sectional area is used to calculate the stress. If instantaneous cross-sectional area is used the curve would rise as shown in **figure- 3 (a)** . For a material
with low ductility there is no definite yield point and usually off-set yield points are defined for convenience. This is shown in **figure-3**.b For a brittle material stress increases linearly with strain till fracture occurs.

4-Yield criteria

There are numerous yield criteria, going as far back as Coulomb (1773). Many of these were originally developed for brittle materials but were later applied to ductile materials. Some of the more common ones will be discussed briefly here.

4.1 Maximum principal stress theory (Rankine theory)

According to this, if one of the principal stresses σ1 (maximum principal stress), σ2 (minimum principal stress) or σ3 exceeds the yield stress, yielding would occur. In a two dimensional loading situation for a ductile material where tensile and compressive yield stress are nearly of same magnitude

$$\sigma_1 = \pm \sigma_y$$

$$\sigma_2 = \pm\sigma_y$$

Using this, a yield surface may be drawn, as shown in **figure- 4.1**
Yielding occurs when the state of stress is at the boundary of the rectangle.
Consider, for example, the state of stress of a thin walled pressure vessel. Here σ1= 2σ2, σ1 being the circumferential or hoop stress and σ2 the axial stress. As the pressure in the vessel increases the stress follows the dotted line. At a point (say) a, the stresses are still within the elastic limit but at b, σ1reaches σy although σ2 is still less than σy. Yielding will then begin at point b.
This theory of yielding has very poor agreement with experiment.

However,the theory has been used σ_2 successfully for brittle materials.

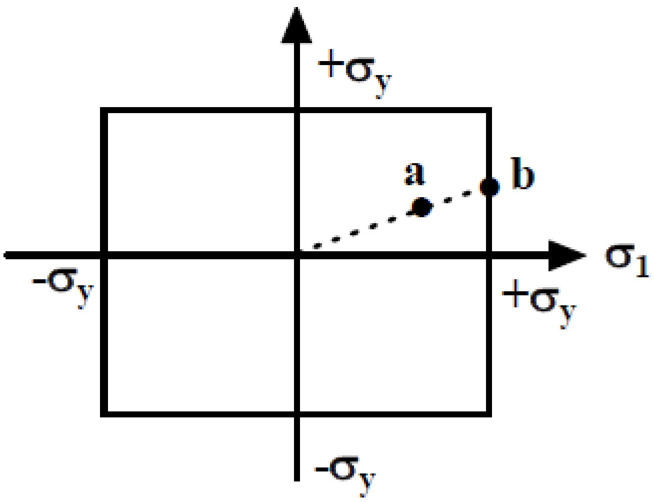

figure- 4.1 *Yield surface corresponding to maximum principal stress theory*

3.1.4.2 Maximum principal strain theory (St. Venant's theory)
According to this theory, yielding will occur when the maximum principal strain just exceeds the strain at the tensile yield point in either simple tension or compression. If ε1 and ε2 are maximum and minimum principal strains corresponding to σ1 and σ2, in the limiting case

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) \qquad |\sigma_1| \geq |\sigma_2|$$

$$\varepsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1) \qquad |\sigma_2| \geq |\sigma_1|$$

$$\text{This gives, } E\varepsilon_1 = \sigma_1 - \nu\sigma_2 = \pm\sigma_0$$

$$E\varepsilon_2 = \sigma_2 - \nu\sigma_1 = \pm\sigma_0$$

The boundary of a yield surface in this case is thus given as shown in **figure-**4-2

