computing of column design strength:

<u>Longitudinal reinforcement</u>

Check ρ_g within acceptable limits:

$$A_g = 400^2 = 160\ \ 000\, mm^2$$

$$A_{st} = \frac{\pi \times 30^2}{4} \times 8 = 5\ \ 652\, mm^2$$

$$0.01 < \rho_g = \frac{5\ 652}{160\ 000} = 3.53\% < 0.08$$

Check minimum number of longitudinal bars:

$8 > 4 \quad \therefore Ok.$

Check minimum distance between longitudinal bars:

$S_{Minimum} = Maximum[1.5 \times 30^{mm}, 40^{mm}]$

$S_{Minimum} = 45^{mm} < 110^{mm} \;\therefore Ok.$

<u>Lateral reinforcement (Ties)</u>

Checking of Lateral Reinforcement (Ties):

Ties diameter:

$\because\ \emptyset = 30^{\text{mm}} < 32^{\text{mm}}, \therefore$ we can use $\emptyset = 10^{\text{mm}}$ for ties

Ties spacing:

$S_{\text{Maximum}} = \min[16 \times 30^{\text{mm}}, 48 \times 10^{\text{mm}}, 400^{\text{mm}}] = 400^{\text{mm}} = S_{\text{Provided}} \;\therefore \text{Ok.}$

Ties arrangement:

$\because S_{\text{Spacing between longitudinal bars}} < 150^{\text{mm}}$

Then, alternate longitudinal bars are supported.

<u>Column Design Strength</u>

Strength of axially loaded columns are not related to whether reinforcement are distributed on two faces or on four faces nor related to γ value. To emphasize this fact, two extremes interaction diagrams, the first one for reinforcement distributed on four faces and with γ value of 0.6 while the other with reinforcement on two faces and with γ value of 0.9, have been compared in below.

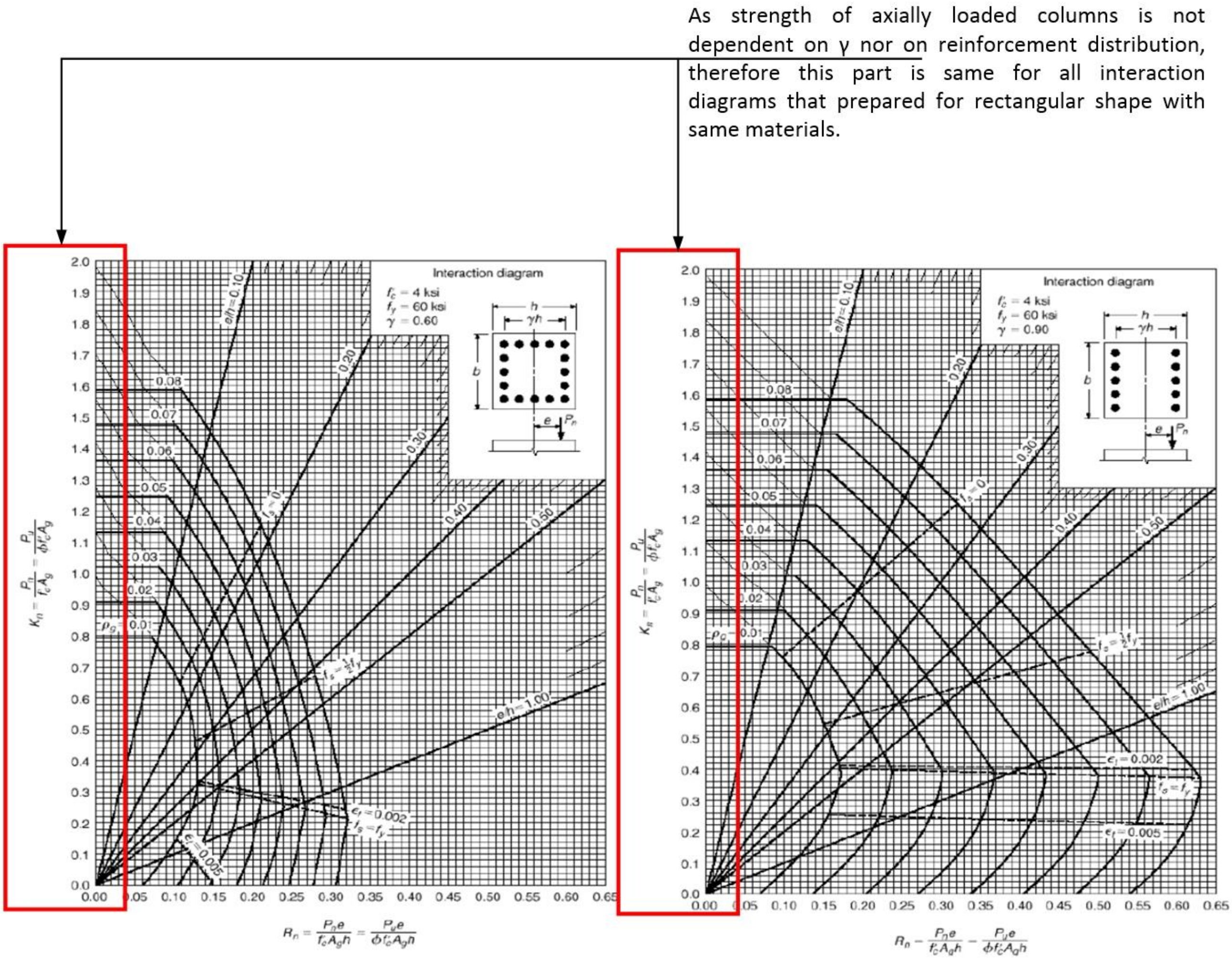


Figure 9.5-12: Two interaction diagrams that are equally applicable to solve the axially loaded column of Example 9.5-2.

Adopting any one of interaction diagrams for rectangular columns with $f_c' = 4\,ksi$ and $f_y = 28\,MPa$ will leads to:

For

$$\rho_g = \frac{5\,652}{160\,000} = 3.53\%$$

$$K_n = \frac{P_n}{f_c' A_g} \approx 1.06$$

$$P_n = \frac{(1.06 \times 27.5 \times 400 \times 400)}{1000} = 4664\,kN$$

For compression control region and with tied columns:

$$\phi = 0.65$$

$$\phi P_n = 0.65 \times 4664 = 3032\,kN$$

This design strength is close to that computed based on equations in Example 9.2-1, $\emptyset P_{nMaximum} = 3110\,kN$

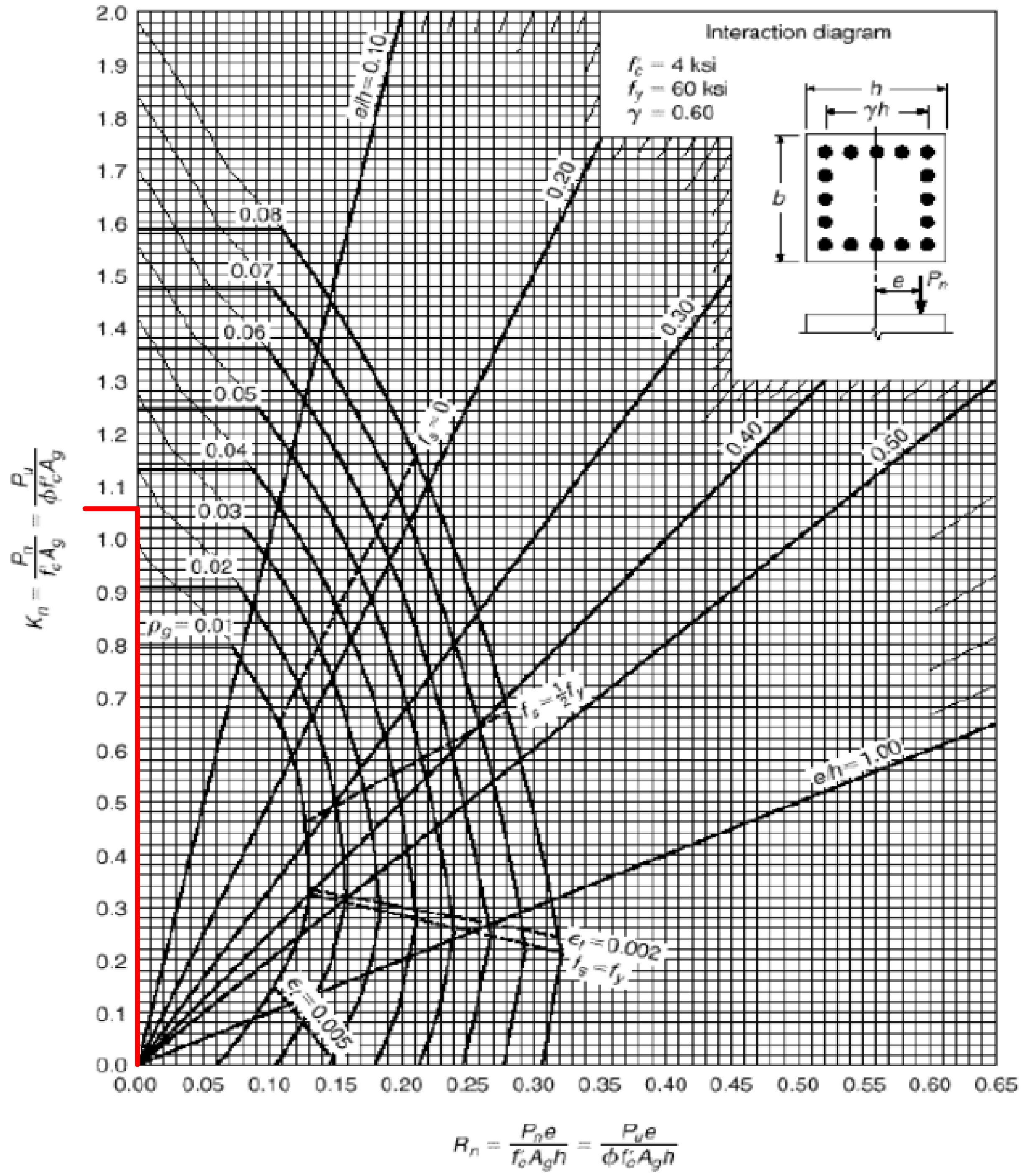


Figure 9.5-13: Sample interaction diagram adopted to solve the axially loaded column of Example 9.5-2.

Example 9.5-3

In ***Example 9.2-2*** above, it was required to check the column shown in ***Figure 9.2-6*** to general requirements of ACI code and then determine whether this column is adequate to carry a factored load of P_u= 2250 kN. Material properties where $f_c' = 27.5\,\mathrm{MPa}$, and $f_y = 420\mathrm{MPa}$. Resolve this example based on interaction diagram instead of equations for axially loaded columns.

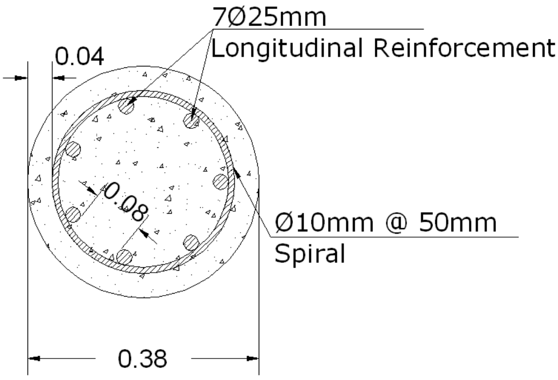


Figure 9.2-6: Spiral column of Example 9.2-2. Reproduce for convenience.

Solution

<u>Checking for General Requirements of ACI Code</u>

<u>Longitudinal reinforcement</u>

Check ρ_g within acceptable limits:

$$A_g = \frac{\pi \times 380^2}{4} = 113\ \ 354\ mm^2$$

$$A_{st} = \frac{\pi \times 25^2}{4} \times 7 = 3\ 434\ mm^2$$

$$\rho_g = \frac{3\ \ 434}{113\ \ 354} = 3.0\%$$

$0.01 < \rho_g < 0.08\ \therefore Ok.$

Check minimum number of longitudinal bars

$7 > 6\ \ \therefore Ok.$

Check minimum distance between longitudinal bars

$S_{Minimum} = Maximum[1.5 \times 25^{mm}, 40^{mm}]$

$S_{Minimum} = 40.0^{mm} < 80^{mm}\ \therefore Ok.$

<u>Check the lateral reinforcement (Spiral):</u>

Check Spiral Diameter:

$\phi_{Spiral} = 10mm\ Ok.$

Check Spiral Steel Ratio:

$$A_{sp} = \frac{\pi \times 10^2}{4} = 78.5^{mm^2}$$

$$\rho_{sProvided} = \frac{4 \times 78.5^{mm^2}}{(380 - 2 \times 40)^{mm} \times 50^{mm}} = 0.0209$$

$$\rho_{sMinimum} = 0.45 \times \left(\frac{113\ \ 354}{\frac{\pi \times 300^2}{4}} - 1\right) \times \frac{27.5}{420} = 0.0178 < 0.0209\ \therefore Ok.$$

Check the Clear Spacing:

$25^{mm} < [S_{Clear\ Provided} = 50^{mm} - 10^{mm} = 40^{mm}] < 80^{mm}\ \therefore Ok.$

<u>Axial Design Strength of the Column</u>

With any of interaction diagrams for circular columns,

With

$$\rho_g = \frac{3\ \ 434}{113\ \ 354} = 3.0\%$$

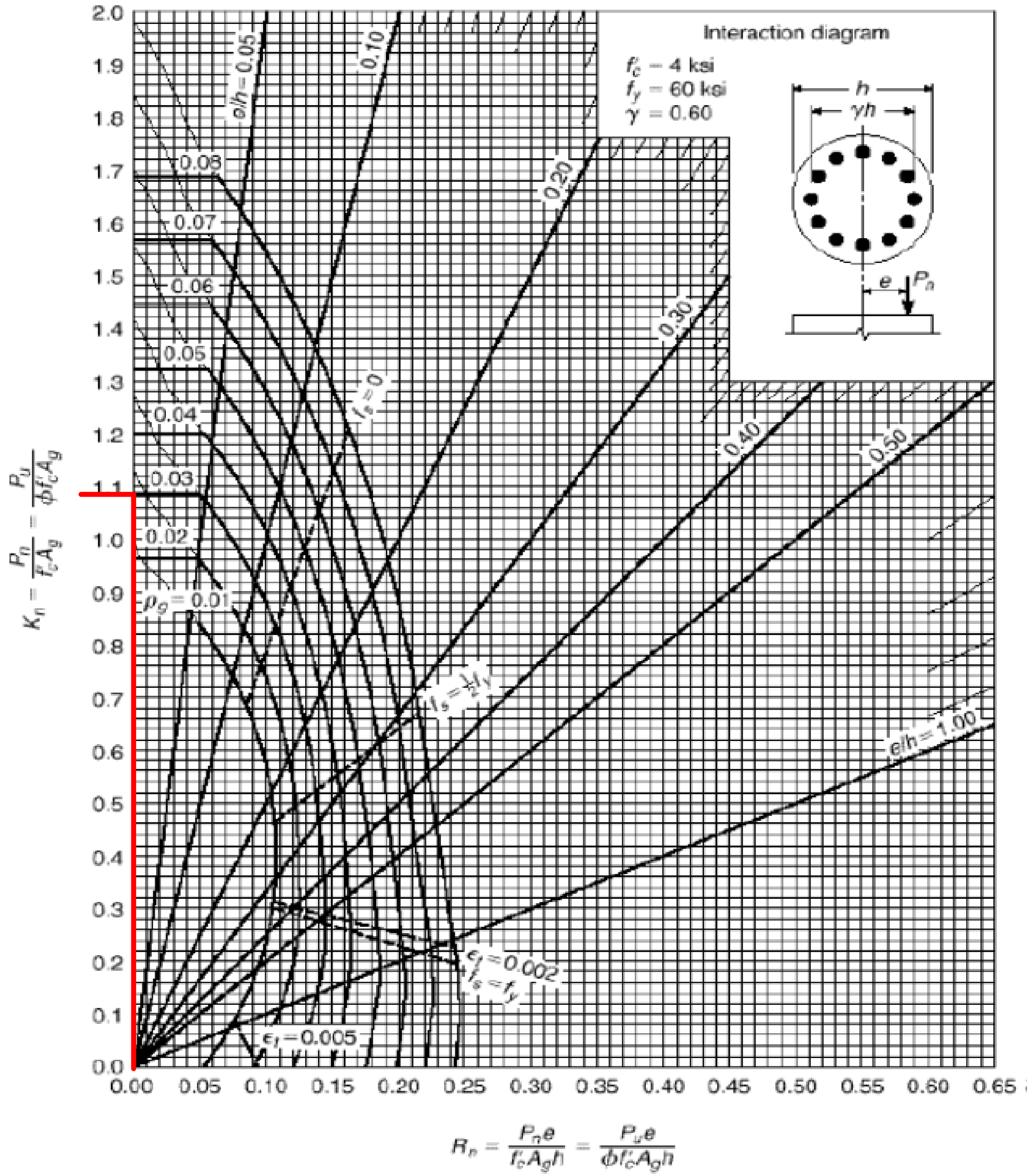


Figure 9.5-14: A sample interaction diagram adopted to solve the Example 9.5-3.

$$K_n = \frac{P_n}{f'_c A_g} \approx 1.1$$

$$P_n = \frac{\left(1.1 \times 27.5 \times \left(\frac{\pi \times 380^2}{4}\right)\right)}{1000} = 3431\ kN$$

As axially loaded columns are located in compression-controlled regions, therefore ϕ of 0.75 is adopted for this spiral column:

$$\phi P_n = 0.75 \times 3431 = 2573\ kN$$

This value is close to that of 2557 kN which computed based on relations for axially loaded columns in ***Example 9.2-2***. The proposed column is adequate as:

$$\phi P_n = 2573\ kN > P_u = 2250\ \text{kN}$$

9.6 DESIGN OF A COLUMN WITH COMPRESSION LOAD PLUS UNIAXIAL MOMENT

9.6.1 General Guides for Columns Design

- The following guides that related to columns design have been proposed by ***J. G. MacGregor*** in his book "***Reinforced Concrete: Mechanics and Design, 4th Edition***):
- Type of Column:
  - For eccentricity, e/h, greater than 0.2, a tied column with bars in the faces farthest from axis of bending is most efficient. Even more efficiency can be obtained by using of a rectangular column.
  - Tied columns with bars in four faces are used for e/h ratios of less than about 0.2 and also when moments exist about both axes. ***Many designers prefer this arrangement because there is less possibility of construction error in the field if there are equal numbers of rebars in each face of the column***.
  - Spiral columns are relatively infrequent in non-seismic areas. In seismic areas or in other situations where ductility is important, spiral columns are used frequently.
- Estimating the Column Size:
  - The initial stage in column design involves estimating the required size of column. There is no simple rule for doing this, since the axial-load capacity of a given cross section varies with the moment acting on section. For very small moments following relations can be used (these relations similar to that derived in ***Article 9.3***):
  - For Tied Columns:

    $$A_{gTrail} \geq \frac{P_U}{0.4[f_c' + f_y \rho_g]}$$
  - For spiral column:

    $$A_{gTrail} \geq \frac{P_U}{0.5[f_c' + f_y \rho_g]}$$
  - Both of these relations will tend to underestimate the column size if there are moments present.
- Column Thickness "b":
  - The Fire Codes usually specified minimum column size as follows:

Table 9.6-1: Minimum column thickness for fire rating requirements, adopted from

Fire Rating (hours)	*Minimum Column Thickness (mm)*
1 hour	$b \geq 225\ mm$
2-3 hours	$b \geq 300\ mm$

  - Although the ACI Code does not specify a minimum column size, ***the minimum dimension of cast-in-place tie column should not be less than 200mm and preferably not less than 250mm***.
  - ***The diameter of a spiral column should not be less than about 300mm***.

9.6.2 Using Interaction Charts in Design Process

Conventional design charts permit the direct design of eccentrically loaded columns throughout the common range of strength and geometric variables. They may be used in one of two ways as presented in ***Article 9.6.2.1*** and ***Article 0*** below.

