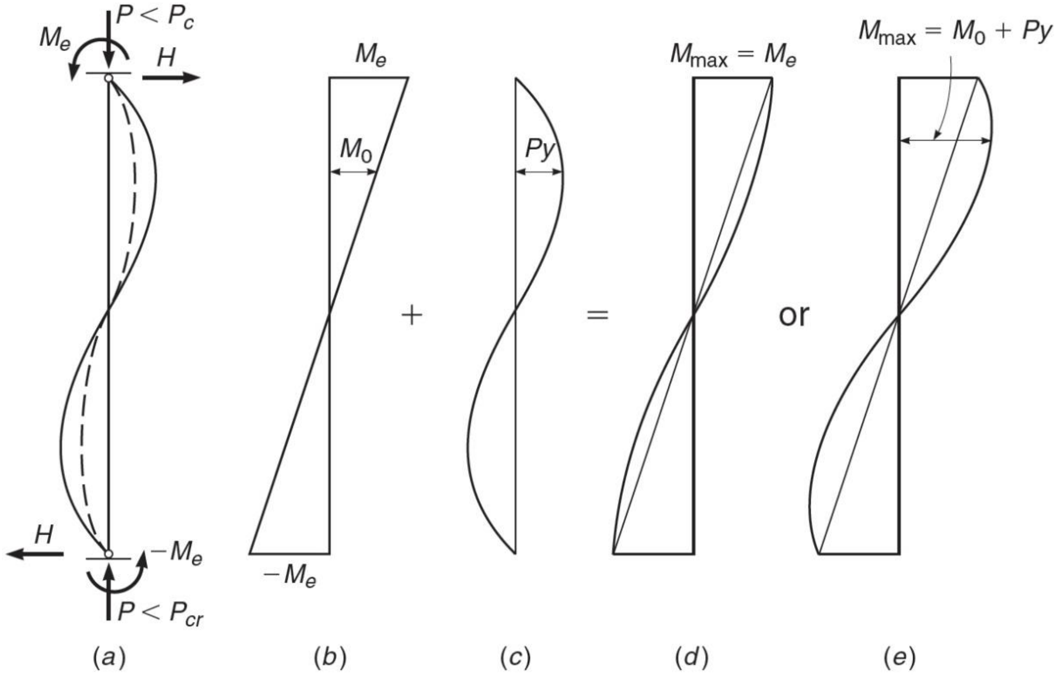



**Figure 10.1-15: Moments in slender members with compression plus bending, bent in double curvature.**

- Then the moment magnification factor
$$\frac{1}{1-\frac{P}{P_c}}$$
must be modified to be able to represent the difference between a column that has single curvature and a column that has double curvature.
- This can be done by including the $C_m$ factor:
$$Moment\ Magnification\ Factor\ = \frac{C_m}{1-\frac{P}{P_c}}$$ **Eq. 10.1-20**
where $C_m$ factor can be computed as follows (***ACI Code 6.6.4.5.3***), see ***Figure 10.1-16*** below
- It is very useful to note that ***in addition to its direct effect on the strength of an axially loaded column, Euler or Bucking load*** $\mathrm{P_c}$ ***has an indirect effect on the strength of a column subjected to an axial force and bending moment through its effect on the moment magnification factor***.

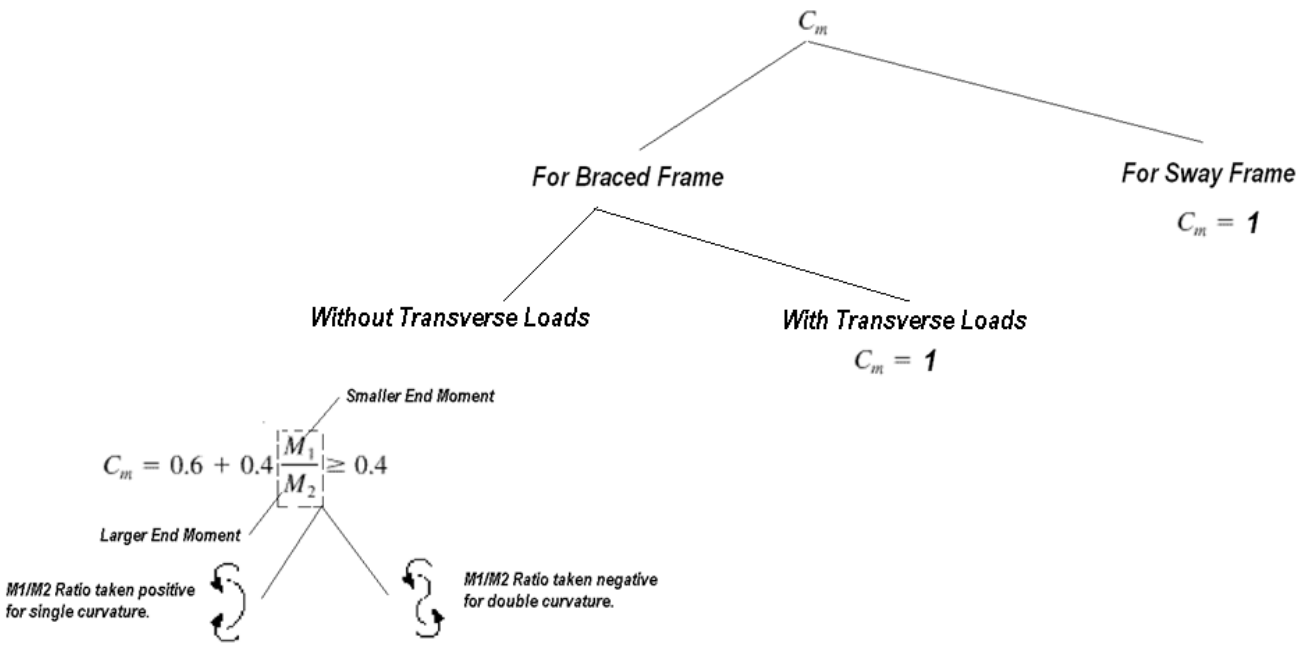



**Figure 10.1-16: Computing factor $C_m$ according to Article 6.6.4.5.3 of the code.**

## 10.2 ACI STRATEGIES FOR SLENDER COLUMNS

This article summary ACI strategy to deal with slender columns based on following steps:

- First step in ACI strategy:
  - A criterion should be imposed to check if the effect of slenderness is important and should be included or it is minor and can be neglected (***Article 6.2.5*** of the ACI Code). See ***Article 10.2.2*** below.
  - As this criterion depends on pre-classification of building into ***sway*** or ***non-sway*** building, then this article include a general guide to classify the building into sway or non-sway, ***Article 6.6.4*** of the code or ***Article 10.2.3*** below .
- Second step in ACI strategy:
  - When the effect of slenderness is classified important according to above first step, ACI offers following ***three different methods*** indicated in ***Figure 10.2-1*** below to compute this effect.
  - ***First and second methods are out the scope of our course***. Therefore, ***the course will focus on the third method only*** (***moment magnification method***).
  - According to ACI Code (***6.2.6***), ***total moment including secondary moment shall not exceed 1.4 times the main moment that compute based on first order analysis*** (i.e., the analysis that based on undeformed shape). According to this limitation, ACI Code considers secondary moments that have values greater that 40% of the corresponding main moments as an indication on the instability of the building.

1. Elastic Second Order Analysis
   Article 6.7 of the code

2. Inelastic Second Order Analysis
   Article 6.8 of the code

In these two methods, equilibrium equations are written in terms of the deflected shape. Therefore, the computed moments already include the secondary effects and they can be adopted directly in the design process.

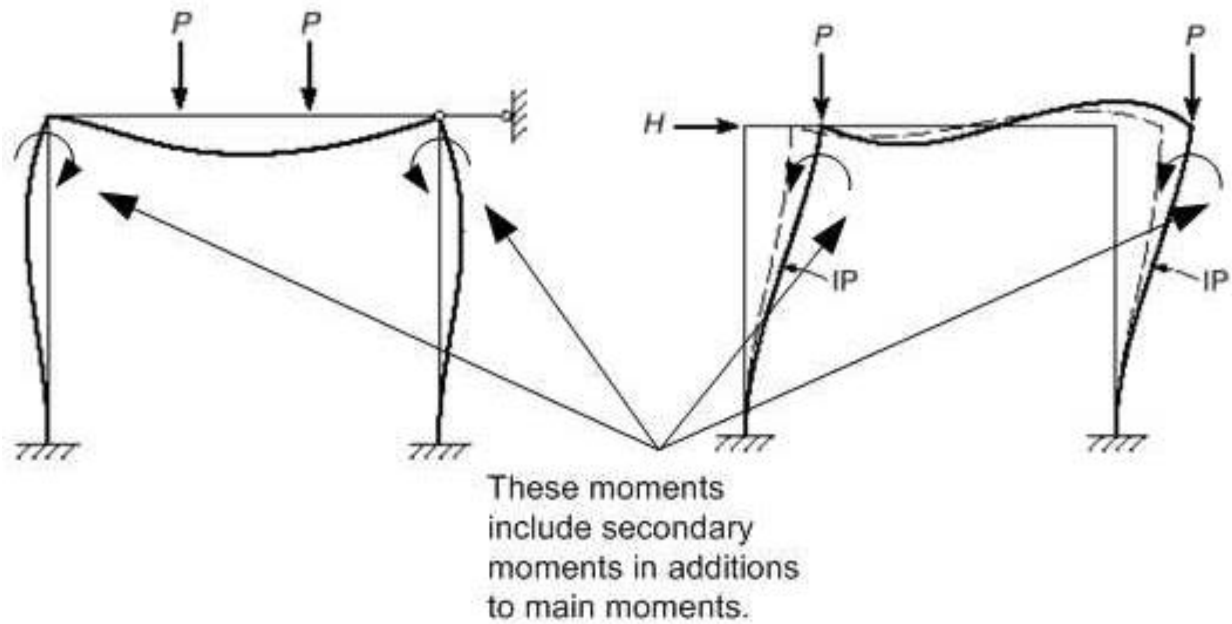



**Figure 10.2-1: ACI procedures to deal with the secondary moments in slender columns.**

Moment Magnification Method
Article 6.6 of the code

In this method, equilibrium equations are written in terms of the un-deflected shape. Therefore, the computed moments include the main moments only. These moment should be magnified with appropriate factors to include the secondary effects to be adopted in the design process.

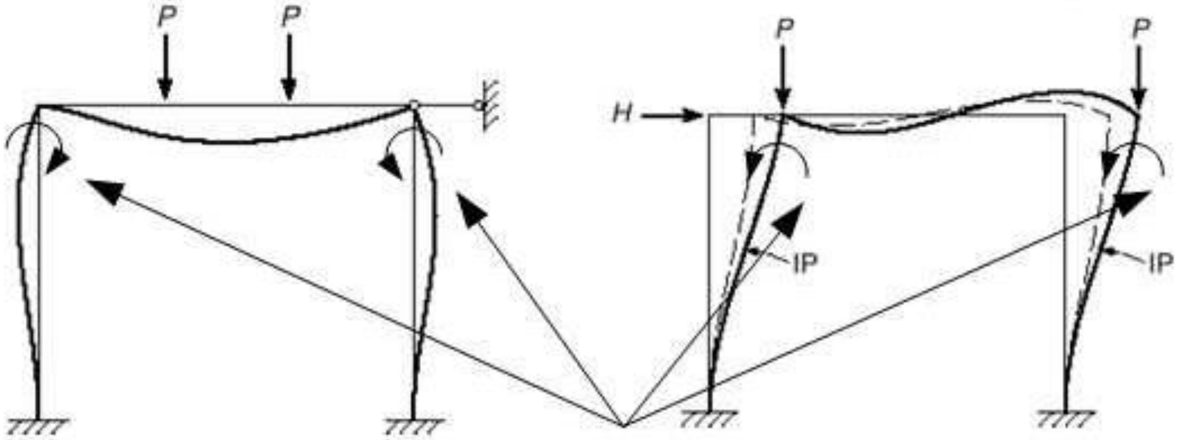



**Figure 10.2-1: ACI procedures to deal with the secondary moments in slender columns. Continued.**

- As it is clear from above discussions of ACI methods, ACI strategy focuses on the effects of slenderness on columns that subjected to an axial force and uniaxial moment. Columns that subjected to concentric load will be treated indirectly to predicate the effect of slenderness (this will be discussed in ***Article 10.3***).

### 10.2.1 ACI Procedure in a Flowchart Form

Aforementioned discussed procedures to deal with secondary effects of column slenderness have been summarized in a flowchart form as indicated in ***Figure 10.2-2*** below.

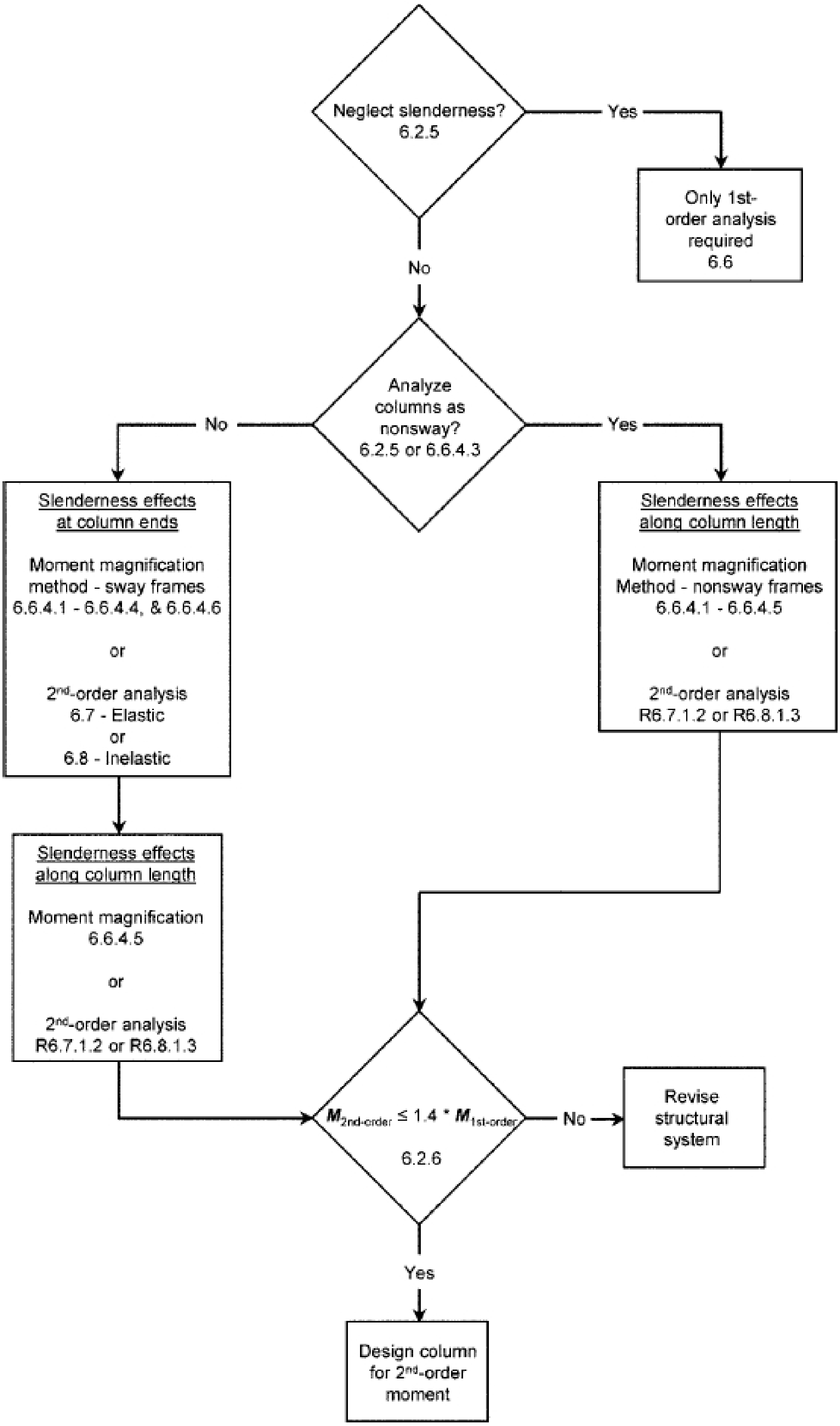



**Figure 10.2-2: Flowchart for determining column slenderness effects.**

### 10.2.2 ACI Criteria for Neglecting of Slenderness Effects

- To permit the designer to dispense with the complicated analysis required for slender column design for the ordinary cases in which the Slenderness Effect can be neglected, ***ACI Code (6.2.5)*** provides limits below which the effects of slenderness are insignificant and may be neglected.
- These limits are adjusted to result in a ***maximum unaccounted reduction in column capacity of no more than 5 percent***.
- Separate limits are applied to braced and unbraced frames. The Code provisions are as follows:
- For compression members in ***nonsway frames***, the effects of slenderness may be neglected when:

$$\frac{kl_u}{r} \leq 34 - 12\frac{M_1}{M_2} \qquad \textbf{Eq. 10.2-1}$$

where

$$34 - 12\frac{M_1}{M_2} \leq 40 \qquad \textbf{Eq. 10.2-2}$$

- For compression members in ***sway frames***, the effects of slenderness may be neglected when:

$$\frac{kl_u}{r} \leq 22 \qquad \textbf{Eq. 10.2-3}$$

- In these provisions:
  - k is the effective length factor (computed as Discusses in ***Article 10.1.2.5***).
  - $l_u$ is the unsupported length, taken as the clear distance between floor slabs, beams, or other members providing lateral support.
  - $M_1$ is the smaller factored end moment on the compression member.
  - $M_2$ is the larger factored end moment on the compression member.
  - Sign for Ratio $\frac{M_1}{M_2}$ is determined based on ***Figure 10.2-3*** below.


**Figure 10.2-3: Sign convention for the ratio $M_1/M_2$.**

**Example 10.2-1**

For a hall-braced frame shown in ***Figure 10.2-4*** below, classify indicated column into short or slender column when:

- Working in xz plane.
- Working in yz plane.

In your solution, assume that all foundations are behave as perfect hinges.

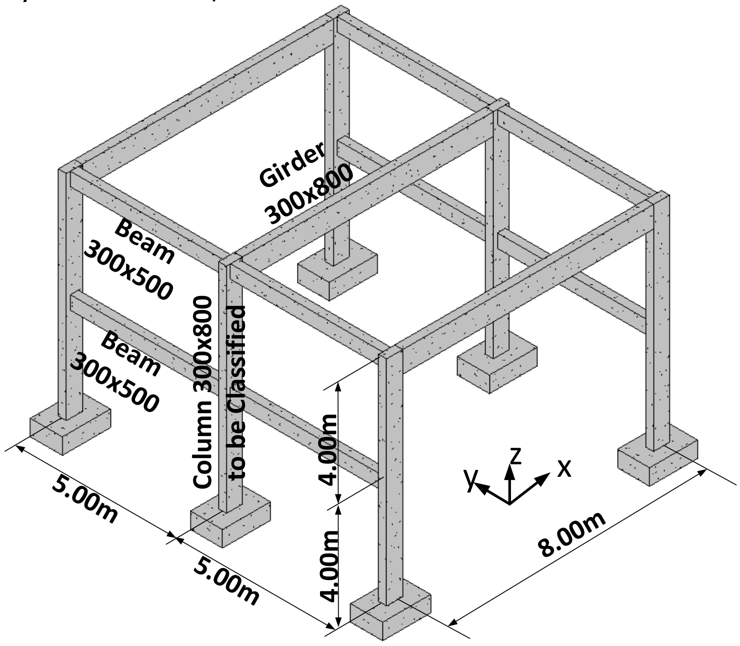



**Figure 10.2-4: Hall-braced Frame of Example 10.2-1.**

**Solutions**

**Working in xz plane**

Effective length factor:

$\psi_{Bottom} \approx \infty$

$$\psi_{Top} = \frac{\Sigma \frac{EI}{L}_{Columns}}{\Sigma \frac{EI}{L}_{Beams}} = \frac{\frac{0.7 \times \frac{0.3 \times 0.8^3}{12}}{8 - \frac{0.8}{2}}}{\frac{0.35 \times \frac{0.3 \times 0.8^3}{12}}{8}} = 2.1$$

Based on alignment chart for braced frame, see ***Figure 10.2-5*** below.

$k = 0.92$

Slenderness Ratio:

$$\frac{kl_u}{r} = \frac{0.92 \times (8.0 - 0.8)}{0.3 \times 0.8} = 27.6$$

ACI Classification:

$$\frac{kl_u}{r} \;?\; 34 - 12\frac{M_1}{M_2} \leq 40$$

With hinge support, $M_1 = 0$:

$$\frac{kl_u}{r} = 27.6 < 34$$

Then column is short.

**Working in yz plane:**

Effective length factor:

With a lower hinge support, column lower part is more critical than upper part.

$\psi_{Bottom} \approx \infty$

$$\psi_{Top} = \frac{\Sigma \frac{EI}{L}_{Columns}}{\Sigma \frac{EI}{L}_{Beams}} = \frac{2 \times \frac{0.7 \times \frac{0.8 \times 0.3^3}{12}}{4}}{2 \times \frac{0.35 \times \frac{0.3 \times 0.5^3}{12}}{5}} = 1.44$$

Based on alignment chart for braced frame, see ***Figure 10.2-5*** below.

$k = 0.88$

<u>Slenderness Ratio:</u>

$$\frac{kl_u}{r} = \frac{0.88 \times (4.0 - 0.5)}{0.3 \times 0.3} = 34.2$$

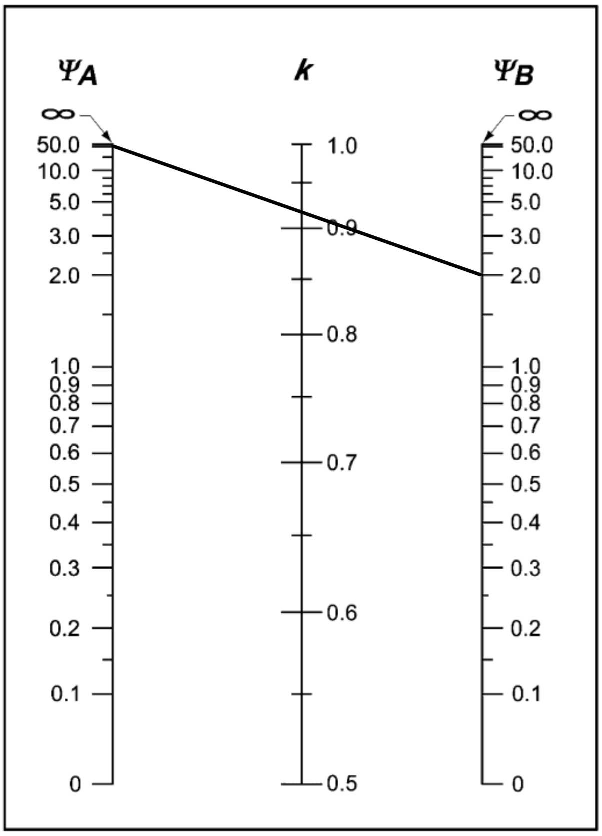
<u>ACI Classification:</u>

$$\frac{kl_u}{r} \;?\; 34 - 12\frac{M_1}{M_2} \le 40$$

With hinge support, $M_1 = 0$:

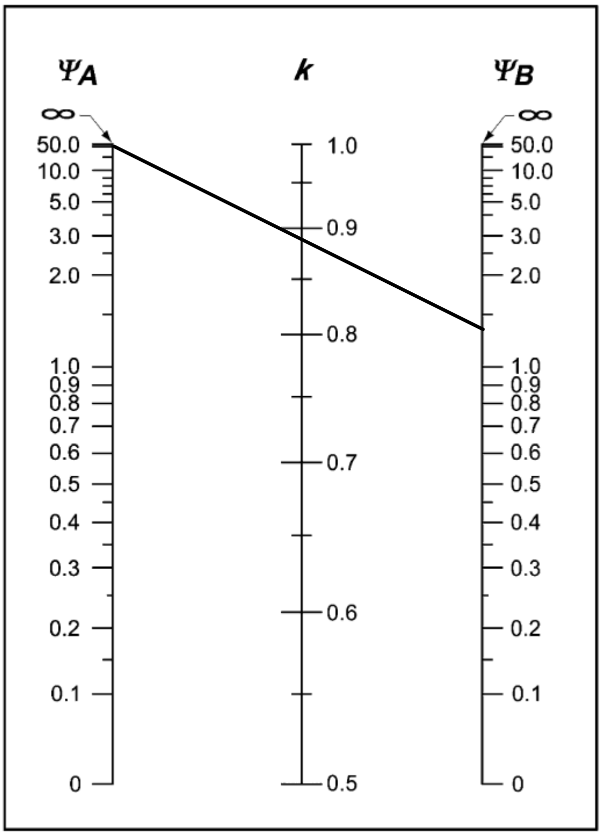
$$\frac{kl_u}{r} = 34.2 > \; 34$$

Then column is slender.




