College Of Engineering
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*Dept. of Chem. & Petrochemical Engineering*
*Subject : Physics*
*First Stage*

# Physics

# Chapter 6 - Universal Gravitation

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## Newton's Law of Universal Gravitation

Every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the distance between them.

$$F_g = G\frac{m_1 m_2}{r^2}$$

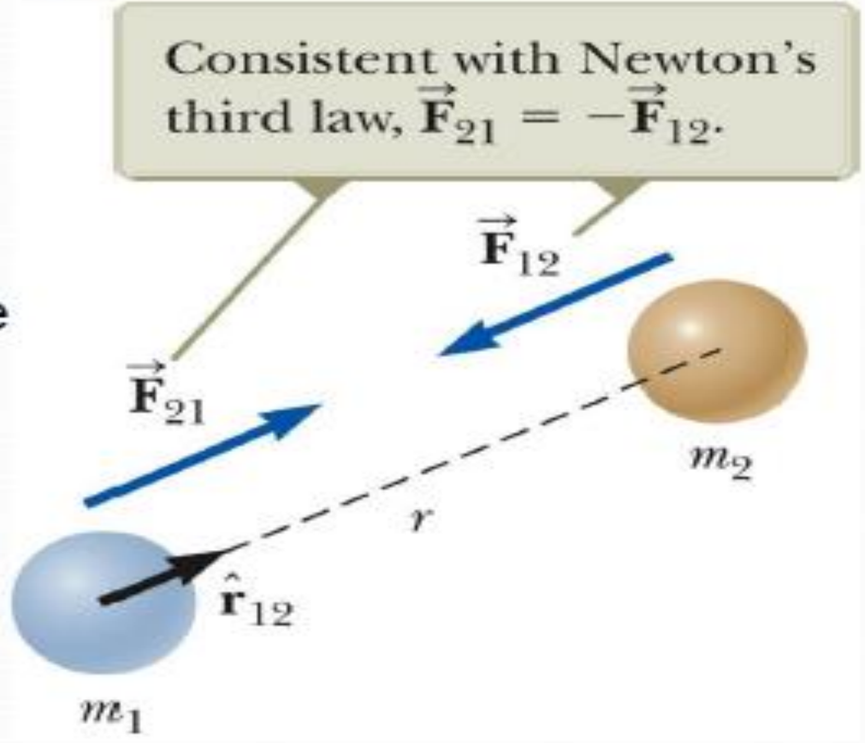
G is the universal gravitational constant and equals $6.673 \times 10^{-11}$ N·m$^2$ / kg$^2$.

This is an example of an ***inverse square law.***

The law can also be expressed in vector form

$$\vec{\mathbf{F}}_{12} = -G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}_{12}$$

- The negative sign indicates an attractive force.




$\vec{\mathbf{F}}_{12}$ is the force exerted by particle 1 on particle 2.

$\vec{\mathbf{F}}_{21}$ is the force exerted by particle 2 on particle 1.

# More About Forces

Gravitation is a field force that always exists between two particles, regardless of the medium between them.

The force decreases rapidly as distance increases.

The gravitational force exerted by a finite-size, spherically symmetric mass distribution on a particle outside the distribution is the same as if the entire mass of the distribution were concentrated at the center.

For example, the force exerted by the Earth on a particle of mass m near the surface of the Earth is

$$F_g = G\frac{M_E m}{R_E^2}$$

## Example

Two stars of masses $M$ and $4M$ are separated by distance $d$. Determine the location of a point measured from $M$ at which the net force on a third mass would be zero.