9.6.2.1 Selection of Reinforcement for Column of Given Size

For a given factored load $\mathrm{P_u}$ and equivalent eccentricity $\mathrm{e} = \frac{\mathrm{M_u}}{\mathrm{P_u}}$ and given cross section this direct procedure can be summarized as follows:

<u>Design of Longitudinal Reinforcement:</u>

- Calculate the ratio γ based on required cover distances to the bar centroid, and select the corresponding column design chart.
- Calculate $K_n = \frac{\mathrm{P_u}}{\emptyset \mathrm{f_c' A_g}}$ and $R_n = \frac{P_u e}{\emptyset f_c' A_g h}$ where $\mathrm{A_g}$ is section gross area.
- Strength reduction value is selected based on type of section (i.e. is the member a compression controlled member or a tension controlled member or in the transition region).
- From the graph, for the values found in above, read the required reinforcement ratio ρ_g.
- Calculate the total steel area $\mathrm{A_{st}}$.
- Compute the required number of longitudinal bars:
  $\text{No. of Longitudinal Bars} = \frac{\mathrm{A_{st}}}{\mathrm{A_{Bar}}}$
- The limitations on the number and arrangement of longitudinal bars are as discussed in the design of columns for axial loads.

<u>Design of Lateral Reinforcement</u>

Design of lateral reinforcement is exactly as discussed in the design of columns for axial loads, ***Article 9.3***. For convenience, these procedures have been represented in below:

Ties:

- Select ties diameter:
- If $\varphi_{Longitudinal} \leq 32^{mm}$ then:
  $\phi_{Ties} = 10^{mm}$
  Else
  $\phi_{Ties} = 13^{mm}$
- Select ties spacing:
  $S_{Required} \leq Minimum[16\varphi_{Bar}\ , 48\varphi_{ties}, Least\ Column\ Dimensions]$
- Arrange the ties according to requirements of the ACI for maximum spacing between longitudinal bars (use the standard arrangements of ***Figure 9.2-2*** above).

Spiral:

- Spiral Diameter
  $\phi_{Spiral} \geq 10^{mm}$
- Compute $\rho_{sMinimum}$
  $\rho_{sMinimum} = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f_c'}{f_{sy}}$
  Let $\rho_s = \rho_{sMinimum}$ to compute the required $S_{Required}$:
  $S_{Required} = \frac{4A_{sp}}{D_c\ \rho_{sMinimum}}$
- Check with Limitation for Clear Spacing
  The clear spacing $S_{Required\ Clear}$ between turns of the spiral must be:
  $25 \leq S_{Clear}\ \leq 80^{mm}$

Example 9.6-1

In a two-story building that shown in ***Figure 9.6-1*** below an exterior column is to be designed for the following loading:

- First Load Pattern:
  $P_{Dead} = 987\ kN$
  $P_{Live} = 1\ 481\ kN$
  $M_{Dead} = 220\ kN.m$
  $M_{Live} = 315\ kN.m$
- Second Load Pattern:
  $P_{Dead} = 987\ kN$
  $P_{Live} = 738\ kN$
  $M_{Dead} = 220\ kN.m$
  $M_{Live} = 315\ kN.m$

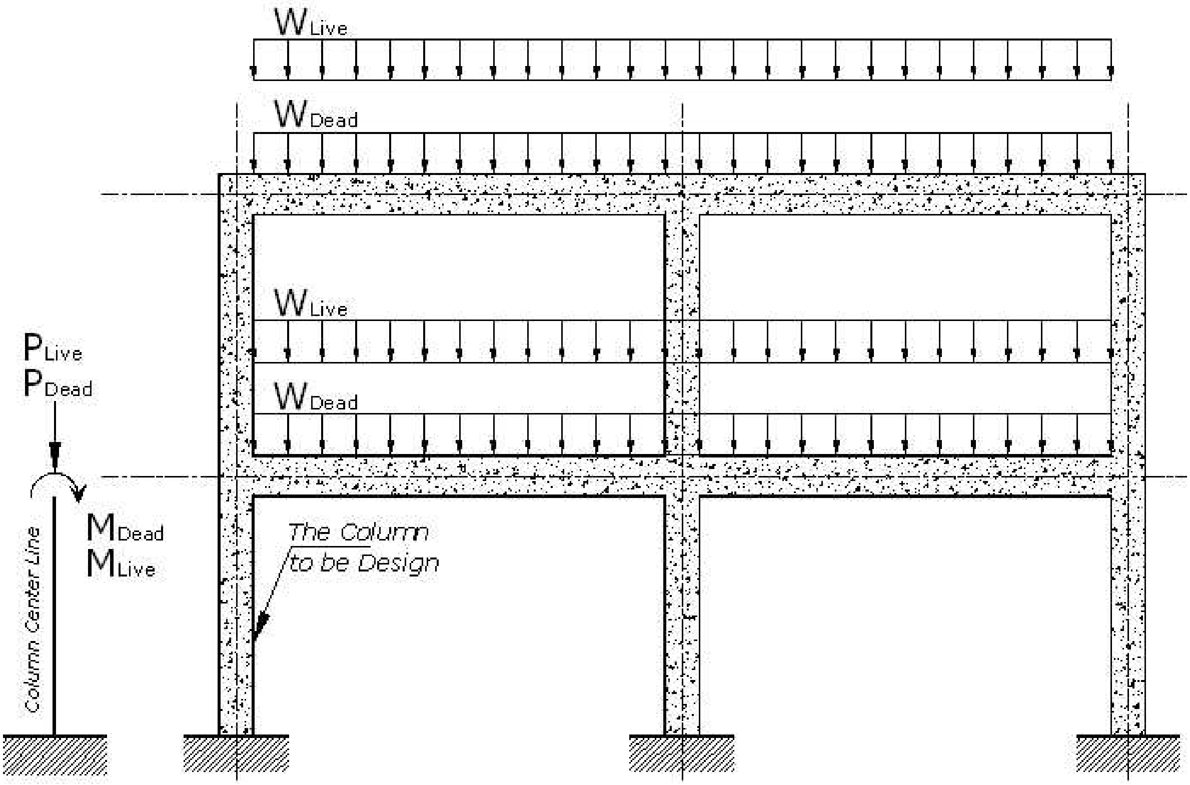
Architectural considerations required that a rectangular column to be used, with dimensions:
$b = 500^{mm}$ and $h = 625^{mm}$

Materials:
$f_c' = 28\ MPa$
$f_y = 420\ MPa$

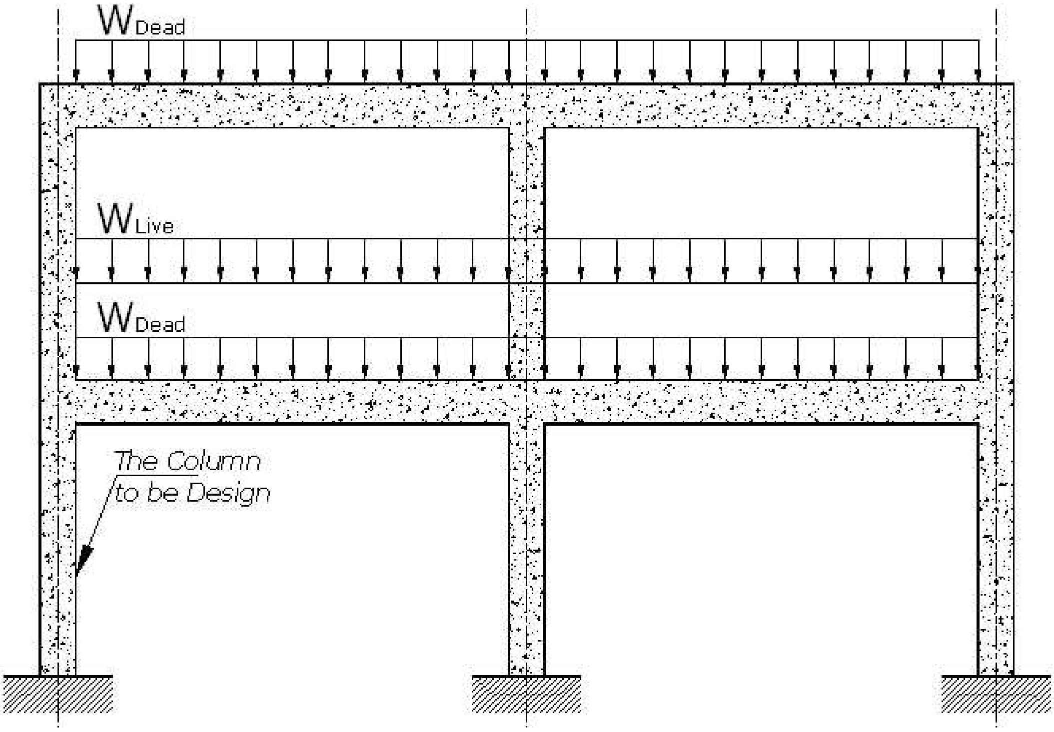
Reinforcement:
Try $\varphi = 32^{mm}$ for longitudinal reinforcement ($A_{Bar} = 819\ mm^2$).
Try $\varphi = 10^{mm}$ for lateral reinforcement.

Based on above data

- Design the column for first load pattern.
- Check to ensure that the column is adequate for the second load pattern.



First Load Pattern
Figure 9.6-1: Building and the edge column for the Example 9.6-1.



Second Load Pattern
Figure 9.6-1: Building and the edge column for the Example 9.6-1. Continued.

Solution

Design of Column for First Load Pattern

The column will be designed initially for full load, then it would be checked for adequacy when live load is partially removed.

According to the ACI safety provisions, the column must be designed for a factored load:

$P_{u\ Maximum} = 1.2 \times 987 + 1.6 \times 1\,481 = 3\,554\ \mathrm{kN}$

$M_u = 1.2 \times 220 + 1.6 \times 315 = 768\ kN.m$

Design of Longitudinal Reinforcement:

- Calculate the ratio γ based on required cover distances to the bar centroid, and select the corresponding column design chart.

  $\gamma h = 625 - 2 \times 40 - 2 \times 10 - 32 = 493\ mm$

  $$\gamma = \frac{\gamma h}{h} = \frac{493}{625} = 0.79$$

  Say $\gamma = 0.80$ and assume that the reinforcement will be distributed on four faces. Then the interaction diagram that used in the design is that shown in ***Figure 9.6-2*** below.

- Calculate $K_n = \frac{P_u}{\emptyset f'_c A_g}$ and $R_n = \frac{P_u e}{\emptyset f'_c A_g h}$:

$$e = \frac{M_u}{P_u} = \frac{768\ kN.m}{3\,554\ \mathrm{kN}} = 0.216\ m$$

$$\frac{e}{h} = \frac{0.216m}{0.625m} = 0.35$$

- Based on $\frac{e}{h}$ ratio, one can see that the tensile strain for this column under proposed loads is less than 0.002. Therefore the section is compression controlled section and strength reduction factor is $\emptyset = 0.65$.

$$K_n = \frac{P_u}{\emptyset f'_c A_g} = \frac{3\,554\,000\ \mathrm{N}}{0.65 \times 28 \frac{\mathrm{N}}{\mathrm{mm}^2} \times (625 \times 500)\mathrm{mm}^2} = 0.625$$

$$R_n = \frac{P_u e}{\emptyset f'_c A_g h} = \frac{768\ \times 10^6\ N.mm}{0.65 \times 28 \frac{N}{mm^2} \times (625 \times 500)\mathrm{mm}^2 \times 625mm} = 0.216$$

- From the graph, the required reinforcement ratio, ρ_g, would be:
  $\rho_g = 0.04$
- Calculate the total steel area A_{st}:
  $A_{st} = 0.04 \times 625 \times 500 = 12\,500\ mm^2$
- Compute the required number of longitudinal bars:
  $$\text{No. of Longitudinal Bars} = \frac{A_{st}}{A_{Bar}} = \frac{12\,500\ \text{mm}^2}{819\ \text{mm}^2} = 15.3$$
  Try 16 $\emptyset$ 32.

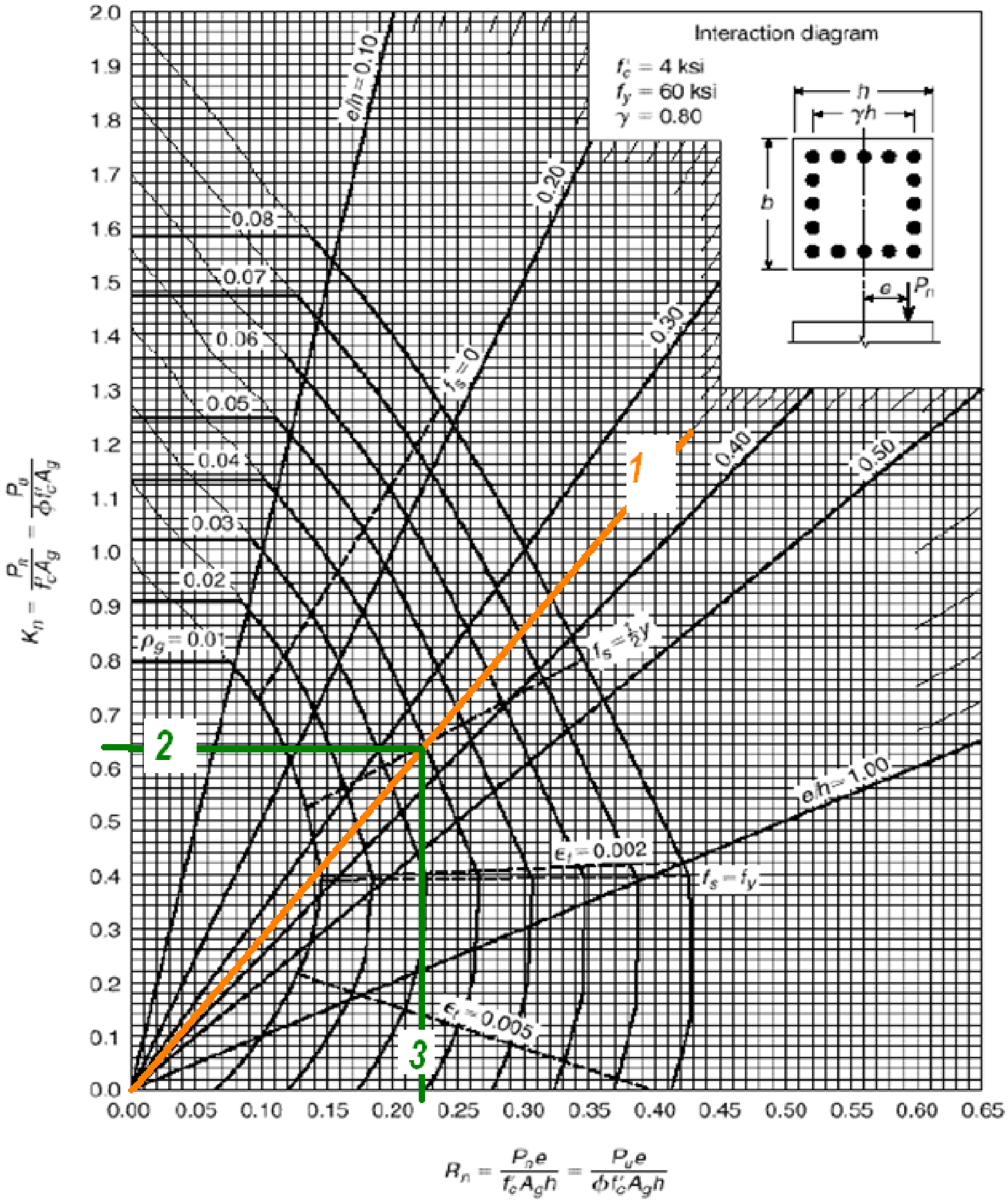


Figure 9.6-2: Interaction diagram for Example 9.6-1, first load pattern.

<u>Design of Lateral Reinforcement:</u>

- Ties diameter:
  $\because\ \emptyset = 32^{\text{mm}}, \therefore$ we can use $\emptyset = 10^{\text{mm}}$ for ties
- Ties spacing:
  $S_{\text{Required}} = \min[16 \times 32^{\text{mm}}, 48 \times 10^{\text{mm}}, 500^{\text{mm}}] = 480^{\text{mm}}$
  Use $\emptyset 10$mm @ 475 mm
- Ties arrangement:
  The following arrangement can be used for our column:

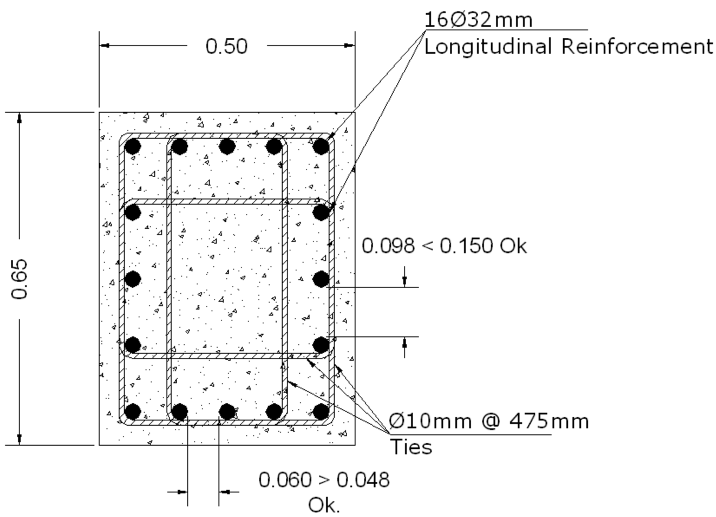


Figure 9.6-3: Design section for the column of Example 9.6-1 based on first load pattern.

Column Checking for Second Load Pattern

Aim of Checking for the Second Load Pattern:

- Before starting the checking, it is useful to discuss the aim of this checking.
- At first sight this checking seems unnecessary as the column that designed with live load acting on all floors and roofs of course will be adequate when live loads acting on the floor under consideration only.
- Unfortunately, the problem is not so simple as appear, i.e. some columns that are adequate for full live loads may be not adequate for partially live load, this strange fact can be explained as follows:
  - Assume that required reinforcement has been selected based on full live loads as was done in previous article and assume that load path for dead and full live loads will be as shown in Figure below.
  - As the axial force in a column resulting from accumulation of loads acting on the floor under consideration and on above floors and roof, then removing live loads from above floors and roof will decrease the axial force in that column.
  - For gravity loads, bending moments in a column are mainly resulting from negative moments of beams that connected directly to the column, the removing of live loads from above floors and roof does not change the bending moments in the column. Based on this reasoning, bending moments have been assumed the same in first and second load patterns.
- Then with second load pattern, load path will move vertically in downward direction (as we have negative ΔP and zero ΔM). With this movement, load case that was inside the interaction diagram may move to be outside it. Therefore, the section that was pass under full live load may fail under partial live load!

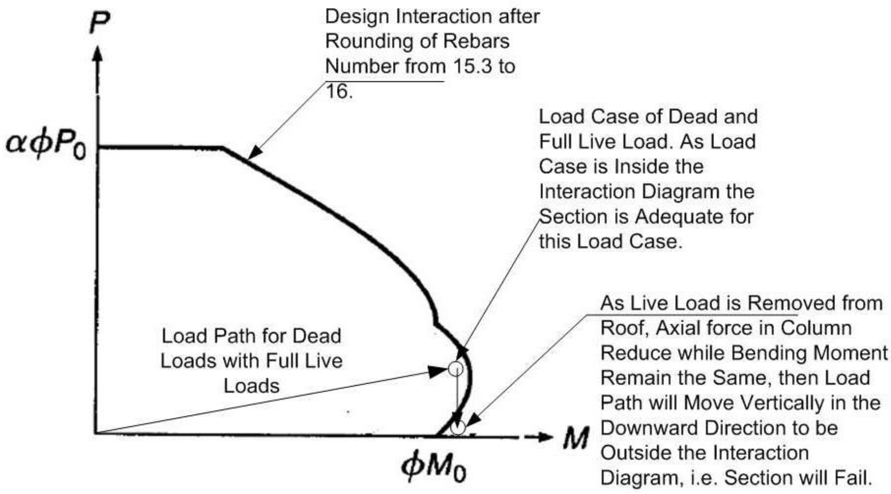


Figure 9.6-4: Schematic integration diagram to show the aim of checking for the second pattern.

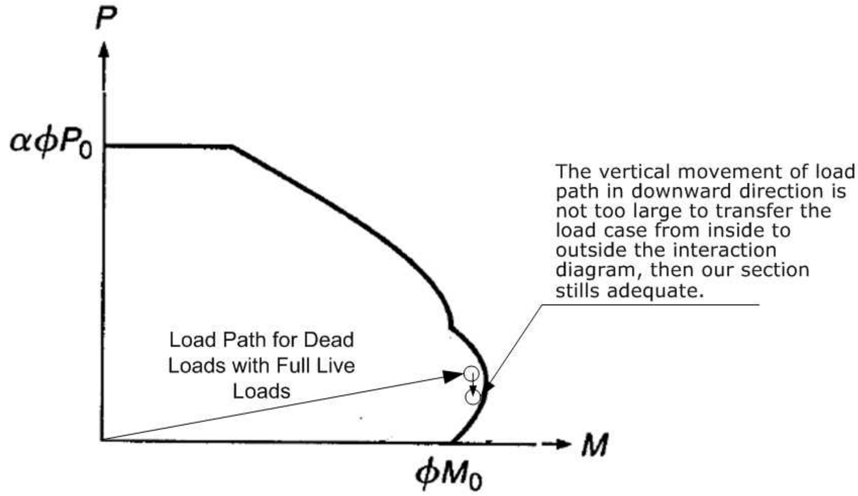


Figure 9.6-4: Schematic integration diagram to show the aim of checking for the second pattern. Continued.