(a)
Nonsway Frames

K factor for working with xz plane.




(a)
Nonsway Frames

K factor for working with yz plane.

**Figure 10.2-5: Effective length factor for Example 10.2-1.**

---

**Example 10.2-2**

For the braced column presented in ***Figure 10.2-6*** below,

- Relative to plane XZ,
  - Is the column classified as short or slender?
  - What is column critical load in this plane?
- Relative to plane YZ, from foundation to first levels,
  - Is the column classified as short or slender?
  - What is column critical load in this plane?

In your solution, assume that footing behaves as a perfect hinge, sustained load is 90% of the total load, the girder has a span of 12m, and beams have spans of 6m.

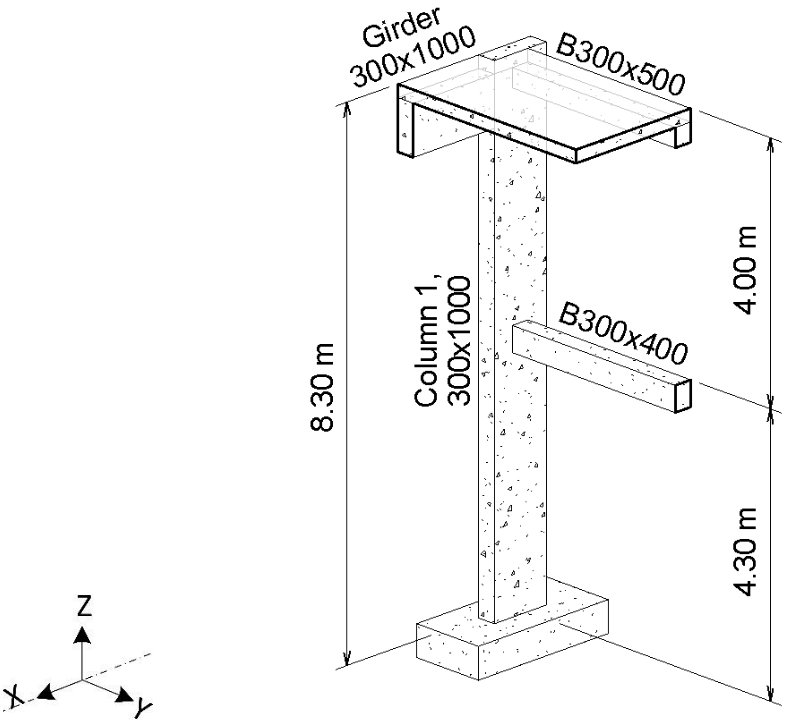



**Figure 10.2-6: Braced column for Example 10.2-2.**

## Solution

Plane XZ;

$\psi_{A\ at\ hinge\ end} = \infty$

$$\psi_{B\ @\ top\ of\ column} = \frac{\dfrac{0.7 \times \dfrac{300 \times 1000^3}{12}}{8300 - \dfrac{1000}{2}}}{\dfrac{0.35 \times \dfrac{300 \times 1000^3}{12}}{12000}} \approx 3.0$$

Then, effective length factor, $k$, is, see ***Figure 10.2-7*** below:

$k = 0.94$

Column unsupported length is,

$l_u = 8000 + 300 - 1000 = 7300\ mm$

Slenderness ratio is,

$$\frac{kl_u}{r} = \frac{0.94 \times 7300}{0.3 \times 1000} = 22.9\ ?\ 34 - 12\frac{M_1}{M_2}$$

With hinge support,

$M_1 = 0$

$$\frac{kl_u}{r} = 22.9 < 34$$

Then the column could be classified as short.

Column critical load is,

$$P_{Critical} = \frac{(\pi^2 EI)}{(kl_u)^2}$$

As nothing is mentioned about reinforcement, then

$$EI = \frac{0.4E_cI_g}{1 + \beta_{dns}} = \frac{0.4 \times (4700 \times \sqrt{28}) \times \dfrac{300 \times 1000^3}{12}}{1 + 0.9} = 131 \times 10^{12}\ N.mm^4$$

$$P_{Critical} = \frac{(\pi^2 \times 131 \times 10^{12})}{(0.94 \times 7300)^2} = 27458\ kN$$

Relative to plane YZ and from foundation to first levels,

$\psi_{A\ at\ hinge\ end} = \infty$

$$\psi_{B\ @\ top\ of\ column} = \frac{\dfrac{0.7 \times \dfrac{1000 \times 300^3}{12}}{4300}}{\dfrac{0.35 \times \dfrac{300 \times 400^3}{12}}{6000}} = 3.9$$

Then, effective length factor, $k$, is, see ***Figure 10.2-7*** below:

$k = 0.95$

Column unsupported length is,

$l_u = 4000 + 300 - 400 = 3900\ mm$

Slenderness ratio is,

$$\frac{kl_u}{r} = \frac{0.95 \times 3900}{0.3 \times 300} = 41.2\ ?\ 34 - 12\frac{M_1}{M_2}$$

With hinge support,

$M_1 = 0$

$$\frac{kl_u}{r} = 41.2 > 34$$

Then the column could be classified as slender. Column critical load is,

$$P_{Critical} = \frac{(\pi^2 EI)}{(kl_u)^2}$$

As nothing is mentioned about reinforcement, then

$$EI = \frac{0.4E_c I_g}{1 + \beta_{dns}} = \frac{0.4 \times (4700 \times \sqrt{28}) \times \frac{1000 \times 300^3}{12}}{1 + 0.9} = 11.8 \times 10^{12}\ N.mm^4$$

$$P_{Critical} = \frac{(\pi^2 \times 11.8 \times 10^{12})}{(0.95 \times 3900)^2} = 8481\ kN$$

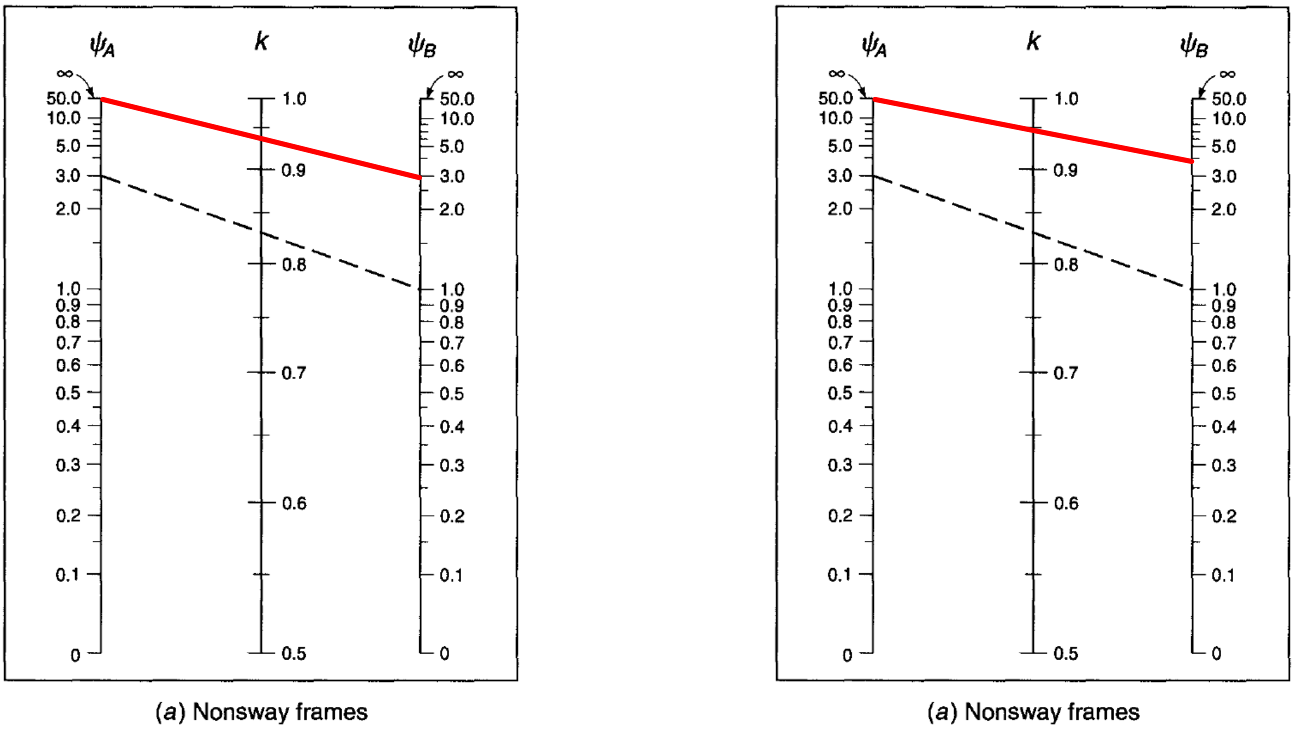



**Figure 10.2-7: Alignment chart applied for Example 10.2-2.**

**Example 10.2-3**

With referring to building of ***Figure 10.2-8*** below, is a typical interior column classified as short or slender when analyzed:

- In a plane along Sec. 1-1.
- In a plane along Sec. 2-2.

In your solution, assume braced building and assume footings behave as hinges.

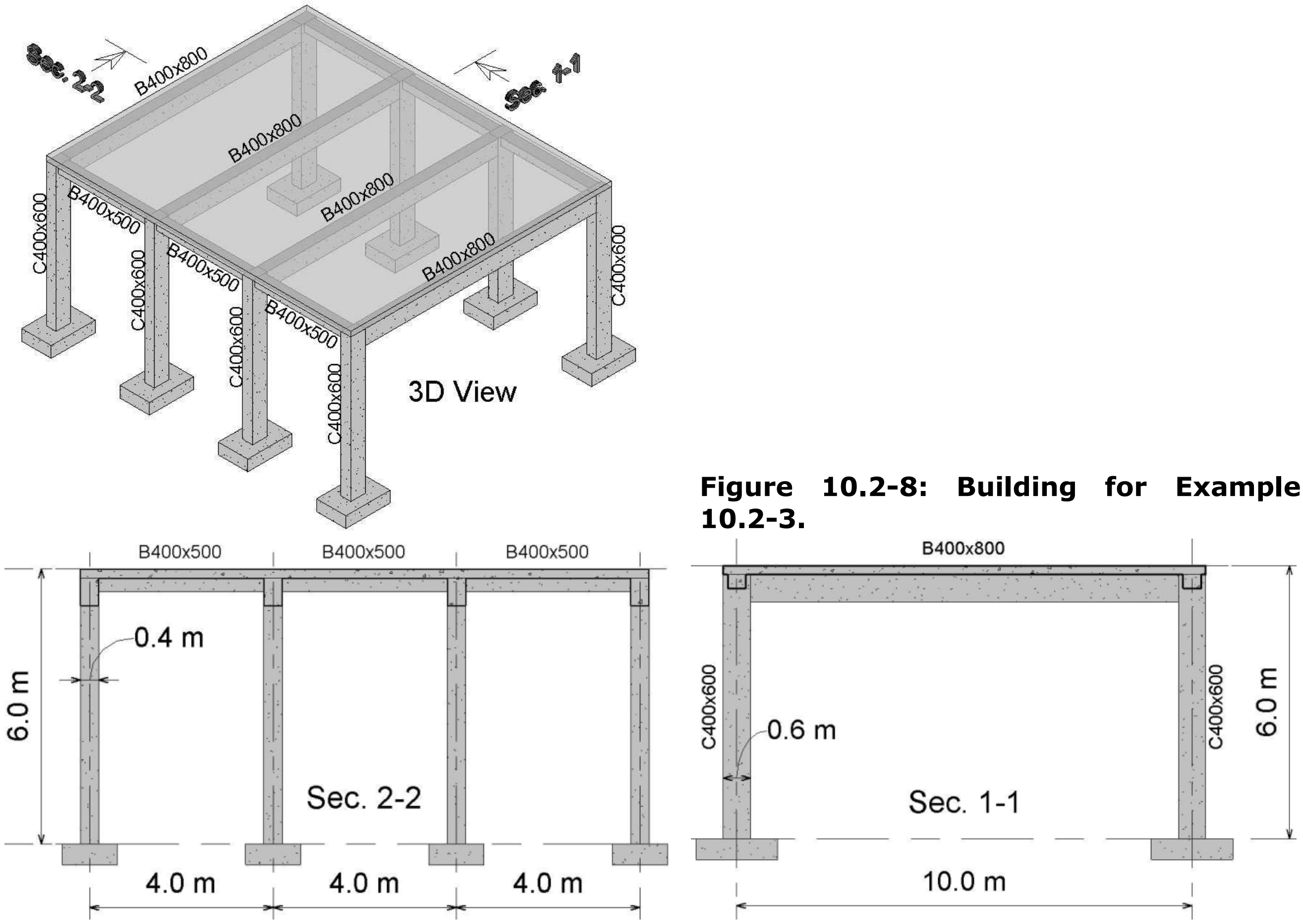



**Figure 10.2-8: Building for Example 10.2-3.**

**SOLUTION**

**IN PLANE ALONG SEC. 1-1**

$\psi_{A\ at\ hinge\ end} = \infty$

$$\psi_{B\ @\ top\ of\ column} = \frac{\dfrac{0.7 \times \dfrac{400 \times 600^3}{12}}{6000 - \dfrac{800}{2}}}{\dfrac{0.35 \times \dfrac{400 \times 800^3}{12}}{10000}} = 1.51$$

Then, effective length factor, $k$, is, see ***Figure 10.2-9*** below:

$k = 0.9$

Column unsupported length is,

$l_u = 6000 - 800 = 5200\ mm$

Slenderness ratio is,

$$\frac{kl_u}{r} = \frac{0.9 \times 5300}{0.3 \times 600} = 26.5\ ?\ 34 - 12\frac{M_1}{M_2}$$

With hinge support,

$M_1 = 0$

$$\frac{kl_u}{r} = 26.2 < 34$$

Then the column could be classified as short.

## IN PLANE ALONG SEC. 2-2

$\psi_{A\ at\ hinge\ end} = \infty$

$$\psi_{B\ @\ top\ of\ column} = \frac{\dfrac{0.7 \times \dfrac{600 \times 400^3}{12}}{\left(6000 - \dfrac{500}{2}\right)}}{2 \times \left(\dfrac{0.35 \times \dfrac{400 \times 500^3}{12}}{4000}\right)} = 0.53$$

Then, effective length factor, $k$, is see ***Figure 10.2-9*** below:

$k = 0.82$

Column unsupported length is,

$l_u = 6000 - 500 = 5500\ mm$

Slenderness ratio is,

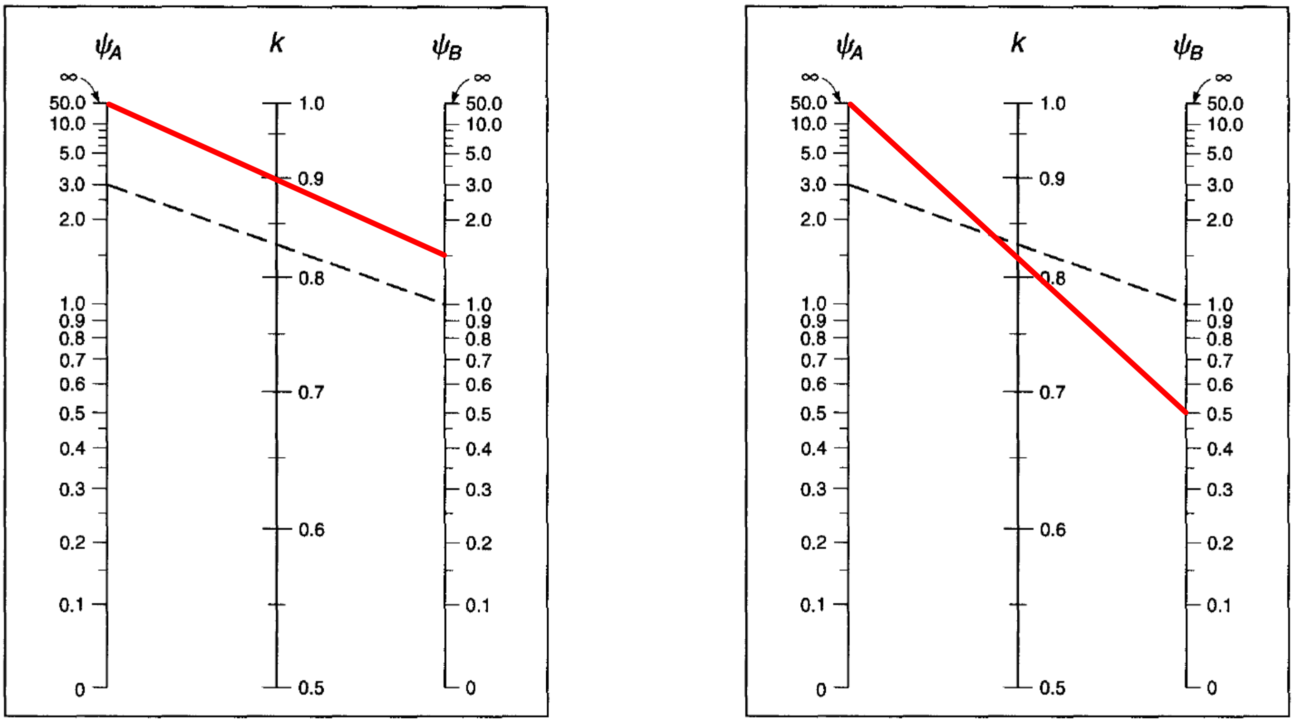
$$\frac{kl_u}{r} = \frac{0.82 \times 5500}{0.3 \times 400} = 37.6\ ?\ 34 - 12\frac{M_1}{M_2}$$

With hinge support,

$M_1 = 0$

$$\frac{kl_u}{r} = 37.6.34$$

The column is classified as slender.




(*a*) Nonsway frames (*a*) Nonsway frames

**Figure 10.2-9: Alignment chart applied for Example 10.2-3.**

### 10.2.3 ACI Criteria for Nonsway versus Sway Frames

- The previous discussion shows clearly significant difference between the behavior of slender columns in nonsway (braced) frames and the corresponding columns in sway (unbraced) frames.
- ACI Code provisions for the approximate design of slender columns reflect this difference and there are separate provisions for nonsway versus sway frames.
- In actual structures, a frame is seldom either completely braced or completely unbraced. It is necessary, therefore, to determine in advance if bracing provided by shear walls, elevator and utility shafts, stairwells, or other elements is adequate to restrain the frame against significant sway effects, see ***Figure 10.2-10***.

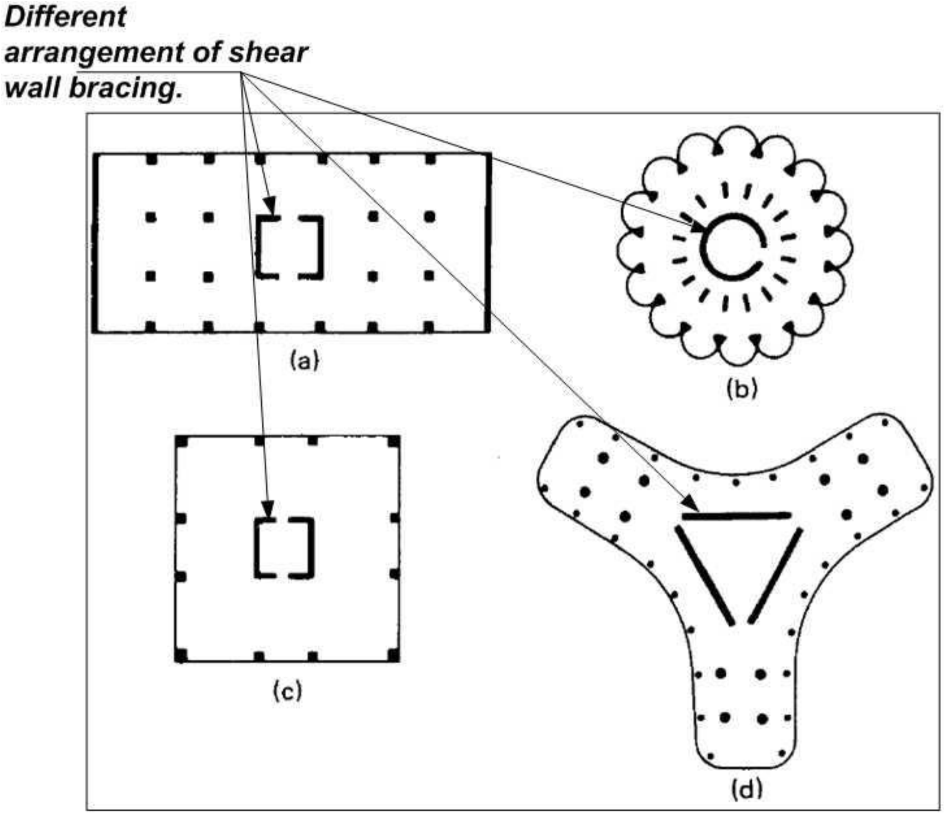



**Figure 10.2-10: Different arrangement of shear walls that may provide frame bracing.**

- Determination of bracing system effectiveness can be executed based any one of the following methods:
  - By Inspection (***ACI Commentary 6.6.4.1***)
    - i. By engineering judgment, the engineer may decide if the stiffness of shear wall or a steel bracing system is adequate to classify the frame under consideration as a braced frame or not.
    - ii. According to ***previous code provisions***, it shall be permitted to consider ***compression members braced against sidesway when bracing elements have a total stiffness, resisting lateral movement of that story, of at least 12 times the gross stiffness of the columns within the story***.
  - Based on Stability Index Concept (ACI code ***Article 6.6.4.3***)
    - i. If the effectiveness of a shear wall or bracing system is questionable, a frame can be classified to a braced or nonbraced based on the concept of ***Stability Index*** which computed as follows, see ***Figure 10.2-11*** below.

$$Stability\ Index = Q = \frac{\sum P_u \Delta_0}{V_u l_c} \quad \textbf{Eq. 10.2-4}$$

where:

$\sum P_u$ and $V_u$ are the total factored vertical load and story shear, respectively, for the story.