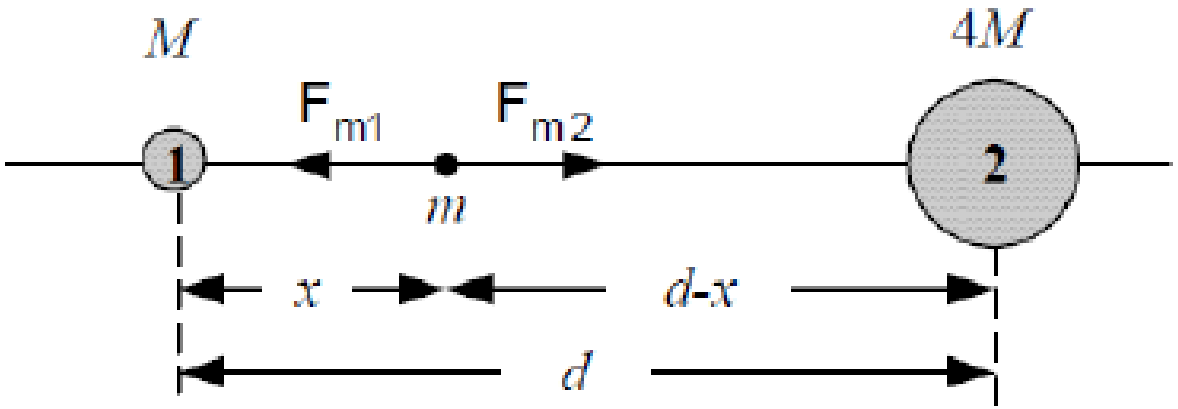
$$\vec{F}_{m2} = -\vec{F}_{m1}$$

$$G\frac{m4M}{(d-x)^2} = G\frac{mM}{(x)^2}$$

$$\frac{4}{(d-x)^2} = \frac{1}{(x)^2}$$

Solving for $x$ then,

$$x = \frac{d}{3}$$



# Finding g from G

The magnitude of the force acting on an object of mass m in free fall near the Earth's surface is the weight mg.

This can be set equal to the force of universal gravitation acting on the object.

$$mg = G\frac{M_E m}{R_E^2} \rightarrow g = G\frac{M_E}{R_E^2}$$

Substitute for the mass of earth $M_E$ = 5.98 × $10^{24}$ kg and the radius of the earth $R_E$ = 6.38 × 10^6m

$$\therefore g = G\frac{M_e}{R_e{}^2} = 6.67\times10^{-11}\frac{5.98\times10^{24}}{6.38\times10^6} = 9.8 m/s^2$$

If an object is some distance $h$ above the Earth's surface, $r$ becomes $R_E + h$.

$$g = \frac{GM_E}{(R_E + h)^2}$$

This shows that $g$ decreases with increasing altitude.

As $r \rightarrow \infty$, the weight of the object approaches zero.

# Variation of *g* with Height

**Example**

**Determine the magnitude of the acceleration of gravity at an altitude of 500km.**

$$g' = G\frac{M_e}{(R_e + h)^2}$$

$$g' = 6.67\times10^{-11}\frac{5.98\times10^{24}}{(6.38\times10^{6} + 0.5\times10^{6})^2}$$

$$= 8.43 m/s^2$$

**TABLE 13.1**

*Free-Fall Acceleration g at Various Altitudes Above the Earth's Surface*

| Altitude *h* (km) | *g* ($m/s^2$) |
|---|---|
| 1 000 | 7.33 |
| 2 000 | 5.68 |
| 3 000 | 4.53 |
| 4 000 | 3.70 |
| 5 000 | 3.08 |
| 6 000 | 2.60 |
| 7 000 | 2.23 |
| 8 000 | 1.93 |
| 9 000 | 1.69 |
| 10 000 | 1.49 |
| 50 000 | 0.13 |
| ∞ | 0 |

# The Gravitational Field

A **gravitational field** exists at every point in space.

When a particle of mass *m* is placed at a point where the gravitational field is $\vec{\mathbf{g}}$, the particle experiences a force.

The gravitational field is defined as

$$\vec{\mathbf{g}} \equiv \frac{\vec{\mathbf{F}}_g}{m}$$

Where $\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$ the field exerts a force on the particle.

The gravitational field is the gravitational force experienced by a *test particle* placed at that point divided by the mass of the test particle.

The presence of the test particle is not necessary for the field to exist.

The *source particle* creates the field.

