Dept. of Chem. & Petrochemical Engineering
Subject : Physics
First Stage

Physics

Chapter-4 Static Equilibrium and Elasticity

lecturer
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Static Equilibrium

Equilibrium implies that the object moves with both constant velocity and constant angular velocity relative to an observer in an inertial reference frame.

Will deal now with the special case in which both of these velocities are equal to zero

This is called static equilibrium.

Static equilibrium is a common situation in engineering.

The principals involved are of particular interest to civil engineers, architects, and mechanical engineers.

Rigid Object in Equilibrium

In the particle in equilibrium model a particle moves with constant velocity because the net force acting on it is zero.

- The objects often cannot be modeled as particles.

For an extended object to be in equilibrium, a second condition of equilibrium must be satisfied.

- This second condition involves the rotational motion of the extended object.

Torque

$$\vec{\tau} = \vec{\mathbf{F}} \times \vec{\mathbf{r}}$$

- The tendency of the force to cause a rotation about O depends on F and the moment arm d. The net torque on a rigid object causes it to undergo an angular acceleration.

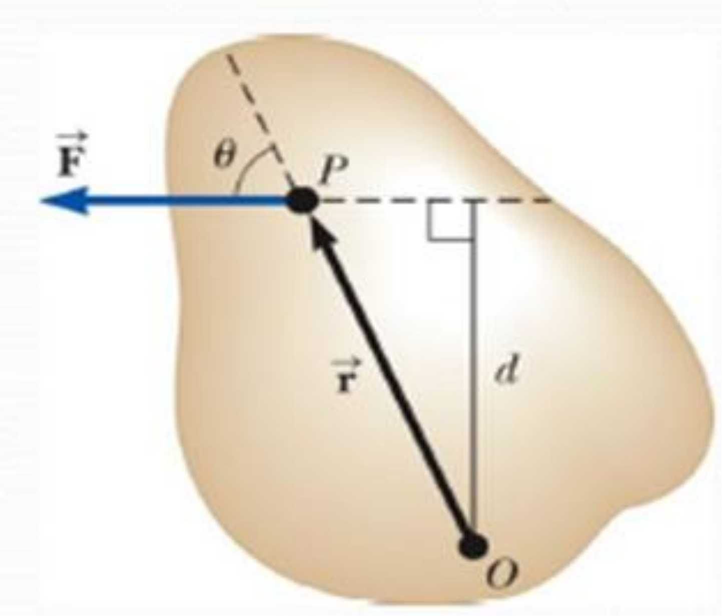
The net external force on the object must equal zero.

$$\sum \vec{\mathbf{F}}_{ext} = 0$$

- If the object is modeled as a particle, then this is the only condition that must be satisfied.

The net external torque on the object about any axis must be zero.

$$\sum \vec{\tau}_{ext} = 0$$



- This is needed if the object cannot be modeled as a particle.

These conditions describe the rigid object in equilibrium analysis model. We will restrict the applications to situations in which all the forces lie in the xy plane. There are three resulting equations:

$$\Sigma F_x = 0\ ,\quad \Sigma F_y = 0$$

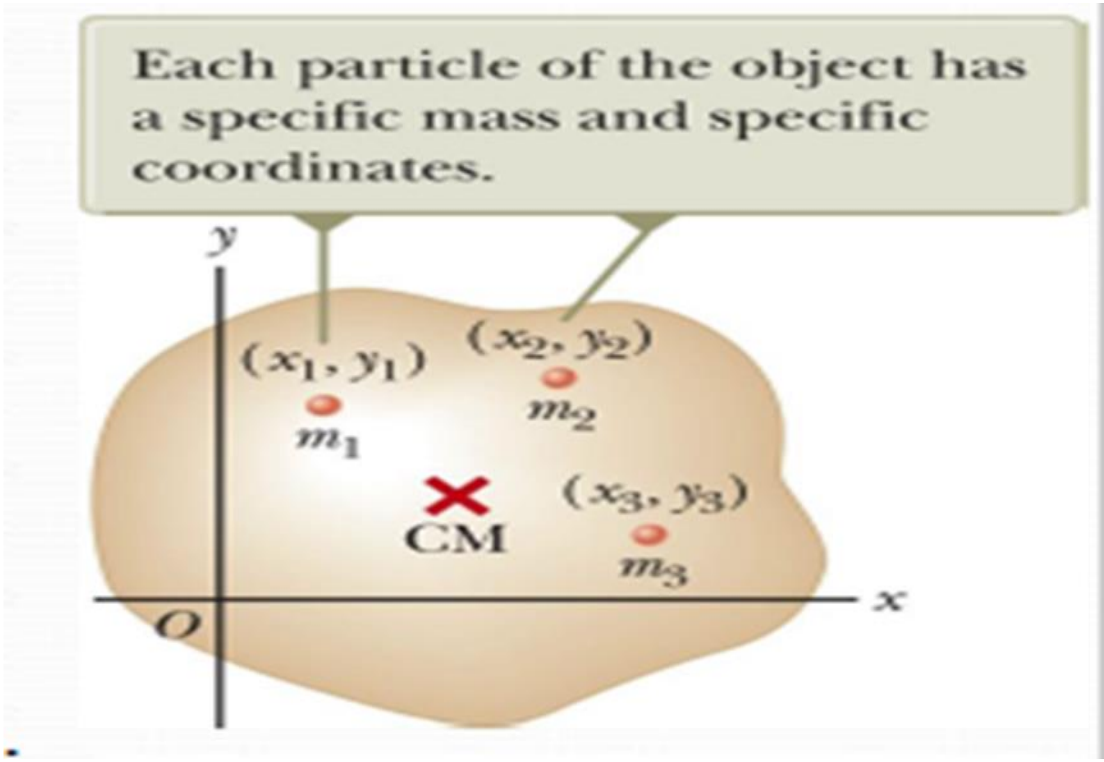
$$\Sigma \tau_z = 0$$

- **<u>Center of Mass</u>**
- An object can be divided into many small particles.
- □ Each particle will have a specific mass and specific coordinates.

The x coordinate of the center of mass will be

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

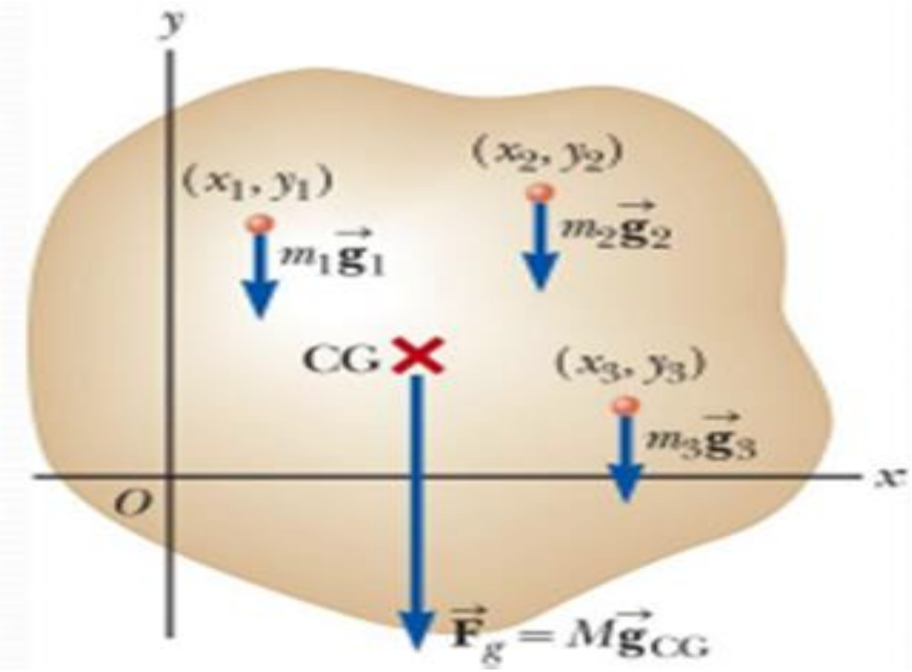
Similar expressions can be found for the y and z coordinates.