$\Delta_0$ is the first-order relative deflection between the top and the bottom of the story due to $\mathrm{V_u}$.

$l_c$ is the length of the compressive member measured center-to-center of the joints in the frame.

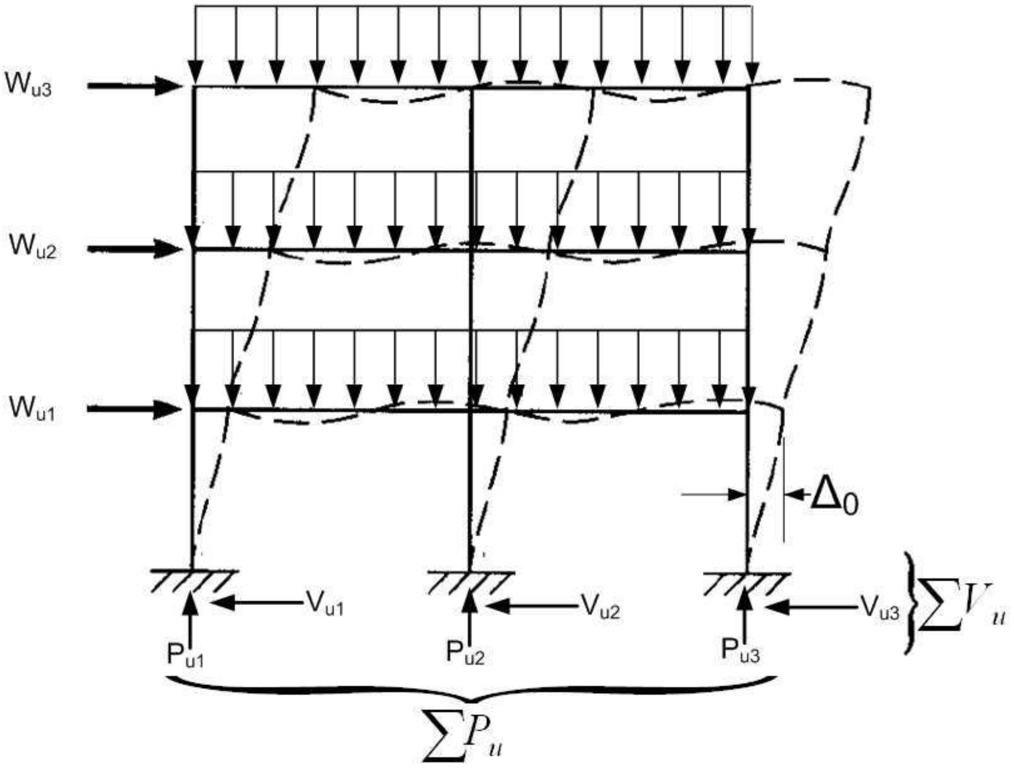



**Figure 10.2-11: Parameters adopted in computing of the Stability Index.**

ii. A _story_ that has a ***stability index not greater 0.05 can be classified as a braced and vice versa***.

iii. According to ***ACI Code 6.6.3.1***, section properties may be represented using the modulus of elasticity, $E$, of:

$E = E_c$

and section properties as indicated in ***Table 10.2-1*** below.

**Table 10.2-1: Moment of inertia and cross sectional area permitted for elastic analysis at factored load level, Table 6.6.3.1.1(a).**

| Member and condition | | Moment of Inertia | Cross-sectional area |
|---|---|---|---|
| Columns | | $0.70I_g$ | $1.0A_g$ |
| Walls | Uncracked | $0.70I_g$ | |
| | Cracked | $0.35I_g$ | |
| Beams | | $0.35I_g$ | |
| Flat plates and flat slabs | | $0.25I_g$ | |

**Example 10.2-4**

For the building frame shown in ***Figure 10.2-12*** below, based on an elastic first order analysis with ACI stiffnesses of ***Table 10.2-1*** above and with neglecting of frame selfweight, lateral deflections have been computed for ground and first stories and summarized in ***Figure 10.2-13*** below. Use ACI ***stability index method*** to classify ground and first stories as braced or sway.

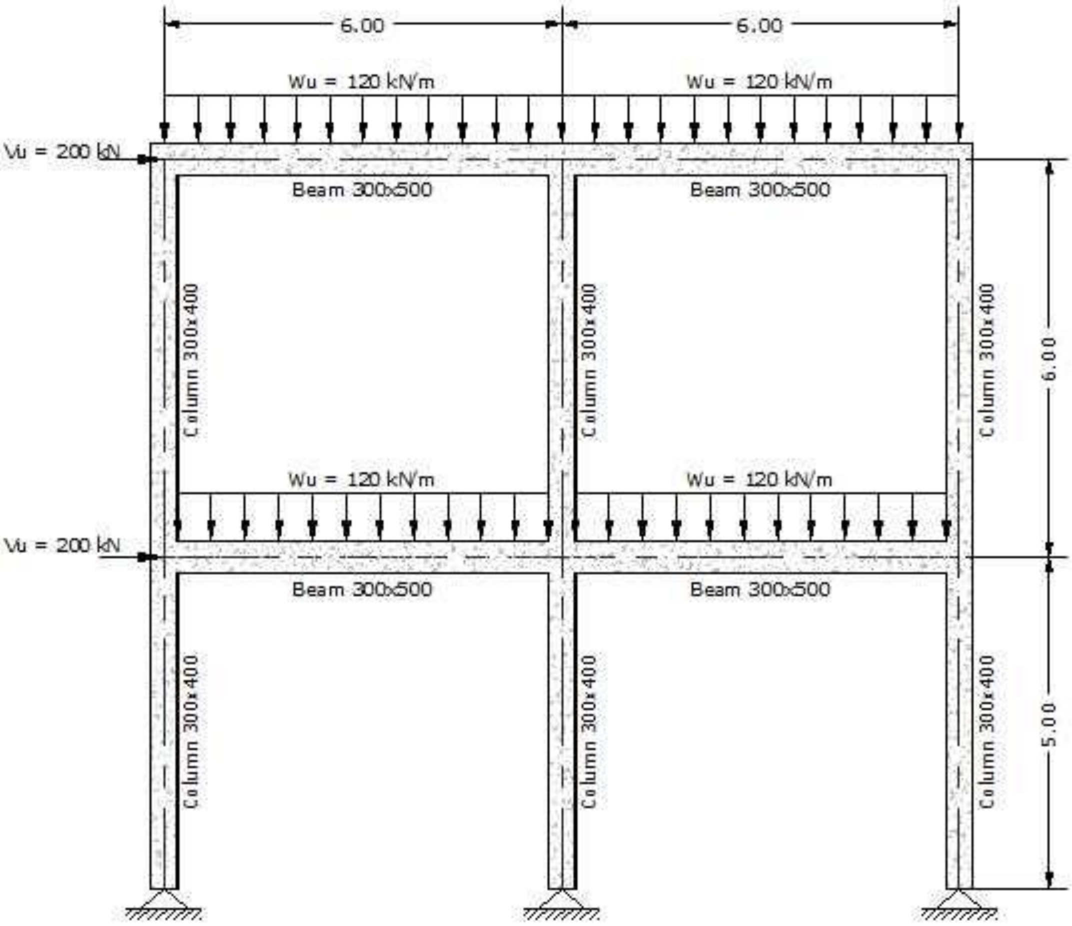



**Figure 10.2-12: Frame for Example 10.2-4.**

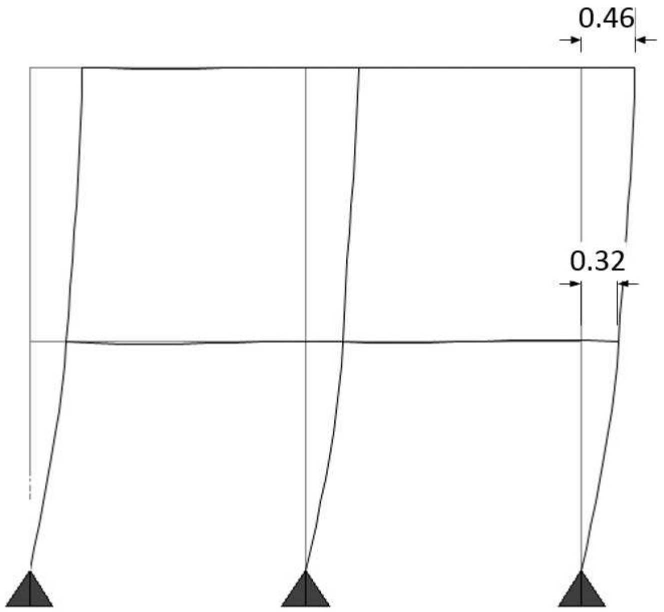



**Figure 10.2-13: Lateral deflections of the ground and first stories for the frame of Example 10.2-4.**

**Solution**

- For first story:

$$Stability\ Index = Q = \frac{\sum P_u \Delta_0}{V_u l_c}$$

$$\sum P_u = \text{the total factored vertical for the story} = 120 \frac{\text{kN}}{\text{m}} \times 6\text{m} \times 2 = 1440\ \text{kN}$$

$V_u =$ total factored story shear $= 200$ kN
$\Delta_0 =$ the first order relative deflection between the top and the bottom of the story due to $V_u$
$\Delta_0 = 0.46 - 0.32 = 0.14$ m
$l_c =$ the length of the compressive member measured center − to − center of the joints in the frame
$l_c = 6$m

$$Stability\ Index = Q = \frac{1440\ kN \times 0.14m}{200\ kN \times 6m} = 0.168 > 0.05$$

Then, the story is unbraced.

- For the ground story:

$$\sum P_u = \text{the total factored vertical for the story} = \left(120\frac{\text{kN}}{\text{m}} \times 6\text{m} \times 2\right) \times 2 = 2880\ \text{kN}$$

$V_u =$ total factored story shear $= 200$ kN $\times 2 = 400$ kN
$\Delta_0 =$ the first order relative deflection between the top and the bottom of the story due to $V_u$
$\Delta_0 = 0.32$ m
$l_c =$ the length of the compressive member measured center − to − center of the joints in the frame
$l_c = 5$m

$$Stability\ Index = Q = \frac{2880\ \text{kN} \times 0.32m}{400\ kN \times 5m} = 0.46 > 0.05$$

Then, the story is unbraced.

---

**Example 10.2-5**

Re-solve ***Example 10.2-4*** above, but when the building is stiffened with a shear wall that shown in ***Figure 10.2-14*** below. the deformation would be as indicated in ***Figure 10.2-15*** below.

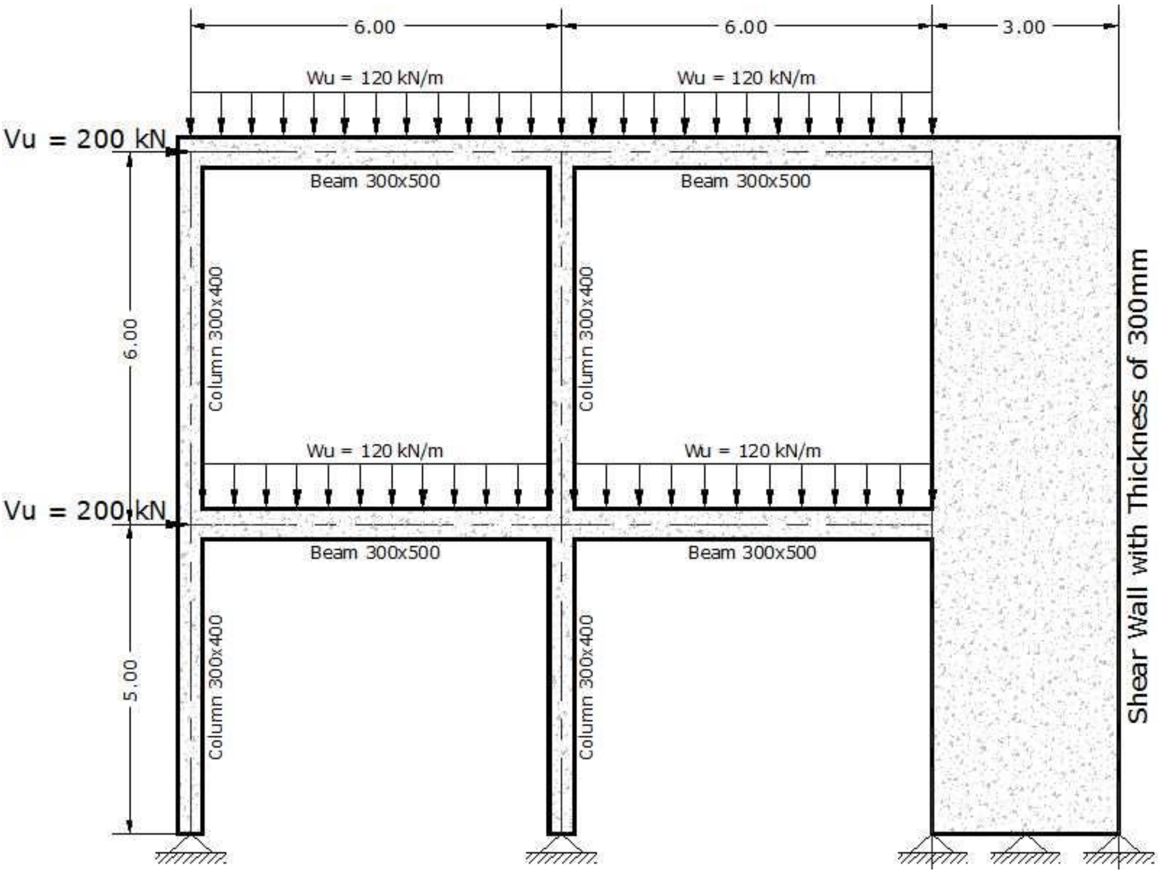



**Figure 10.2-14: Frame for Example 10.2-5.**

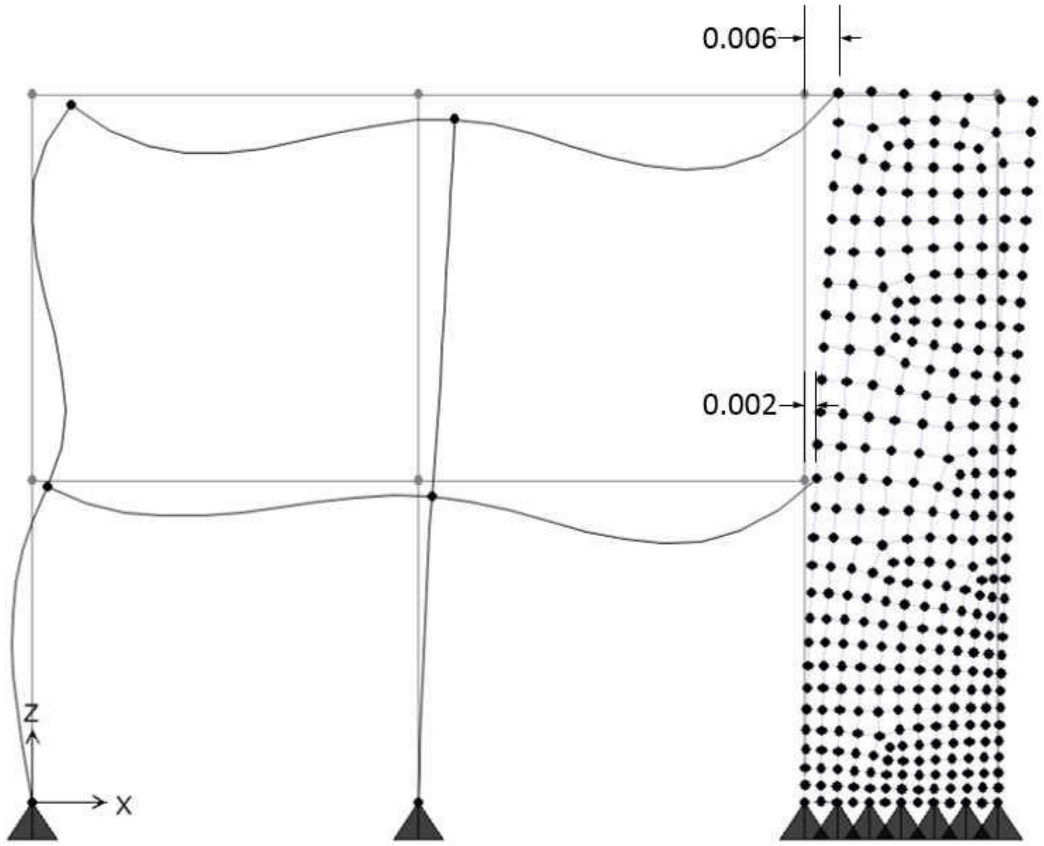



**Figure 10.2-15: Lateral deflections of the ground and first stories for the frame of Example 10.2-5.**

## Solution

- For First Story:

  $\Delta_0 = 0.006 - 0.002 = 0.004\text{ m}$

  $$Stability\ Index = Q = \frac{\sum P_u \Delta_0}{V_u l_c} = \frac{1440\ kN \times .004m}{200\ kN \times 6m} = 0.005 < 0.05$$

  Then, the story is braced.
- Ground Story:

  $\Delta_0 = 0.002\text{ m}$

  $$Stability\ Index = Q = \frac{\sum P_u \Delta_0}{V_u l_c} = \frac{2880\text{ kN} \times 0.002m}{400\ kN \times 5m} = 0.003 < 0.05$$

  Then, the story is braced.

---

## 10.3 SUMMARY OF ACI MOMENT MAGNIFIER METHOD FOR NONSWAY FRAMES

- For a column that may be a slender column in a braced story, ACI checking procedure can be summarized as follows:
  1. Select a trial column section to carry the factored axial load $P_u$ and moment $M_u = M_2$ (where $M_2$ is that maximum moment that occurs at one of column two ends) that computed from a first-order frame analysis (i.e. an analysis that based on undeformed shape and that predicate main moments only), assuming short column behavior and following the procedures of ***Chapter 9***.
  2. Determine if the frame should be considered as nonsway or sway, either based on intuition or based on stability index and as discussed in ***Article 10.2.3***.
  3. Find the unsupported length $l_u$ (as discussed in ***Article 10.1.2.4***).
  4. For the trial column, check for consideration of slenderness effects using the criteria of ***Article 10.2.2*** with k = 1.0.
  5. If slenderness is tentatively found to be important, refine the calculation of k based on the alignment chart in ***Article 10.1.2.5***.
  6. If moments from the frame analysis are small, check to determine if the following minimum moment controls.
     a. If
        $$M_2 < M_{2,min} = P_u(15 + 0.03h) \quad \textbf{Eq. 10.3-1}$$
        where 15 and h are in mm.
     b. Then use:
        $$M_2 = M_{2,min} = P_u(15 + 0.03h) \quad \textbf{Eq. 10.3-2}$$
  7. Calculate the equivalent uniform moment factor $C_m$ (as was discussed in ***Article 10.1.3.2***.
  8. Calculate $\beta_{dns}$, EI and $P_c$ as discussed in ***Article 10.1***.
  9. Calculate the moment magnification factor $\delta_{ns}$ and magnified moment $M_c$ based on following relations:
     $$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} \geq 1.0 \quad \textbf{Eq. 10.3-3}$$
     $$M_c = \delta_{ns} M_2 \quad \textbf{Eq. 10.3-4}$$
  10. Check the adequacy of the column to resist axial load and magnified moment using the column design charts of ***Chapter 9*** in the usual way. Revise the column section and reinforcement if necessary.
  11. If column dimensions are altered, repeat the calculations for k, $I_{eff}$ . and $P_c$ based on the new cross section. Determine the revised moment magnification factor and check the adequacy of the new design.

---

**Example 10.3-1**

Forces and moments acting on column shown in Figure 10.3-1 have been computed based on a ***First Order Analysis*** (i.e. equilibrium relations have been written based on un-deflected shape):

- Check to see if this column is classified short or slender based on ACI criterion.
- If the column is slender, magnify the applied moment based on ACI Moment Magnification Method.

In your solution, assume that:

- Frame is braced.
- The column has an effective length factor (k) equal to 0.9.
- Sustained load is 60% of the total load.
- $f_c' = 21\ MPa$.

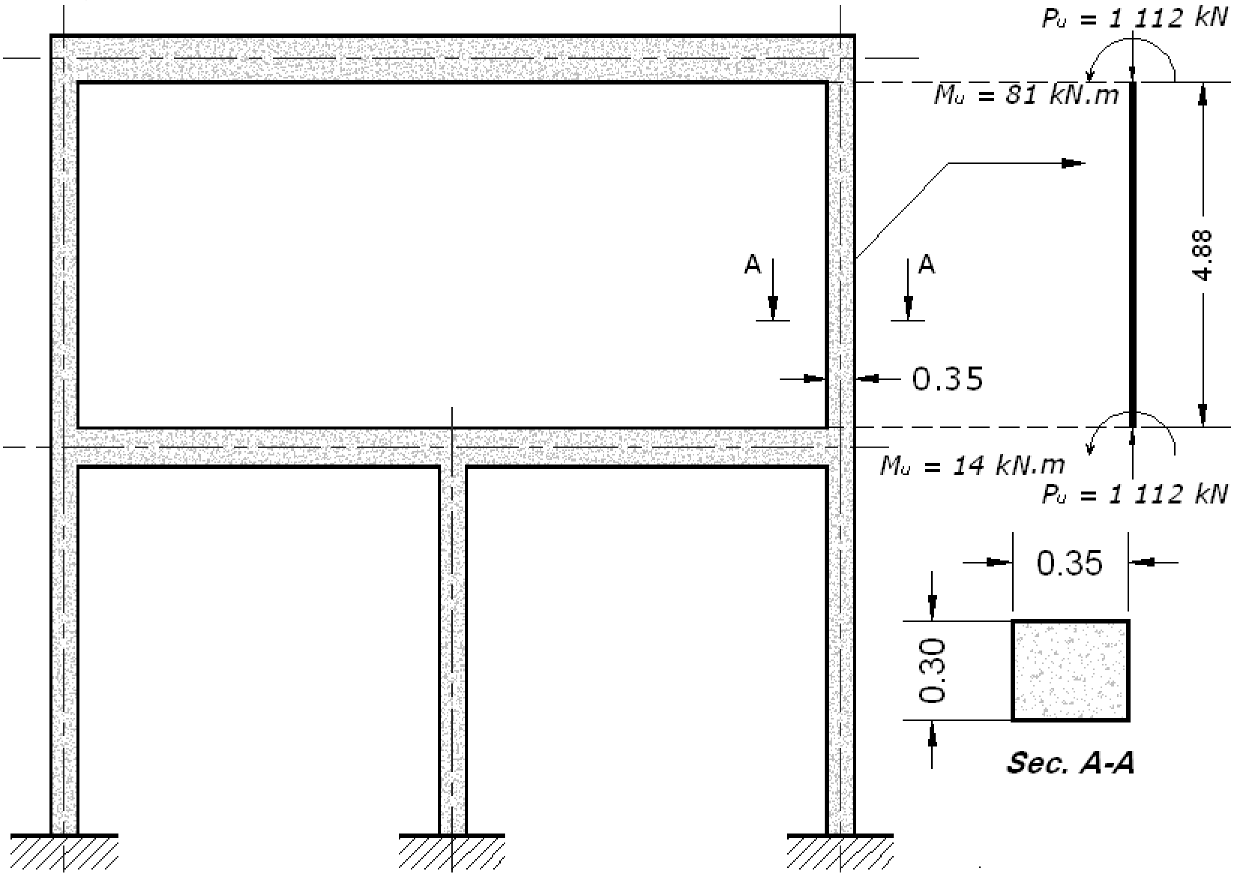



**Figure 10.3-1: Frame and column forces for Example 10.3-1.**

## Solution

- Check to see if the column is classified short or slender according to ACI criterion:

  $$\frac{kl_u}{r}\ ?\ 34 - 12\frac{M_1}{M_2} \leq 40$$

- Sign of $M_1/M_2$ can be conclude as follows:
  - Both ends of the column will rotate in anti-clockwise direction according to applied moments.
  - Then based on continuity principle, deflected shape will be as indicated in ***Figure 10.3-2*** below.
  - Based on above reasoning, one can conclude that the column is under double curvature and sign of $M_1/M_2$ ***should be negative according to ACI sign convention***.