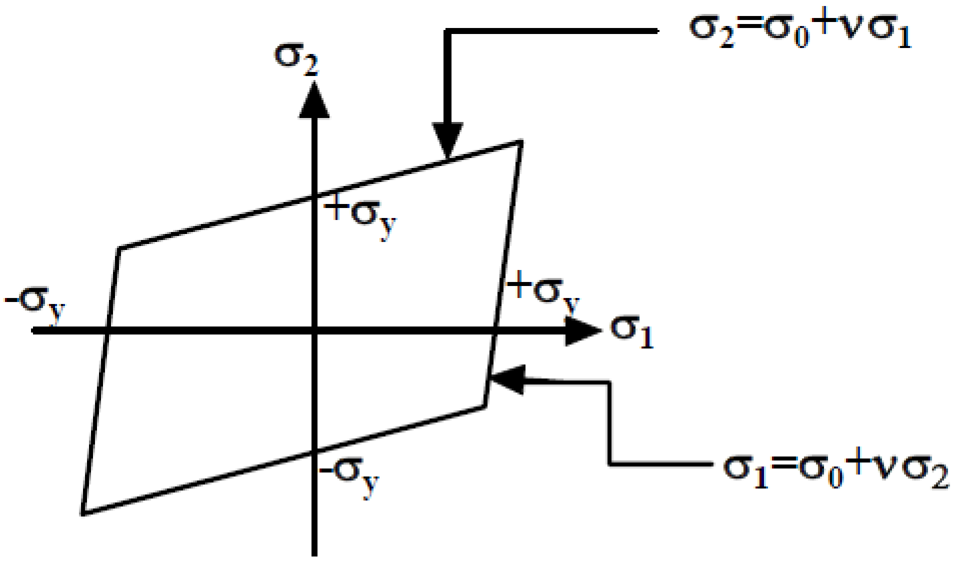

figure-4-2 *Yield surface corresponding to maximum principal strain theory*

3.1.4.3 Maximum shear stress theory (Tresca theory)

According to this theory, yielding would occur when the maximum shear stress just exceeds the shear stress at the tensile yield point. At the tensile yield point σ2= σ3 = 0 and thus maximum shear stress is σy/2. This gives us six conditions for a three-dimensional stress situation:

$$\sigma_1 - \sigma_2 = \pm\sigma_y$$

$$\sigma_2 - \sigma_3 = \pm\sigma_y$$

$$\sigma_3 - \sigma_1 = \pm\sigma_y$$

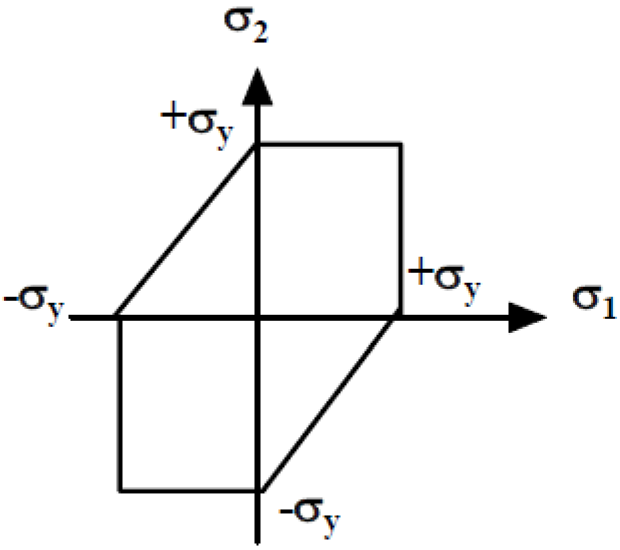

figure-4-3 *Yield surface corresponding to maximum shear stress theory*

In a biaxial stress situation **figure-**4-3 case, σ3 = 0 and this gives

$$\sigma_1 - \sigma_2 = \sigma_y \qquad \text{if } \sigma_1 > 0, \sigma_2 < 0$$

$$\sigma_1 - \sigma_2 = -\sigma_y \qquad \text{if } \sigma_1 < 0, \sigma_2 > 0$$

$$\sigma_2 = \sigma_y \qquad \text{if } \sigma_2 > \sigma_1 > 0$$

$$\sigma_1 = -\sigma_y \qquad \text{if } \sigma_1 < \sigma_2 < 0$$

$$\sigma_1 = -\sigma_y \qquad \text{if } \sigma_1 > \sigma_2 > 0$$

$$\sigma_2 = -\sigma_y \qquad \text{if } \sigma_2 < \sigma_1 < 0$$

This criterion agrees well with experiment.
In the case of pure shear, σ1 = - σ2 = k (say), σ3 = 0
and this gives σ1- σ2 = 2k= σy

This indicates that yield stress in pure shear is half the tensile yield stress and this is also seen in the Mohr's circle (figure- 4.4) for pure shear.

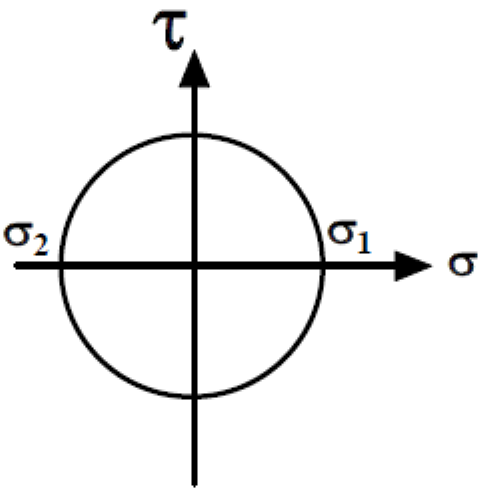

figure- 4.4- *Mohr's circle for pure shear*