<u>Checking Details:</u>

- Check to ensure that the column is adequate for the second load pattern:
  $P_{u\ Minimum} = 1.2 \times 987 + 1.6 \times 738 = 2\ 365\ kN \qquad M_u = 768\ kN.m$
  Say $\gamma = 0.80$
  $e = \frac{M_u}{P_u} = \frac{768\ kN.m}{2\ 365\ \text{kN}} = 0.325\ m \quad \frac{e}{h} = \frac{0.325\ m}{0.625 m} = 0.52$
  $\text{K}_\text{n} = \frac{\text{P}_\text{u}}{\text{Ø}\text{f}'_\text{c}\text{A}_\text{g}} = \frac{2\ 365\ \ 000\ \text{N}}{0.65 \times 28 \frac{\text{N}}{\text{mm}^2} \times (625 \times 500)\text{mm}^2} = 0.416$
  $\text{R}_\text{n} = \frac{\text{P}_\text{u}\text{e}}{\text{Ø}\text{f}'_\text{c}\text{A}_\text{g}\text{h}} = \frac{768\ \times 10^6\ N.mm}{0.65 \times 28 \frac{N}{mm^2} \times (625 \times 500)\text{mm}^2 \times 625 mm} = 0.216$
- From ***Figure 9.6-5*** with $\rho_{\text{g Required}} = 0.028 < \rho_{\text{Provided}}$, one concludes that vertical movement of load path in downward direction is not too large to transfer the load case from inside to outside the interaction diagram, then the section stills adequate.

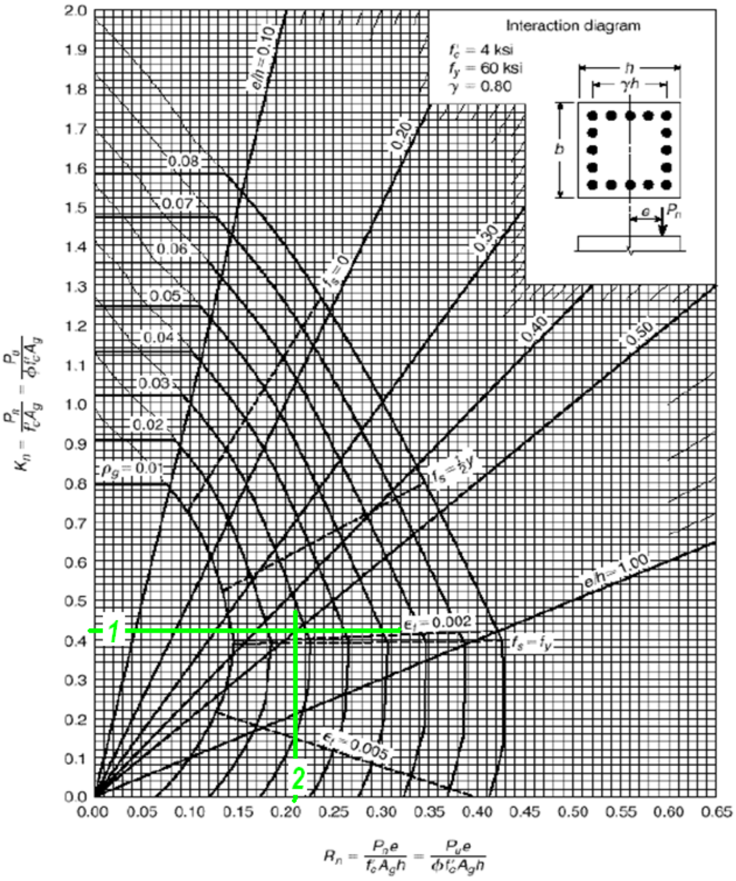


Figure 9.6-5: Interaction diagram to check the second load pattern for the column of Example 9.6-1.

9.6.2.2 Selecting of Column Size for a Given Reinforcement Ratio

Example 9.6-2

A column is to be designed to carry factored loads of:

$P_u = 2\,139\ kN$

$M_u = 667\ kN.$

Assume that:

- Bending moment about major or strong axis.
- Material strengths $f_y = 420MPa$ and $f_c' = 28\ MPa$ are specified.
- Cost studies for the particular location indicate that a reinforcement ratio of about 0.03 is optimum.
- Column depth: h=625mm.
- $\emptyset = 36mm$ for longitudinal reinforcements ($A_{Bar} = 1\,006\ mm^2$).
- Steel with bars concentrated in two layers, adjacent to the outer faces of the column and parallel to the axis of beading, will be used.

Find the required column width "b" and design the longitudinal lateral reinforcements.

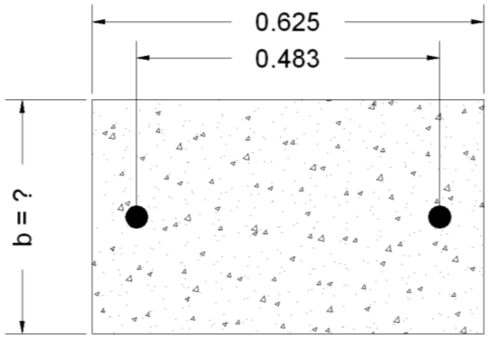


Figure 9.6-6: Proposed section for Example 9.6-2.

Solution

Column Width "b" and Design of Longitudinal Reinforcement

- Calculate the ratio γ based on required cover distances to the bar centroid, and select the corresponding column design chart.

  $\gamma h = 483\ mm$

  $$\gamma = \frac{\gamma h}{h} = \frac{483}{625} = 0.78$$

- Say $\gamma = 0.80$ and as steel is assumed to be concentrated in two layers, then the design interaction diagram will be as ***Figure 9.6-7*** below.

  As

  $$e = \frac{M_u}{P_u} = \frac{667}{2\,139} = 0.31$$

  and

  $$\frac{e}{h} = \frac{0.31}{0.625} = 0.496 \approx 0.5$$

  then (from Figure above)

  $$K_n = \frac{P_u}{\emptyset f_c' bh} = 0.51$$

- As we working in the compression controlled region, then the strength reduction factor ∅ is 0.65.

  $$b = \frac{2\,139\,000\ \mathrm{N}}{0.65 \times 28 \frac{\mathrm{N}}{\mathrm{mm}^2} \times 0.51 \times 625\mathrm{mm}} = 369\ \mathrm{mm}$$

  Use 375mm by 625mm section.

  $$A_{st\ Required} = 0.03 \times 625\mathrm{mm} \times 375\mathrm{mm} = 7\,031\ \mathrm{mm}^2$$

  $$\text{No. of Rebars} = \frac{7\,031\ \mathrm{mm}^2}{1\,006\ \mathrm{mm}^2} = 6.99$$

  Use $8\emptyset 36mm$ rebars.

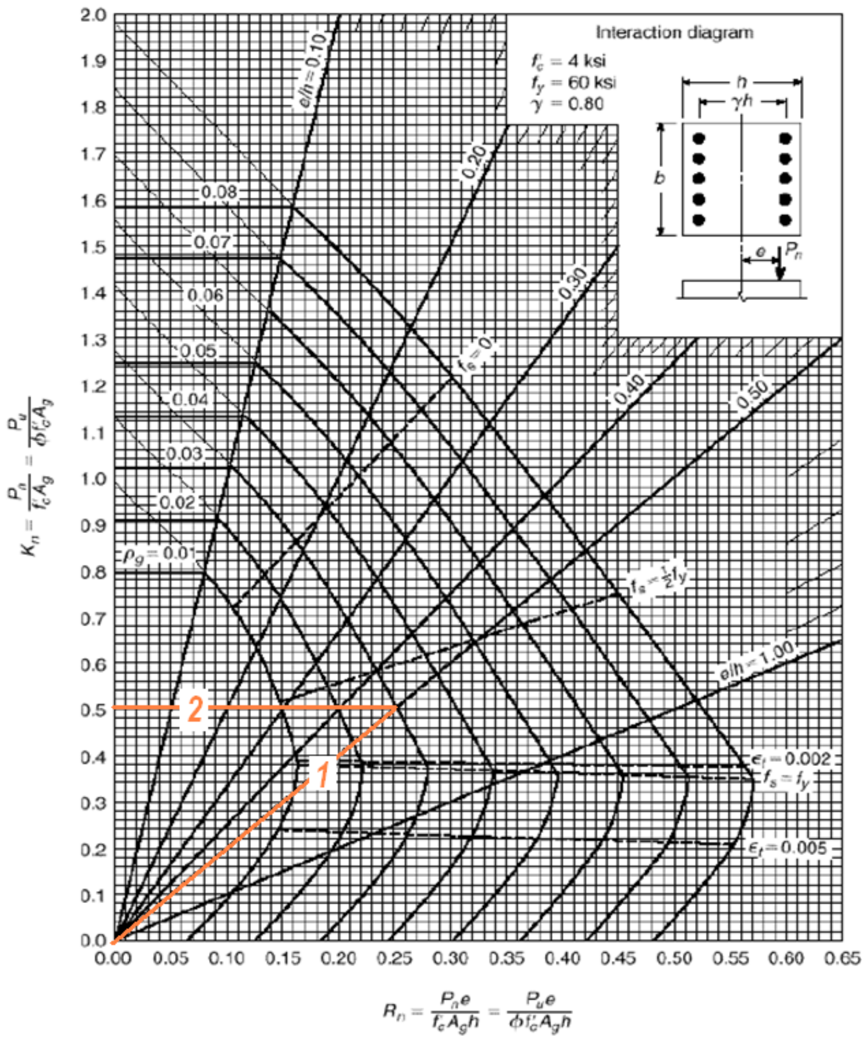


Figure 9.6-7: Interaction diagram to design the column of Example 9.6-2.

Design of Lateral Reinforcement

- Ties diameter:
  $\because\ \emptyset = 36^{\text{mm}} > 32^{\text{mm}}, \therefore$ we must use $\emptyset = 13^{\text{mm}}$ for ties
- Ties spacing:
  $S_{\text{Required}} = \min[16 \times 36^{\text{mm}}, 48 \times 13^{\text{mm}}, 375^{\text{mm}}] = 375^{\text{mm}}$, Use $\emptyset 13\text{mm}$ @ 375 mm
- Ties arrangement:
  As can be shown from ***Figure 9.6-8*** below, the proposed distribution does not satisfy the ACI Code requirements related to minimum spacing between longitudinal rebars. Then bundled rebars must be used in our design.

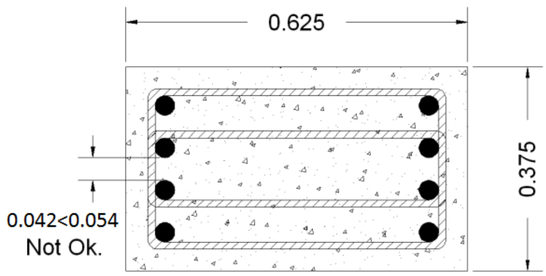


Figure 9.6-8: Final design section for the column of Example 9.6-2.

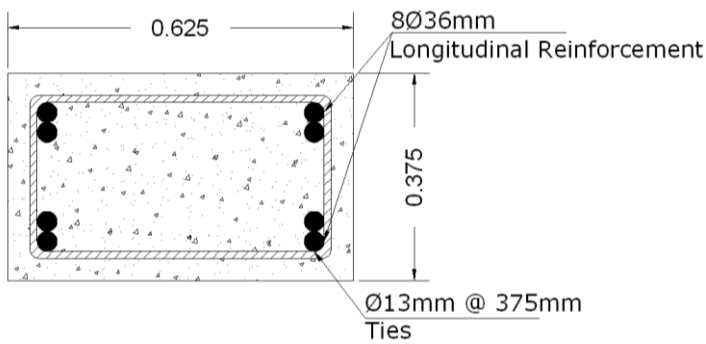


Figure 9.6-8: Final design section for the column of Example 9.6-2. Continued.

9.7 HOMEWORK PROBLEMS: ANALYSIS AND DESIGN OF A COLUMN UNDER AXIAL LOAD AND UNIAXIAL MOMENT

Problem 9.7-1

Using an appropriate interaction curve, determine the value of P_n for the short tied column shown in ***Figure 9.7-1*** below. Assume that $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.

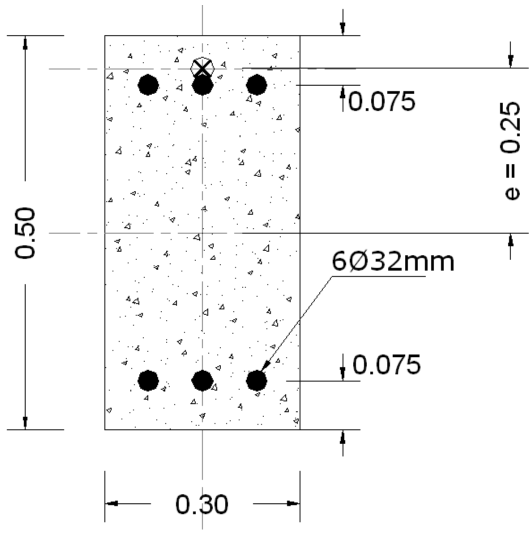


Figure 9.7-1: Column for Problem 9.7-1.

Answers

- Compute γ:
  $\gamma = 0.70$
- As the reinforcement is distributed along two faces only and as $\gamma = 0.07$, then use corresponding interaction diagram:
  $\because \frac{e}{h} = 0.50$
  $A_g = 150\ 000\ mm^2, A_{Bar} = 804\ mm^2, A_{st} = 4\ 824\ mm^2, \therefore\ \rho_g = 0.032$
  $K_n = \frac{P_n}{f_c{}'A_g} = \ 0.51 \Longrightarrow P_n = 2\ 142\ kN$ ■

Problem 9.7-2

For the column shown in ***Figure 9.7-2*** below, based on structural calculations a designer has proposed the attached section. Check the adequacy of this section to ACI Code requirements and to the applied load. Assume that selfweight can be neglected.

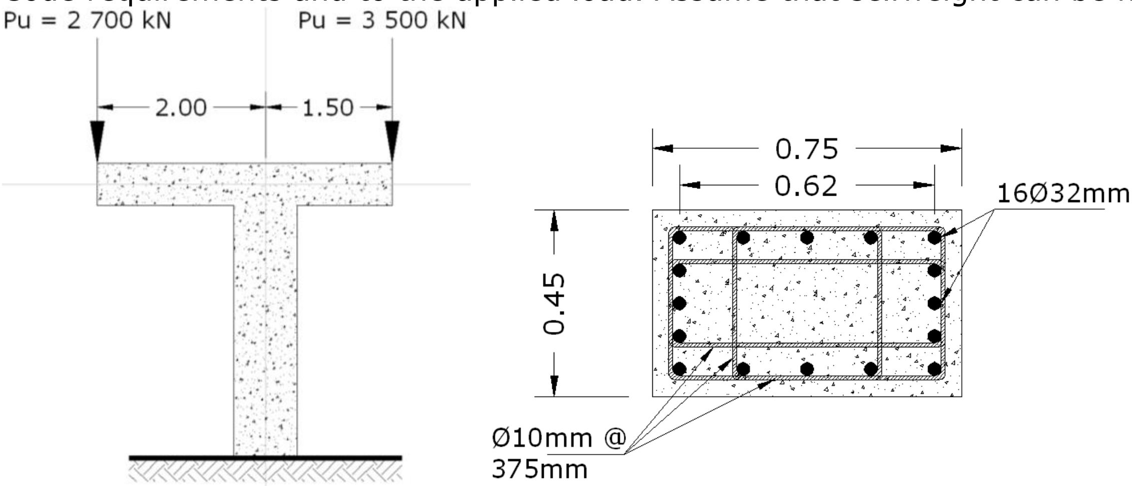


Figure 9.7-2: Frame and proposed column section for Problem 9.7-2.

Answers

<u>Checking of Longitudinal Reinforcement:</u>

- Check if ρ_g within acceptable limits:
  $A_g = 337\ 500\ mm^2, A_{st} = 12\ 864\ mm^2 \Longrightarrow 0.01 < \rho_g = 3.81\ \% < 0.08$
- Check minimum number of longitudinal bars:
  $16 > 4 \quad \therefore Ok.$
- Check minimum distance between longitudinal bars:

$S_{Minimum} = 48^{mm}, S_{Provided} = 47.5\ mm\ \approx 48mm\ Ok.$

Section Strength

- Calculate the design axial load strength and bending moment for given eccentricity $(\emptyset P_n, \emptyset M_n)$:

  $P_u = 6\,200\ kN, M_u = 150\ kN.m \Longrightarrow e = 0.024\ m, \because \frac{e}{h} = 0.032 < 0.10$
- Then this column can be analyzed as an axially loaded column, i.e. the applied moment can be neglected.

  $\emptyset P_{nMaximum} = 6\,827\ kN > P_u\ Ok.$

Checking of Ties:

- Ties diameter:

  $\because \varphi_{Longitudinal\ Bars} = 32^{mm}$

  Then using of $\varphi_{Ties} = 10^{mm}$ is okay.
- Tie spacing

  $S_{Required} = 450^{mm} > S_{Provided}\ Ok.$
- Ties arrangement:

  The proposed distribution is adequate according to ACI requirements.

Problem 9.7-3

The short tied column shown in ***Figure 9.7-3*** below is to be used to support the following loads and moments:

$P_D = 556\ kN,\ \ P_L = 623\ kN, M_D = 102\ kN.m\ , and\ \ M_L = 122\ kN.m$

Select longitudinal bars to be placed in its end faces only using appropriate ACI column interaction diagram, and design the ties.

Assume: Short column, $\emptyset\, 32\ mm$ for longitudinal reinforcement, $f_c' = 28\ \text{MPa}$, and $f_y = 420\ \text{MPa}$.

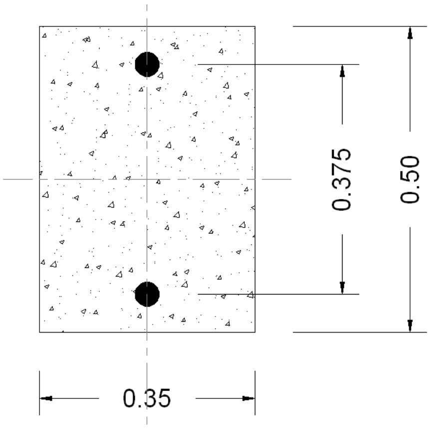


Figure 9.7-3: Column section for Problem 9.7-3.

Answers

Applied Factored Loads:

$P_u = 1\,664\ kN, M_u = 318\ kN.m \Longrightarrow e = 0.191\ m\ \Longrightarrow \frac{e}{h} = 0.38$

Longitudinal reinforcement:

- Compute γ:

  $\gamma = 0.75$
- Based on $\frac{e}{h}$, the strength reduction factor "$\emptyset$" can assumed to be 0.65:

  $K_n = \frac{P_u}{\emptyset f_c' A_g} = 0.52$
- Steel ratio ρ_g can be computed from interpolation from curves of $\gamma = 0.70$ and $\gamma = 0.80$.

γ	0.70	0.75	0.80
ρ_g	2.2%	2.1%	2.0%

$A_{st} = 3\,675\ mm^2 \Longrightarrow No.of\ Rebars = 4.57$, Try $6\,\emptyset\, 32\text{mm}$.

Design of Lateral Reinforcement (Ties):

- Ties diameter:

  $\because\ \emptyset = 32^{\text{mm}}, \therefore$ we can use $\emptyset = 10^{\text{mm}}$ for ties
- Ties spacing:

  $S_{\text{Maximum}} = 350^{\text{mm}}$

- Ties arrangement:
  $\because S_{\text{Spacing between longitudinal bars}} < 150^{\text{mm}}$
  Then, alternate longitudinal bars will be supported by corner bars.