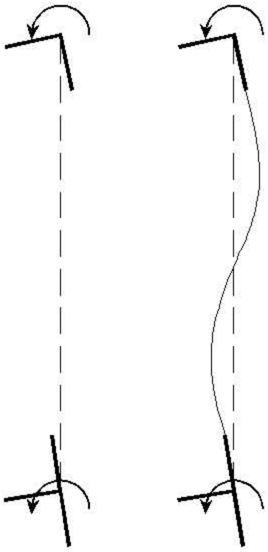


**Figure 10.3-2: Deflected shape for columns of Example 10.3-1, deduced based on continuity conditions.**

- Then
  $$\frac{\mathrm{kl_u}}{\mathrm{r}} = \frac{0.9 \times 4.88}{0.3 \times 0.35} \ ? \ 34 - 12\left(-\frac{14}{81}\right) \leq 40$$
  $$\frac{\mathrm{kl_u}}{\mathrm{r}} = 41.8 > (36.1 \ \leq 40\,)$$
  Therefore, ***column is classified long according to ACI criterion***.
- Moments of first order analysis should be modified to include the slenderness effect:
  $$M_c = \delta_{ns} M_2$$
  where:
  $$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}}$$
  $$C_m = 0.6 + 0.4\frac{M_1}{M_2} = 0.6 + 0.4\left(-\frac{14}{81}\right) = 0.53$$
  $$P_c = \frac{\pi^2 EI}{(kl_u)^2}$$
  $$EI_{eff} = \frac{0.4E_cI_g}{1+\beta_{dns}} = \frac{0.4\left(4700\sqrt{21}\right)\left(\frac{350^3 \times 300}{12}\right)}{1.6}$$
  $$EI_{eff} = \frac{0.4(21.5 \ \times 10^3)(1.07 \times 10^9)}{1.6} = 5.75 \times 10^{12}\ N \times mm^2$$
  $$P_c = \frac{\pi^2 \times 5.75 \times 10^{12}}{(0.9 \times 4\,880)^2} = 2\,939\ kN$$
  $$\delta_{ns} = \frac{0.53}{1 - \frac{1\,112}{0.75 \times 2\,939_c}} = 1.07$$
- This means that the primary moment that compute based on first order analysis should be increased by 7% to include the secondary moment.
  $$M_2 = 81_{Larger\ one\ of\ end\ moments} ?\ M_{2Min} = P_u(15mm + 0.03h)$$
  $$M_2 = 81\ kN_{Larger\ one\ of\ end\ moments} ?\ M_{2Min} = 1\,112\,000\ N(15mm + 0.03 \times 350)$$
  $$M_2 = 81\ kN.m\ _{Larger\ one\ of\ end\ moments} > M_{2Min} = 28.4\ kN.m$$
  $$M_2 = 81\ kN.m$$
  $$M_c = 1.07 \times 81 = 86.7\ kN.m\ \blacksquare$$

---

**Example 10.3-2**

Figure 10.3-3 below shows an elevation view of a multistory concrete frame building. Assumptions and Data:

- 1.2m wide by 0.3 deep beams on all column lines.
- Interior columns are tentatively dimensioned at 0.45m by 0.45m.
- Exterior columns are 0.40m by 0.40m.
- The frame is effectively braced against sway by stair and elevator shafts having concrete walls that are monolithic with the floors located in the building corners (not shown in the figure).
- Live loads have been subjected based on load pattern shown. This pattern has been selected based on a first-order analysis. Full roof live load distribution on roof and upper floors has been used.
- Service internal forces (axial force and bending moments at column two ends) for the typical interior column C3 have been computed based on a first order analysis (usually done by a computer program) and as shown below:

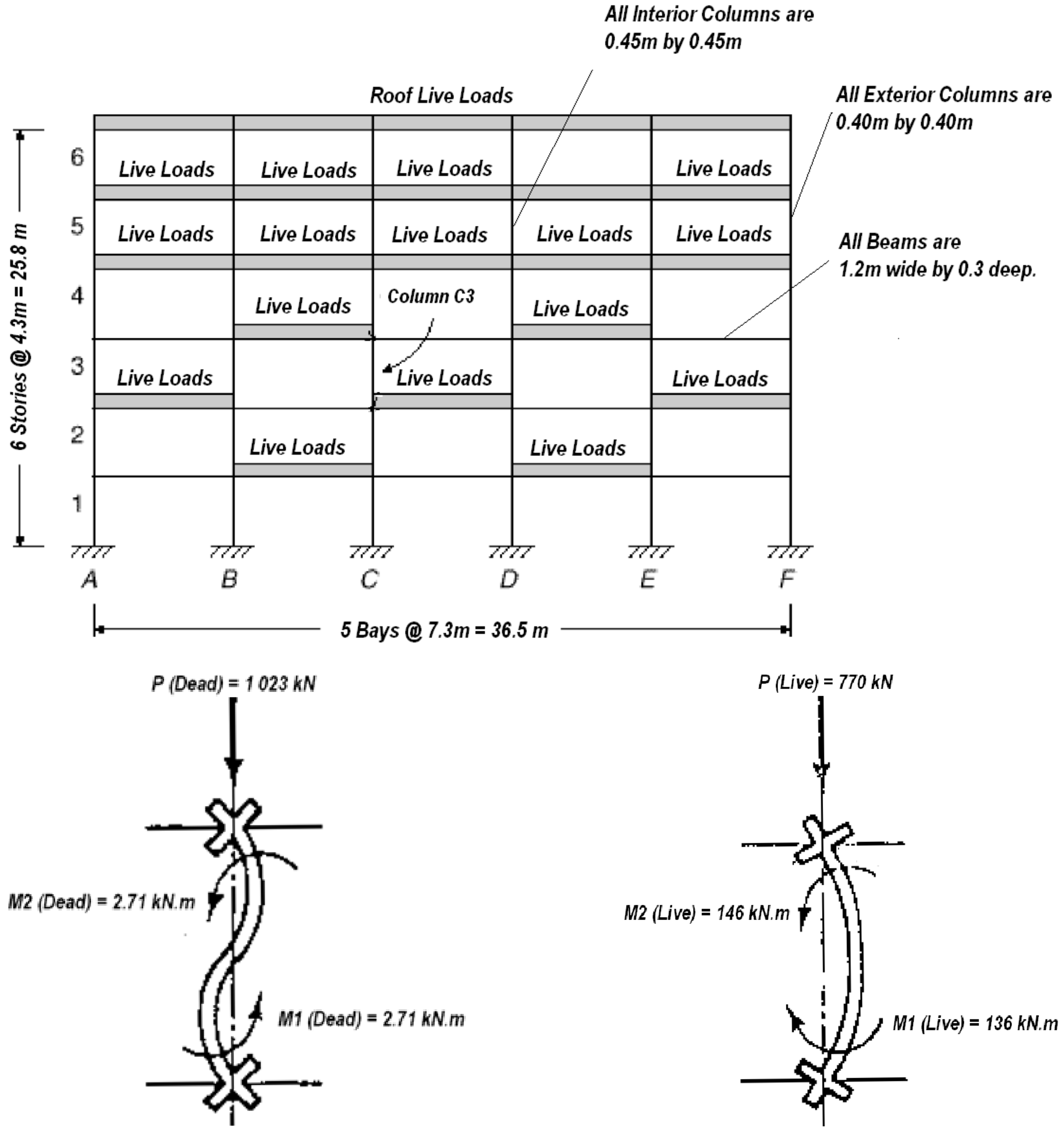
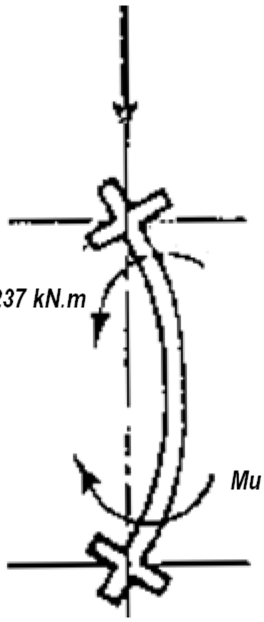



**Figure 10.3-3: Frame for Example 10.3-2.**

Based on above data and assumptions, design column C3 using the ACI moment magnifier method. Use $f_c' = 28MPa$ and $f_y = 420\ MPa$.

**Solution**

- Aim of this example is to show the direct application of ACI procedure for design a slender column in a braced frame (the column is not in the ground floor).
- Select a trial column: dimensions of a trial column have been assumed in the problem statement (0.45m by 0.45m).
- Factored axial force and bending moments on Column C3 can be computed as follows:
  $P_u = 1.2 \times 1\,023\ kN + 1.6 \times 770\ kN = 2\,460\ kN$
  $M_{u2} = 1.2 \times 2.71\ kN.m + 1.6 \times 146\ kN.m = 237\ kN.m$
  $M_{u1} = 1.2 \times (-2.71)\ kN.m + 1.6 \times 136\ kN.m = 214\ kN.m$
- Resultant of factored loads is shown in Figure below. Then column C3 will have single curvature due to factored loads.




- Determine if the frame should be considered as nonsway or sway:
  It is clear from problem statement that based on intuition the stiffness of shear wall has been assumed to be adequate to classify the building frame as a braced frame. If this decision is questionable, it can be checked based on stability index Q.
- Find the unsupported length $l_u$:
  $l_u = 4.3m - 0.3m = 4.0m$
- For the trial column, check for consideration of slenderness effects using the k = 1.0.
  $$\frac{kl_u}{r} = \frac{1.0 \times 4.0\ m}{0.3 \times 0.45m} = 29.6\ ?\ \ 34 - 12\frac{M_1}{M_2} = 34 - 12 \times \frac{214}{237} = 23.2 < 40$$
  $$\frac{kl_u}{r} = 29.6 > \ 34 - 12\frac{M_1}{M_2} = 23.2$$
  Then, the column C3 is a slender column.
- As slenderness is tentatively found to be important, refine the calculation of k based on the alignment chart in ***Article 10.1.2.5***:
  $$\Psi = \frac{\sum \frac{EI}{l}_{Columns}}{\sum \frac{EI}{l}_{Beam}}$$
  Because $E_c$ is the same for column and beams, it will be canceled in the stiffness calculations.
  $$\Psi = \frac{\sum \frac{I}{l}_{Columns}}{\sum \frac{I}{l}_{Beam}}$$

$$\Psi_a = \Psi_b = \frac{\dfrac{0.7 \times \dfrac{0.45^4}{12}}{4.3} \times 2}{\dfrac{0.35 \times (\dfrac{1.2 \times 0.3^3}{12} \times 2)}{7.3} \times 2} = \frac{1.112 \times 10^{-3}}{0.518 \times 10^{-3}} = 2.15$$

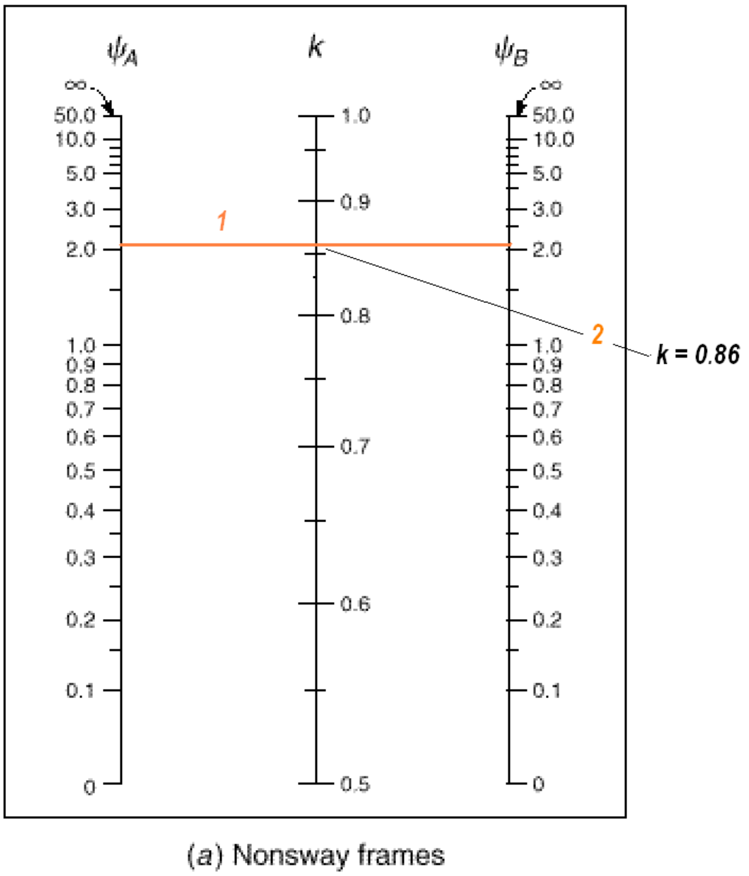



**Figure 10.3-4: Alignment chart applicable for Example 10.3-2.**

$$\frac{kl_u}{r} = \frac{0.86 \times 4.0\, m}{0.3 \times 0.45m} = 25\ >\ 34 - 12\frac{M_1}{M_2} = 23.2$$

$$\frac{kl_u}{r} = 29.6 >\ 34 - 12\frac{M_1}{M_2} = 23.2$$

This is still above the limit value of 23.2, conforming that slenderness must be considered.

- If moments from the frame analysis are small, check to determine if the following minimum moment controls:

  $M_2?\ M_{2,min} = P_u(15 + 0.03h)$

  $M_2 = 237\ kN.m\ ?\ M_{2,min} = 2\,460\,000\ (15 + 0.03 \times 450mm) = 70.1\ kN.m$

  $M_2 = 237\ kN.m > M_{2,min} = 70.1\ kN.m\ \ Ok.$

- Calculate the equivalent uniform moment factor $C_m$ (as was discussed in ***Article 10.1.3.2***).

  $$C_m = 0.6 + 0.4\frac{M_1}{M_2} = 0.6 + 0.4 \times \frac{214}{237} = 0.96$$

- Calculate $\beta_{dns}$, EI and $P_c$ as discussed in ***Article 10.1***.

  $$\beta_d = \frac{\text{maximum factored axial sustained load}}{\text{maximum factored axial load associated}} = \frac{1.2\ \times 1\,023}{2\,460} = 0.50$$

  $$EI = \frac{0.4E_cI_g}{1+\beta_d} = \frac{0.4 \times 4700 \times \sqrt{28} \times \dfrac{450^4}{12}}{1.5} = 2.27\ \times 10^{13}\ N.mm^2$$

  $$P_c = \frac{\pi^2 EI}{kl^2} = \frac{\pi^2 \times 2.27\ \times 10^{13}\ N.mm^2}{(0.86 \times 4\,000\ mm)^2} = 18\,913\ \text{kN}$$

- Calculate the moment magnification factor $\delta_{ns}$ and magnified moment $M_c$ based on following relations:

  $$\delta_{ns} = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}} = \frac{0.96}{1 - \dfrac{2\,460}{0.75 \times 18\,913}} = 1.16\ \geq 1.0\ Ok.$$

$M_c = \delta_{ns} M_2 = 1.16 \times 237 = 275\ kN.m$

- Finally, column C3 can be designed for an axial force of $P_u = 2\,460\ kN.m$ and a bending moment of $M_u = 275\ kN.m$ and according to procedures of Chapter 9 to obtain the design results that shown in Figure below:

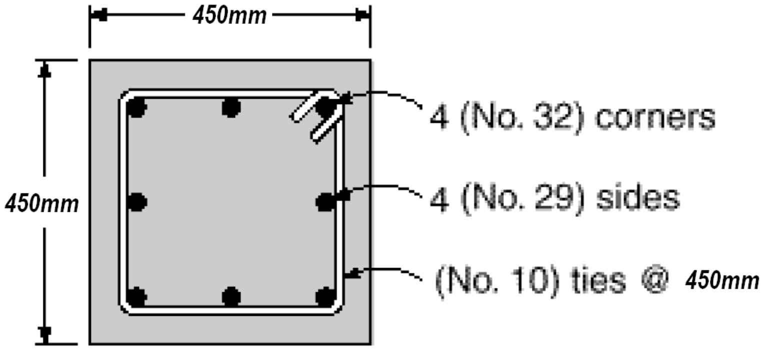



**Figure 10.3-5: Final design section for Example 10.3-2.**

---

**Example 10.3-3**

The frame shown in ***Figure 10.3-6*** below is a part of a building that can be considered braced by presence of stiff concrete walls surrounding the elevator shafts.

The soil under the footings is soft with a relative stiffness of $\psi = 5$[1] is considered appropriate at the base. Other values of $\psi$ have been computed and are given in ***Figure 10.3-6*** below.

Structural factored load for column B between points 0 and 1 is $P_u = 2\,000\ kN$. The concrete modulus of elasticity is $E_c = 30\,000\ MPa$. All columns are 400 mm x 400 mm square. Assume that $\beta_{dns} = 0.5$.

- Calculate the buckling load;
- Calculate the moment that should be used to design column B in the ground floor.

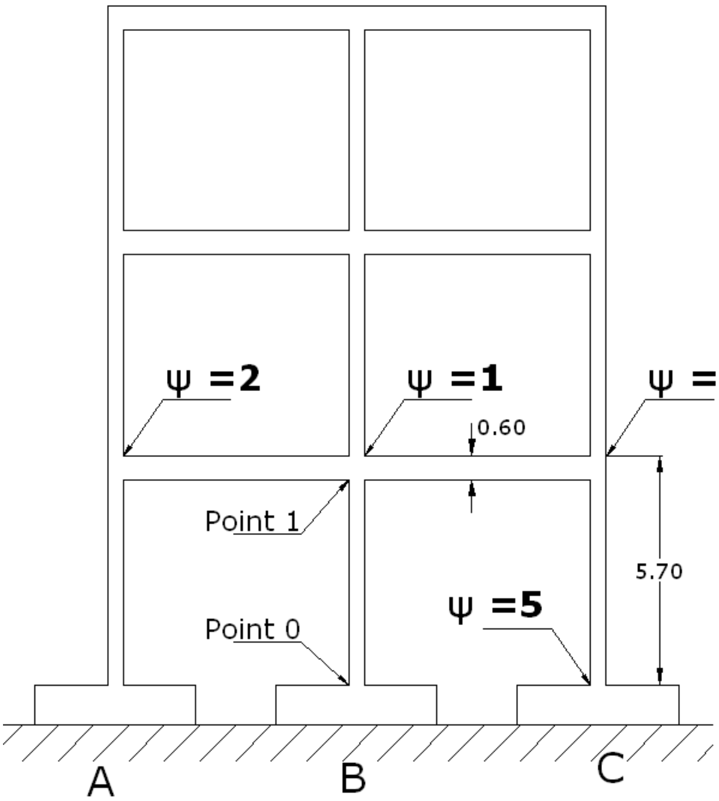



**Figure 10.3-6: Frame for Example 10.3-3.**

---

[1] For more details about computing $\psi$ for foundation, see "Reinforced Concrete: Mechanics and Design", 4th Edition by J. G. Macgregor (Page 568).

**Solution**

**Practical Aspects of the Problem**

- This problem simulates a very common case that faces the structural engineer in his daily work with the interior columns. Generally, moment in an interior column of a multistory building has a negligible value due to the length and load symmetry between different spans. As the approach of structural stability of the slender columns is based on the amplification of the actual moments only, then it cannot be able to simulate inverse proportionality between the strength of an axially loaded column and its slenderness ratio.
- To extend this approach to include this common and important case, the ACI Code states that (as was previously discussed) the design moment in a slender column of a braced frame must not be taken less than the following minimum value:

$M_{2,\text{Min}} = P_u(15\text{mm} + 0.03\text{hmm})$

- When this moment is substituted into the relation of the ACI moment magnification method, one will obtain the following relation:

$$M_c = \frac{1}{1 - \frac{P_u}{0.75P_c}} \times [P_u(15\text{ mm} + 0.03\text{h mm})]$$

- If the applied factored load $P_u$ approaching the Euler Buckling Load $P_c$, the design moment $M_c$ will approach the infinite. Then the ACI moment magnification method has been extended to simulate the well-known fact of the inverse proportionality between the strength of the axially loaded column and its slenderness ratio (Euler Law).

**Compute the Buckling Load $P_{cr}$:**

Compute the effective length factor from the alignment chart of braced frames, with $\psi_0 = 5$ and $\psi_1 = 1$, see ***Figure 10.3-7*** below.

$k = 0.84$

$l_u = 5.7m - 0.6m = 5.1m$

$$P_{cr} = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2 \times \frac{30\,000}{1+0.5} \times 0.4 \times \frac{(400^4)}{12}}{(0.84 \times 5\,100)^2} = 9\,169\,kN$$

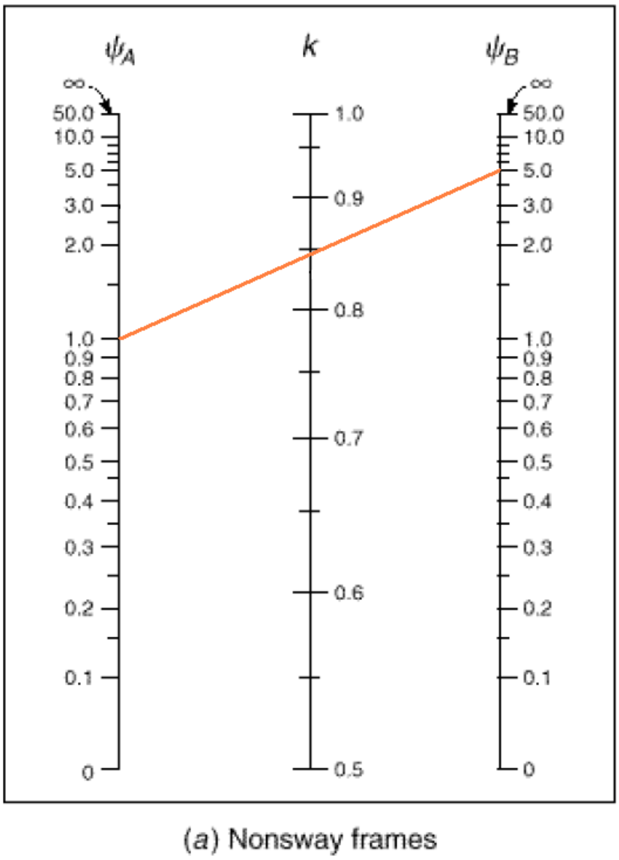



**Figure 10.3-7: Alignment chart applied to Example 10.3-3.**

## Compute the Design Moment

- Check if the column is short or slender column:

$$\frac{kl_u}{r} = \frac{0.85 \times 5.1\text{m}}{0.3 \times 0.4\text{m}} = 36.125 \ ? \ 34 - 12\frac{M_1}{M_2}$$

As $M_1 = 0$ and $M_2 = M_{2\,Min}$, then the above ratio will be:

$$\frac{kl_u}{r} = 36.2 > 34 - 12 \times 0$$

$$\frac{kl_u}{r} = 36.2 > 34$$

Then, the column is a slender column.

$$M_c = \delta_{ns} M_{2\,Min}$$

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}}$$

- For member in which $M_{2\,Min}$ exceeds $M_2$ (as for our problem), ACI states that the $C_m$ can be either taken equal to 1.0 or shall be based on the ratio of the computed end moments $M_1$ and $M_2$. Our solution will be based on the more conservative value of $C_m = 1.0$.

