College Of Engineering
University Of Anbar

كليـــة الهندسة
جامعـة الأنبـــار

Dept. of Chem. & Petrochemical Engineering
Subject : Physics
First Stage

Physics

Chapter 2 –Vectors

lecturer
Dr. Mohammed Jasim

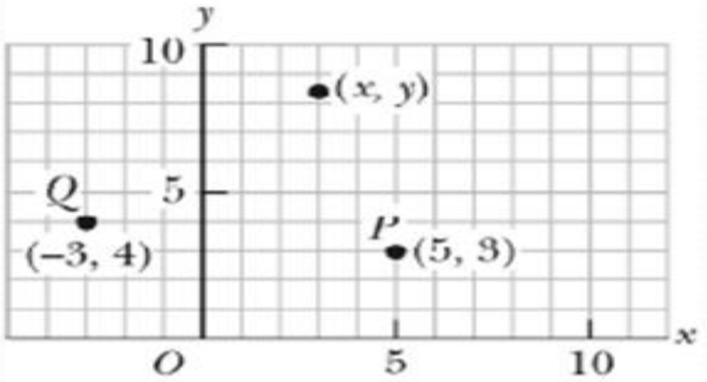
Coordinate Systems

Used to describe the position of a point in space

Common coordinate systems are:

Cartesian Coordinate System

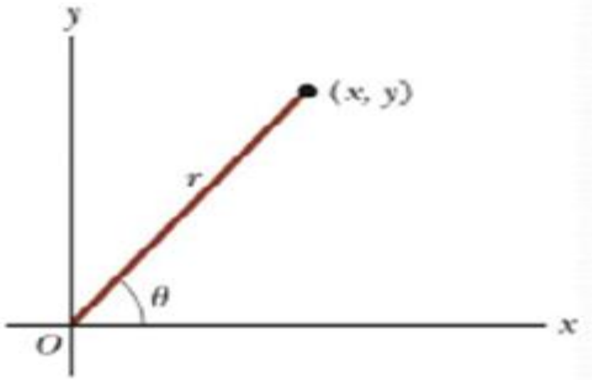
In cartesin (Also called rectangular) coordinate system: x- and y- axes intersect at the origin Points are labeled (x,y)



Polar Coordinate System

Origin and reference line are noted

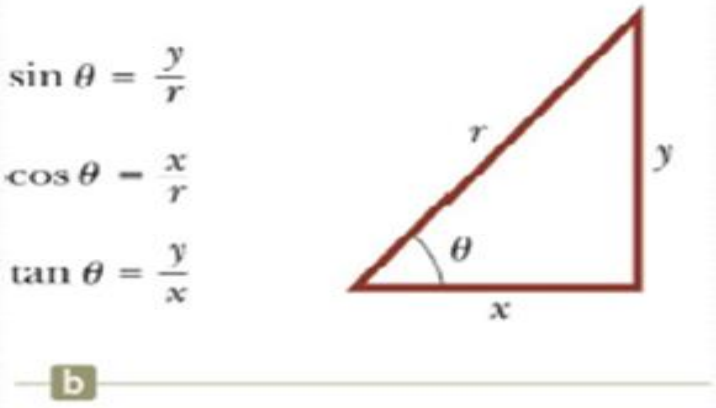
Point is distance *r* from the origin in the direction of angle θ, from reference line. The reference line is often the x-axis. Points are labeled (r, θ). Based on forming a right triangle from *r* and θ



$x = r\cos\theta$ *and* $y = r\sin\theta$

If the Cartesian coordinates are known: $\tan\theta = \dfrac{y}{x}$

$$r = \sqrt{x^2 + y^2}$$



Example

The Cartesian coordinates of a point in the xy plane are (x,y) = (-3.50, -2.50) m, as shown in the figure. Find the polar coordinates of this point. Solution: From Equation 3.4,