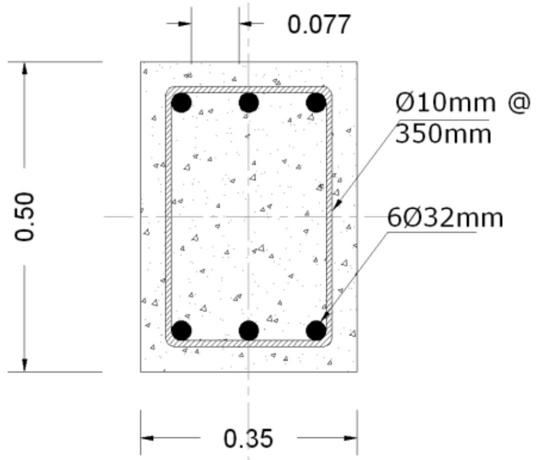


Figure 9.7-4: Final design section for Problem 9.7-4.

Problem 9.7-4

Design the spiral column that supports four girders of bridge shown in ***Figure 9.7-5*** below. In your design assume that.

- Each girder has a dead load reaction of 150 kN and has a live load reaction of 100 kN.
- Assume that live load acting on right span only.
- f_c' = 28 MPa and f_y = 420 MPa.
- Rebar No. 25 for longitudinal reinforcement (A_{bar} = 510mm^2) and No. 10 for spiral reinforcement.
- Column has a height of 4m, and it is assumed short.
- Column and cap selfweight should be included in your solution.

As was discussed previously, your solution can be based on the equivalent circular section instead of actual hexagonal section, see ***Figure 9.4-5***.

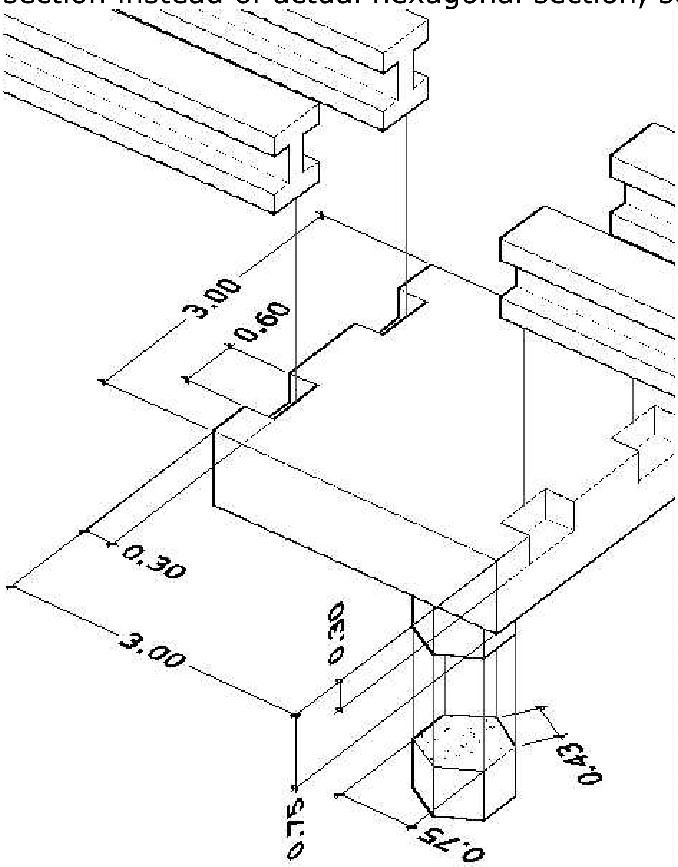


Figure 9.7-5: Bridge girders and column for Problem 9.7-4.

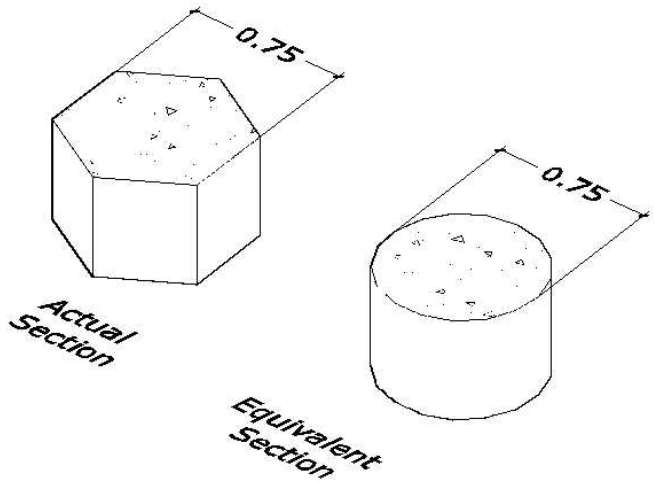


Figure 9.4-5: Transformation of an octagonal column into the equivalent circular section. Reproduced for convenience

Answers

Applied Factored Loads:

$$P_D = (150\ kN \times 4)_{Girders\ Reactions} + \left((3.0^2 \times 0.75 - 0.3^2 \times 0.6 \times 4)m^3 \times 24\frac{kN}{m^3}\right)_{Cap\ Selfweight} + \left(\left(\frac{\pi 0.75^2}{4} \times 4\right)m^3 \times 24\frac{kN}{m^3}\right)_{Column\ Selfweight}$$

$$P_D = (600\ )_{Girders\ Reactions} + (157)_{Cap\ Selfweight} + (42)_{Column\ Selfweight} = 799\ kN$$

As all dead loads are symmetric, then M_{Dead} is zero.

$$P_L = (100\ kN \times 2)_{Live\ Load\ Reactions\ from\ Right\ Side\ Span} = 200\ kN$$

$$M_L = P_L \times Arm\ = 200\ kN \times \left(\frac{3.0}{2} - \frac{0.3}{2}\right)m = 270\ kN.m$$

Then factored forces will be:

$$P_u = \ 1\ 279\ kN, M_u = 432\ kN.m \Longrightarrow e = 0.338\ m \Longrightarrow \frac{e}{h} = 0.45$$

Longitudinal reinforcement:

- Compute γ:

  $$\gamma h = 750 - 40 \times 2 - 10 \times 2 - 25 = 625 \Longrightarrow \gamma = \frac{\gamma h}{h} = \frac{625}{750} = 0.83$$

  Say γ = 0.8.
- Based on $\frac{e}{h}$, the strength reduction factor "$\emptyset$" can be taken equal to 0.75:

  $$K_n = \frac{P_u}{\emptyset f_c' A_g} = \frac{1\ 279\ 000\ N}{0.75 \times 28 \times \frac{750^2 \times \pi}{4}} = 0.138$$
- Based on interaction diagram shown ***Figure 9.7-6*** below, it seems that required ratio ρ_g is less than 1%, then ACI minimum reinforcement ratio should adopted:

  $$\rho_g = 0.01 \Longrightarrow A_{st} = \ 0.01 \times \frac{750^2 \times \pi}{4} = 4\ 416\text{mm}^2 \Longrightarrow No.of\ Rebars\ = \ \frac{4\ 416}{510} = 8.65$$

  Then use ***9Ø25mm***.

Spiral Design:

- Spiral diameter:

  $$\because \varphi_{Spiral} = 10^{mm}\ \ \therefore Ok.$$
- Compute $\rho_{SMinimum}$:

  $$D_c = 750^{mm} - 2 \times 40^{mm} = 670^{mm}$$

  $$A_c = \frac{\pi \times 670^2}{4} = 352\ 386\ mm^2, A_g = \frac{\pi \times 750^2}{4} = 441\ 562\ mm^2$$

  $$\rho_{sMinimum} = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f_c'}{f_{sy}} = 0.45\left(\frac{441\ 562}{352\ 386} - 1\right) \times \frac{28}{420} = 7.59 \times 10^{-3}$$

  $$A_{sp} = \frac{\pi \times 10^2}{4} = 78.5^{mm^2}$$

  $$S_{Required} = \frac{4A_{sp}}{D_c\ \rho_{sMinimum}} \Longrightarrow \therefore S_{Required} = \frac{4 \times 78.5^{mm^2}}{670^{mm} \times 7.59 \times 10^{-3}} = 61.7^{mm}$$

Try $\varphi 10^{mm}$ @ 60^{mm}

Use $\boldsymbol{\varphi 10^{mm}}$ @ $\boldsymbol{60^{mm}}$

- The final section of the column is shown in ***Figure 9.7-7*** below.

$\because S_{Clear} = 50^{mm} < 80^{mm} \;\; \therefore Ok.$

$\because S_{Clear} = 50mm > 25^{mm} \;\; \therefore Ok.$

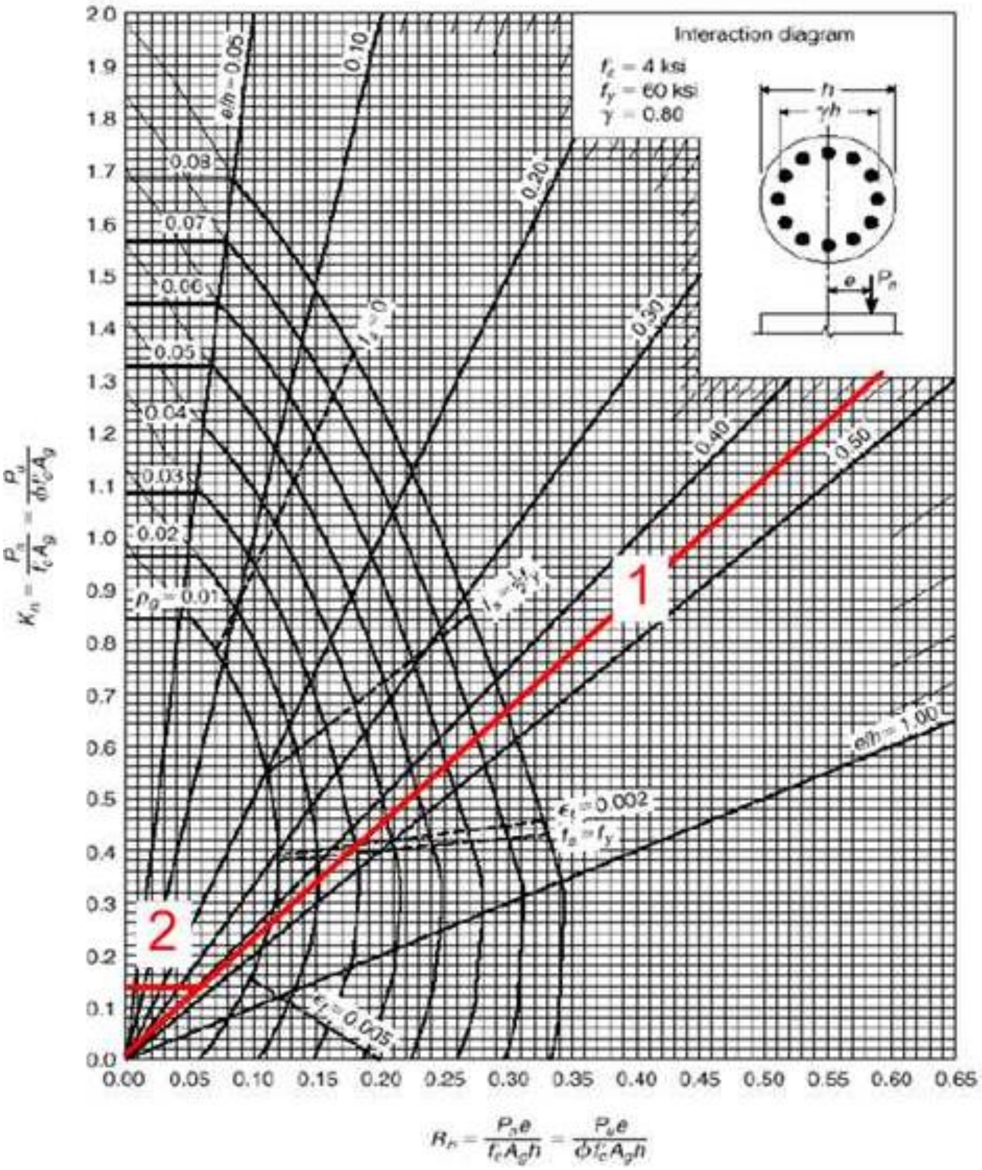


Figure 9.7-6: Inteaction diagram for the equivalent circular column of Problem 9.7-4.

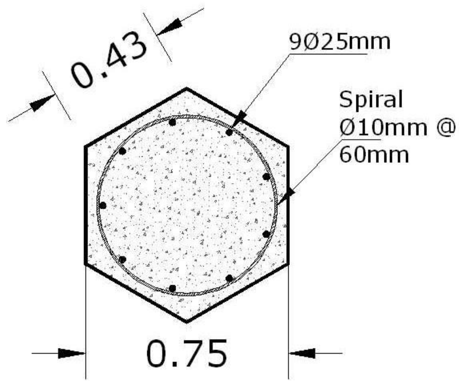


Figure 9.7-7: Final column section for the column of Problem 9.7-4.

9.8 ANALYSIS OF COLUMNS SUBJECTED TO COMPRESSION FORCE AND BIAXIAL MOMENTS

9.8.1 A Circular Column under an Axial Force and Biaxial Moments

- Circular columns have polar symmetry and thus the same ultimate capacity in all directions.
- Then if a circular column is subjected to biaxial moments, these moments can be transformed into an equivalent uniaxial bending moment that computed based on the following relations:

  $e = \sqrt{e_x^2 + e_y^2}$

  or

  $M_u = \sqrt{M_{ux}^2 + M_{uy}^2}$

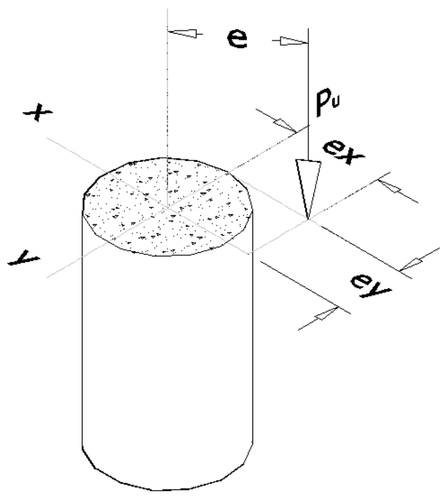


Figure 9.8-1: Circular column under axial force and biaxial moments.

Example 9.8-1

Use an appropriate interaction diagram to determine P_n value that can be supported by circular column shown in ***Figure 9.8-2*** below.
Assume that: $f_y = 420MPa$, $f_c' = 28\ MPa$ and $A_{Bar} = 645\ mm^2$.

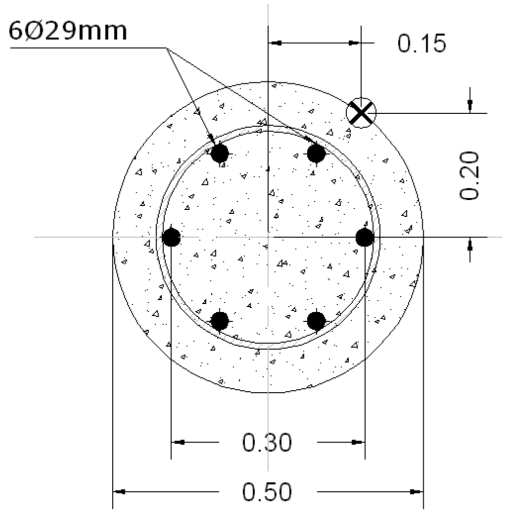


Figure 9.8-2: Circular column of Example 9.8-1.

Solution

- The equivalent eccentricity for the resultant moment can be determined based on the following relation.

  $e = \sqrt{e_x^2 + e_y^2} = \sqrt{0.15^2 + 0.20^2} = 0.25 \Longrightarrow \frac{e}{h} = \frac{0.25}{0.50} = 0.5$

  $\gamma = \frac{0.30}{0.50} = 0.6$

- Based on γ value and as the column is a circular column, then the interaction diagram of ***Figure 9.8-3*** below has been adopted.

$$A_{st} = 6 \times 645\ mm^2 = 3\,870\ mm^2, A_g = \frac{\pi \times 500^2}{4} = 196\,250\ mm^2 \Longrightarrow \rho_g = \frac{3\,870\ mm^2}{196\,250\ mm^2} \approx 2.0\%$$

$$K_n = 0.25 = \frac{P_n}{A_g f_c'} \Longrightarrow P_n = 0.25 \times 196\,250\ mm^2 \times 28 \frac{N}{mm^2} = 1\,374\ kN$$

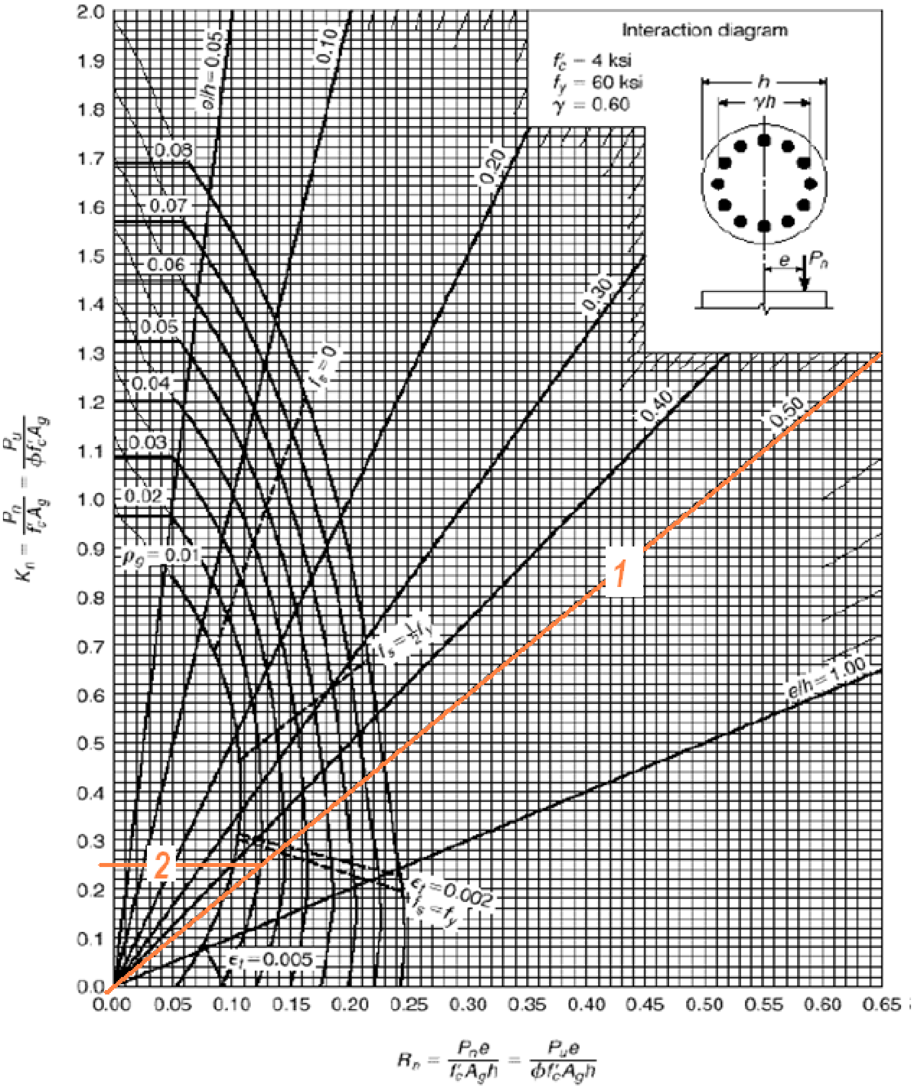


Figure 9.8-3: Interaction diagram adopted for circular column of Example 9.8-1.

9.8.2 Analysis of a Rectangular Column under an Axial Force and Biaxial Moments

9.8.2.1 Basic Concepts

- The article aims to analyze a rectangular column under a compression force and biaxial moments.
- With analysis problem, column is assumed to has known dimensions and known reinforcement and to be checked for resisting a force set consists of a compression force and biaxial moments (see ***Figure 9.8-4*** below).

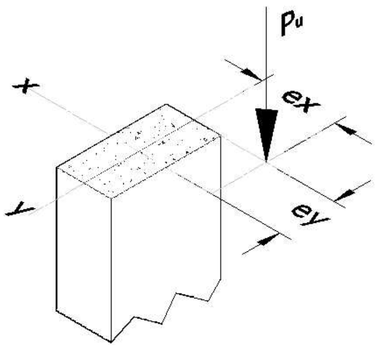


Figure 9.8-4: A Rectangular column under an axial force and biaxial moments.

- Criterion for Including or Negating the Effect of the Smaller Moment:
- According (Nilson, Design of Concrete Structures, 14th Edition, 2011), following criterion can be adopted to consider the minor bending moment into consideration: "***In general, biaxial bending should be taken into account when the estimated ratio of smaller to larger bending moments approaches or exceed 0.2***".
- As for a column with compression force and uniaxial moment, analysis of a column with a compression force and biaxial moments starts with construction of column interaction diagram.
- If a specific load set is located inside or on the interaction diagram, then this column is adequate to resist applied load set safely and vice versa.
- Typical interaction diagram for a rectangular column under a compression force and biaxial moments is shown in ***Figure 9.8-5*** below.