Then:

$$\delta_{ns} = \frac{1}{1 - \frac{P_u}{0.75P_c}}$$

$$P_u = 2\,000 \text{ kN}$$

$$\delta_{ns} = \frac{1}{1 - \frac{2\,000}{0.75 \times 9\,169}} = 1.42$$

As the ***magnification factor is in the range of maximum value of 1.4, hence column dimensions should be revised or additional bracing should be adopted***.

- Finally:

$$M_c = 1.42 \times [2\,000 \times 10^3 \times (15\text{ mm} + 0.03 \times 400\text{mm})] = 1.42 \times 54 \text{ kN.m} = 76.7 \text{ kN.m} \ ■$$

---

**Example 10.3-4**

A column is loaded as shown in ***Figure 10.3-8*** below. Check if the column is short or slender, and then compute the moment that must be used in the design. Assume $f_c' = 28\text{ MPa}$ and $\beta_{dns} = 0.4$.

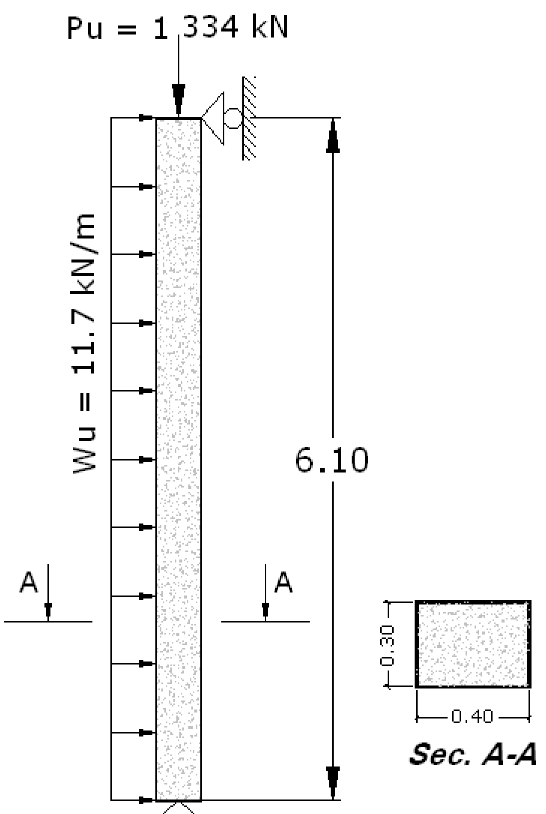



**Figure 10.3-8: Column for Example 10.3-4.**

## Solution

- The problem shows how to use the moment magnification method in a column that has no end moments $M_1 = M_2 = 0$ but has a large mid-span bending moment.
- As ACI design procedure in written in terms of end moment $M_2$, then this problem requires a special consideration.
- It can be shown that the ACI for procedure for a column with mid-span moment can be rewritten in the following form[2]:
  $M_c = M_{Mid\ Span} \times \delta_{ns}$
- Checking to see if the column is short or slender:
  $$\frac{kl_u}{r} = \frac{1.0 \times 6.1m}{0.3 \times 0.4m} = 50.8\ ?\ 34 - 12\frac{M_1}{M_2} = 34$$
  $$\frac{kl_u}{r} = 50.8 > 34$$
  Then the column is classified as a long column according to ACI Code.
- Compute the Design Moment $\boldsymbol{M_c}$:
  $$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}}$$
  As the column is subjected to transverse loads, then:
  $C_m = 1.0$
  $$EI = \frac{4700\sqrt{28}}{1 + 0.4} \times \left(0.4 \times \frac{400^3 \times 300}{12}\right) = 1.14 \times 10^{13}\ N.mm^2$$
  $$P_{cr} = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2 \times 1.14 \times 10^{13}\ N.mm^2}{(1.0 \times 6\ 100)^2\ mm^2} = 3\ 021\ kN$$
  $$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} = \frac{1.0}{1 - \frac{1\ 334}{0.75 \times 3\ 021}} = 2.43$$
  As the ***magnification factor is significantly larger than the maximum value of 1.4, hence column dimensions should be revised or additional bracing should be adopted***.
  $$M_c = M_{Mid\ Span} \times \delta_{ns} = \frac{11.7 \times 6.1^2}{8} \times 2.43 = 132\ kN.m\ ■$$

---

[2] See "Design of Reinforced Concrete" 7th Edition, by J. C. McCormac (Page 329).

## 10.4 SUMMARY OF ACI MOMENT MAGNIFIER METHOD FOR SWAY FRAMES

For a column that may be a slender column in a sway story, ACI checking procedure can be summarized as follows:

- Check to see if the story can be classified as sway or braced. This can be done either based on inspection or based on the concept of stability index "Q".
- Check if the column is short or slender based on the following limitation:
  $\frac{kl_u}{r} \leq 22$
- Classify the applied loads into:
  - That don't produce sway (for example dead and live loads),
  - That produces sway (for example wind load, earthquake load, and loads due to lateral earth pressure.
- Based on first order structural analysis compute the axial forces "$P_u$" and bending moment ("$M_s$" and "$M_{ns}$") due to sway and nonsway loads respectively.
- Compute the design moments based on the following relation:
  $M_1 = M_{1ns} + \delta_s M_{1s}$ $\quad\quad$ $M_2 = M_{2ns} + \delta_s M_{2s}$
  where the moment magnification factor for a sway story can be computed based on one of the following two approaches:
  - First Method:

$$\delta_s = \frac{1}{1 - Q} \geq 1.0 \quad \textbf{Eq. 10.4-1}$$

    If $\delta_s$ calculated exceeds 1.4, $\delta_s$ shall be re-calculated by second method below.
  - Second Method:

$$\delta_s = \frac{1}{1 - \frac{\sum P_u}{0.75 \sum P_c}} \geq 1.0 \quad \textbf{Eq. 10.4-2}$$

    where

  i. $\sum P_u$ is the summation for all the factored vertical loads in a story,
  ii. $\sum P_c$ is the summation for all sway-resisting columns in a story,
  iii. $P_c$ is the Euler load computed as discussed in ***Article 10.1*** with k from alignment chart for sway frame and with $\beta_{ds}$ defined as ***the ratio of the maximum factored sustained shear within a story to the maximum factored shear in that story***. Lateral forces due to wind or earthquake cannot be sustained loads. Then $\beta_{ds}$ for these loads is always equal to zero. While lateral forces due to earth pressure can be sustained loads and $\beta_{ds}$ for these loads is not equal to zero (See Figures below).

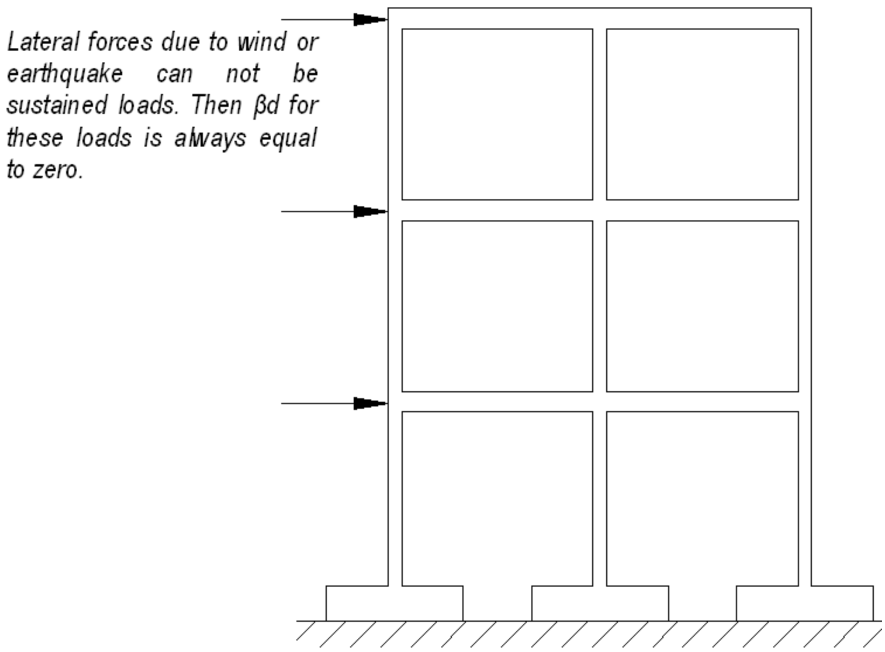



**Figure 10.4-1: Transient lateral force with $\beta_{ds} = 0$.**

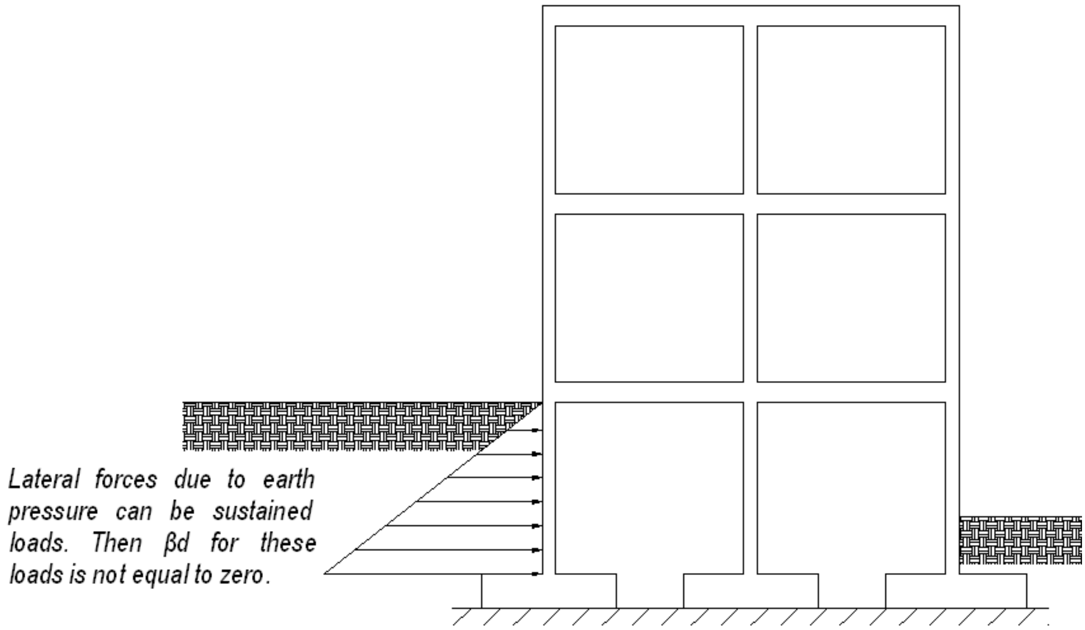



**Figure 10.4-2: Sustained lateral force with $\beta_{ds} \neq 0$.**

**Example 10.4-1**

For the sway frame shown Figure 10.4-3 below:

- Check to see if column AB is classified as short or slender according to ACI criterion?
- If column is found to be slender, use ACI ***Moment Magnification Method*** to magnify the first order moments to include the slenderness effects.

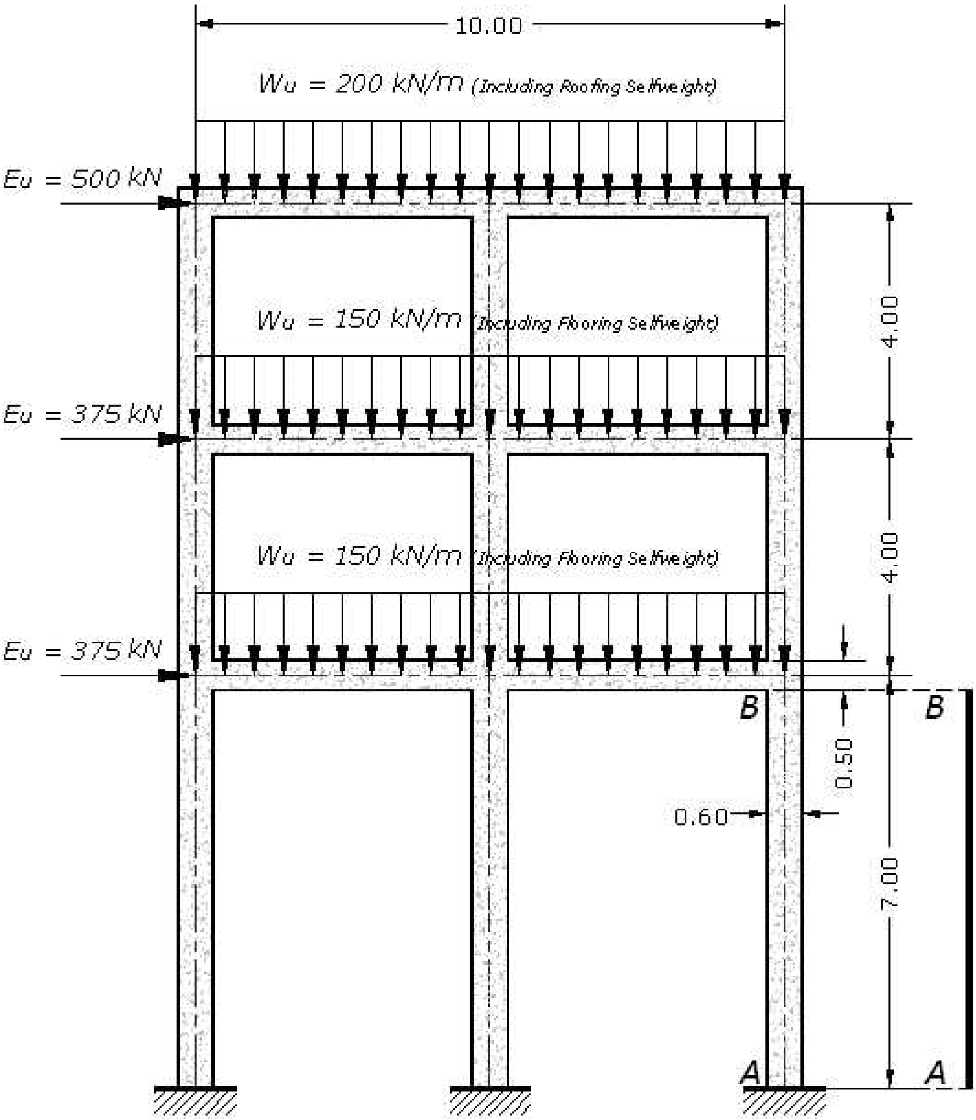



**Figure 10.4-3: Frame for Example 10.4-1.**

In your solution assume that:

- All columns are 0.6m by 0.3m with $f_c'$ = 28 MPa,
- All supports are fixed,
- Columns selfweight can be neglected,
- First order moment due to gravity loads (nonsway load) and due to lateral loads (sway loads) are as indicated in ***Figure 10.4-4*** below,
- Under sway loads, end B has zero rotation (***Shear Building Assumption***).

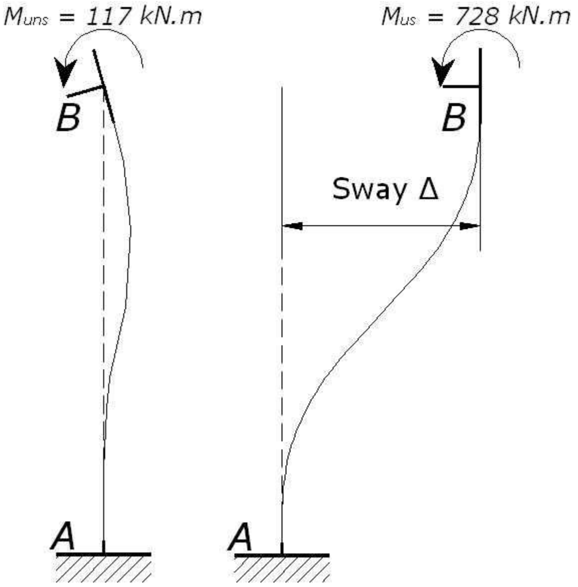



**Figure 10.4-4: Moments from a first order analysis for Column AB of Example 10.4-1.**

**Solution**

- Check to see if the story can be classified as sway or braced. This can be done either based on intuition or based on the concept of stability index "Q":
  Based on problem statement, column is part from unbraced story.
- Check if the column is short or slender based on the following limitation:

$$\frac{kl_u}{r} \; ? \; 22$$

$$l_u = 7.0 - \frac{0.5}{2} = 6.75m$$

- Based on assuming that end B has zero rotation, column boundary conditions will be similar to those presented in ***Figure 10.4-5*** below.

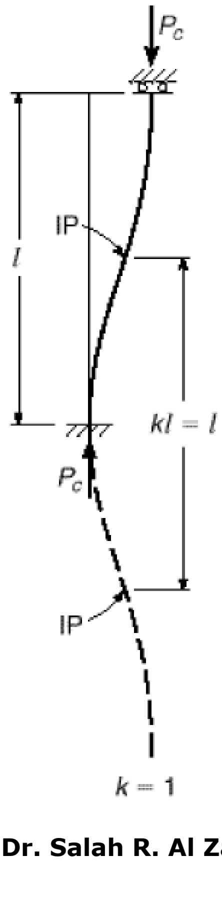



**Figure 10.4-5: Column boundary conditions with shear building assumption.**

Then k = 1.0

$r = 0.3h = 0.3 \times 0.6m = 0.18m$

$\frac{kl_u}{r} = \frac{1.0 \times 6.75}{0.18} = 37.5 > 22$

Then the column is a slender column.

- Classify the applied loads into a load that don't produce sway and that produces sway:
  Loads are already classified in the problem statement.
- Compute the design moments based on the following relation:
  $M_2 = M_{2ns} + \delta_s M_{2s}$
  As we have no information about stability index, the $\delta_s$ can only be computed based on second method:
  $$\delta_s = \frac{1}{1 - \frac{\sum P_u}{0.75 \sum P_c}}$$
  As selfweight of columns can be neglected according to problem statement, then resultant of vertical loads $\sum P_u$ will be computed based on factored loads acting on roof and floors:
  $$\sum P_u = \left(200 \frac{kN}{m} \times 10m\right)_{Roof\ Loads} + 2\left(150 \frac{kN}{m} \times 10m\right)_{Floors\ Loads}$$
  $$\sum P_u = 5000\ kN$$
  Based on assumption of rigid flooring systems, all columns will have same boundary conditions, and as all columns have same dimensions, then they will have same critical load $P_C$.
  $$\sum P_c = 3 \times P_{c\ for\ column\ AB}$$
  $$P_{c\ for\ column\ AB} = \frac{\pi^2 EI}{(kl_u)^2}$$
  $$EI = \frac{4700\sqrt{28}}{1 + 0_{\beta_{ds}\ is\ Zero\ for\ Lateral\ Loads}} \times \left(0.4 \times \frac{600^3 \times 300}{12}\right)$$
  $$EI = 24.9 \times 10^3 \frac{N}{mm^2}(2.16 \times 10^9 mm^4) = 53.8 \times 10^{12}\ N.mm^2$$
  $$P_{c\ for\ column\ AB} = \frac{\pi^2 \times 53.8 \times 10^{12}\ N.mm^2}{(1.0 \times 6\,750\ mm)^2} = 11\,642\ kN$$
  $$\sum P_c = 3 \times 11\,642\ kN = 34\,926\ kN$$
  $$\delta_s = \frac{1}{1 - \frac{\sum P_u}{0.75 \sum P_c}} = \frac{1}{1 - \frac{5000}{0.75 \times 34\,926}} = 1.24 < 1.4 \therefore Ok.$$
  $$M_2 = M_{2ns} + \delta_s M_{2s} = 117\ kN.m + 1.24 \times 728\ kN.m = 1\,020\ kN.m$$

---

# Contents

# CHAPTER 11

# ANALYSIS OF INDETERMINATE BEAMS AND FRAMES

## 11.1 CONTINUITY

- Continuity in steel structures:
  - The individual members that compose a steel structure are fabricated or cut separately and joined together by rivets, bolts, welds, or nails.
  - Unless the joints are specially designed for rigidity, they are ***too flexible to transfer moments of significant magnitude from one member to another***.
- Continuity in reinforced concrete structures:
  - In contrast, in reinforced concrete structures:
    - As much of the concrete as is practical is placed in one single operation.
    - Reinforcing steel is not terminated at the ends of a member but is extended through the joints into adjacent members.
    - At construction joints, special care is taken to bond the new concrete to the old by carefully cleaning the latter, by extending the reinforcement through the joint, and by other means.

    As a result, reinforced concrete structures usually represent ***monolithic***, or ***continuous***, units.
  - A load applied at one location causes deformation and stress at all other locations.
  - Even in precast concrete construction, which resembles steel construction in that individual members are brought to the job site and joined in the field, connections are often designed to provide for the transfer of moment as well as shear and axial load, producing at least partial continuity.
- The effect of continuity:
  - In continuous beams:
    - The effect of continuity is most simply illustrated by a continuous beam, as shown in ***Figure 11.1-1a***.
    - With simple spans, such as provided in many types of steel construction, only the loaded member CD would deform, and all other members of the structure would remain straight.
    - But with continuity from one member to the next through the support regions, as in a reinforced concrete structure, the distortion caused by a load on one single span is seen to spread to all other spans, although ***the magnitude of deformation decreases with increasing distance from the loaded member***.
    - All members of the six-span structure are subject to curvature, and thus also to bending moment, as a result of loading span CD.
  - In rigid-jointed frame subjected to gravity forces:
    - Similarly, for the rigid-jointed frame of ***Figure 11.1-1b***, the distortion caused by a load on the single member GH spreads to all beams and all columns, although, as before, ***the effect decreases with increasing distance from the load***.
    - All members are subject to bending moment, even though they may carry no transverse load.

- In rigid-jointed frame subjected to horizontal forces:
  - If horizontal forces, such as forces caused by ***wind*** or ***seismic action***, act on a frame, it deforms as illustrated by ***Figure 11.1-1c***.
  - Here, too, all members of the frame distort, even though the forces act only on the left side; the amount of distortion is ***seen to be the same for all corresponding members, regardless of their distance from the points of loading, in contrast to the case of vertical loading***.
  - A member such as EH, even without a directly applied transverse load, will experience deformations and associated bending moment.