$$r = \sqrt{x^2 + y^2}$$

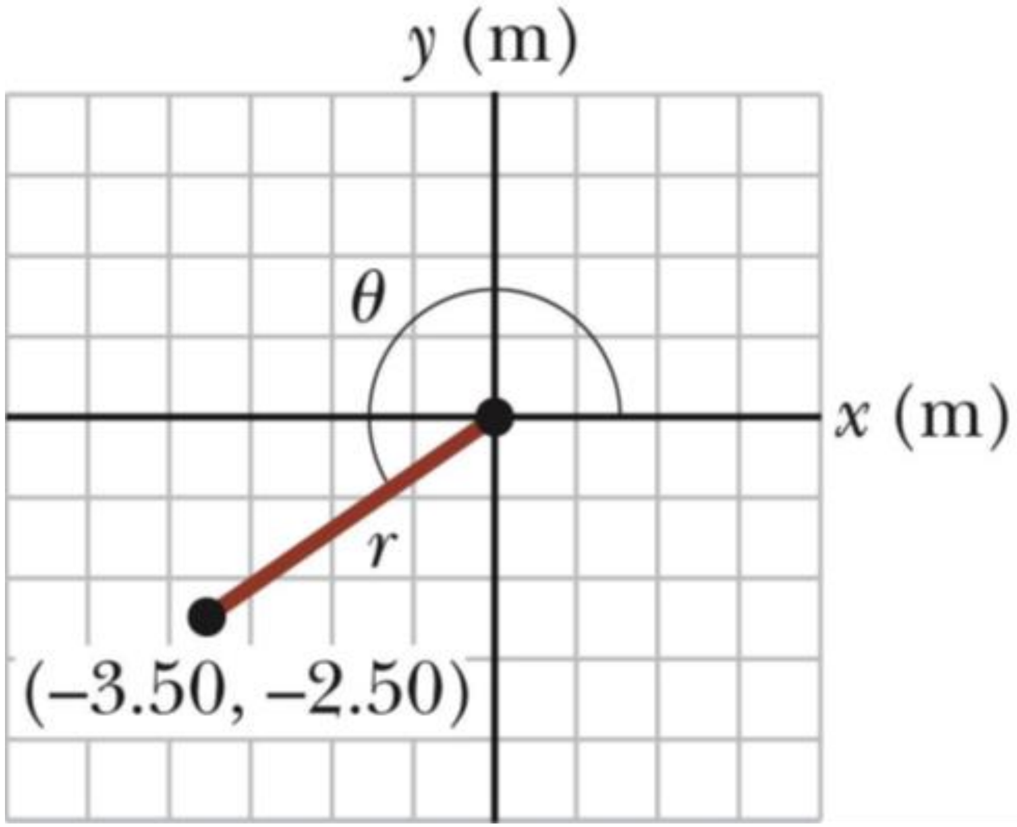
$$= \sqrt{(-3.50 \text{ m})^2 + (-2.50 \text{ m})^2}$$

$$= 4.30 \text{ m}$$

and from Equation 3.3,

$$\tan\theta = \frac{y}{x} = \frac{-2.50 \text{ m}}{-3.50 \text{ m}} = 0.714$$

$$\theta = 216^\circ \quad \text{(signs give quadrant)}$$



Vectors and Scalars

A scalar quantityis completely specified by a single value with an appropriate unit and has no direction.

- Many are always positive
- Some may be positive or negative
- Rules for ordinary arithmetic are used to manipulate scalar quantities.

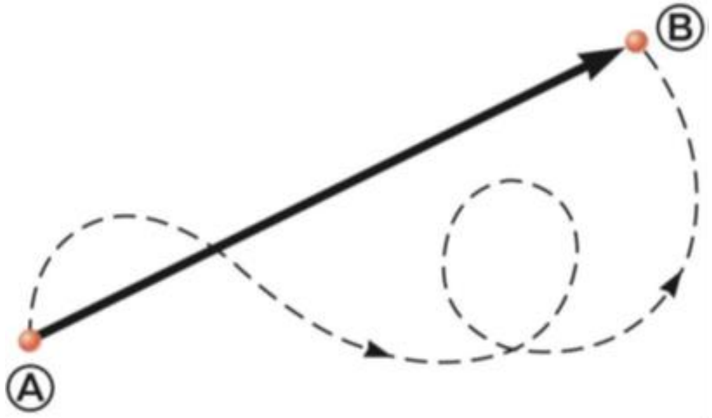
A vector quantityis completely described by a number and appropriate units plus a direction.

Example: A particle travels from A to B along the path shown by the broken line.

- This is the distance traveled and is a scalar.

The displacementis the solid line from A to B

- The displacement is independent of the path taken between the two points.
- Displacement is a vector.