The field can be detected and its strength measured by placing a test particle in the field and noting the force exerted on it.

# The Gravitational Field, 2

The gravitational field vectors point in the direction of the acceleration a particle would experience if placed in that field.

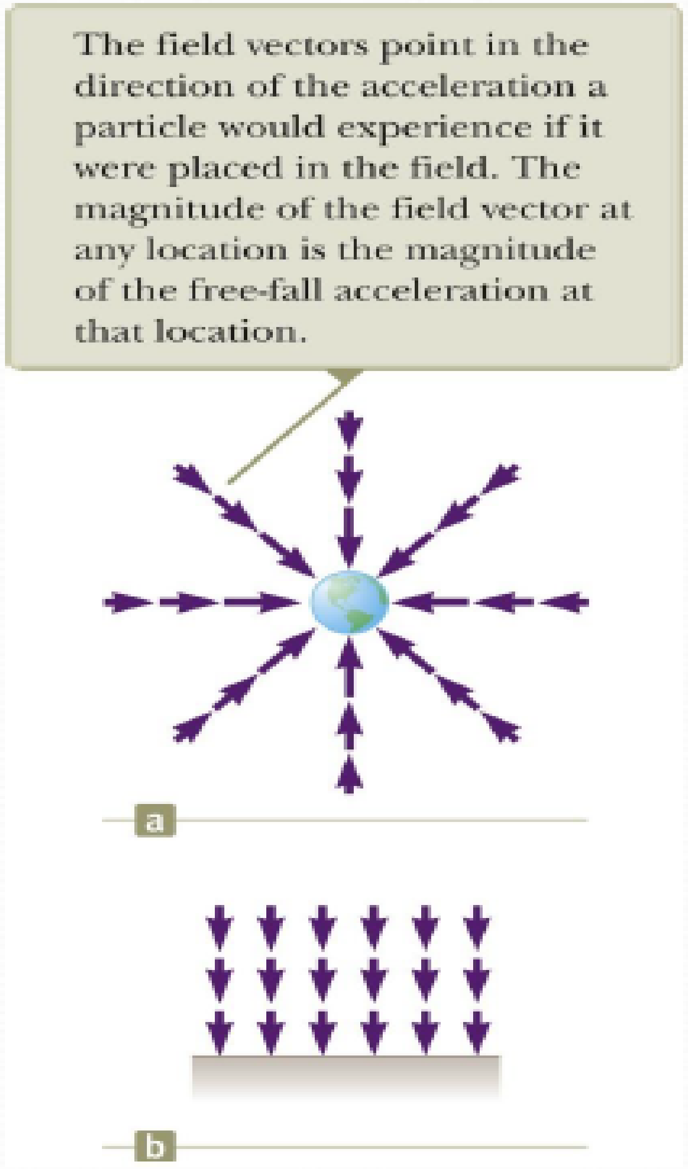
The magnitude is that of the free-fall acceleration at that location.

Part B of the figure shows the gravitational field vectors in a small region near the Earth's surface.

- The vectors are uniform in both magnitude and direction.

The gravitational field describes the "effect" that any object has on the empty space around itself in terms of the force that *would* be present *if* a second object were somewhere in that space.

$$\vec{\mathbf{g}} = \frac{\vec{\mathbf{F}}_g}{m} = -\frac{GM}{r^2}\hat{\mathbf{r}}$$



# Gravitational Potential Energy

Near the Earth's surface, the gravitational potential energy function was U = $mgh$ for a particle at a distance h from the earth surface.

- This was valid only when the particle is near the Earth's surface, where the gravitational force is constant.

The gravitational force is conservative.