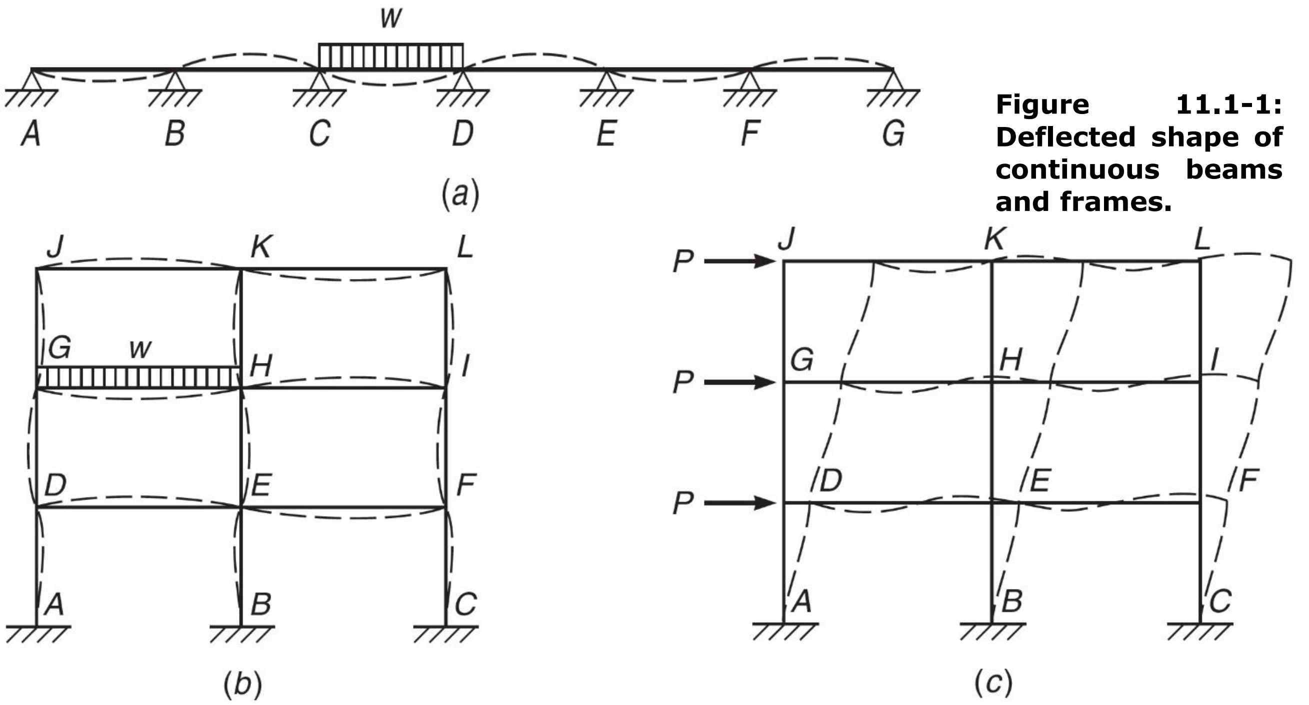



**Figure 11.1-1: Deflected shape of continuous beams and frames.**

- Statically determinate versus statically indeterminate structures:
  - In ***statically determinate structures***, such as simple-span beams, ***the deflected shape and the moments and shears depend only on the type and magnitude of the loads and the dimensions of the member***.
  - In contrast, inspection of the ***statically indeterminate structures*** in Figure 11.1-1 shows that ***the deflection curve of any member depends not only on the loads but also on the joint rotations, whose magnitudes in turn depend on the distortion of adjacent, rigidly connected members***.
  - For a rigid joint such as joint H in the frame shown in Figure 11.1-1b or c, all the rotations at the near ends of all members framing into that joint must be the same.
  - For a correct design of continuous beams and frames, it is evidently necessary to determine moments, shears, and thrusts considering the effect of continuity at the joints.
- Analysis of statically indeterminate structures
  - Elastic analysis:

    The determination of these internal forces in continuously reinforced concrete structures is usually based on elastic analysis of the structure at factored loads with methods that will be described in Sections 11.2 through 11.5.
  - Prerequisite data for an elastic analysis:

    Such analysis requires knowledge of the cross-sectional dimensions of the members.

- Preliminary estimation of member dimensions for analysis purpose:
  Member dimensions are initially estimated during preliminary design, which is described in ***Section 11.6*** along with guidelines for establishing member proportions.
- Approximate methods and their usefulness:
  - For ***checking the results of more exact analysis***, the approximate methods of ***Section 11.7*** are useful.
  - ACI coefficient method:
    ***For many structures***, ***a full elastic analysis is not justified***, and ***the ACI coefficient method of analysis described in Section 11.8 provides an adequate basis for design moments and shears***.

## 11.2 LOADING*

- The individual members of a structural frame must be designed for the worst combination of loads that can reasonably be expected to occur during its useful life.
- Internal moments, shears, and thrusts are brought about by the combined effect of dead and live loads, plus other loads, such as wind and earthquake, as discussed in ***Chapter 1***.
- Dead versus live loads:
  - While dead loads are constant, live loads such as floor loads from human occupancy can be placed in various ways, some of which will result in larger effects than others.
  - In addition, the various combinations of factored loads specified in ***Chapter 1*** must be used to determine the load cases that govern member design. The subject of load placement will be addressed first.

### 11.2.1 PLACEMENT OF LOADS

- Influence lines for maximum positive moments through imagination of deflected shape:
  - In ***Figure 11.2-1a*** only span CD is loaded by live load. The distortions of the various frame members are seen to be largest in, and immediately adjacent to, the loaded span and to decrease rapidly with increasing distance from the load.
  - Since bending moments are proportional to curvatures, the moments in more remote members are correspondingly smaller than those in, or close to, the loaded span.
  - However, the loading shown in ***Figure 11.2-1a*** does not produce the maximum possible positive moment in CD. In fact, if additional live load were placed on span AB, this span would bend down, BC would bend up, and CD itself would bend down in the same manner, although to a lesser degree, as it is bent by its own load.
  - Hence, the positive moment in CD is increased if AB and, by the same reasoning, EF are loaded simultaneously.
  - ***By expanding the same reasoning to the other members of the frame, it is easy to see that the <u>checkerboard pattern</u> of live load shown in Figure 11.2-1b produces the largest possible positive moments***, not only in CD but also in all loaded spans.
  - Hence, two such checkerboard patterns are required to obtain the maximum positive moments in all spans.

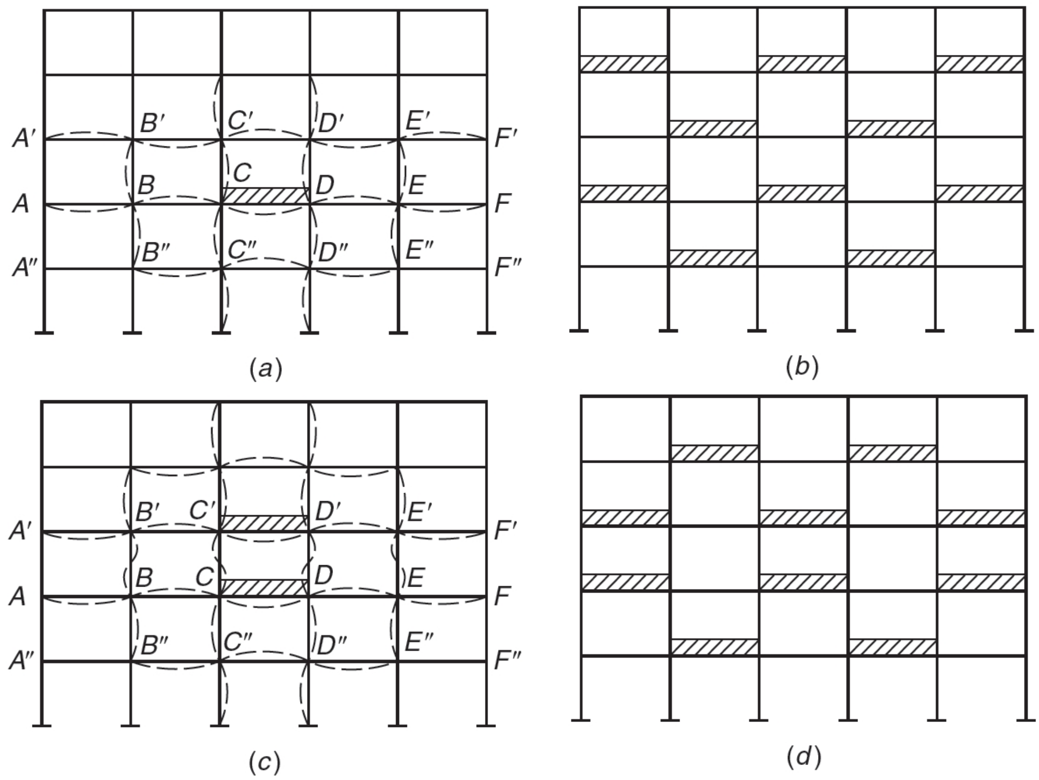



**Figure 11.2-1: Alternate live loadings for maximum effects.**

- Influence lines for minimum span moments:
  - In addition to maximum span moments, i***t is often necessary to investigate minimum span moments***.
  - Dead load, acting as it does on all spans, usually produces only positive span moments.
  - However, live load, placed as in ***Figure 11.2-1a***, and even more so in ***Figure 11.2-1b***, is seen to bend the unloaded spans upward, that is, to produce negative moments in the span.
  - ***If these negative live load moments are larger than the generally positive dead load moments, a given girder, depending on load position, may be subject at one time to positive span moments and at another to negative span moments***. It must be designed to withstand both types of moments; that is, it must be furnished with tensile steel at both top and bottom.
  - Thus, the loading of ***Figure 11.2-1b***, in addition to giving maximum span moments in the loaded spans, gives minimum span moments in the unloaded spans.
- Influence lines for maximum negative moments at the supports:
  - Maximum negative moments at the supports of the girders are obtained, on the other hand, if loads are ***placed on the two spans adjacent to the particular support, and in a corresponding pattern on the more remote girders***.
  - A separate loading scheme of this type is then required for each support for which maximum negative moments are to be computed.
- Influence lines for maximum moments in columns:
  - In each column, the largest moments occur at the top or bottom.
  - While the loading shown in ***Figure 11.2-1c*** results in large moments at the ends of columns $CC'$ and $DD'$, the reader can easily be convinced that these moments are further augmented (i.e. increased) if additional loads are placed as shown in ***Figure 11.2-1d***.

### 11.2.2 LOAD COMBINATIONS

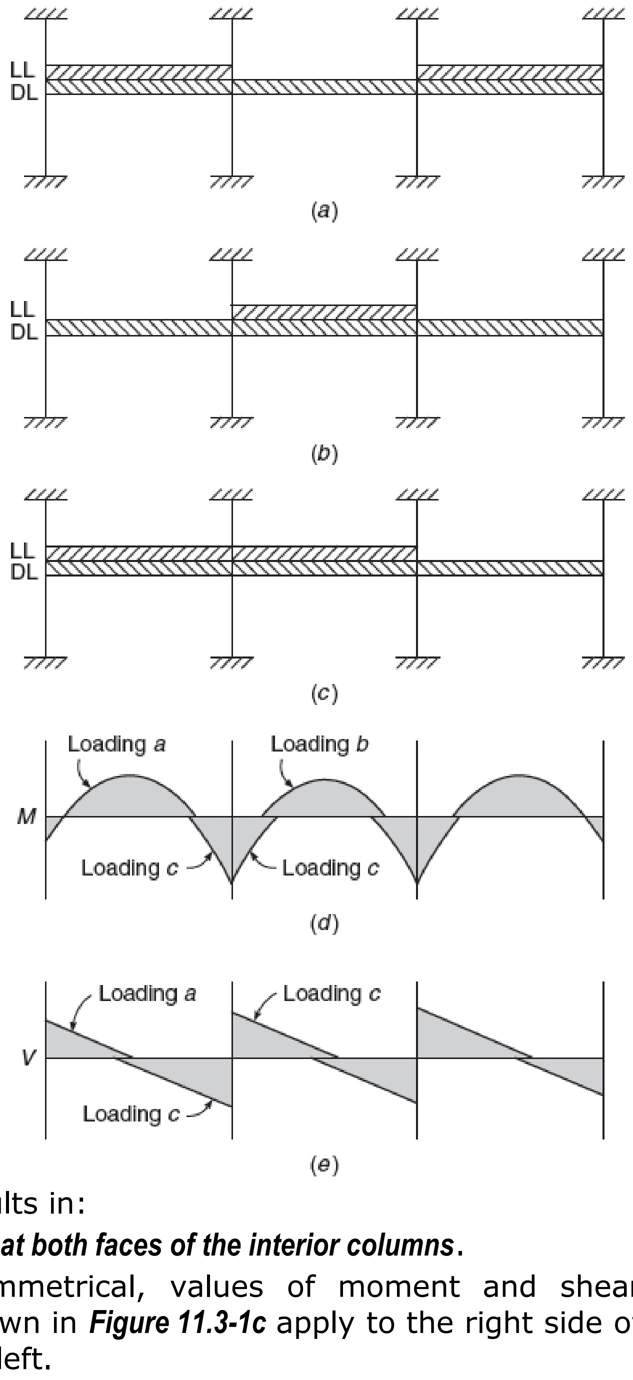
- The ACI Code requires that structures be designed for a number of load combinations, as discussed in ***Chapter 1***.
- Thus, for example, factored load combinations might include:
    - Dead plus live load;
    - Three possible combinations that include dead, live, and wind load;
    - Two combinations that include dead load, live load, and earthquake load, with some of the combinations including snow, rain, and roof live load.
- Load Case and Load Combination:
    - While each of the combinations may be considered as an individual loading condition, ***experience has shown that the most efficient technique involves separate analyses for each of the basic loads without load factors***, that is, a full analysis for unfactored dead load only, separate analyses for the various live load distributions described in Section 11.2.1, and separate analyses for each of the other loads (wind, snow, etc.).
    - Once the separate analyses are completed, it is a simple matter to combine the results using the appropriate load factor for each type of load.
    - This procedure is most advantageous because, for example, live load may require a load factor of 1.6 for one combination, a value of 1.0 for another, and a value of 0.5 for yet another. Once the forces have been calculated for each combination, the combination of loads that governs for each member can usually be identified by inspection.

## 11.3 SIMPLIFICATIONS IN FRAME ANALYSIS*

- Considering the complexity of many practical building frames and the need to account for the possibility of alternative loadings, there is evidently a need to simplify.
- This can be done by means of certain approximations that allow the ***determination of moments with reasonable accuracy while substantially reducing the amount of computation***.
- Simplification for Regular Frames:
    - Definition of Regular Frames:
      Regular frames can be defined as the frames that have no unusual asymmetry of loading or shape.
    - Concept of Subframe:
        - Definition of Subframe:
          In case of regular frames, moments due to vertical loads can be determined with sufficient accuracy by dividing the entire frame into simpler subframes. Each of these consists of one continuous beam, plus the top and bottom columns framing into that particular beam.
        - Load acting on Subframe:
          Placing the live loads on the beam in the most unfavorable manner permits sufficiently accurate determination of all beam moments, as well as the moments at the top ends of the bottom columns and the bottom ends of the top columns.
        - Boundary conditions for subframe:
          For this partial structure, the far ends of the columns are considered fixed, except for such first-floor or basement columns where soil and foundation conditions dictate the assumption of hinged ends.
        - Subframe in ACI code:
          Subframe approach is ***explicitly permitted by ACI Code 6.3.1.2 and 6.4***, which allow the following assumptions for floor and roof members under gravity load:
        - To calculate moments and shears in columns, beams, and slabs, the structural model may be limited to the members in the level being

considered and the columns above and below that level; the far ends of columns built integrally with the structure may be considered fixed.
- The maximum positive moment near midspan occurs with the factored live load on the span and on alternate spans, and the maximum negative moment at a support occurs with the factored live load on adjacent spans only.

- An example subframe:
  - Figure 11.3-1 demonstrates the application of the ACI Code requirements for live load on a three-span subframe.
  - The loading in ***Figure 11.3-1a*** results in:
    - The ***maximum positive moments in the exterior spans***,
    - The ***minimum positive moment in the center span***,
    - The ***maximum negative moments at the interior faces of the exterior columns***.
  - The loading shown in ***Figure 11.3-1b*** results in:
    - The ***maximum positive moment in the center span***,
    - The ***minimum positive moments in the exterior spans***.




**igure 11.3-1: Subframe loading as required y ACI Code 6.4: Loading for (a) maximum ositive moments in the exterior spans, the ninimum positive moment in the center pan, and the maximum negative moments t the interior faces of the exterior columns; b) maximum positive moment in the center pan and minimum positive moments in the xterior spans; (c) maximum negative noment at both faces of the interior olumns; (d) envelope moment diagram; and e) envelope shear diagram. (DL and LL epresent factored dead and live loads, espectively).**

  - The loading in ***Figure 11.3-1c*** results in:
    - The ***maximum negative moment at both faces of the interior columns***.
    - Since the structure is symmetrical, values of moment and shear obtained for the loading shown in ***Figure 11.3-1c*** apply to the right side of the structure as well as the left.

    - Due to the simplicity of this structure, joints away from the spans of interest are not treated as fixed.
  - Envelopes versus diagrams:
    - Moments and shears used for design are determined by combining the moment and shear diagrams for the individual load cases to obtain the maximum values along each span length.
    - The resulting envelope moment and shear diagrams are shown in ***Figure 11.3-1***d and ***e***, respectively.
    - Critical sections and cutoff points:

      The moment and shear envelopes (note the range of positions for points of inflection and points of zero shear) are used not only to design the critical sections but also to determine cutoff points for flexural reinforcement and requirements for shear reinforcement.
- Forces in columns:

  About columns, the ACI Code indicates:
  - The factored axial load and factored moment occurring simultaneously for each applicable factored load combination shall be considered (ACI Code 10.2.4.1).
  - For frames or continuous construction, consideration shall be given to the effect of floor and roof load patterns on the transfer of moment to exterior and interior columns and of eccentric loading due to other causes (ACI Code 6.6.2.2).
  - In computing moments in columns due to gravity loading, the far ends of columns built integrally with the structure may be considered fixed (ACI Code 6.3.1.2).
  - Floor or roof level moments shall be resisted by distributing the moment between columns immediately above and below the given floor in ***proportion to the relative column stiffnesses considering conditions of restraint*** (ACI Code 6.5.5 and 6.6.2.1).
  - Concept of Tributary Areas:
    - Although it is not addressed in the ACI Code, axial loads on columns are usually determined based on the column ***tributary areas***, which are defined based on ***the midspan of flexural members framing into each column***.
    - The axial load from ***the tributary area is used in design***, with the exception of ***first interior columns***, which are typically designed for ***an extra 10 percent axial load to account for the higher shear expected in the flexural members framing into the exterior face of first interior columns***.
    - The use of this procedure to determine axial loads due to gravity is conservative (note that the total vertical load exceeds the factored loads on the structure) and is adequately close to the values that would be obtained from a more detailed frame analysis.

## 11.8 ACI MOMENT COEFFICIENTS

### 11.8.1 BASIC CONCEPTS

- ***ACI Code 6.5*** includes expressions that may be used for the ***approximate calculatio***n of ***maximum moments*** and ***shears*** in:
  - Continuous beams,
  - One-way slabs.
- The expressions for moment take the form of:

  $M_u = Coefficient\ w_u \ell_n^2$ **Eq. 11.8-1**

  where:

  $w_u$ is the total factored load per unit length on the span,

  $\ell_n$ is the ***clear span from face to face of supports for positive moment***, or the ***average of the two adjacent clear spans for negative moment***.
- Shear is taken equal to:

  $V_u = Coefficient\ w_u \ell_n$ **Eq. 11.8-2**
- ***Nondimensional analysis*** can be adopted to show the validity of ***Eq. 11.8-1*** and ***Eq. 11.8-2***.
- The coefficients, found in ***ACI Code 6.5.2*** and ***6.5.4***, are shown in ***Table 11.8-1*** and summarized in ***Figure 11.8-1***.

**Table 11.8-1: Moment and shear values using ACI coefficient, Table 6.5.2 of ACI code.**

| Moment | Location | Condition | $M_u$ |
|---|---|---|---|
| Positive | End span | Discontinuous end integral with support | $w_u\ell_n^2/14$ |
| | | Discontinuous end unrestrained | $w_u\ell_n^2/11$ |
| | Interior spans | All | $w_u\ell_n^2/16$ |
| Negative[1] | Interior face of exterior support | Member built integrally with supporting spandrel beam | $w_u\ell_n^2/24$ |
| | | Member built integrally with supporting column | $w_u\ell_n^2/16$ |
| | Exterior face of first interior support | Two spans | $w_u\ell_n^2/9$ |
| | | More than two spans | $w_u\ell_n^2/10$ |
| | Face of other supports | All | $w_u\ell_n^2/11$ |
| | Face of all supports satisfying (a) or (b) | (a) slabs with spans not exceeding 3 m<br>(b) beams where ratio of sum of column stiffnesses to beam stiffness exceeds 8 at each end of span | $w_u\ell_n^2/12$ |

[1]To calculate negative moments, $\ell_n$ shall be the average of the adjacent clear span lengths.

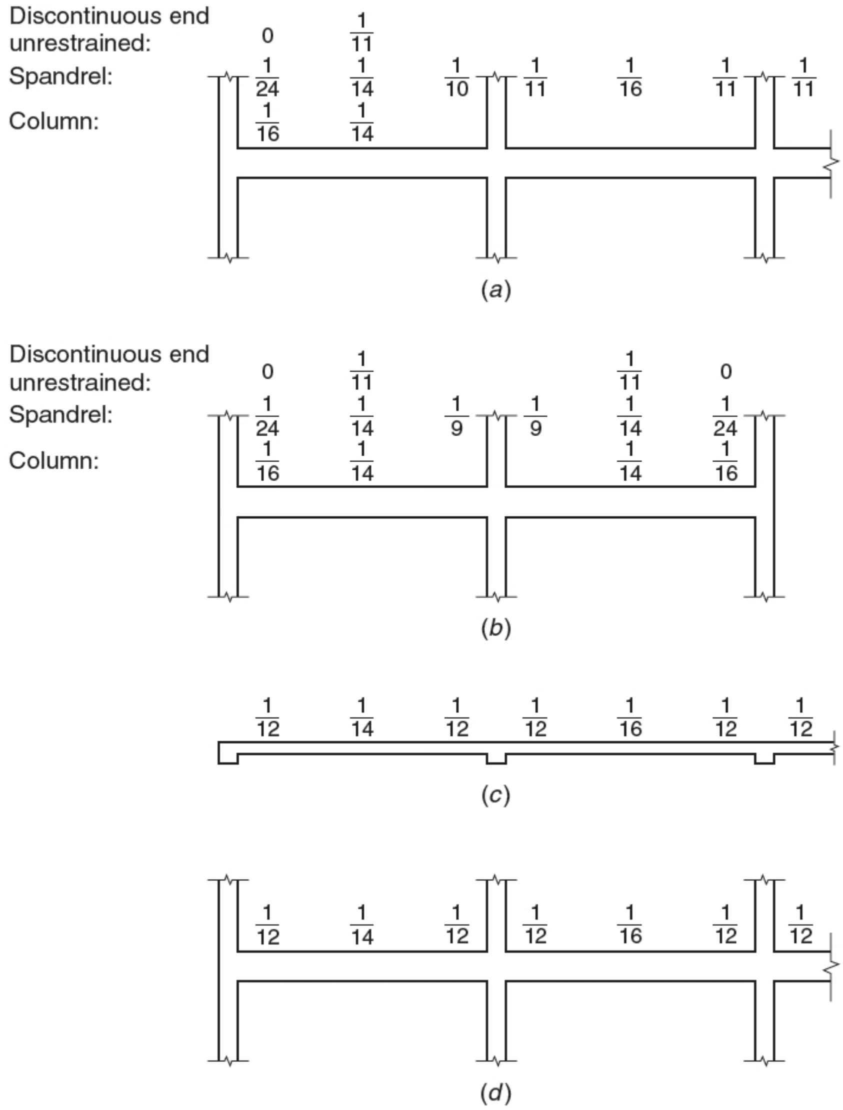



**Figure 11.8-1: Summary of ACI moment coefficients: (a) beams with more than two spans; (b) beams with two spans only; (c) slabs with spans not exceeding 3m; and (d) beams in which the sum of column stiffnesses exceeds 8 times the sum of beam stiffnesses at each end of the span.**

### 11.8.2 DIFFERENT TYPES OF DISCONTINUOUS END

Three types for discontinues supports of ***Table 11.8-1*** are presented and discussed in below.