Vector Notation

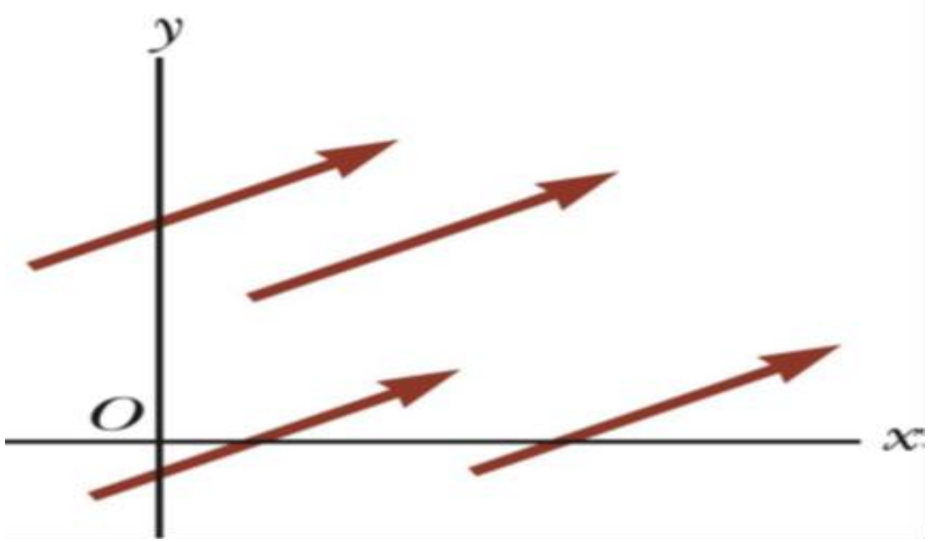
Text uses bold with arrow to denote a vector :

Also used for printing is simple bold print: **A**

When dealing with just the magnitude of a vector in print, an italic letter will be used: *A* or $\left|\vec{A}\right|$

- The magnitude of the vector has physical units.
- The magnitude of a vector is always a positive number.

When handwritten, use an arrow:



Equality of Two Vectors

Two vectors are equal if they have the same magnitude and the same direction.

if A= B and they point along parallel lines

All of the vectors shown are equal.

Allows a vector to be moved to a position parallel to itself

Adding Vectors

Vector addition is very different from adding scalar quantities.

When adding vectors, their directions must be taken into account.

Choose a scale.

Draw the first vector, A , with the appropriate length and in the direction specified, with respect to a coordinate system.

Draw the next vector with the appropriate length and in the direction specified, with respect to a coordinate system whose origin is the end of vector A and parallel to the coordinate system used for A .

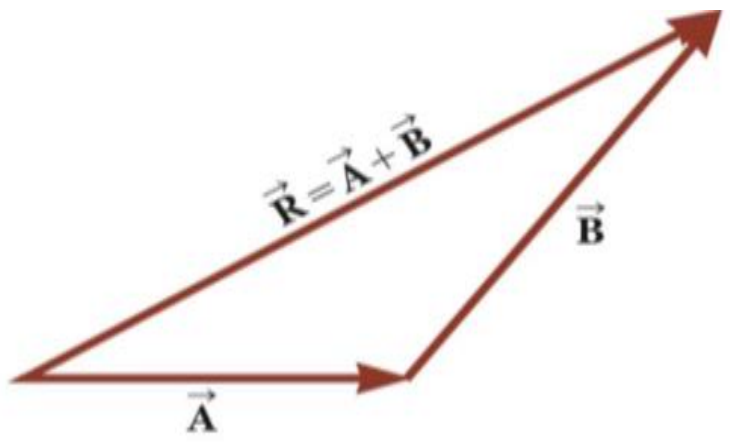
Adding Two Vectors Graphically

Continue drawing the vectors “tip-to-tail” or “head-to-tail”.

The resultant is drawn from the origin of the first vector to the end of the last vector.

Measure the length of the resultantand its angle.

Use the scale factor to convert length to actual magnitude.



Vectors, Rules

When two vectors are added, the sum is independent of the order of the addition.

□This is the Commutative Law of Addition.

$$\vec{A}+\vec{B}=\vec{B}+\vec{A}$$

When adding three or more vectors, their sum is independent of the way in which the individual vectors are grouped.

□This is called the Associative Property of Addition.

$$\vec{A}+\left(\vec{B}+\vec{C}\right)=\left(\vec{A}+\vec{B}\right)+\vec{C}$$

When added to the original vector, gives a resultant of zero.