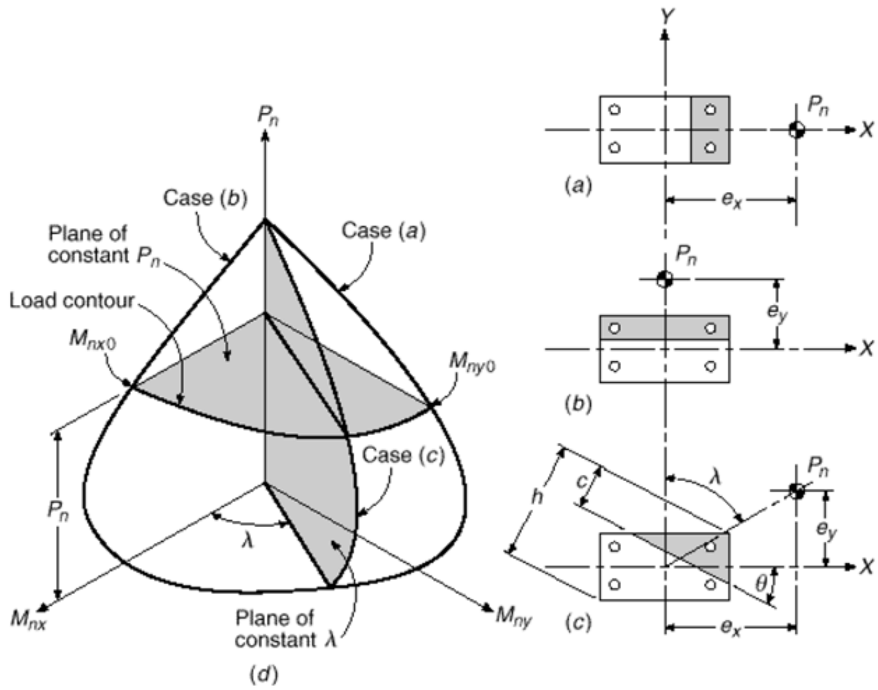


Figure 9.8-5: Interaction diagram for compression plus biaxial bending: (a) uniaxial bending about Y axis; (b) uniaxial bending about X axis; (c) biaxial bending about diagonal axis; (d) interaction surface.

- It is difficult to draw or represent of a three-dimension interaction diagram (especially without a computer program), then for practical applications curve of ***Figure 9.8-5*** is usually approximated based on one of two methods presented in Articles ***9.8.2.2*** and ***9.8.2.3*** below.

9.8.2.2 Reciprocal Load Method

- It is a simple approximate design method developed by Bresler.
- It has been satisfactorily verified by comparison with results of extensive tests and accurate calculations.
- The method can be summarized as follows:
  - Re-draw the interaction diagram in terms of ($\frac{1}{P_n}$, e_x, and e_y) instead of (P_n, M_x, and M_y) to obtain the surface "S" that shown in ***Figure 9.8-6*** below. Based on the new terms, the main unknown in an analysis problem is $\frac{1}{P_n}$.
  - Use a plane S_1 that defined by points A, B, and C to approximate the original surface S. Then the approximate value of unknown $\frac{1}{P_n}$ can be computed based on the following relation:
    $$\frac{1}{P_n} = \frac{1}{P_{nx0}} + \frac{1}{P_{ny0}} - \frac{1}{P_0}$$
    where
    P_n is approximate value of nominal load in biaxial bending with eccentricities e_x and e_y
    P_{ny0} is nominal load when only eccentricity e_x is present $(e_y = 0)$ (can be computed from a specific interaction diagram for an axial force and uniaxial bending moment).
    P_{nx0} is nominal load when only eccentricity e_y is present $(e_x = 0)$ (can be computed from a specific interaction diagram for an axial force and uniaxial bending moment).
    P_0 is nominal load for concentrically loaded column (can be computed from a specific interaction diagram for an axial force and uniaxial bending moment or may be computed based on relations given in article 2 but without factors of 0.8 for tied columns and 0.85 for spiral columns).
  - Finally, column adequacy can be checked based on the following comparison:
    If
    $P_u \leq \emptyset P_n$
    Then the column is adequate. Else the column is inadequate to support a factored applied load of P_u acting at eccentricities e_x and e_y.

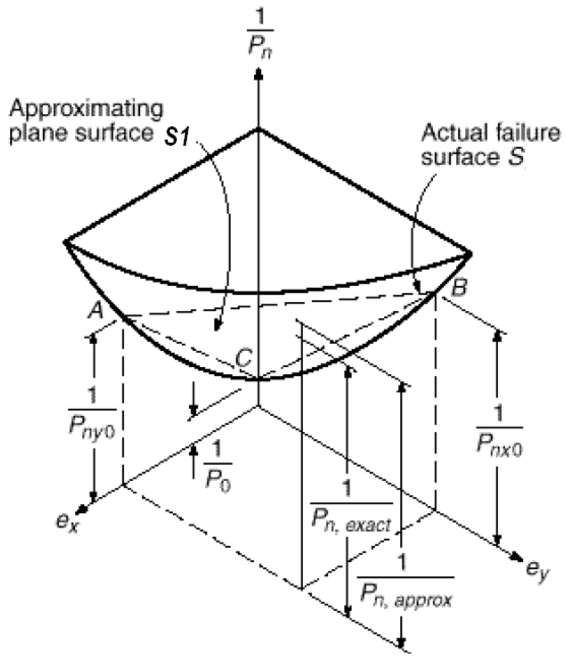


Figure 9.8-6: Interaction surfaces for the reciprocal load method.

- Notes on Reciprocal Load Method:
  The reciprocal load method is very simple to use, ***but the representation of the curved failure surface by an approximating plane is not reliable in the range of large eccentricities, where failure is initiated by steel yielding***.

Example 9.8-2

The 300 by 500mm column shown in ***Figure 9.8-7*** below is reinforced with eight No. 29 bars ($A_{Bars} = 645mm^2$) arranged around the column perimeter. A factored load P_u of $1\,134\,kN$ is to be applied with eccentricities $e_y = 75mm$ and $e_x = 150mm$. Material strengths $f_y = 420MPa$ and $f'_c = 28\,MPa$ are specified. Check the adequacy of the column using the reciprocal load method.

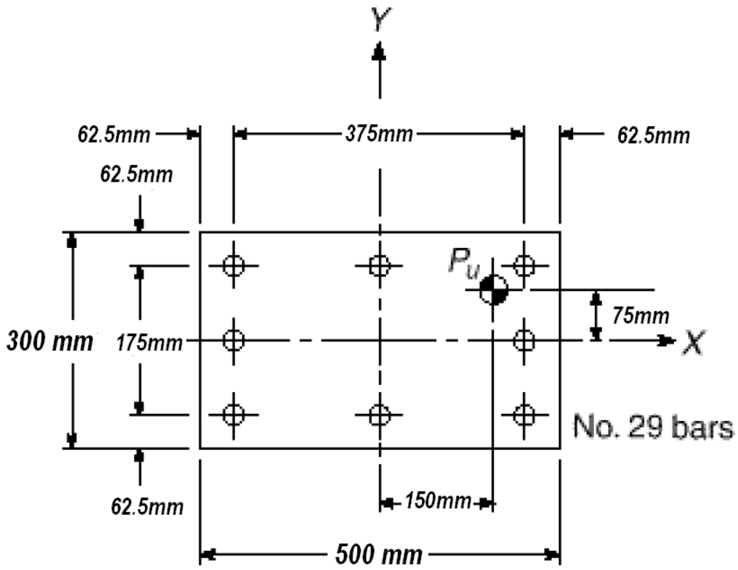


Figure 9.8-7: Column for Example 9.8-2.

Solution

- Considering the bending moment about y-axis (To compute P_{ny0}):
  $A_{st} = 8 \times 645mm^2 = 5\,160\,mm^2, A_g = 500 \times 300 = 150\,000\,mm^2 \Longrightarrow \rho_g = 3.44\,\%$
  $\frac{e}{h} = \frac{150}{500} = 0.3, \gamma = \frac{375}{500} = 0.75$
  As we don't have an interaction diagram with $\gamma = 0.75$, then we'll use the average value for $\gamma = 0.70$ and $\gamma = 0.80$, see ***Figure 9.8-8*** below.
  $K_{n\,avg.} = \frac{P_{ny0}}{A_g f'_c} = \frac{0.62 + 0.66}{2} = 0.64$
  $P_{ny0} = 0.64 \times 150\,000 \times 28 = 2\,688\,kN$
- Considering the bending moment about x-axis (To compute $\mathbf{P_{xy0}}$):
  $\frac{e}{h} = \frac{75}{300} = 0.25, \gamma = \frac{175}{300} = 0.58$
  Say $\gamma = 0.60$
  $K_n = \frac{P_{nx0}}{A_g f'_c} = 0.65$
  $P_{nx0} = 0.65 \times 150\,000 \times 28 = 2\,730\,kN$
- Consider the case of axially load column (To compute $\boldsymbol{P_0}$):
  $P_0 = 0.85 \times 28 \times (150\,000 - 5\,160) + 420 \times 5\,160 = 5\,614\,kN$
- Compute the approximate column strength when it is subjected to an axial force and biaxial moments:
  $\frac{1}{P_n} = \frac{1}{P_{nx0}} + \frac{1}{P_{ny0}} - \frac{1}{P_0} = \frac{1}{2\,730} + \frac{1}{2\,688} - \frac{1}{5\,614} = 5.60 \times 10^{-4}$
  $P_n = 1\,785\,kN\ ?\, P_{nmax} = 0.80 \times P_0 = 0.8 \times 5\,614\,kN = 4\,491\,kN$
  $P_n = 1\,785\,kN < P_{nmax} = 4\,491\,kN \quad Ok.$
- Finally, check column adequacy based on following comparison:
  $\emptyset P_n = 0.65 \times 1\,785\,kN = 1\,160\,kN > 1\,134\,kN$
  The column is adequate according to reciprocal load method.

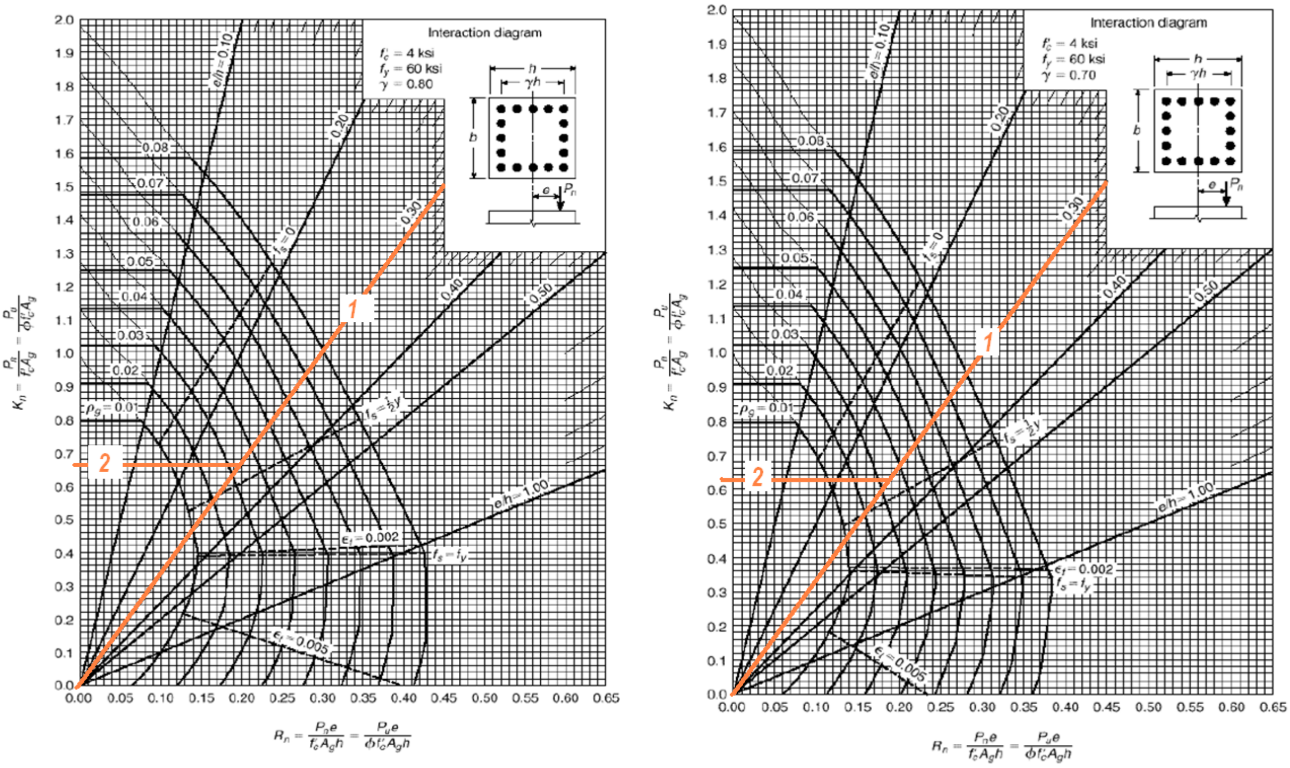


Figure 9.8-8: Interaction diagrams adopted in Example 9.8-2.

9.8.2.3 Load Contour Method

- Checking of a column adequacy for an axial force and biaxial moments (i.e., checking if the load state is inside or outside the column interaction diagram) can also be done based on checking if the load state is inside or outside the ***Load Contour*** for a plane of constant force P_n.

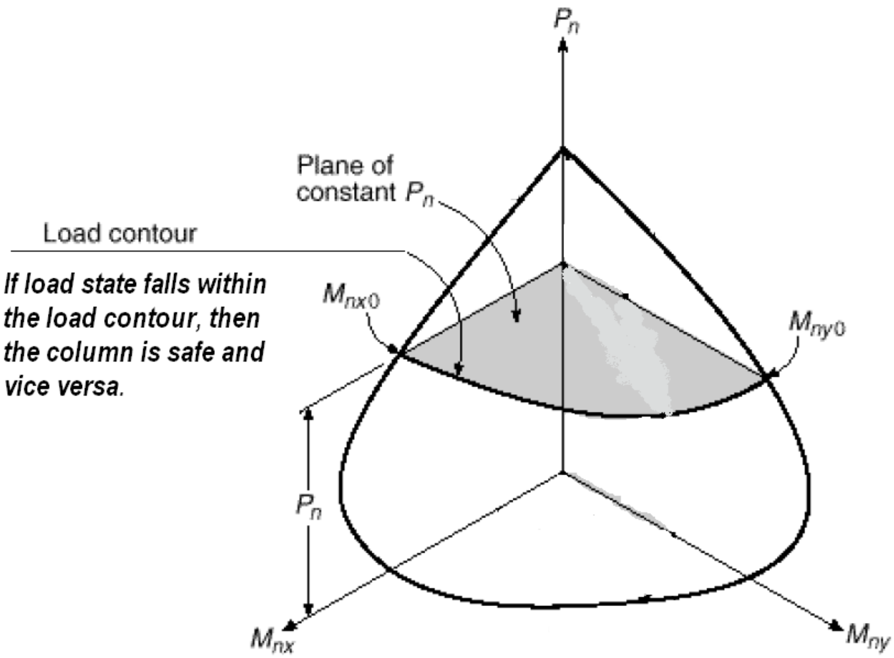


Figure 9.8-9: Interaction contours at constant P_n.

- If load state falls within the *Load Contour*, then the column is safe and vice versa.
- General form of load contour curve can be approximated by a nondimensional interaction equation:
  $$\left(\frac{M_{nx}}{M_{nx0}}\right)^{\alpha 1} + \left(\frac{M_{ny}}{M_{ny0}}\right)^{\alpha 2} = 1.0$$
  where
  $M_{nx} = P_n e_y$
  $M_{nx0} = M_{nx}$ when $M_{ny} = 0.0$
  $M_{ny0} = M_{ny}$ when $M_{nx} = 0.0$
  α_1 and α_2 are exponents depending on:
  - Column dimensions.
  - Amount and distribution of steel reinforcement.
  - Stress-strain characteristics of steel and concrete.
  - Amount of concrete cover.
  - Size of lateral ties or spiral.
- When $\alpha_1 = \alpha_2 = \alpha$, the shapes of such interaction contours are as shown in ***Figure 9.8-10*** below for specific values.
- Values of $\boldsymbol{\alpha}$:
  - α values fall in the range from 1.15 to 1.55 for square and rectangular columns.
  - Values near the lower end of that range are the more conservative.

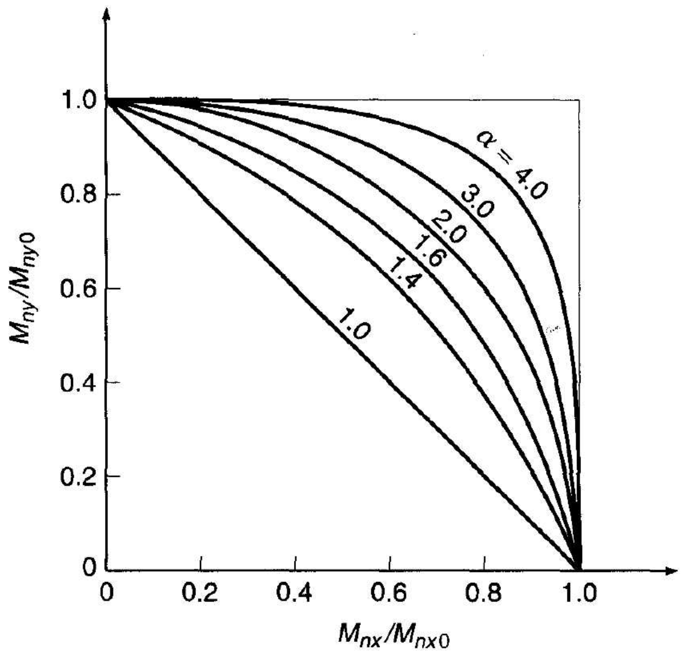


Figure 9.8-10: Interaction contours at constant P_n for varying α.

- More Useful Form of Load Contour:
  - Introducing of the ACI factors for reducing nominal axial and flexure strengths to design strength presents no difficulty. With the appropriate ϕ factors applied to P_n , M_{nx}, and M_{ny}, a new failure surface is defined:
    $$\left(\frac{\phi M_{nx}}{\phi M_{nx0}}\right)^{\alpha} + \left(\frac{\phi M_{ny}}{\phi M_{ny0}}\right)^{\alpha} = 1.0$$
  - The above equation can be rewritten in terms of applied moments:
    $$\left(\frac{M_{ux}}{\phi M_{nx0}}\right)^{\alpha} + \left(\frac{M_{uy}}{\phi M_{ny0}}\right)^{\alpha} = 1.0$$
- How to Use Load Contour:
  - In practice, the values of P_u, M_{ux}, M_{uy} are known from the analysis of the structure.
  - For a trial column section, the values of M_{nx0} and M_{ny0} corresponding to the load P_u can easily be found by the usual methods for uniaxial beading.
  - It can be confirmed that a particular combination of factored moments falls within the load contour (safe design) or outside the contour (failure), and the design modified if necessary.
- Notes on Load Contour Method:
  - Selection of the appropriate value of the exponent $\propto$ is made difficult by a number of factors relating to column shape and bar distribution.
  - For many cases, the usual assumption that $\alpha_1 = \alpha_2$ is a poor approximation.

Example 9.8-3

Re-check the column of ***Example 9.8-2*** by the Load Contour Method. Assume that the exponent ∝ conservatively taken equal to 1.15.