Center of Gravity

All the various gravitational forces acting on all the various mass elements are equivalent to a single gravitational force acting through a single point called the center of gravity (CG). Each particle contributes a torque about an axis through the origin equal in magnitude to the particle's weight multiplied by its moment arm.

The center of gravity of the object coincides with its center of mass.



Problem-Solving Strategy- Equilibrium Problems Conceptualize

 Identify all the forces acting on the object.

 Image the effect of each force on the rotation of the object if it were the only force acting on the object. Categorize

 Confirm the object is a rigid object in equilibrium.

 The object must have zero translational acceleration and zero angular acceleration.

Analyze

 Draw a diagram.

 Show and label all external forces acting on the object.

 Particle under a net force model: he object on which the forces act can be represented in a free body diagram as a dot because it does not matter where on the object the forces are applied.

 Rigid object in equilibrium model: Cannot use a dot to represent the object because the location where the forces act is important in the calculations.

Problem-Solving Strategy –Equilibrium Problems, 2

Analyze, cont

- Establish a convenient coordinate system.
- Find the components of the forces along the two axes.
- Apply the first condition for equilibrium (SF=0). Be careful of signs.
- Choose a convenient axis for calculating the net torque on the rigid object.
- Choose an axis that simplifies the calculations as much as possible.
- Apply the second condition for equilibrium.
- The two conditions of equilibrium will give a system of equations.
- Solve the equations simultaneously.

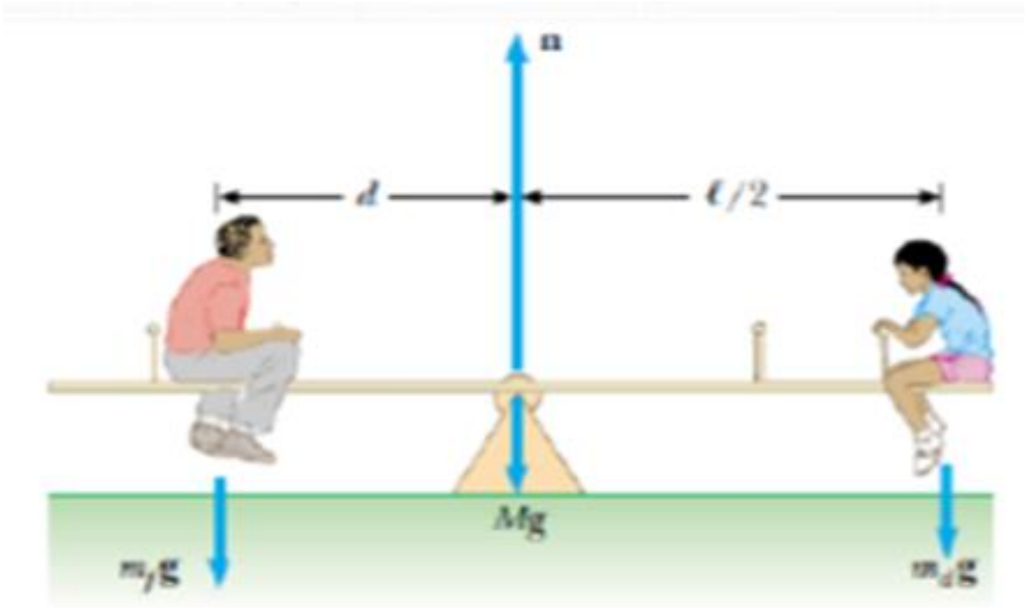
Finalize

- Make sure your results are consistent with your diagram.
- If the solution gives a negative for a force, it is in the opposite direction to what you drew in the diagram.
- Check your results to confirm $\Sigma Fx = 0, \Sigma Fy = 0, \Sigma \tau = 0$

The Seesaw Revisited Example

A seesaw consisting of a uniform board of mass M and length L supports a father and daughter with masses mf and md, respectively, as shown in Figure. The support (called the fulcrum) is under the center of gravity of the board, the father is a distance dfrom the center, and the daughter is a distance L/2from the center.