- Represented as $-\vec{A}$
- $\vec{A}+\left(-\vec{A}\right)=0$

The negative of the vector will have the same magnitude, but point in the opposite direction.

To subtract two vectors use $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ as $\vec{\mathbf{A}} + \left(-\vec{\mathbf{B}}\right)$

Continue with standard vector addition procedure

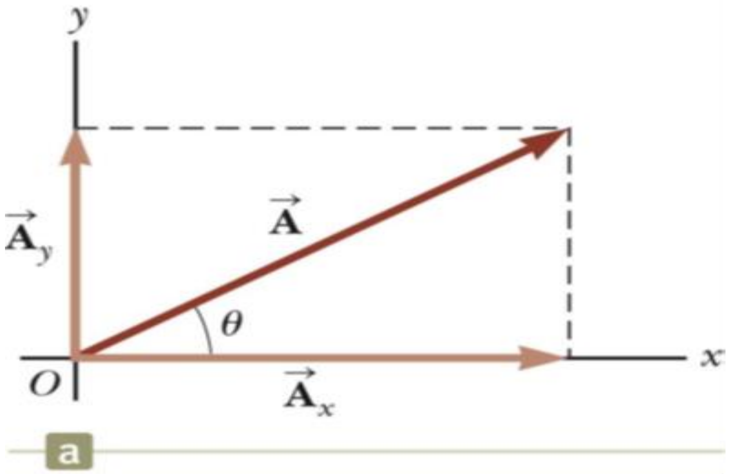
<u>Components of a Vector, Introduction</u>

A component is a projection of a vector along an axis.

Any vector can be completely described by its components.

It is useful to use rectangular components.

These are the projections of the vector along the x-and y-axes.



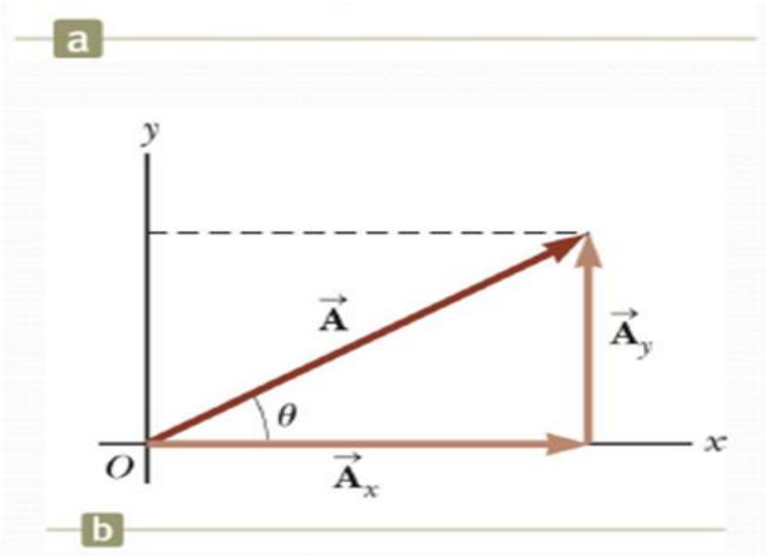
$\vec{A}_x$ and $\vec{A}_y$ are the ***component vectors*** of $\vec{A}$

- They are vectors and follow all the rules for vectors.

A_x and A_y are scalars, and will be referred to as the ***components*** of $\vec{A}$

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

These three vectors form a right triangle.



Components of a Vector, 2

The x-component of a vector is the projection along the x-axis.

$$A_x = A\cos\theta$$

The y-component of a vector is the projection along the y-axis.

$$A_y = A\sin\theta$$

This assumes the angle θ is measured with respect to the x-axis.

□ If not, do not use these equations, use the sides of the triangle directly.

The components are the legs of the right triangle whose hypotenuse is the length of A.

- $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \tan^{-1} \frac{A_y}{A_x}$
- May still have to find θ with respect to the positive x-axis

In a problem, a vector may be specified by its components or its magnitude and direction.

Unit Vectors

A unit vector is a dimensionless vector with a magnitude of exactly 1. Unit vectors are used to specify a direction and have no other physical significance.

The symbols $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ represent unit vectors

They form a set of mutually perpendicular vectors in a right-handed coordinate system.