Solution

- Nominal Bending Strength about y-axis ($\emptyset M_{ny0}$):

$$\gamma = \frac{0.375^m}{0.5^m} = 0.75$$

$$\mathrm{A_g} = 500^{\mathrm{mm}} \times 300^{\mathrm{mm}} = 150\ \ 000\ \mathrm{mm}^2$$

$$A_{st} = \ 8 \times 645 mm^2 = 5\ 160\ mm^2$$

$$\rho_g = 3.44\ \%$$

Based on above definition of ($\emptyset M_{ny0}$), ***one must start the solution with*** $\mathrm{K_n}$ ***value to compute the required*** $\mathrm{R_n}$ ***value based on steel reinforcement ratio***. Required R_n can't be computed based on e/h ratio as this solution will not be consistent with the definition of ($\emptyset M_{ny0}$). Based on interaction diagrams presented in ***Figure 9.8-11*** below, column strength $\emptyset M_{ny0}$ would be:

$$K_n = \frac{P_u}{\emptyset f_c' A_g} = \frac{1\ 134\ 000}{0.65 \times 28 \times 150000} = 0.41$$

$$R_{n\ avg.} = \left(\frac{\emptyset M_{ny0}}{f_c' A_g h}\right)_{avg} = \frac{0.21 + 0.24}{2} = 0.22$$

$$\emptyset M_{ny0} = 0.65(0.22 \times 28 \times 150\ 000 \times 500) = 300\ kN.m$$

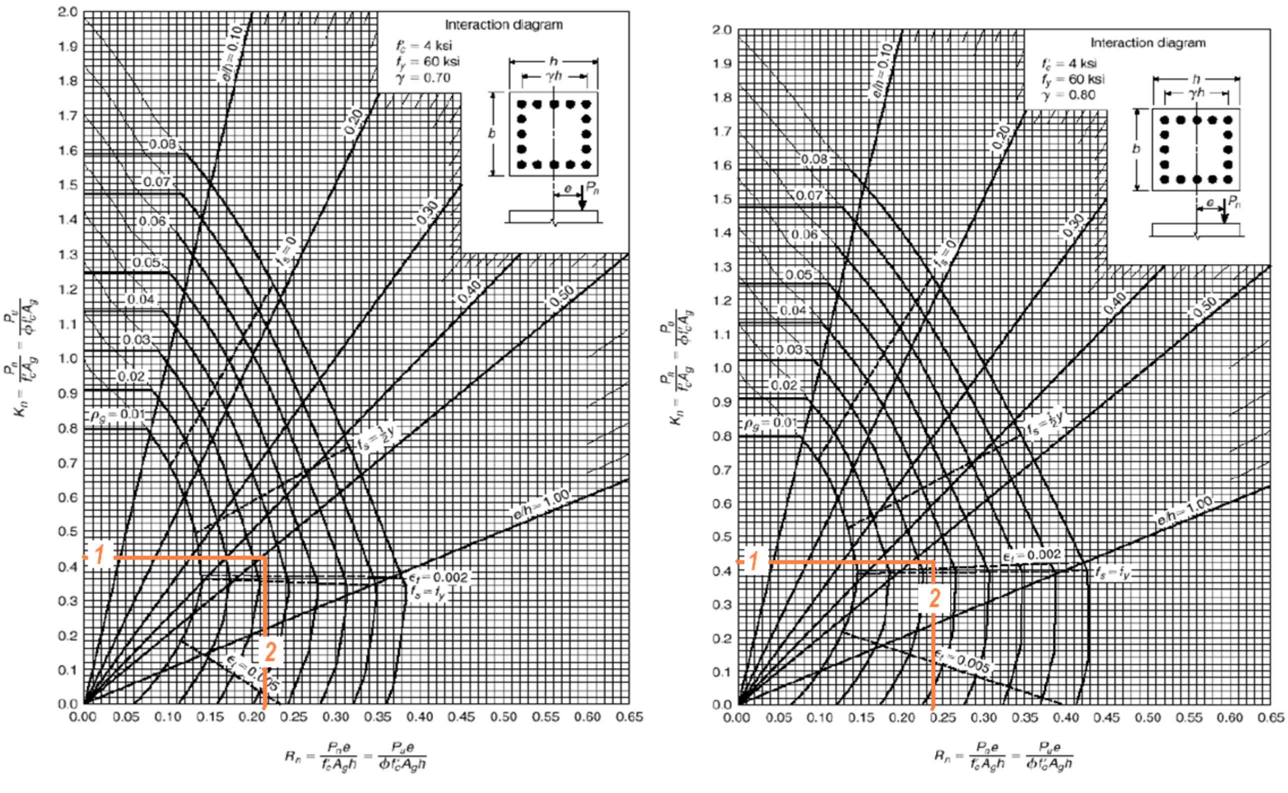


Figure 9.8-11: Interaction diagrams adopted to compute $\emptyset M_{ny0}$ of Example 9.8-3.

- Nominal Bending Strength about x-axis ($\emptyset M_{nx0}$):

$$\gamma = \frac{0.175^m}{0.3^m} = 0.58$$

Say $\gamma = 0.60$, and based on interaction diagram of ***Figure 9.8-12*** below.

$$K_n = \frac{P_u}{\emptyset f_c' A_g} = 0.41\ (as\ befor)$$

$$R_n = \left(\frac{\emptyset M_{nx0}}{f_c' A_g h}\right) = \ \ 0.19$$

$$\emptyset M_{nx0} = 0.65(0.19 \times 28 \times 150\ 000 \times 300) = 156\ kN.m$$

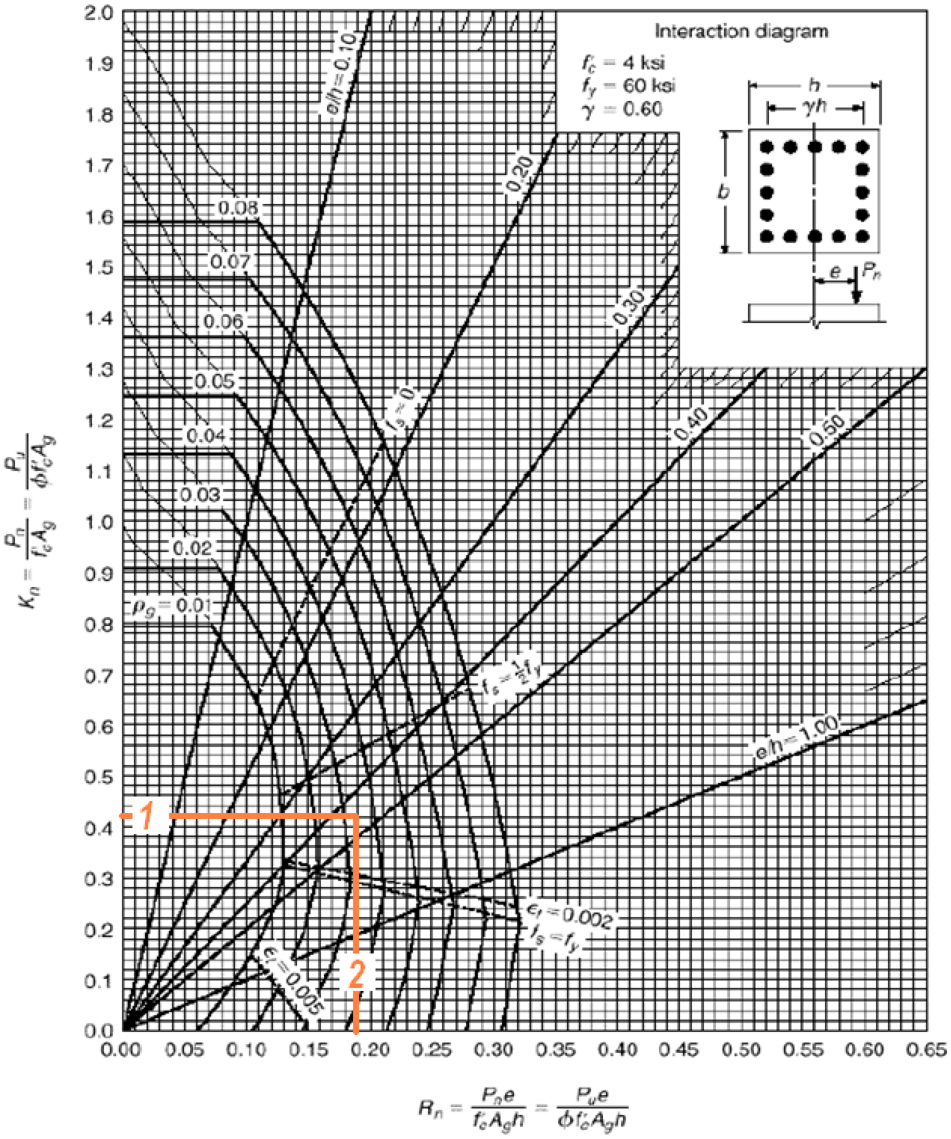


Figure 9.8-12: Interaction diagram adopted to compute $\emptyset M_{nx0}$ of Example 9.8-3.

- Check column adequacy based on Load Contour Method:

$M_{uy} = 1\,134\; kN \times 0.150m = 170\; kN.m$

$M_{ux} = 1\,134\; kN \times 0.075m = 85\; kN.m$

$$\left(\frac{M_{ux}}{\phi M_{nx0}}\right)^{1.15} + \left(\frac{M_{uy}}{\phi M_{ny0}}\right)^{1.15} \; ? \; 1.0$$

$$\left(\frac{85}{156}\right)^{1.15} + \left(\frac{170}{300}\right)^{1.15} \; ? \; 1.0$$

$0.548 + 0.566 = 1.1 \approx 1.0 \quad Ok.$

The column is adequate according to Load Contour Method.

Table of Contents

10.1 INTRODUCTION AND BASIC CONCEPTS

10.1.1 Definition of Slender Columns

- A column is said to be slender if its cross-sectional dimensions are small compared with its length.
- The degree of slenderness is generally expressed in terms of the slenderness ratio ℓ_u/r, where ℓ_u is the unsupported length of the member and r is the radius of gyration of its cross section, equal to:

$$r = \sqrt{\frac{I}{A}}$$ **Eq. 10.1-1**

- According to ***ACI 6.2.5.1***, the radius of gyration r for rectangular column can be determined from ***Eq. 10.1-2***.

$$r_{For\ a\ Rectangular\ Section} = 0.3h$$ **Eq. 10.1-2**

while for circular columns it may be taken as in ***Eq. 10.1-3***.

$$r_{For\ a\ Circular\ Section} = 0.25D$$ **Eq. 10.1-3**

- It has long been known ***that a member of great slenderness will collapse under a smaller compression load than a stocky member with the same cross-sectional dimension***.

Example 10.1-1

With referring to gross homogenous sections, show that Eq. 10.1-2 and Eq. 10.1-3 are rational in nature and can be derived from definition of ***Eq. 10.1-1***.

Solution

For rectangular section:

$$r_{rectangular} = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{bh^3}{12}}{bh}} = \frac{1}{\sqrt{12}}h = 0.228h \approx 0.3h$$

For circular section:

$$r_{circular} = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi D^4}{64}}{\frac{\pi D^2}{4}}} = \frac{1}{4}D = 0.25D$$

- This article aims to discuss the effects of slenderness on:
  - The strength of axially loaded columns,
  - The strength of columns that subjected to axial force and bending moment.

10.1.2 Effect of Slenderness Ratio on Strength of Axially Loaded Columns

10.1.2.1 Basic Concepts

- Based on experimentally work, the relation between column strength and its slenderness ratio is as shown in ***Figure 10.1-1*** below.

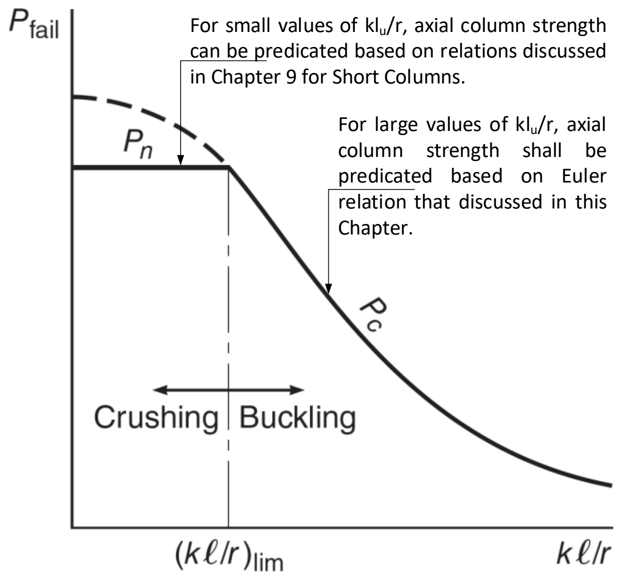


Figure 10.1-1: Effect of slenderness on strength of axially loaded columns.

- It can been shown that, for lower values of kl_u/r (values less than $(kl_u/r)_{Limit}$ in ***Figure 10.1-1*** above) column strength can be predicated by the relation derived in Chapter 9:

$$P_n = 0.85 f_c' \left(A_g - A_{st}\right) + A_{st} f_y \qquad \textbf{Eq. 10.1-4}$$

- For larger slenderness ratio, column strength can be predicated based on the following relation that derived by Euler more than 200 years ago:

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} \qquad \textbf{Eq. 10.1-5}$$

where kl_u is defined as the ***effective length*** and it ***represents the distance between the inflection points***.

- Correspondingly, there is a limiting slenderness ratio $(kl_u/r)_{Limit}$:
  - For values smaller than $(kl_u/r)_{Limit}$ this, failure occurs by simple crushing, regardless of kl_u/r;
  - For values larger than $(kl_u/r)_{Limit}$ failure occurs by buckling, the buckling load or stress decreasing for greater slenderness.

10.1.1.1 Physical Meaning of Euler Load or Critical Load

- An axially loaded column similar to that shown below should be designed for the dominated acting force (axial force P_u in this case).
- However, this column is a part from structure that should be adequate for many decades. During that long age, this column may be subjected to a temporary lateral force due to a minor cause that can't be accounted in the design process. Then this column will be displaced laterally as shown below:

Figure 10.1-2: A column subjected to dominate axial force.

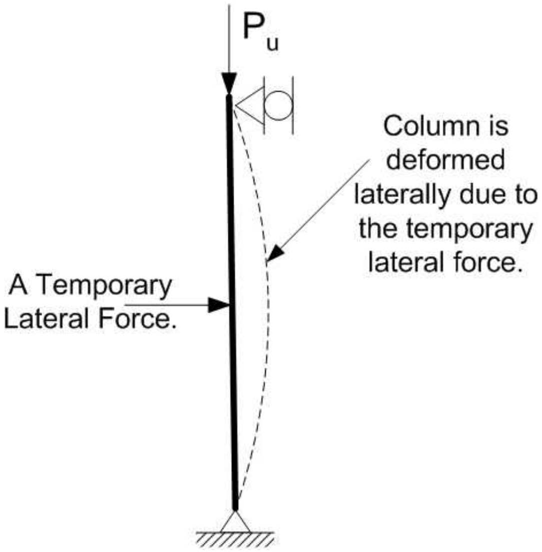


Figure 10.1-3: A column subjected to dominate axial force and to minor or temporary lateral forces.

- It has been noted experimentally, and has been approved analytically, that each column has a critical load (P_c) that when the column is loaded with an axial load P_u less than P_c and subjected to a lateral temporary force at the same time, it will return to its undeform shape when this temporary lateral load remove and vice versa.

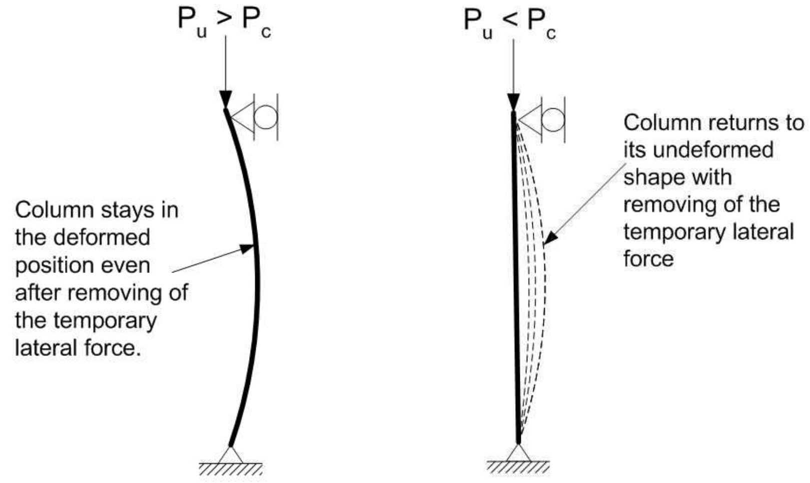


Figure 10.1-4: Behavior of axially loaded column when P_u is less or/and greater than P_c.

- As we have no control on the occurring of such temporary lateral force, then we cannot accept a column that loaded with an axial force equal to or greater than its critical load. Such column is classified as ***unstable column*** in engineering practice.
- Then critical or Euler load represents a very important limit on axial load in columns:
  - For short columns:

    $P_{crashing} < P_c$ **Eq. 10.1-6**

    then

    $\mathrm{P_n = P_{crashing} = 0.85f_c'(A_g - A_{st}) + A_{st}f_y}$ **Eq. 10.1-7**
  - For long or slender columns:

    $P_{crashing} \geq P_c$ **Eq. 10.1-8**

    then

    $$P_n = P_c = \frac{\pi^2 EI}{(kl_u)^2}$$ **Eq. 10.1-9**

10.1.2.2 Computing of Buckling Load or Euler Load

- To compute or estimate critical load (or Euler Load) one should compute or estimate following quantities:
  1. Member stiffness or rigidity (EI)
  2. Member unsupported length (l_u).
  3. Effective length factor or k factor.
- Each one of above quantities will be discussed briefly below:

10.1.2.3 ACI Procedure for Computing (EI) to be used in Euler Formula

- In homogeneous elastic members such as steel columns, EI is easily obtained from Young's modulus and the usual moment of inertia.
- Reinforced concrete columns, however, are
  - Nonhomogeneous, since they consist of both steel and concrete,
  - Steel is substantially elastic, concrete is not and is in addition subject to creep and to cracking if tension occurs on the convex side of the column.
- All of these factors affect the effective value of (EI) for a reinforced concrete member.
- According to ***Article 6.6.4.4.4*** of the ACI code effective value of (EI) or, EI_{eff}, as called by the code, can be determined based on any one of the following relations:

  $$EI_{eff} = \frac{0.4E_cI_g}{1+\beta_{dns}}$$ **Eq. 10.1-10**

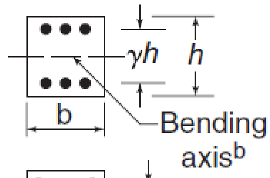
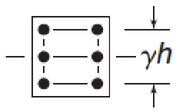
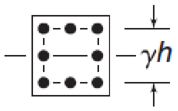
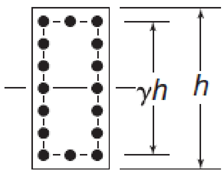
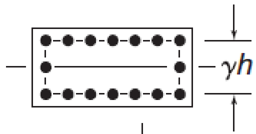
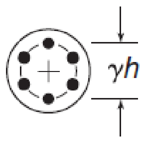
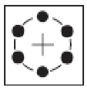
  $$EI_{eff} = \frac{0.2E_cI_g + E_sI_{se}}{1+\beta_{dns}}$$ **Eq. 10.1-11**

  $$EI_{eff} = \frac{E_c\mathrm{I}}{1+\beta_{dns}}$$ **Eq. 10.1-12**

  where E_c is modulus of elasticity of concrete.
  - I_g is moment of inertia of gross section of column.

- E_s is modulus of elasticity of steel.
- I_{se} is moment of inertia of reinforcement about centroidal axis of member cross section. According to (Wight, 2016), calculation of I_{se} can be simplified with refereeing to Table 10.1-1.

Table 10.1-1: Calculations of I_{se}, adopted from (Wight, 2016).

Type of Column	Number of Bars	I_{se}
	—	$0.25A_{st}(\gamma h)^2$
	3 bars per face 6 bars per face	$0.167A_{st}(\gamma h)^2$ $0.117A_{st}(\gamma h)^2$
	8 bars (3 per face) 12 bars (4 per face) 16 bars (5 per face)	$0.187A_{st}(\gamma h)^2$ $0.176A_{st}(\gamma h)^2$ $0.172A_{st}(\gamma h)^2$
	$h = 2b$ 16 bars as shown About strong axis	$0.128A_{st}(\gamma h)^2$
	$b = 2h$ About weak axis	$0.219A_{st}(\gamma h)^2$
	—	$0.125A_{st}(\gamma h)^2$
	—	$0.125A_{st}(\gamma h)^2$

- I is the effective moment of inertial computed based on ***Table 10.1-2*** below.
- β_{dns} is ratio of maximum factored axial sustained axial load to maximum factored axial load associated with the same load combination.

Table 10.1-2: Alternative moments of inertia for elastic analysis at factored load, *Table 6.6.3.1.1(b)* **of the ACI code.**

Member	Alternative value of I for elastic analysis: Minimum	I	Maximum
Columns and walls	$0.35I_g$	$\left(0.80+25\frac{A_{st}}{A_g}\right)\left(1-\frac{M_u}{P_u h}-0.5\frac{P_u}{P_o}\right)I_g$	$0.875I_g$
Beams, flat plates, and flat slabs	$0.25I_g$	$(0.10+25\rho)\left(1.2-0.2\frac{b_w}{d}\right)I_g$	$0.5I_g$

- Notes on Computing of EI_{eff}:
  - Creep due to sustained loads will increase the lateral deflections of a column and, hence, the moment magnification.