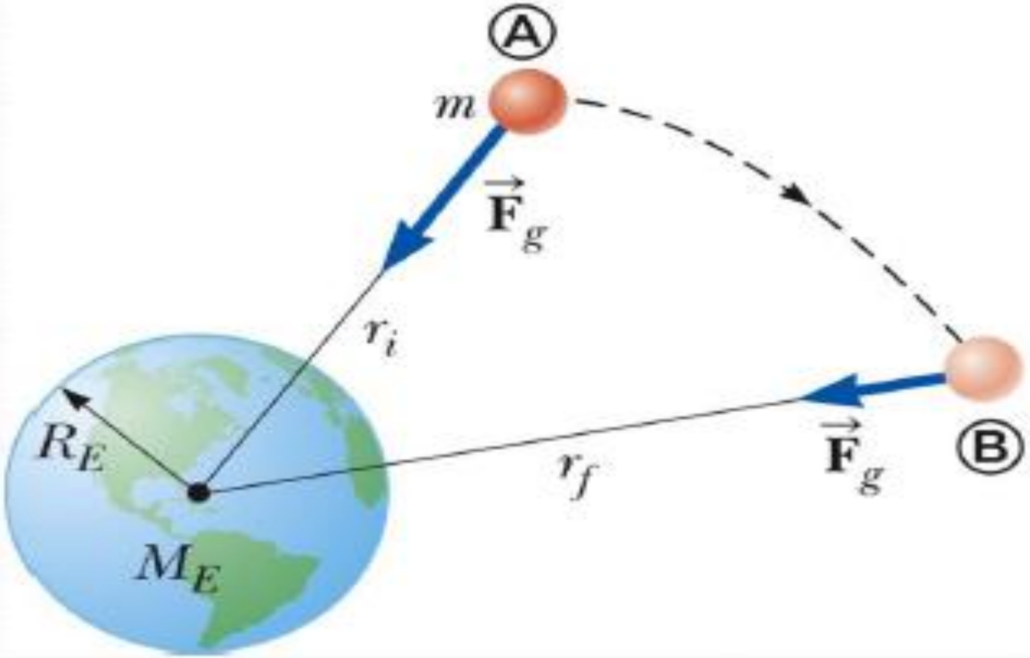
The change in gravitational potential energy of a system associated with a given displacement of a member of the system is defined as the negative of the internal work done by the gravitational force on that member during the displacement.

$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)\, dr$$

$$U_f - U_i = -GM_e m \int_{r_i}^{r_f} \frac{dr}{r^2} = GM_e m \left[ -\frac{1}{r} \right]_{r_f}^{r_i}$$

$$U_f - U_i = -GM_e m \left( \frac{1}{r_f} - \frac{1}{r_i} \right)$$

# Gravitational Potential Energy, cont.




As a particle moves from A to B, its gravitational potential energy changes by $\Delta U$.

Choose the zero for the gravitational potential energy where the force is zero.

- This means $U_i = 0$ where $r_i = \infty$

$$U(r) = -\frac{GM_E m}{r}$$

- This is valid only for $r \geq R_E$ and not valid for $r < R_E$.
- $U$ is negative because of the choice of $U_i$.

## Gravitational Potential Energy, General

For any two particles, the gravitational potential energy function becomes

$$U = -\frac{Gm_1m_2}{r}$$

The gravitational potential energy between any two particles varies as $1/r$.

The potential energy is negative because the force is attractive and we chose the potential energy to be zero at infinite separation.

An external agent must do positive work to increase the separation between two objects.

- The work done by the external agent produces an increase in the gravitational potential energy as the particles are separated.
  - $U$ becomes less negative.

The absolute value of the potential energy can be thought of as the ***binding energy.***

If an external agent applies a force larger than the binding energy, the excess energy will be in the form of kinetic energy of the particles when they are at infinite separation.