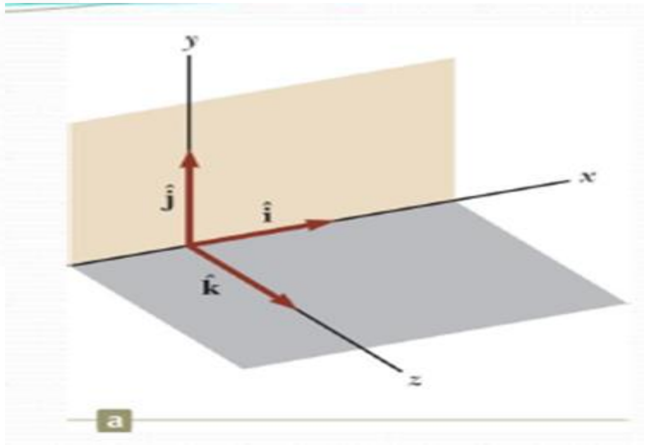
The magnitude of each unit vector is 1

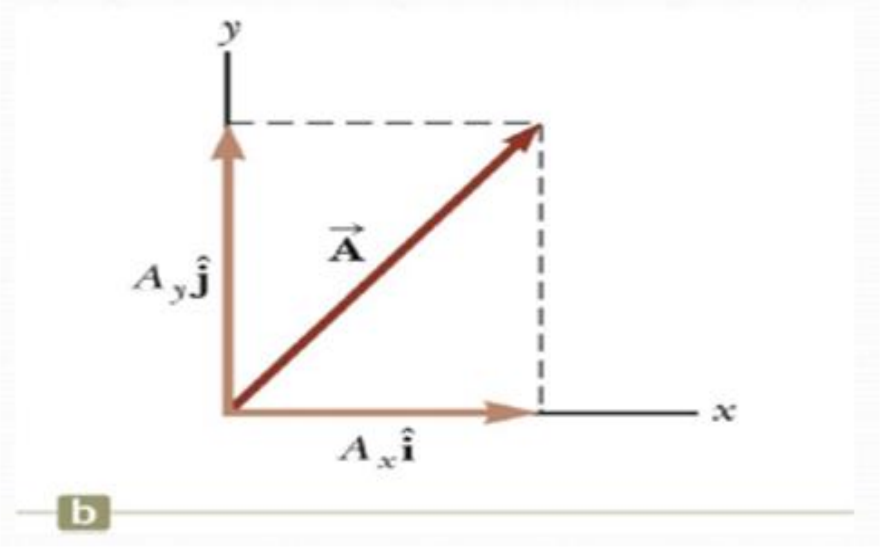
$$|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$$

$\mathbf{A}_x$ is the same as $A_x\,\hat{\mathbf{i}}$ and $\mathbf{A}_y$ is the same as $A_y\,\hat{\mathbf{j}}$ etc.

The complete vector can be expressed as:

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$$





Adding Vectors Using Unit Vectors

Then

$$\vec{\mathbf{R}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}\right) + \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{R}} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}}$$