- Creep effects are approximated in design by reducing the stiffness EI_{eff} by dividing the short-term EI provided by the numerator ***Eq. 10.1-10*** through ***Eq. 10.1-12*** by $(1 + \beta_{dns})$.
- For simplification, ***it can be assumed that*** $\beta_{dns} = 0.6$***. In this case*** Eq. 10.1-10 ***becomes*** $EI_{eff} = 0.25E_cI_g$.
- In reinforced concrete columns subject to sustained loads, creep transfers some of the load from the concrete to the longitudinal reinforcement, increasing the reinforcement stresses. In the case of lightly reinforced columns, this load transfer may cause the compression reinforcement to yield prematurely, resulting in a loss in the effective EI. Accordingly, both the concrete and longitudinal reinforcement terms in ***Eq. 10.1-10*** through ***Eq. 10.1-12*** are reduced to account for creep.
- The equations in ***Table 10.1-2*** above provide more refined values of I considering:
  - Axial load,
  - Eccentricity,
  - Reinforcement ratio,
  - Concrete compressive strength.

10.1.2.4 Column Unsupported Length (ℓ_u)

- The unsupported length of a compression member, ℓ_u, shall be taken as the clear distance between floor slabs, beams, or other members capable of providing lateral support in the direction being considered.
- Where column capitals or haunches are present, ℓ_u shall be measured to the lower extremity of the capital or haunch in the plane considered."

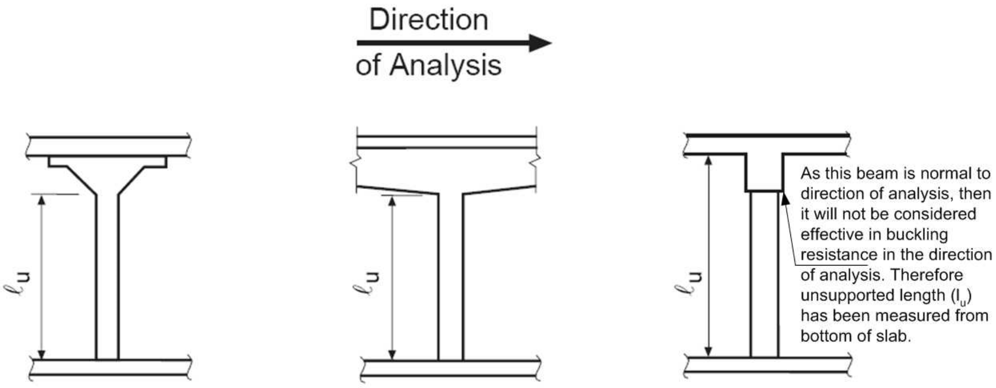


Figure 10.1-5: Unsupported column length, ℓ_u.

10.1.2.5 Computing of Effective Length Factor (or k Factor)

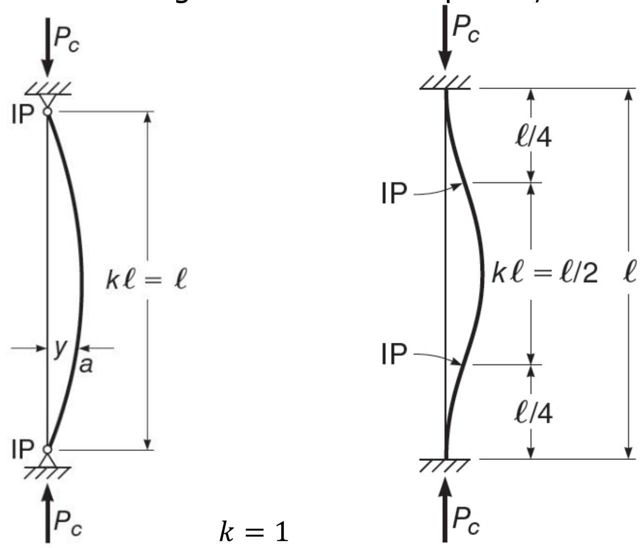
- Meaning of Effective Length Factor (or k Factor):
  - Above Euler relation has been derived originally for simple boundary conditions (i.e. has been derived for a column that has hinge support at both ends). For this column Euler load or (critical load) is:

$$P_c = \frac{\pi^2 EI}{(l_u)^2} \qquad \textbf{Eq. 10.1-13}$$

  - For other boundary conditions, k factor (effective length factor) can be used to transform length of the column under consideration to a length of an equivalent column with both ends are pinned.
  - For example, assume that we intend to compute the critical load for a cantilever column that has 4m height. As cantilever has k = 2 (as will be discussed below), then form buckling analysis point of view, behavior of this cantilever column will be similar to behavior of pinned column with length equal to ($klu = 2x4 = 8m$). Based on this reasoning, one can conclude that:

$$P_{c\ for\ Cantilever\ Column} = \frac{1}{4} of\ P_{c\ for\ the\ pinned\ column\ that\ has\ same\ Length} \qquad \textbf{Eq. 10.1-14}$$

- k Factor for a Isolated Columns with Typical Support Conditions:
  Above transformation of the actual column to an equivalent pinned column is based on the concept of extending or trimming the length of actual column until arriving to the inflection points, see ***Figure 10.1-6*** and ***Figure 10.1-7*** below.



$k = 1$ $\qquad k = \frac{1}{2}$

Figure 10.1-6: Effective length for isolated columns, braced columns.

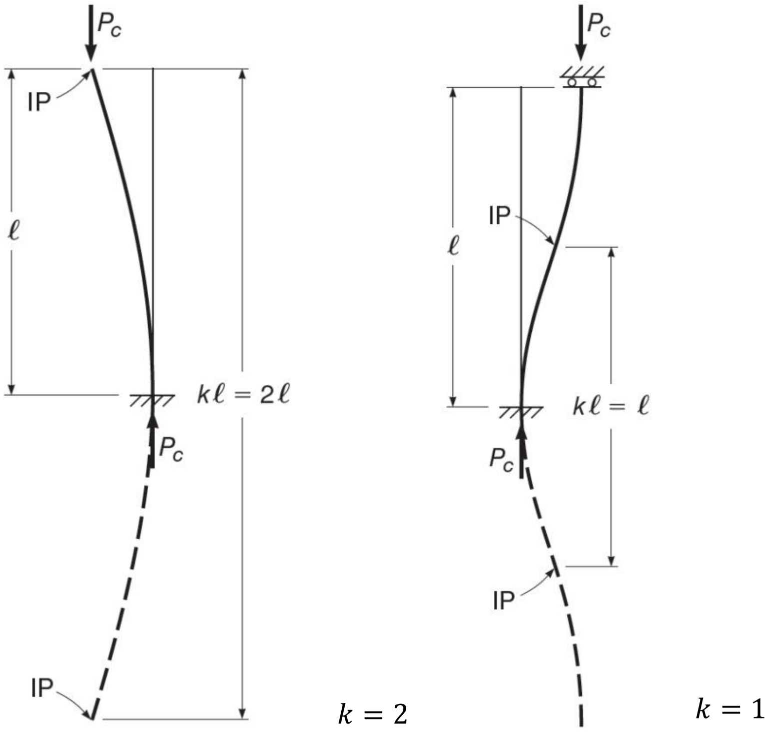
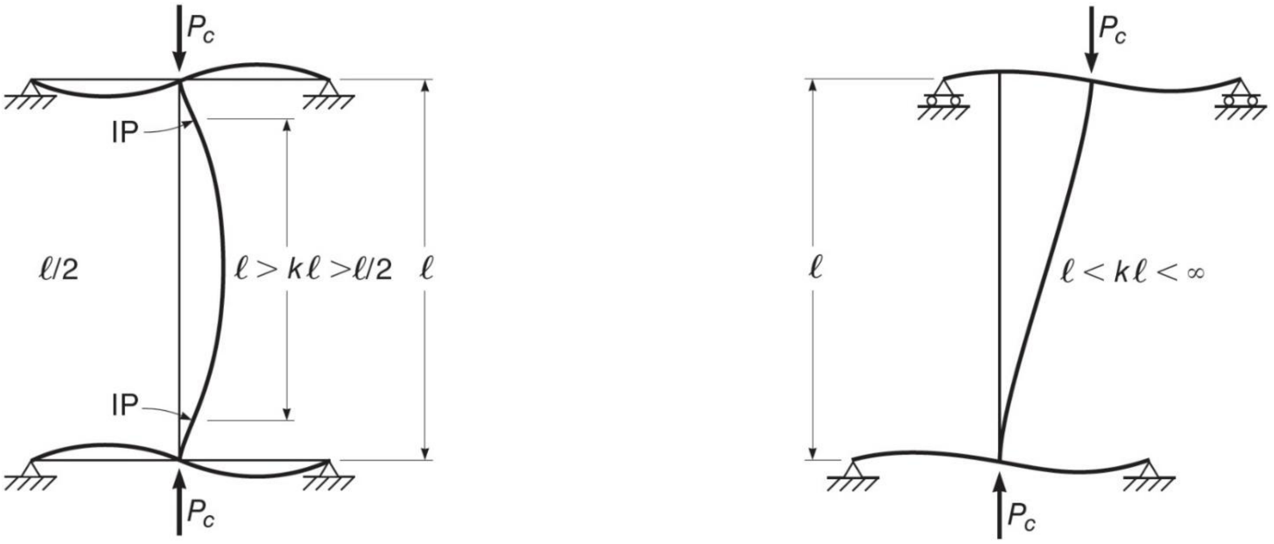


Figure 10.1-7: Effective length for isolated columns, sway columns.

- k Factor for a Column that is Part from a Structure:
  - Columns in real structures are rarely either hinged or fixed but have ends partially restrained against rotation by abutting members. Therefore, the k will be within limits shown in ***Figure 10.1-8*** below.



$\frac{1}{2} < k < 1$ $1 < k < \infty$

(a) Braced frames. (b) Sway frames.

Figure 10.1-8: Effective length factor for columns that are parts from frames.

  - From ***Figure 10.1-8*** above, one concludes that compression members free to buckle in a sway frame are always considerably weaker than when braced against sway.
  - An approximate but generally satisfactory way of determining (k) is by means of ***alignment charts.*** This method can be summarized as follows:
    - Compute the degree of end restraint at each end based on the following relation:

$$\psi = \frac{\sum \frac{EI}{l}_{Columns}}{\sum \frac{EI}{l}_{Beam}} \qquad \textbf{Eq. 10.1-15}$$

    - Based on ψ and frame classification (braced against sway or not), effective length factor (k) can be computed based on alignment charts of ***Figure 10.1-9*** below.

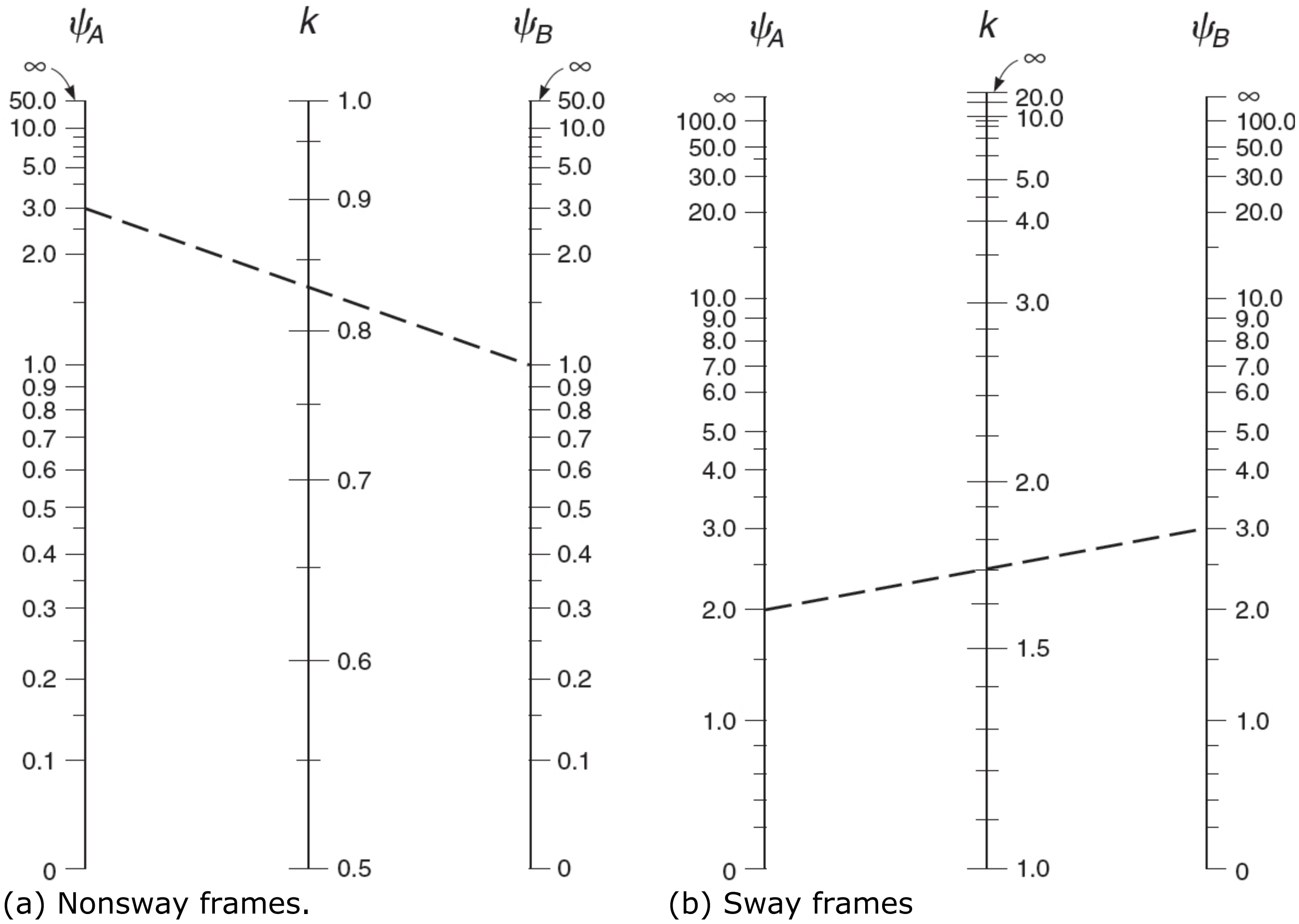


Figure 10.1-9: Alignment charts for effective length factors k.

- ψ for Hinge Support:
  Hinge support can be understood as columns that connected to beams with zero stiffness:

$$\psi_{\text{Hinge}} = \lim_{\frac{EI}{l}_{Beam}\to 0} \frac{\sum \frac{EI}{l}_{Columns}}{\sum \frac{EI}{l}_{Beam}} = \infty \qquad \textbf{Eq. 10.1-16}$$

- In the same approach, ψ_{Fixed} can be interpreted as columns that connected to infinitely rigid beams.

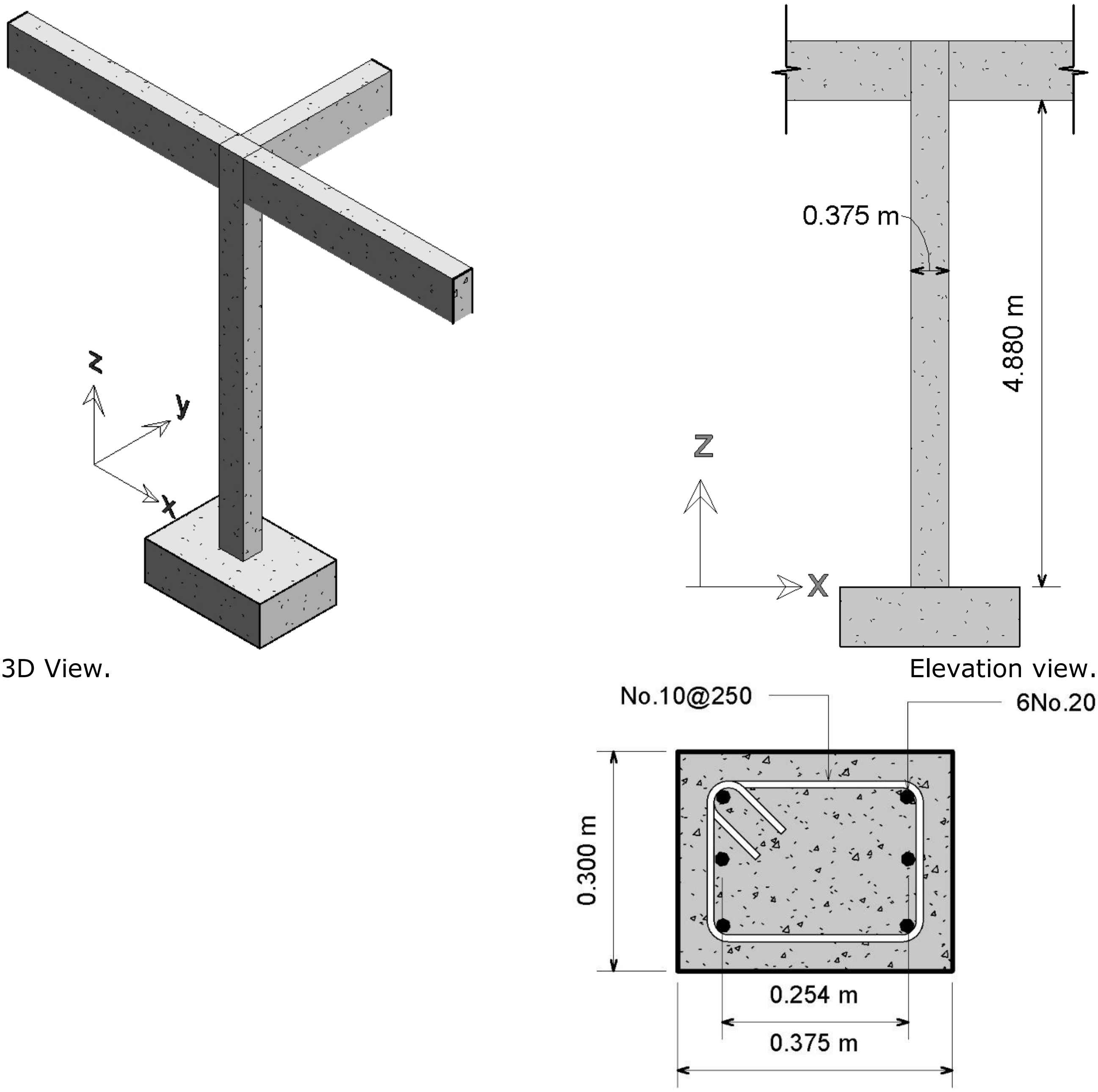
$$\psi_{\text{Fixed}} = \lim_{\frac{EI}{l}_{Beam}\to \infty} \frac{\sum \frac{EI}{l}_{Columns}}{\sum \frac{EI}{l}_{Beam}} = 0 \qquad \textbf{Eq. 10.1-17}$$

10.1.2.6 Analysis Examples for Euler Loads

Example 10.1-2

Using ***Eq. 10.1-10*** to compute critical load about major axis, i.e. buckling in x-z plane, for the column shown in ***Figure 10.1-10*** below. In your solution, assume that:

- k = 0.83.
- f_c' = 28 MPa.
- Assume the sustained load is only 32.7%.
- Column length is 4.88m.



3D View. Elevation view.

Cross sectional of the column

Figure 10.1-10: Column for Example 10.1-2.

Solution

Critical load (or Euler load) can be computed based on following relation:

$$P_c = \frac{\pi^2 EI}{(kl_u)^2}$$

According to example statement, the column shall be adopted to compute EI_{eff}:

$$EI_{\text{eff}} = \frac{0.4E_c I_g}{1+\beta_{dns}}$$

$$E_c = 4700\sqrt{28} = 24870 \text{ MPa}$$

$$I_g = \frac{300 \times 375^3}{12} = 1.32 \times 10^9 \text{mm}^4$$

Based on problem information, $\beta_{dns} = 0.327$.

$$EI_{\text{eff}} = \frac{0.4 \times 24870 \times 1.32 \times 10^9}{1 + 0.327} = 9.89 \times 10^{12}\ N.mm^2$$

Therefore, the critical or Euler load would be:

$$P_c = \left(\frac{\pi^2 \times 9.89 \times 10^{12}\ N.mm^2}{(0.83 \times 4880)^2\ mm^2}\right) \times \frac{1}{1000} = 5944\ \text{kN} \ \blacksquare$$

Example 10.1-3

Resolve ***Example 10.1-2*** above but with determination of EI_{eff} based on ***Eq. 10.1-11*** to take reinforcement into account.