(A) Determine the magnitude of the upward force n exerted by the support on the board.



$$n - m_f g - m_d g - Mg = 0$$

ΣFy= 0 $$n = \boxed{m_f g + m_d g + Mg}$$

ΣFx= 0 the equation also applies, but we do not need to consider it because no forces act horizontally on the board.)

(B) Determine where the father should sit to balance the system.

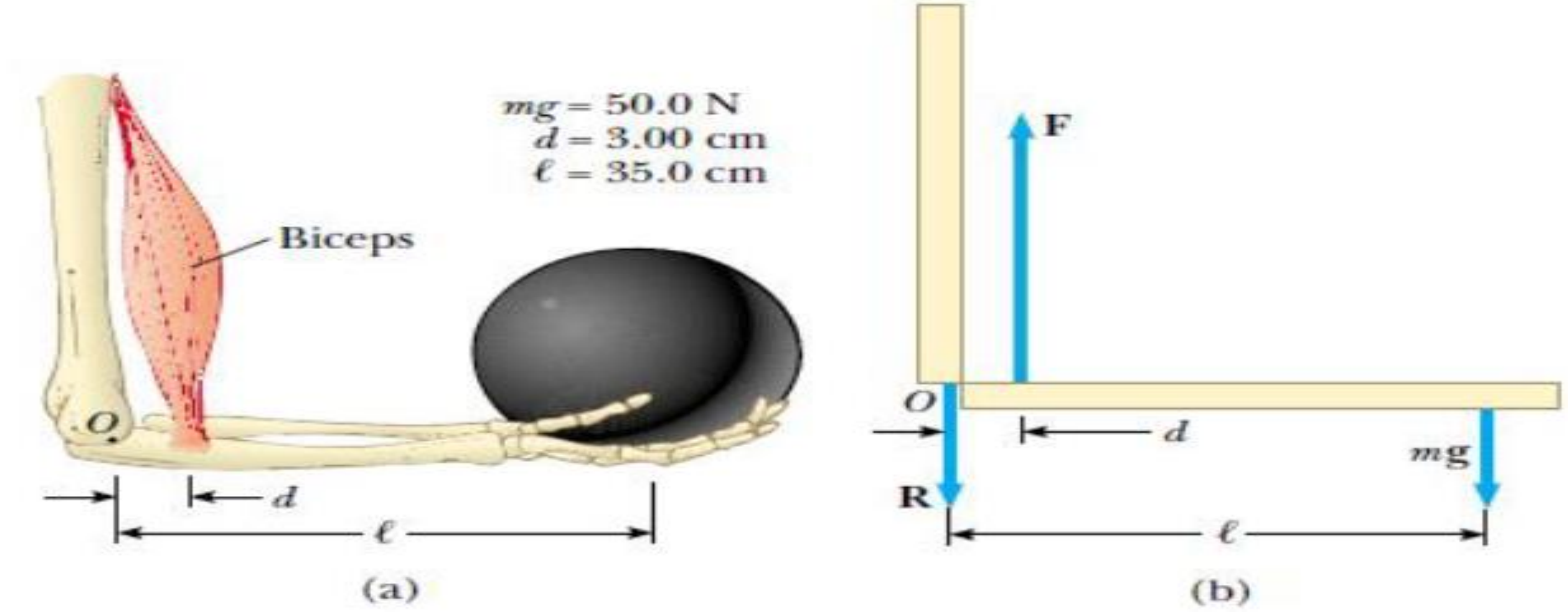
ΣFx= 0, ΣFy= 0, Σtz= 0 that

$$(m_f g)(d) - (m_d g)\frac{\ell}{2} = 0$$

$$d = \boxed{\left(\frac{m_d}{m_f}\right)\frac{1}{2}\ell}$$

Example A Weighted Hand

F is the upward force exerted by the biceps and R is the downward force exerted by the upper arm at the joint.



$$\sum F_y = F - R - 50.0\text{ N} = 0$$

$$\sum \tau = Fd - mg\ell = 0$$

$$F(3.00\text{ cm}) - (50.0\text{ N})(35.0\text{ cm}) = 0$$

$$F = \boxed{583\text{ N}}$$

This value for *F* can be substituted into to give *R* =533 N. As this example shows, the forces at joints and in muscles can be extremely large.