$$\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$$

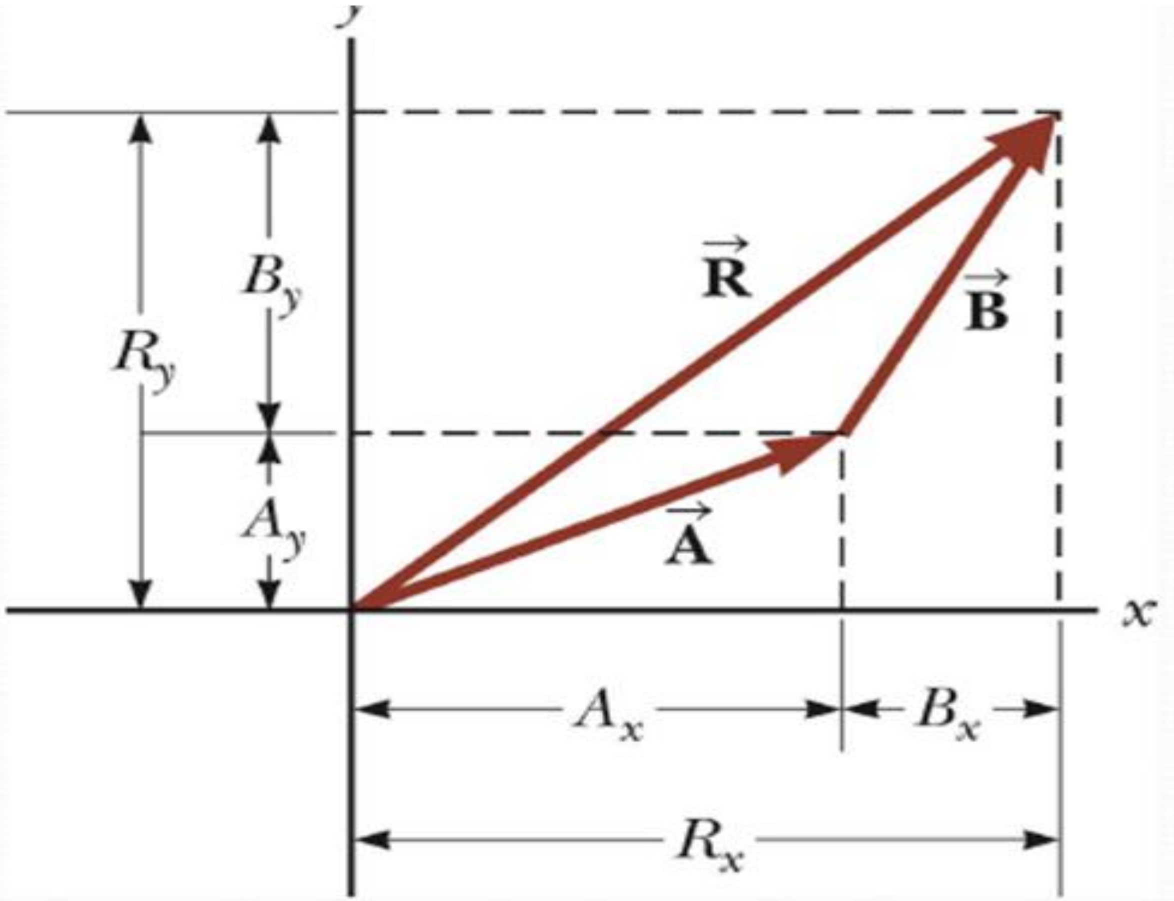
So $R_x = A_x + B_x$ and $R_y = A_y + B_y$

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

Note the relationships among the components of the resultant and the components of the original vectors.

$R_x = A_x + B_x$

$R_y = A_y + B_y$



Three-Dimensional Extension

Using $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ Then

$$\vec{\mathbf{R}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right) + \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$$

$$\vec{\mathbf{R}} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} + (A_z + B_z)\hat{\mathbf{k}}$$

$$\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}} + R_z\hat{\mathbf{k}}$$

So $R_x = A_x + B_x$, $R_y = A_y + B_y$, and $R_z = A_z + B_z$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \qquad \theta_x = \cos^{-1}\frac{R_x}{R}, \text{ etc.}$$

Multiplying or Dividing a Vector by a Scalar

The result of the multiplication or division of a vector by a scalar is a vector.

The magnitude of the vector is multiplied or divided by the scalar

Example

Find the sum of two vectors $\vec{A}$ and $\vec{B}$ given by

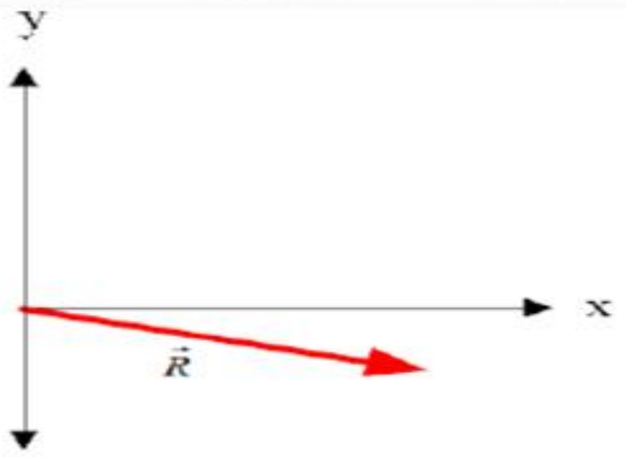
$$\vec{A} = 3i + 4j \qquad \text{and} \qquad \vec{B} = 2i - 5j$$