- **Unrestrained Support:** This type of support occurs when beams or slabs are supported directly on masonry walls or concrete walls without monolithic casting.

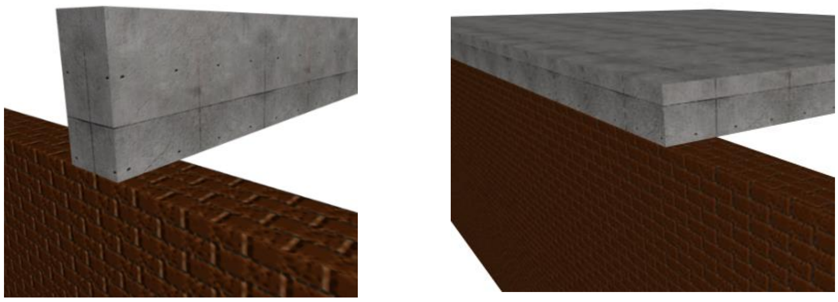


**Figure 11.8-2: Unrestrained Discontinuous Ends.**

- **Spandrel Support:** This type of support occurs when beams or slabs are supported on a monolithically casted edge beam or girder.

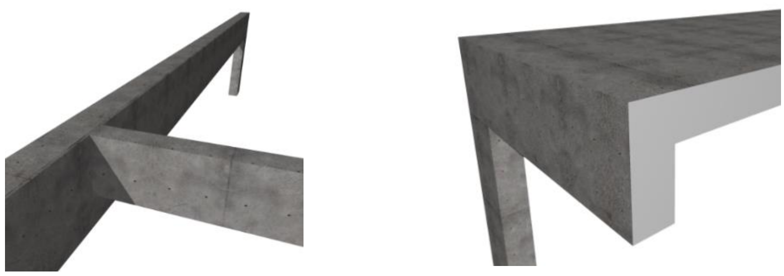


**Figure 11.8-3: Discontinuous ends with spandrel member support.**

- **Column Support:** This type of support occurs when beams supported directly on columns. Slabs that supported directly on columns are out the scope of our course (junior course) and will be studied thoroughly in senior course.

**Figure 11.8-4: Discontinuous ends with column support.**

### 11.8.3 BASES AND CONDITIONS OF ACI COEFFICIENTS

- The ACI coefficients were ***derived by elastic analysis***.
- It is considering ***alternative placement of live load*** to yield maximum negative or positive moments at the critical sections.
- Limitations for ACI coefficients:

  Following limitations have to be satisfied to ensure that the problem to be analyzed is ***similar to that adopted in the elastic analysis of ACI coefficients method*** and to avoid ***overestimation of moments and shear forces***.

  - Members are prismatic, see ***Figure 11.8-5***.
  - Loads are uniformly distributed.
  - The unfactored live load does not exceed 3 times the unfactored dead load.
  - There are two or more spans.
  - The longer of two adjacent spans does not exceed the shorter by more than 20 percent.

    As indicated in ***Figure 11.8-6*** when spans significantly differ, the shortest span may be subjected to negative moments.

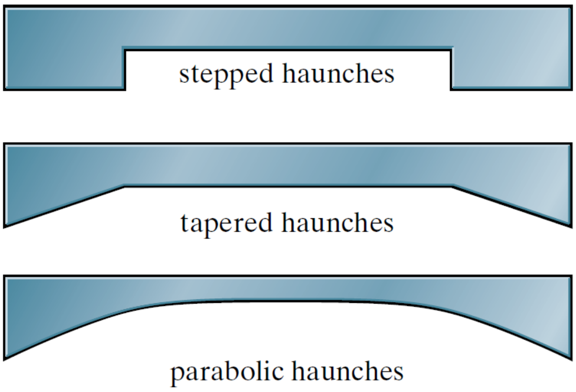



**Figure 11.8-5: Non-prismatic beams.**

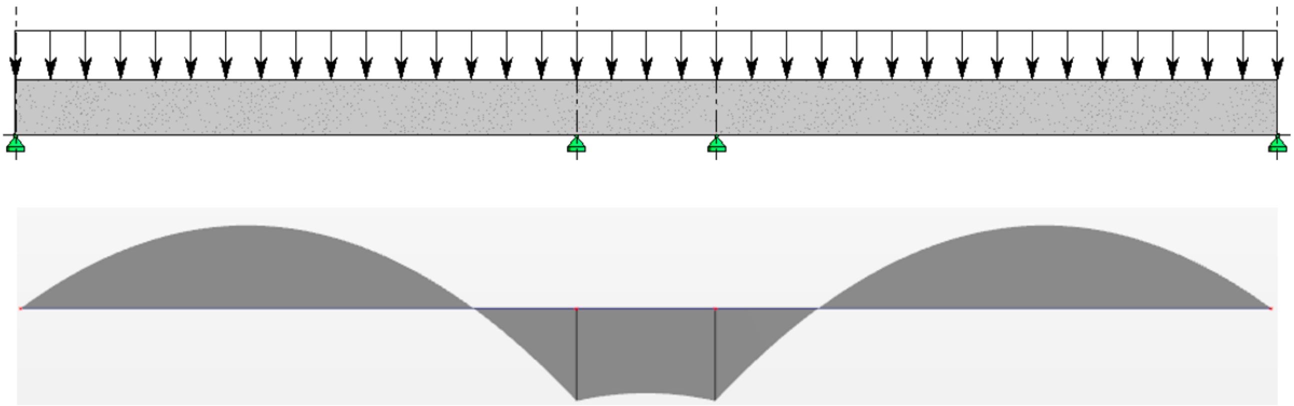


**Figure 11.8-6: A beam with significantly differs spans.**

### 11.8.4 MOMENT DIAGRAM AND MOMENT ENVELOPE

- As alternative loading patterns are considered in applying the Code moment coefficients result in an ***envelope of maximum moments***, as illustrated in ***Figure 11.8-7*** for one span of a continuous frame.
- For maximum positive moment, that span would carry dead and live loads, while adjacent spans would carry dead load only, producing the diagram of ***Figure 11.8-7 a***.
- For maximum negative moment at the left support, dead and live loads would be placed on the given span and that to the left, while the adjacent span on the right would carry only dead load, with the result shown in ***Figure 11.8-7b***.

- ***Figure 11.8-7c*** shows the corresponding results for maximum moment at the right support.
- The composite moment diagram formed from the controlling portions of those just developed (***Figure 11.8-7d***) provides the basis for design of the span.

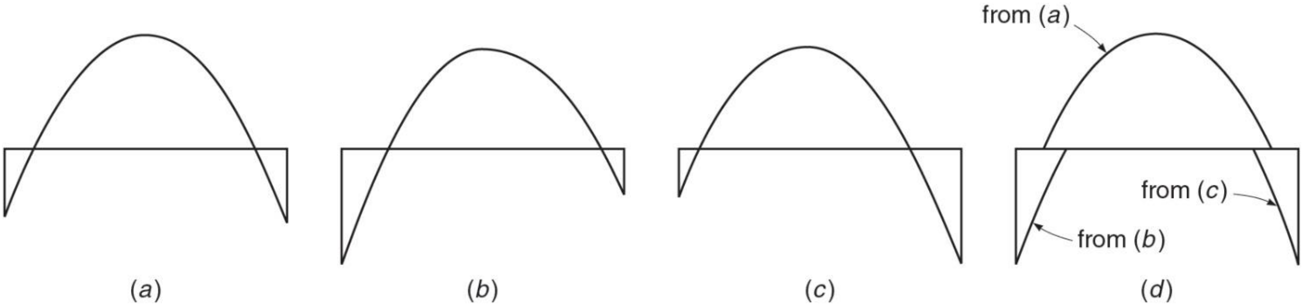



**Figure 11.8-7: Maximum moment diagrams and moment envelope for a continuous beam: (a) maximum positive moment; (b) maximum negative moment at left end; (c) maximum negative moment at right end; and ( d ) composite moment envelope.**

- Inflection Points:
  - As observed in ***Figure 11.8-7***, there is a range of positions for the points of inflection resulting from alternate loadings.
  - In the region of the inflection point, it is evident from ***Figure 11.8-7d*** that there may be a ***reversal of moments for alternative load patterns***. However, within the stated limits for use of the coefficients, there should be no reversal of moments at the critical design sections near midspan or at the support faces.

### 11.8.5 NOTES ON MAXIMUM SHEAR FORCE ACCORDING ACI COEFFICIENTS METHOD

- As indicated in ***Figure 11.8-8***, the shears at the ends of the spans in a continuous frame are modified from the value of $w_u \ell_n / 2$ for a simply supported beam because of the usually unbalanced end moments.

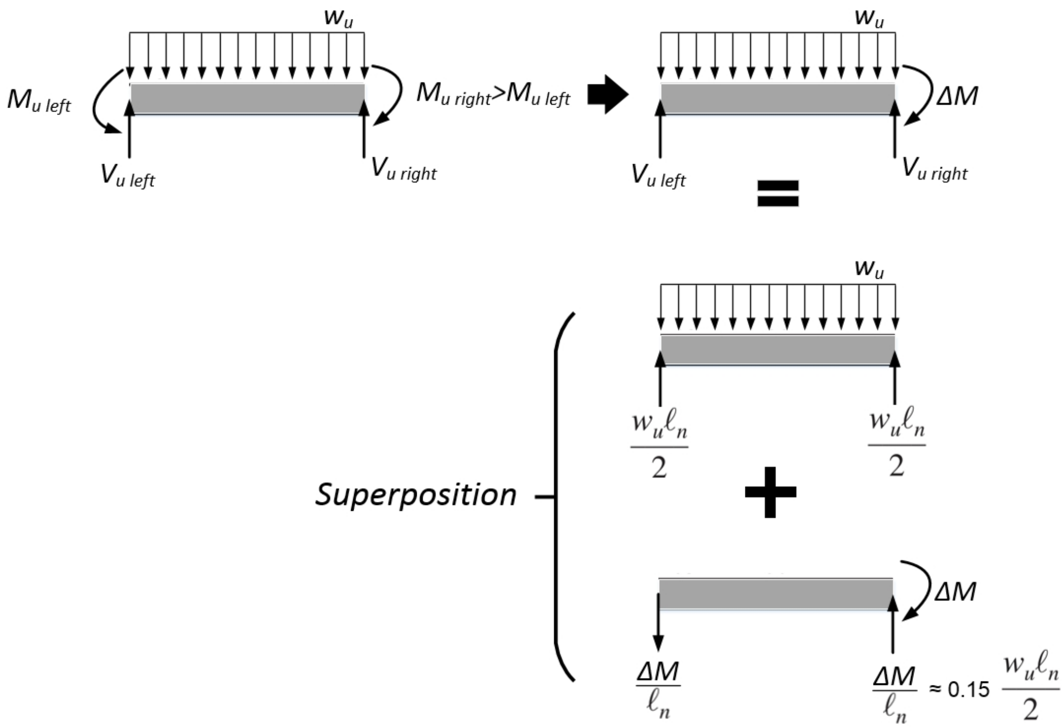



**Figure 11.8-8: Effect of unbalanced moment on shear of end spans.**

- For ***interior spans***, within the limits of the ACI coefficient method, ***this effect will seldom exceed about 8 percent***, and it may be ***neglected***, as suggested in ***Table 11.8-1***.
- However, for end spans, ***at the face of the first interior support***, the additional shear is significant, and a ***15 percent increase*** above the simple beam shear is indicated in ***Table 11.8-1***. The corresponding ***reduction in shear at the face of the exterior support is conservatively neglected***.

### 11.8.6 ACI Coefficient Method versus Closed-from Elastic Analysis

- Comparison of the moments found using the ACI coefficients with those calculated by more exact analysis will usually indicate that the ***coefficient moments are quite conservative***. Actual elastic moments may be ***considerably smaller***.
- Consequently, in many reinforced concrete structures, ***significant economy can be achieved by making a more precise analysis***. This is ***mandatory*** for beams and slabs with spans differing by more than 20 percent, sustaining loads that are not uniformly distributed, or carrying live loads greater than 3 times the dead load.

### 11.8.7 ACI Coefficient Method and Moments in Columns

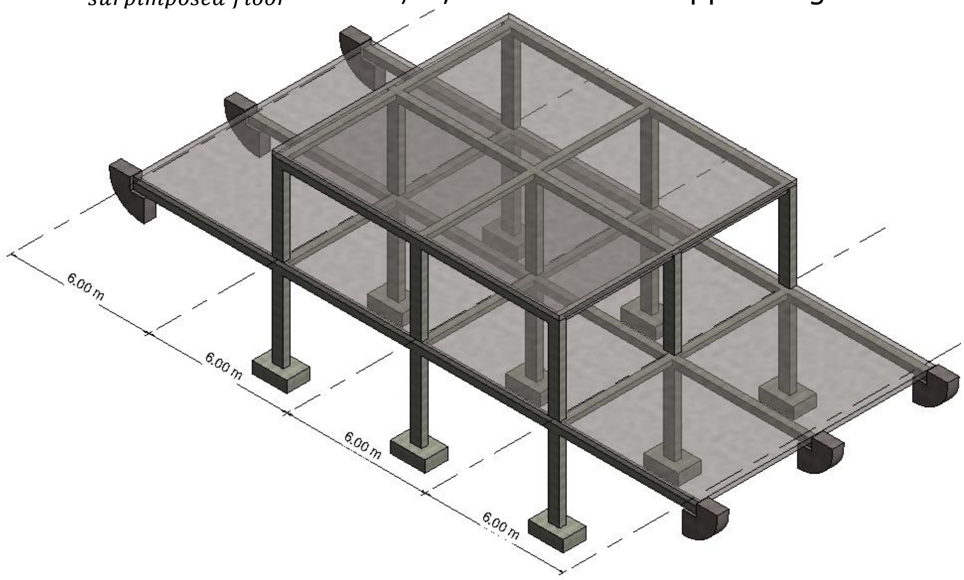
Because the load patterns in a continuous frame that produce critical moments in the columns are different from those for maximum negative moments in the beams, column moments must be found separately.

---

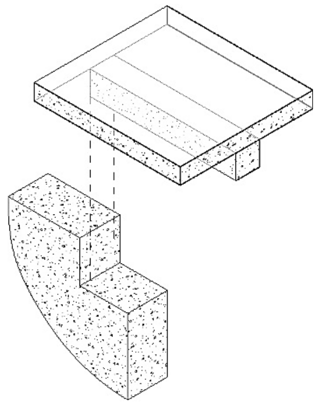
### 11.8.8 ANALYSIS EXAMPLES

**Example 11.8-1**

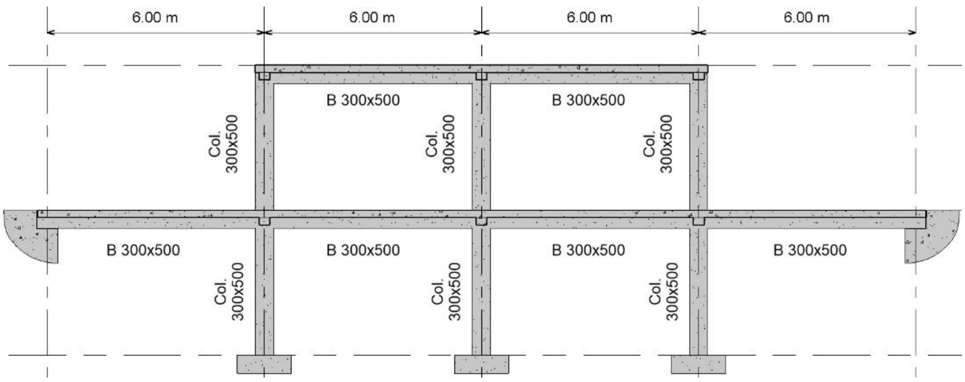
Use ACI coefficients method, if applicable, to determine the factored moments and shear for roof and floor beams of interior frame of building indicated in ***Figure 11.8-9***. In your solution assume that $W_{roof\ live} = 3\ kN/m$, $W_{floor\ live} = 10\ kN/m$, $W_{surpimposed\ roof} = 9\ kN/m$, and $W_{surpimposed\ floor} = 6\ kN/m$, and that the support region has a width of 500mm.




**3D View.**



**Support region.**




**Elevation view for interior frame.**

**Figure 11.8-9: Building frame for Example 11.8-1.**

## Solution

Roof Beams:

$$W_{self\ of\ beam} = 0.5 \times 0.3 \times 24 = 3.6 \frac{kN}{m} \Rightarrow W_{dead\ roof} = 9 + 3.6 = 12.6 \frac{kN}{m}$$

$$W_{u\ roof} = \max(1.4 \times 12.6, 1.2 \times 12.6 + 1.6 \times 3) \approx 20 \frac{kN}{m}$$

Check applicability of ACI coefficients method for roof beams:

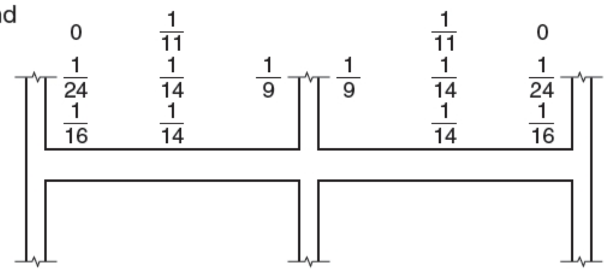
- Members are prismatic, Okay.
- Loads are uniformly distributed, Okay.
- The unfactored live load does not exceed 3 times the unfactored dead load,

  $$W_{dead\ roof} = 12.6 \frac{kN}{m} > W_{roof\ live} = 3 \frac{kN}{m} \therefore Ok.$$
- There are two or more spans:

  As there are two spans, therefore okay.
- The longer of two adjacent spans does not exceed the shorter by more than 20 percent:

  Adjacent spans are equals, therefore okay.

For roof beams, moments can be determined with referring to ***Case b*** of ***Figure 11.8-1*** above.




$$\ell_n = 6.0 - \frac{0.5}{2} \times 2 = 5.5\ m$$

$$M_{u-ve\ int.} = \frac{W_u \ell_n^2}{9} = \frac{20 \times 5.5^2}{9} = 67.2\ kN.m$$

As the discontinuous end is a column support, hence positive and exterior negative moments are:

$$M_{u+ve} = \frac{W_u \ell_n^2}{14} = \frac{20 \times 5.5^2}{14} = 43.2\ kN.m$$

$$M_{u-ve\ ext.} = \frac{W_u \ell_n^2}{16} = \frac{20 \times 5.5^2}{16} = 37.8\ kN.m$$

According to ***Table 11.8-1*** above, factored shear force at exterior face of first interior support is:

$$V_u = 1.15 \frac{W_u \ell_n}{2} = 1.15 \times \frac{20 \times 5.5}{2} = 63.3\ kN$$

<u>Floor Beams:</u>

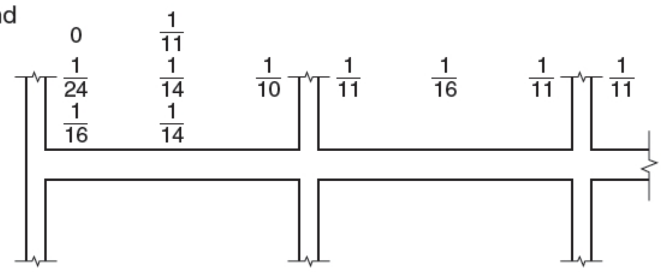
$$W_{self\ of\ beam} = 0.5 \times 0.3 \times 24 = 3.6 \frac{kN}{m} \Rightarrow W_{dead\ floor} = 6 + 3.6 = 9.6 \frac{kN}{m}$$

$$W_{u\ floor} = \max(1.4 \times 9.6, 1.2 \times 9.6 + 1.6 \times 6) \approx 21.2 \frac{kN}{m}$$

Check applicability of ACI coefficients method for floor beams:

- Members are prismatic, Okay.
- Loads are uniformly distributed, Okay.
- The unfactored live load does not exceed 3 times the unfactored dead load,
  $$3W_{dead\ floor} = 3 \times \left(9.6 \frac{kN}{m}\right) > W_{floor\ live} = 6 \frac{kN}{m} \therefore Ok.$$
- There are two or more spans:
  As there are four spans, therefore okay.
- The longer of two adjacent spans does not exceed the shorter by more than 20 percent: Adjacent spans are equals, therefore okay.

For floor beams, moments can be determined with referring to ***Case a*** of ***Figure 11.8-1*** above.




$$\ell_n = 6.0 - \frac{0.5}{2} \times 2 = 5.5\ m$$

For interior spans:

$$M_{u-ve\ for\ interior\ span} = \frac{W_u \ell_n^2}{11} = \frac{21.2 \times 5.5^2}{11} = 58.3\ kN.m$$

$$M_{u+ve\ for\ interior\ span} = \frac{W_u \ell_n^2}{16} = \frac{21.2 \times 5.5^2}{16} = 40.1\ kN.m$$

For exterior spans:

$$M_{u-ve\ int. for\ exterior\ span} = \frac{W_u \ell_n^2}{10} = \frac{21.2 \times 5.5^2}{10} = 64.1\ kN.m$$

As the discontinuous end is simple support, hence positive and exterior negative moments are:

$$M_{u+ve\ for\ exterior\ span} = \frac{W_u \ell_n^2}{11} = \frac{21.2 \times 5.5^2}{11} = 58.3\ kN.m$$

$$M_{u-ve\ ext. for\ exterior\ span} = 0$$

Finally, the shear at exterior face of first interior support is:

$$V_u = 1.15\frac{W_u \ell_n}{2} = 1.15 \times \frac{21.2 \times 5.5}{2} = 67.0\ kN$$

Factored moments and shear forces for roof and floor beams are summarized in ***Figure 11.8-10*** below.