# Systems with Three or More Particles

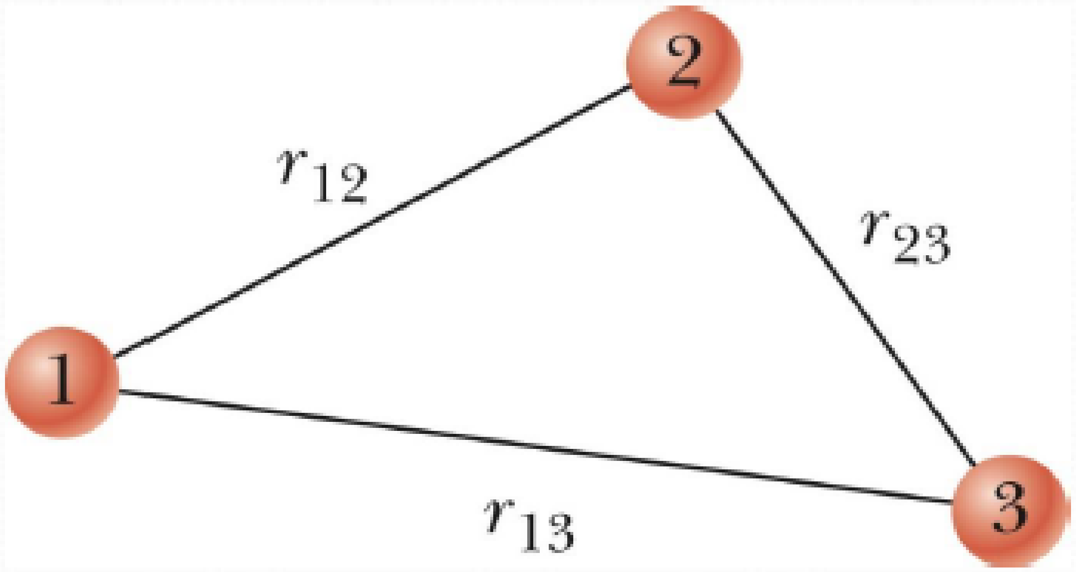
The total gravitational potential energy of the system is the sum over all pairs of particles.

Each pair of particles contributes a term of *U*.

Assuming three particles:

$$U_{total} = U_{12} + U_{13} + U_{23}$$
$$= -G\left(\frac{m_1 m_2}{r_{12}} + \frac{m_1 m_3}{r_{13}} + \frac{m_2 m_3}{r_{23}}\right)$$

The absolute value of $U_{total}$ represents the work needed to separate the particles by an infinite distance.



**Example**

**A system consists of three particles, each of mass 5g, located at the corner of an equilateral triangle with sides of 30cm.**

**(a) Calculate the potential energy of the system.**

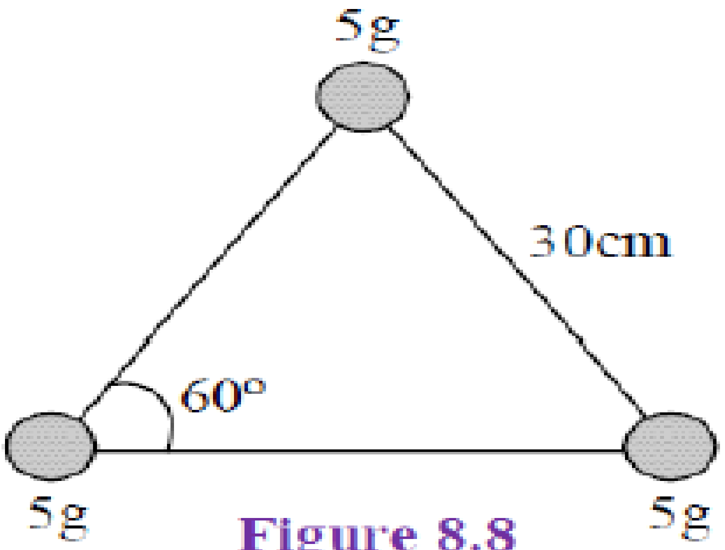


**Figure 8.8**

**Solution**

$$U_{total} = U_{12} + U_{13} + U_{23}$$

$$U_{total} = -G\left(\frac{m^2}{r} + \frac{m^2}{r} + \frac{m^2}{r}\right) = -\frac{3GM^2}{r}$$

$$U_{total} = -\frac{3 \times 6.67 \times 10^{-11} \times (0.005)^2}{0.3} = -1.67 \times 10^{-14}\, J$$