Note that A_x=3, A_y=4, B_x=2, and B_y=-5

$$\vec{R} = \vec{A} + \vec{B} = (3+2)i + (4-5)j = 5i - j$$

The magnitude of vector $\vec{R}$ is

$$R = \sqrt{R_x{}^2 + R_y{}^2} = \sqrt{25+1} = \sqrt{26} = 5.1$$



The direction of $\vec{R}$ with respect to x-axis is

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{-1}{5} = -11^o$$

Example

Two vectors are given by $\vec{A} = 3i - 2j$ and $\vec{B} = -i - 4j$. Calculate (a) $\vec{A} + \vec{B}$, (b) $\vec{A} - \vec{B}$, (c) $|\vec{A} + \vec{B}|$, (d) $|\vec{A} - \vec{B}|$, and (e) the direction of $\vec{A} + \vec{B}$ and $|\vec{A} - \vec{B}|$.

(a) $\vec{A} + \vec{B} = (3i - 2j) + (-i - 4j) = 2i - 6j$

(b) $\vec{A} - \vec{B} = (3i - 2j) - (-i - 4j) = 4i + 2j$

(c) $|\vec{A} + \vec{B}| = \sqrt{2^2 + (-6)^2} = 6.32$

(d) $|\vec{A} - \vec{B}| = \sqrt{4^2 + 2^2} = 4.47$

(e) For $\vec{A} + \vec{B}$, $\theta = \tan^{-1}(-6/2) = -71.6^\circ = 288^\circ$

For $\vec{A} - \vec{B}$, $\theta = \tan^{-1}(2/4) = 26.6^\circ$

Example

A particle moves from a point in the *xy* plane having rectangular coordinates (-3,-5)m to a point with coordinates (-1,8)m. (a) Write vector expressions for the position vectors in unit vector form for these two points. (b) What is the displacement vector?

(a) $\vec{R}_1 = x_1 i + y_1 j = (-3i - 5j)m$

$$\vec{R}_2 = x_2 i + y_2 j = (-i + 8j)m$$

(b) Displacement = $\Delta\vec{R} = \vec{R}_2 - \vec{R}_1$

$$\Delta\vec{R} = (x_2 - x_1)i + (y_2 - y_1)j = -i - (-3i) + 8j - (-5j) = (2i + 13j)m$$

Example

The polar coordinates of a point are r=5.5m and θ=240°. What are the rectangular coordinates of this point?

$$x = r\cos\theta = 5.5 \times \cos 240 = -2.75\ \text{m}$$

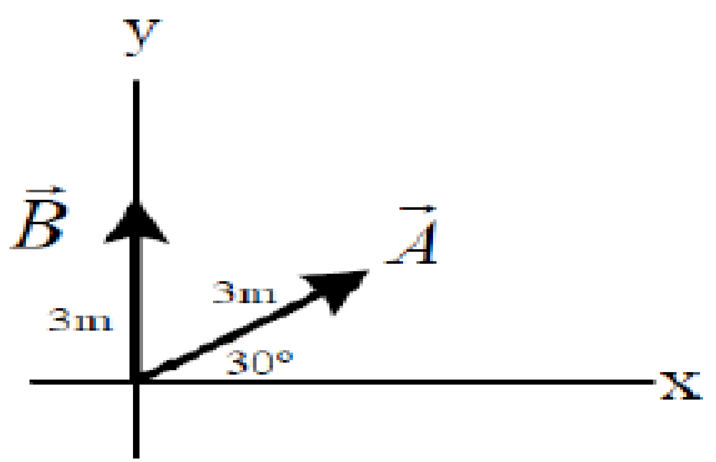
$$y = r\sin\theta = 5.5 \times \sin 240 = -4.76\ \text{m}$$

Homework

A vector has x component of -25 units and a y component of 40 units. Find the magnitude and direction of this vector.

Find the magnitude and direction of the resultant of three displacements having components (3,2) m, (-5, 3) m and (6, 1) m.

Two vector are given by $\vec{A} = 6i - 4j$ and $\vec{B} = -2i + 5j$. Calculate (a) $\vec{A}+\vec{B}$, (b) $\vec{A}-\vec{B}$, $|\vec{A}+\vec{B}|$, (d) $|\vec{A}-\vec{B}|$, (e) the direction of $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$.



Find the x and y components of the vector $\vec{A}$ and $\vec{B}$ shown in Figure . Derive an expression for the resultant vector $\vec{A}+\vec{B}$ in unit vector notation.