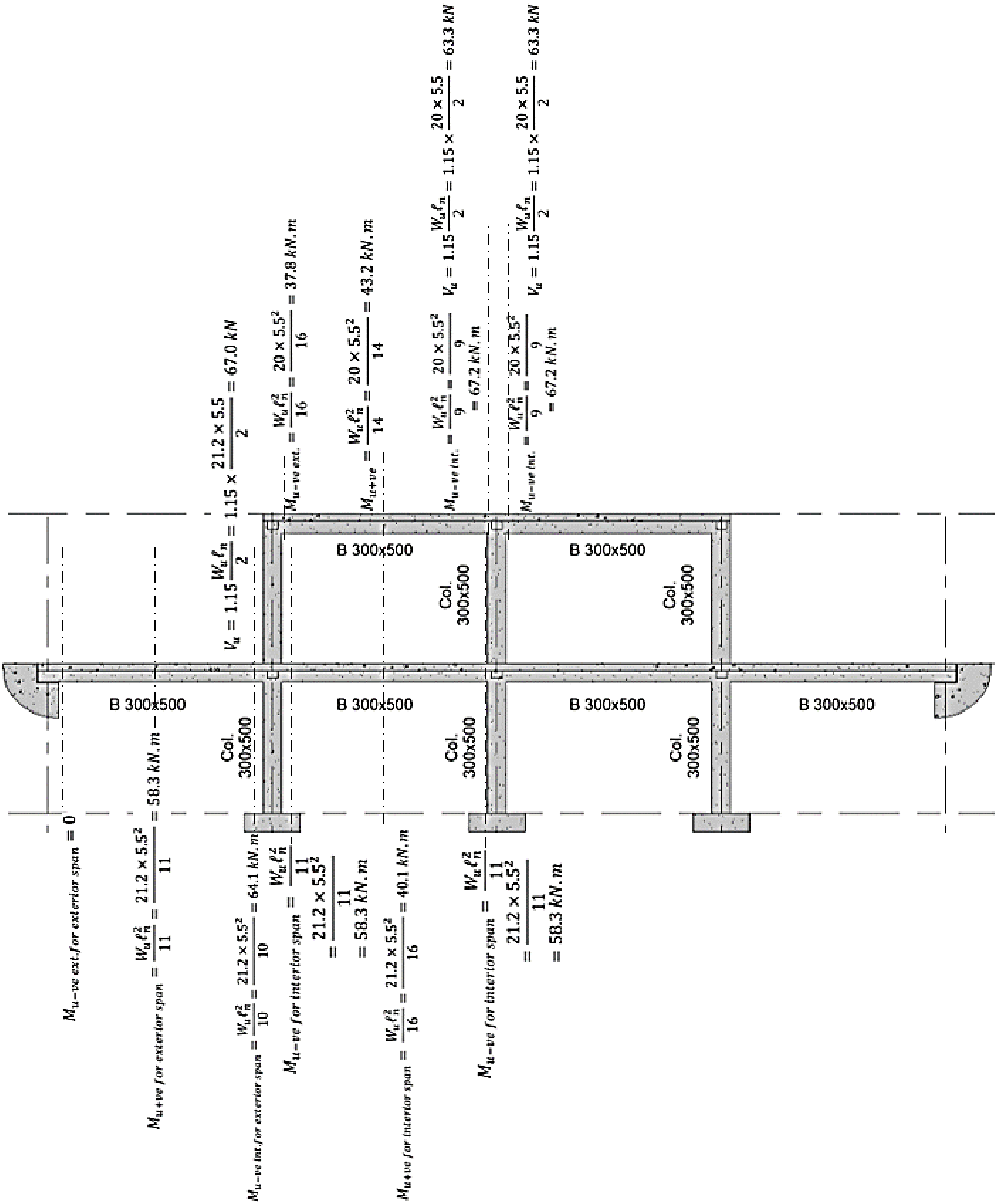
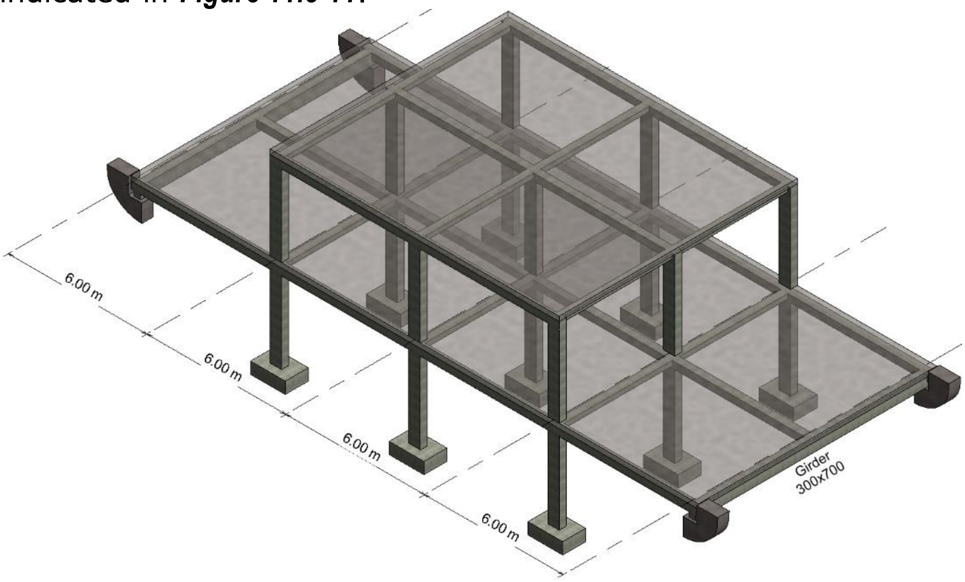



**Figure 11.8-10: Summary of factored moments and shears for Example 11.8-1.**

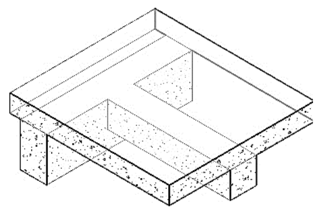
---

**Example 11.8-2**

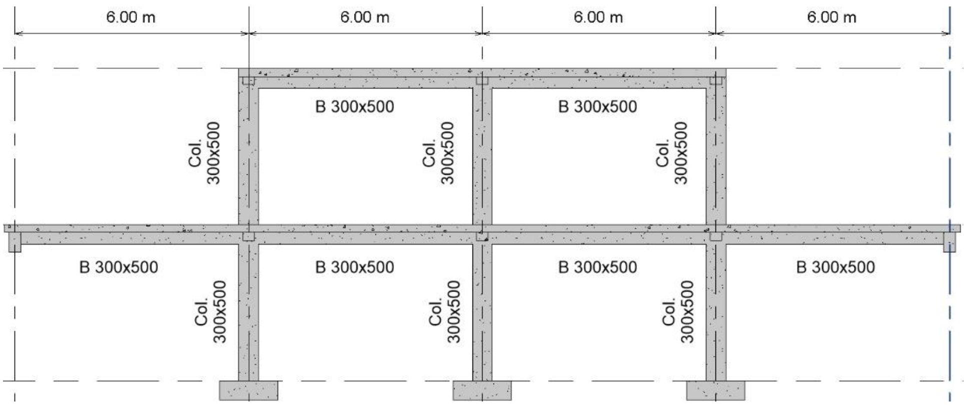
Resolve ***Example 11.8-2*** above when floor beam are supported on spandrel girder as indicated in ***Figure 11.8-11***.




**3D View.**



**Support region.**




**Elevation view for interior frame.**

**Figure 11.8-11: Building frame for Example 11.8-2.**

## Solution

<u>Roof Beams:</u>

Change of exterior support condition for floor beams has no effect on factored forces of roof beams; hence, they would be as indicated in ***Figure 11.8-10*** above.

<u>Floor Beams:</u>

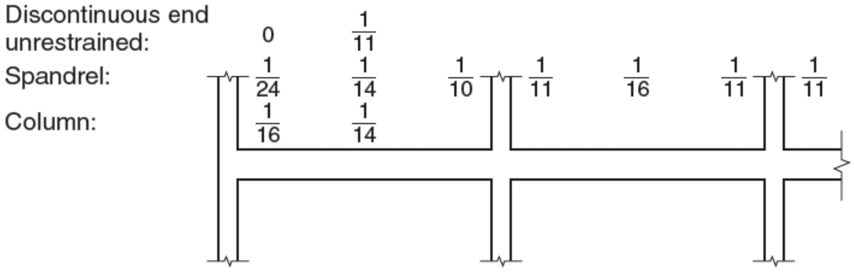
Regarding to floor beams, clear spans for exterior and interior spans are:

$\ell_{n\, interior\, span} = 6.00 - \frac{0.5}{2} \times 2 = 5.5\, m$, $\ell_{n\, exterior\, span} = 6.00 - \frac{0.5}{2} - \frac{0.3}{2} = 5.60\, m$

According to ***Table 11.8-1*** above, $\ell_n$ shall be average of two adjacent span for negative moment:

$$\ell_{n\, avg.} = \frac{5.5 + 5.6}{2} = 5.55\, m$$

With these clear spans, factored moments for an interior span can be determined with referring ***Case a*** of ***Figure 11.8-1*** above:




$$M_{u-ve\, for\, interior\, span} = \frac{W_u \ell_n^2}{11} = \frac{21.2 \times 5.55^2}{11} = 59.4\, kN.m$$

$$M_{u+ve\, for\, interior\, span} = \frac{W_u \ell_n^2}{16} = \frac{21.2 \times 5.5^2}{16} = 40.1\, kN.m$$

While for an exterior span with spandrel exterior support moments would be:

$$M_{u-ve\, int.for\, exterior\, span} = \frac{W_u \ell_n^2}{10} = \frac{21.2 \times 5.55^2}{10} = 65.3\, kN.m$$

$$M_{u+ve\,for\,exterior\,span} = \frac{W_u \ell_n^2}{14} = \frac{21.2 \times 5.6^2}{14} = 47.5\,kN.m$$

$$M_{u-ve\,ext.for\,exterior\,span} = \frac{W_u \ell_n^2}{24} = \frac{21.2 \times 5.6^2}{24} = 27.7\,kN.m$$

It is useful to note that $\ell_{n\,exterior\,span}$ of $5.60\,m$ is used to determine the $M_{u-ve\,ext.for\,exterior\,span}$ as it is governed by its exterior span.

Finally, the shear at exterior face of first interior support is:

$$V_u = 1.15 \frac{W_u \ell_n}{2} = 1.15 \times \frac{21.2 \times 5.6}{2} = 68.3\,kN$$

Factored moments and shear forces for roof and floor beams are summarized in ***Figure 11.8-12*** below.

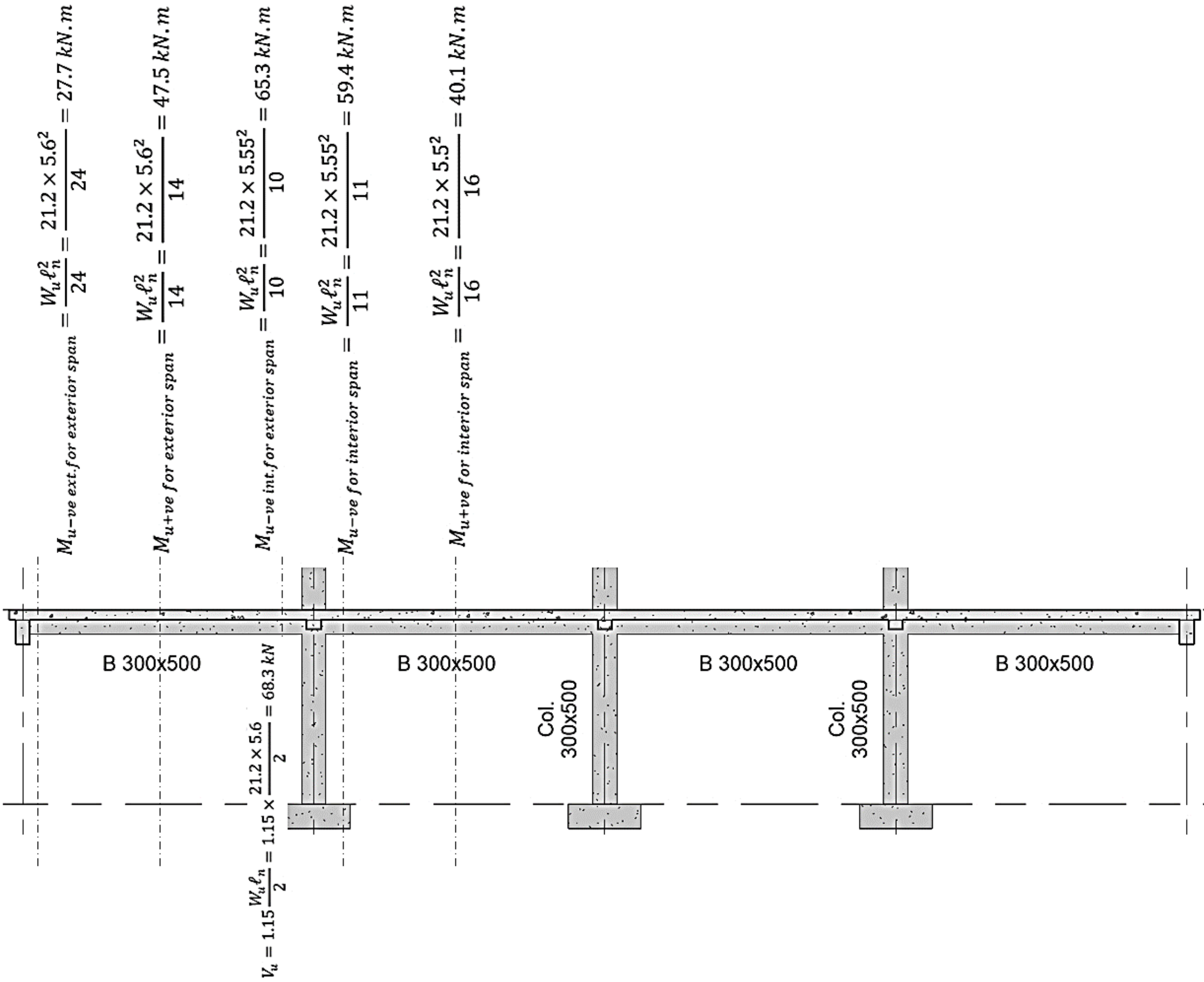
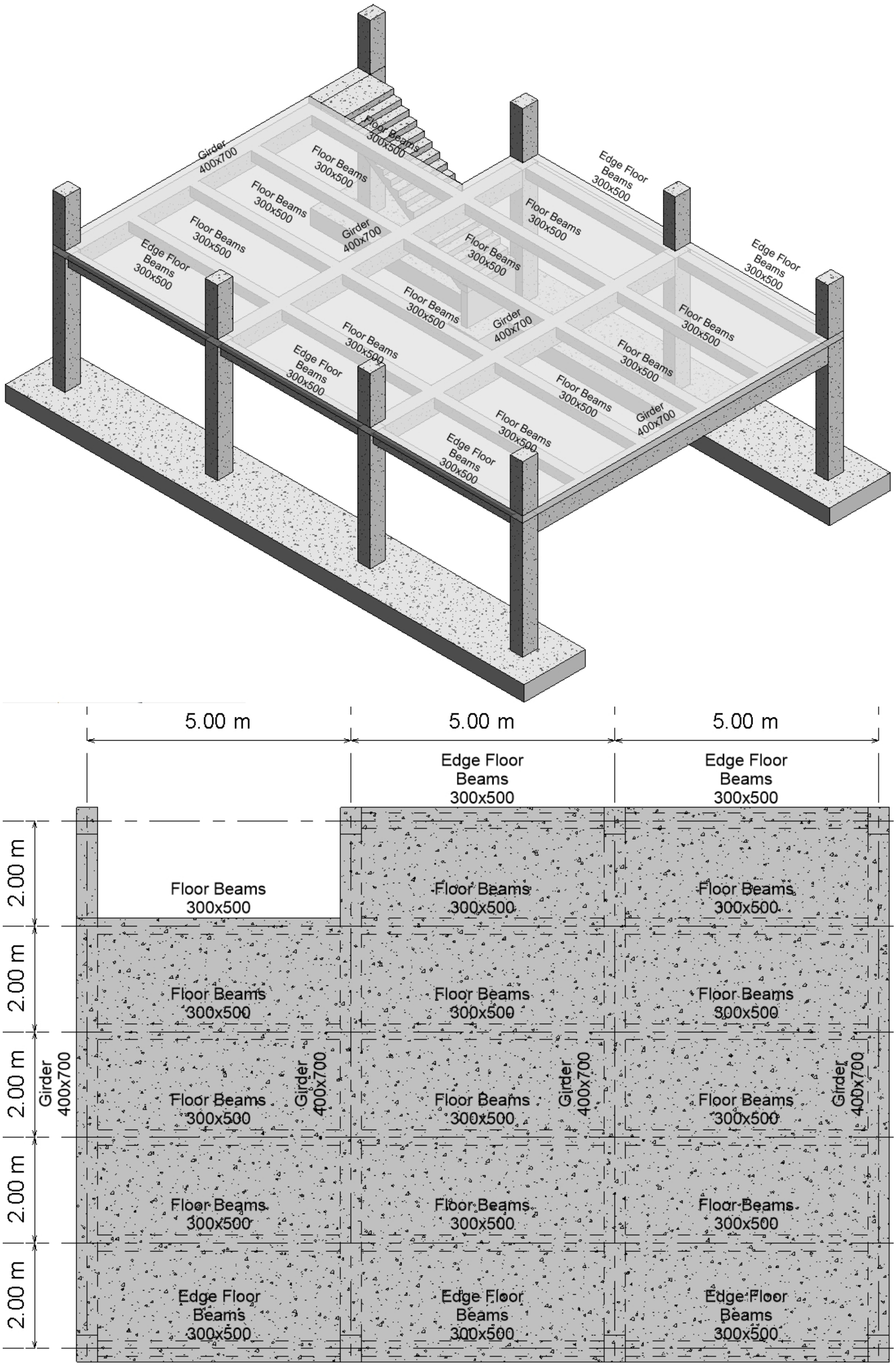



**Figure 11.8-12: Summary of factored moments and shears for Example 11.8-2.**

**Example 11.8-3**

For building frame indicated in ***Figure 11.8-13*** below, edge floor beams and floor beams are subjected to factored loads of $20\,kN/m$ and $40\,kN/m$ respectively. Check if ACI coefficients method is applicable to determine factored forces of a typical floor beam and of the edge floor beam located along stair. In your checking, assume the unfactored live load does not exceed 3 times the unfactored dead load.




**3D View**

**Plan View**

**Figure 11.8-13: Frame for Example 11.8-3.**

## Solution

<u>Applicability of ACI coefficients method:</u>

Checking applicability of ACI coefficients method for the edge floor beams and a typical floor beam:

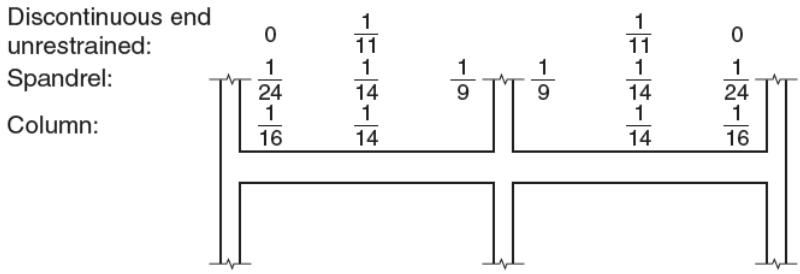
- Members are prismatic, Okay.
- Loads are uniformly distributed, Okay.
- The unfactored live load does not exceed 3 times the unfactored dead load: This condition is satisfied according to example statement.
- There are two or more spans:
  - There are two spans for the edge floor beam located along stair shaft, oaky.
  - There are three spans for a typical floor beam, okay.
- The longer of two adjacent spans does not exceed the shorter by more than 20 percent: Adjacent spans are equals, therefore okay.

<u>Factored Forces for the Edge Floor Beam:</u>

As the edge beam is directly supported on columns (that have width equal to that of the girder), therefore their clear span is:

$$\ell_{n\ for\ edge\ floor\ beams} = 5.00 - \frac{0.4}{2} \times 2 = 4.60\ m$$

With two spans and column exterior support, the factored moments can be determined with referring to ***Case b*** of ***Figure 11.8-1*** above.




$$M_{u-ve\ ext.} = \frac{W_u \ell_n^2}{16} = \frac{20 \times 4.6^2}{16} = 26.5\ kN.m$$

$$M_{u+ve} = \frac{W_u \ell_n^2}{14} = \frac{20 \times 4.6^2}{14} = 30.2\ kN.m$$

$$M_{u-ve\ int.} = \frac{W_u \ell_n^2}{9} = \frac{20 \times 4.6^2}{9} = 47.0\ kN.m$$

$$V_u = 1.15 \frac{W_u \ell_n}{2} = 1.15 \times \frac{20 \times 4.6}{2} = 52.9\ kN$$

The factored forces for edge beams are summarized in see ***Figure 11.8-14*** below.

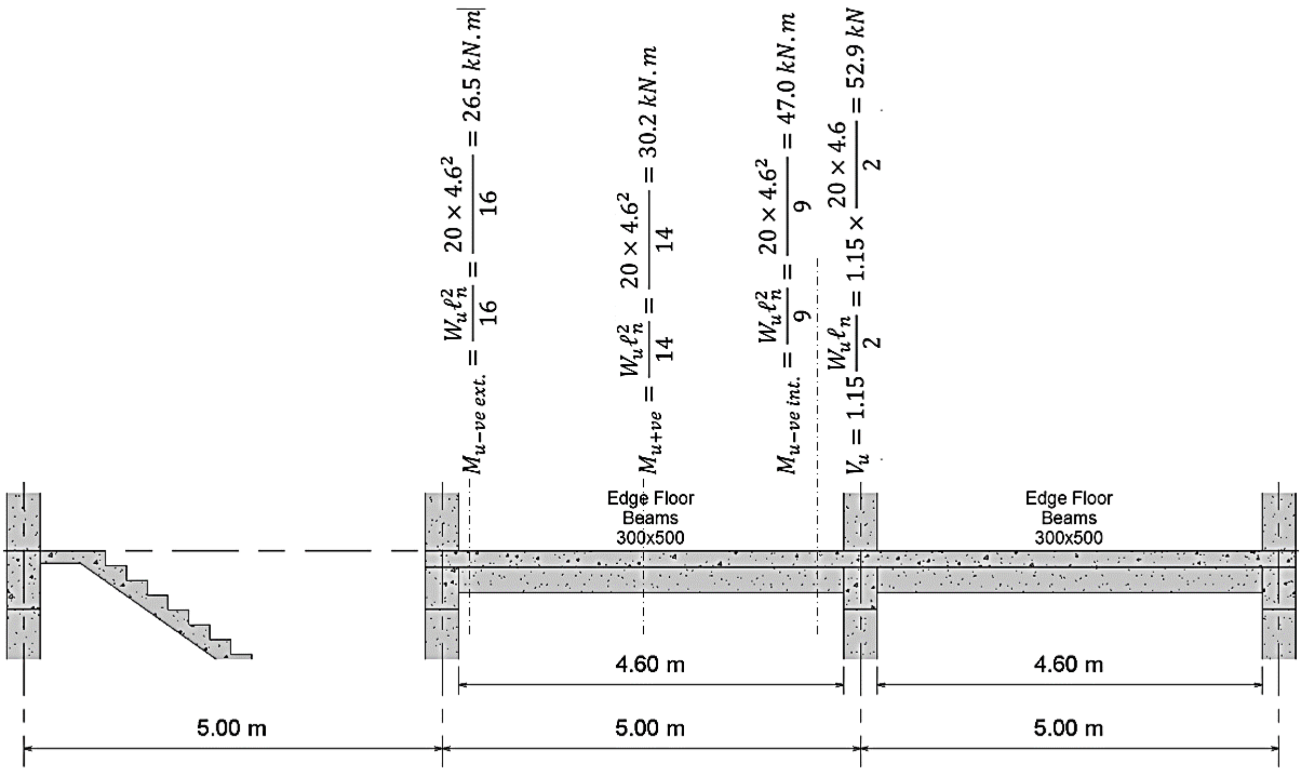



**Figure 11.8-14: Summary of factored forces for edge floor beams Example 11.8-3.**