Solution

According to ***Eq. 10.1-11***, EI_{eff} would be:

$$EI_{eff} = \frac{0.2E_cI_g + E_sI_{se}}{1 + \beta_{dns}}$$

As discussed in ***Chapter 2***,

$E_s = 200000\ MPa$

While based on ***Parallel-Axis Theorem*** of Engineering Mechanics,

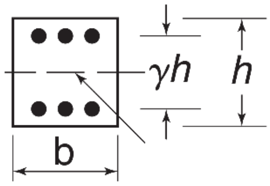
$I_y = \bar{I}_{y'} + Ad^2$

The centroidal moment of inertia for each bar, $\bar{I}_{y'}$, is so small and can be neglected in general:

$\bar{I}_{y'} \approx 0$

$$I_{se} = I_y \approx Ad^2 = \left((314 \times 3) \times \left(\frac{254}{2}\right)^2\right) \times 2 = 0.0304 \times 10^9\ mm^4$$

It can also be determined directly with refereeing to ***Table 10.1-1*** where I_{se} for the indicated case



$$I_{se} = 0.25A_{st}(\gamma h)^2 = 0.25 \times (314 \times 6) \times 254^2 = 0.0304 \times 10^9\ mm^4$$

$$EI_{\text{eff}} = \frac{(0.2 \times 24870 \times 1.32 \times 10^9) + (200000 \times 0.0304 \times 10^9)}{1 + 0.327}$$

$$= 9.53 \times 10^{12}\ N.mm^2$$

$$P_c = \left(\frac{\frac{\pi^2 \times 9.53 \times 10^{12}}{(0.83 \times 4880)^2}}{1000}\right) = 5733\ \text{kN} \ \blacksquare$$

Example 10.1-4

For frame indicated in ***Figure 10.1-11*** below:

- Using the alignment chart of ***Figure 10.1-9***, calculate the effective length factor for column AB of the braced frame shown below.
- Compute the slenderness ratio of column AB.

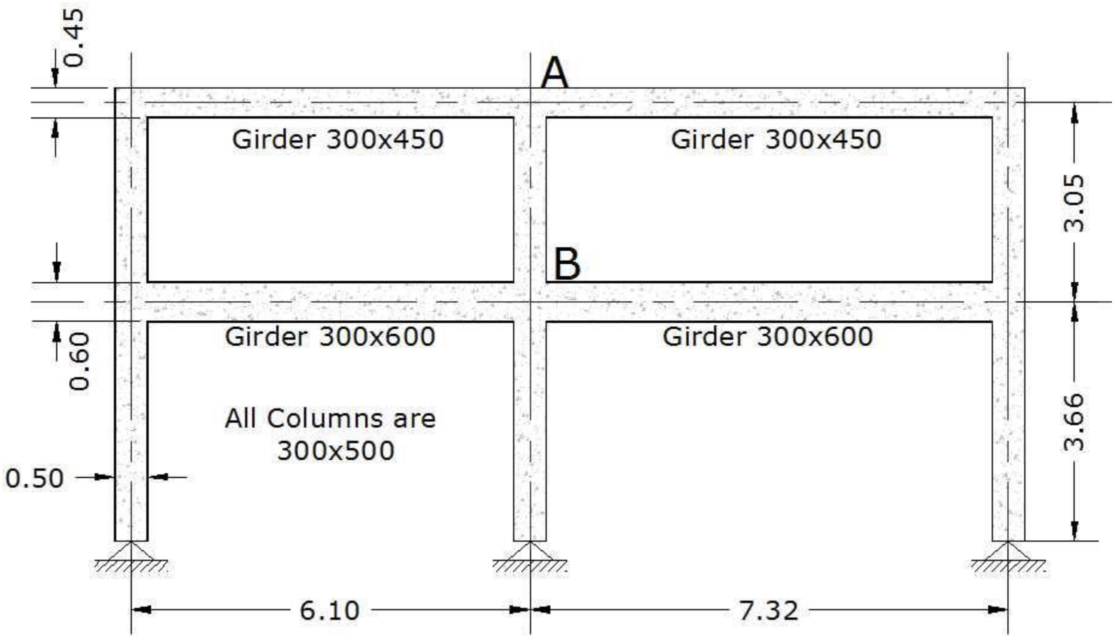


Figure 10.1-11: Frame for Example 10.1-4.

Solution

- Effective Length Factor:

$$\psi = \frac{\sum \frac{EI}{L}_{Columns}}{\sum \frac{EI}{L}_{Beams}}$$

As E_c for columns and beams can be assumed equal, then:

$$\psi_A = \frac{\sum \frac{I}{L_{Columns}}}{\sum \frac{I}{L_{Beams}}}$$

As would be discussed later, according to ACI, I for columns can be taken as $0.7I_g$ and for beams can be taken as $0.35I_g$. These reductions are mainly due to cracking in reinforced concrete.

$$\psi_A = \frac{\frac{0.7 \times \frac{300 \times 500^3}{12}}{3050}}{\frac{0.35 \times \frac{300 \times 450^3}{12}}{6100} + \frac{0.35 \times \frac{300 \times 450^3}{12}}{7320}} = 2.99$$

$$\psi_B = \frac{\frac{0.7 \times \frac{300 \times 500^3}{12}}{3050} + \frac{0.7 \times \frac{300 \times 500^3}{12}}{3660}}{\frac{0.35 \times \frac{300 \times 600^3}{12}}{6100} + \frac{0.35 \times \frac{300 \times 600^3}{12}}{7320}} =$$

$$\psi_B = \frac{1.34 \times 10^6}{0.568 \times 10^6} = 2.36$$

From braced alignment chart, k = 0.875.

- Slenderness Ratio:

$$\frac{kl_u}{r} = \frac{0.875 \times (3.05 - \frac{0.45}{2} - \frac{0.6}{2})}{0.3 \times 0.5} = 14.7$$

Example 10.1-5

With referring to ***Figure 10.1-12***, determine the buckling load for the indicated truss-supporting column when it bends about its major axis. Assume braced story and β_{dns} of 0.7.

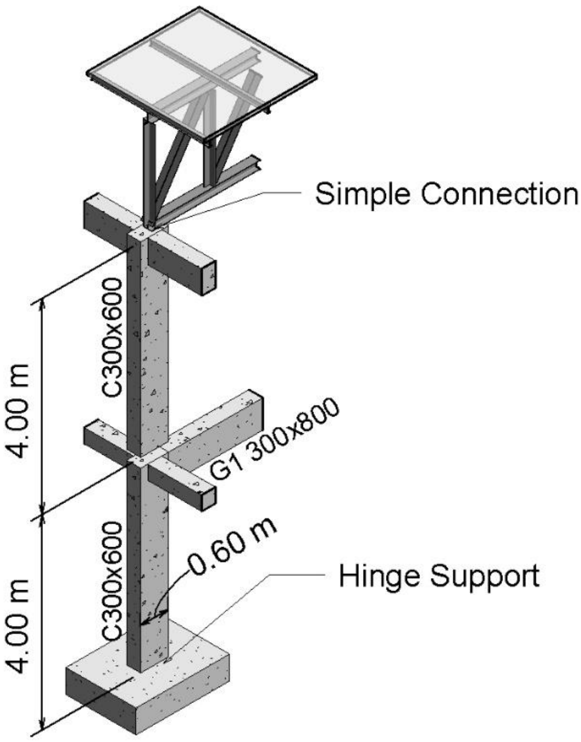


Figure 10.1-12: Truss-supporting column for Example 10.1-5.

Solution

- When column extends from the foundation to the first floor:

According to Euler formula, the buckling load would be:

$$P_c = \frac{\pi^2 EI}{(kl_u)^2}$$

With no information regarding column reinforcement, its rigidity, EI, can be estimated based on the following relation:

$$EI_{eff} = \frac{0.4E_cI_g}{1 + \beta_{dns}}$$

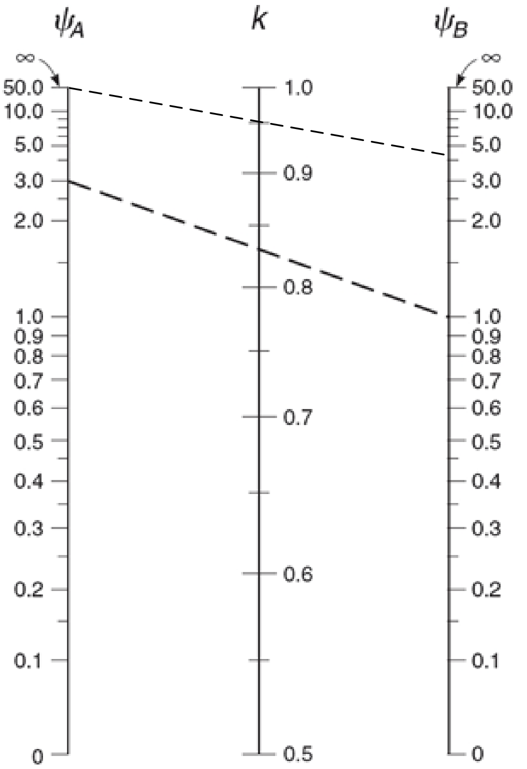
As bending is about the major axis, column moment of inertia, I_g, would be as indicated in below:

$$EI_{eff} = \frac{0.4 \times (4700 \times \sqrt{28}) \times \frac{300 \times 600^3}{12}}{1 + 0.7} = 31.5 \times 10^{12}\ N.mm^2$$

The effective length factor, k, can be determined based on the alignment chart for the braced frame:

$$\psi_{lower} = \psi_{hinge} = \lim_{\frac{EI}{l}_{Beam} \to 0} \frac{\sum \frac{EI}{l}_{Columns}}{\sum \frac{EI}{l}_{Beam}} = \infty$$

$$\psi_{upper} = \psi_A = \frac{\sum \frac{I}{L}_{Columns}}{\sum \frac{I}{L}_{Beams}} = \frac{\dfrac{0.7 \times \frac{300 \times 600^3}{12}}{\left(4000 - \frac{800}{2}\right)} \times 2}{\dfrac{0.35 \times \frac{300 \times 800^3}{12}}{9000}}$$
$$= 4.22$$

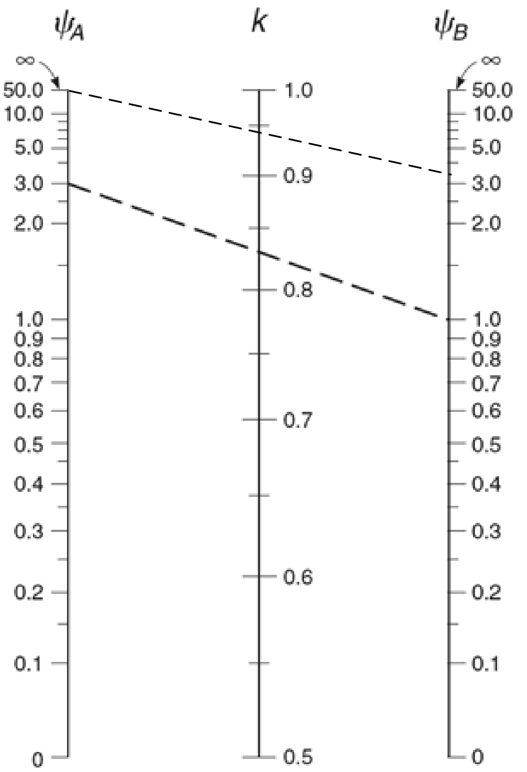


From the alignment chart, one concludes that the effective length factor is:
$k \approx 0.92$
Finally, the unsupported length, l_u, for the column would be:
$l_u = 4000 - 800 = 3200\ mm$
Therefore, the buckling load would be:
$$P_c = \left(\frac{\pi^2 \times (31.5 \times 10^{12})}{(0.95 \times 3200)^2}\right) \times \frac{1}{1000} = 33641\ kN \blacksquare$$

- When column extends from to the first floor to the roof:

$$\psi_{upper} = \psi_{pin} = \lim_{\frac{EI}{l}_{Beam} \to 0} \frac{\sum \frac{EI}{l}_{Columns}}{\sum \frac{EI}{l}_{Beam}} = \infty$$

$$\psi_{lower} = \frac{\sum \frac{I}{L}_{Columns}}{\sum \frac{I}{L}_{Beams}} = \frac{\dfrac{0.7 \times \frac{300 \times 600^3}{12}}{\left(4000 + \frac{800}{2}\right)} \times 2}{\dfrac{0.35 \times \frac{300 \times 800^3}{12}}{9000}} = 3.45$$



From the alignment chart, one concludes that the effective length factor is:
$k \approx 0.94$
$l_u = 4000\ \text{mm}$
$$P_c = \left(\frac{\pi^2 \times (31.5 \times 10^{12})}{(0.94 \times 4000)^2}\right) \times \frac{1}{1000} = 21990\ kN \blacksquare$$

Example 10.1-6

With referring to ***Figure 10.1-13***, determine the buckling load for the indicated truss-supporting column when it bends about its major axis. Assume braced story and β_{dns} of 0.7.

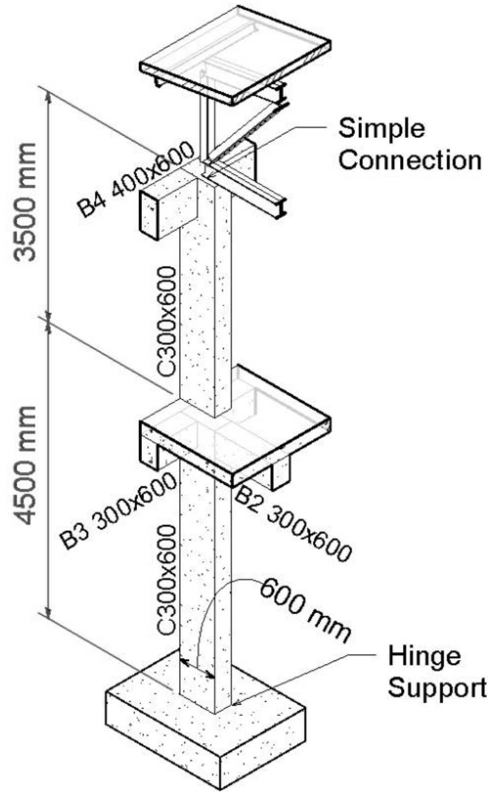


Figure 10.1-13: Truss-supporting column for Example 10.1-6.

Solution

- When column extends from the foundation to the first floor:

  According to Euler formula, the buckling load would be:

  $$P_c = \frac{\pi^2 EI}{(kl_u)^2}$$

  With no information regarding the column reinforcement, its rigidity, EI, can be estimated based on the following relation:

  $$EI_{eff} = \frac{0.4E_cI_g}{1+\beta_{dns}}$$

  As bending is about the major axis, column moment of inertia, I_g, is:

  $$EI_{eff} = \frac{0.4 \times \left(4700 \times \sqrt{28}\right) \times \frac{300 \times 600^3}{12}}{1+0.7}$$
  $$= 31.5 \times 10^{12}\ N.mm^2$$

  The effective length factor, k, can be determined based on the alignment chart for the braced frame:

  $$\psi_{lower} = \psi_{hinge} = \lim_{\frac{EI}{l}_{Beam} \to 0} \frac{\sum \frac{EI}{l}_{Columns}}{\sum \frac{EI}{l}_{Beam}} = \infty$$

  $$\psi_{upper} = \psi_A = \frac{\sum \frac{I}{L}_{Columns}}{\sum \frac{I}{L}_{Beams}} = \frac{\dfrac{0.7 \times \frac{300 \times 600^3}{12}}{\left(4500 - \frac{600}{2}\right)} + \dfrac{0.7 \times \frac{300 \times 600^3}{12}}{\left(3500 + \frac{600}{2}\right)}}{\dfrac{0.35 \times \frac{300 \times 600^3}{12}}{6000}} = 6.0$$

  From the alignment chart, the effective length factor is:

  $k \approx 0.96$

  Finally, the unsupported length, l_u, for the column would be:

  $l_u = 4500 - 600 = 3900\ mm$

  Therefore, the buckling load would be:

  $$P_c = \frac{\pi^2 EI}{(kl_u)^2} = \left(\frac{\pi^2 \times (31.5 \times 10^{12})}{(0.96 \times 3900)^2}\right) \times \frac{1}{1000} = 22179\ kN\ \blacksquare$$

- When column extends from to the first floor to the roof:

  $$\psi_{upper} = \psi_{pin} = \lim_{\frac{EI}{l}_{Beam} \to 0} \frac{\sum \frac{EI}{l}_{Columns}}{\sum \frac{EI}{l}_{Beam}} = \infty$$

  $$\psi_{lower} = \frac{\sum \frac{I}{L}_{Columns}}{\sum \frac{I}{L}_{Beams}} = \frac{\dfrac{0.7 \times \frac{300 \times 600^3}{12}}{\left(4500 - \frac{600}{2}\right)} + \dfrac{0.7 \times \frac{300 \times 600^3}{12}}{\left(3500 + \frac{600}{2}\right)}}{\dfrac{0.35 \times \frac{300 \times 600^3}{12}}{6000}} = 6.0$$

  From the alignment chart, the effective length factor is:

  $k \approx 0.96$,

  $$P_c = \left(\frac{\pi^2 \times (31.5 \times 10^{12})}{(0.96 \times 3500)^2}\right) \times \frac{1}{1000} = 27538\ kN\ \blacksquare$$

10.1.3 Effects of Slenderness on a Column Subjected to a Compression Force and a Moment

- Most reinforced concrete compression members are also subject to simultaneous flexure, caused by transverse loads or by end moments owing to continuity.
- The behavior of members subject to such combined loading also depends greatly on their slenderness.

10.1.3.1 Columns Bent into a Single Curvature

Figure 10.1-14 below shows such a member, axially loaded by P and bent by equal end moments M_e.

- If no axial load were present, the moment M_0 in the member would be constant throughout and equal to the end moments M_e. In this situation, i.e., in simple bending without axial compression, the member deflects as shown by, the dashed curve of ***Figure 10.1-14***.
- When P is applied, the moment at any point increases by an amount equal to P times its lever arm. Then the maximum bending moment due to simultaneous action of flexure and axial force will be:

  $M_{\max} = M_0 + P\Delta$ **Eq. 10.1-18**
- It can be shown that above summation can be re-written in the following multiplication:

  $$M_{\max} = M_0 \times \frac{1}{1 - \frac{P}{P_c}}$$ **Eq. 10.1-19**

  where

  $$\frac{1}{1 - \frac{P}{P_c}}$$

  is known as the ***Moment Magnification Factor***, which reflects the amount by which the moment M_0 is magnified by the presence of simultaneous axial force P.

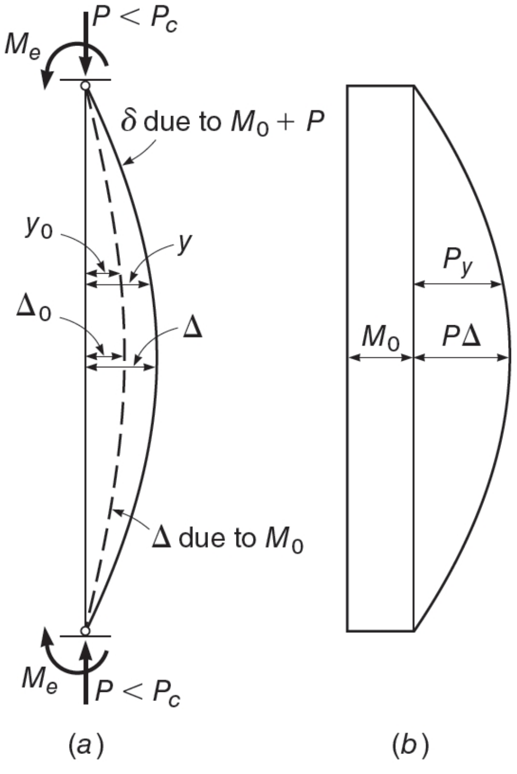


Figure 10.1-14: Moments in slender members with compression plus bending, bent in single curvature.

10.1.3.2 Columns Bent into Double Curvatures

- It clear that the simultaneous effect of the axial force P on the moment magnification for that column shown in ***Figure 10.1-15*** below that has double curvature is ***less than its effect on the moment magnification for a column that has single curvature*** (as the column shown ***Figure 10.1-14*** above).