كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الثالثة

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اسم المادة بالغة العربية : التحليل الرياضي

اسم المادة باللغة الإنكليزية : **Mathematical Analysis**

اسم الحاضرة الأولى باللغة العربية: بديهيات الاعداد الحقيقية

اسم المحاضرة الأولى باللغة الإنكليزية: **Axioms of Real numbers**

xioms of real numbers

1. **The axioms arithmetics**
2. **The axioms of ordered**
3. **The complete Axioms**

*** Let R be a real number and $a, b, c \in R$. Then**

$A_1 : \forall a, b, c \in R \;\; a + (b + c) = (a + b) + c.$

$A_2 : a + b = b + a$

$A_3 : for\ any\ a \in R, \exists!\ element\ 0 \in R\ \ s.t$

$a + (-a) = -a + a = 0$

$A_4 : Ther\ exists\ an\ element\ 0 \in R\ , S.t \qquad a + 0 = 0 + a = a$

Then $(R, +)$ is a commutative group.

$A_5 : a.(b.c) = (a.b).c$

$A_6 : a.b = b.a$

$A_7 :$ $\exists!$ **Element in** $R(1 \in R\ )\ s.t\ a.1 = 1.a = a$

$A_8 :\ \forall a \in R\ , \exists!\ a^{-1} \in R\ , s.t\ a.a^{-1} = a^{-1}.a = 1$

Form $A_5 \to A_8$. $(R, .)$ **commutive ring**

$A_9 : a.(b + c) = (a.b) + (a.c)$

$A_1 \to A_9 \;\; (R, +, .)$ **Is a field**

* Subtraction $a - b = a + (-b)\ , \forall a, b \in R$

* Division $a \div b = a.b^{-1} \ni b \neq 0$

The Axioms of order:

$A_{10}: a \leq b\ or\ b \leq a$

$A_{11}: a \leq b\ and\ b \leq c \to a = b$

$A_{12}: a \leq b\ and\ b \leq c \to a \leq c$

$A_{13}: a \leq b\ , c \in R \to a + c \leq b + c$

$A_{14}: a \leq b\ , c\ is\ not\ negative \to a.c < -b.c$

$A_1 \to A_{14}\ , (R, +\ , .\ , \leq)$ **order field.**

Remark: $R^{+} = \{x \in R\ ; x > 0\}$

$R^{-} = \{x \in R\ ; x < 0\}$

Propositions: **Let $(R,+,.)$ be a field, then prove the following**

1. $\forall a,b,c \in R$, $if\ a+b=b+c$, $then\ a=c$
2. $\forall a,b,c \in R$, $if\ a.b=c.b$, $then\ a=c$
3. $\forall a,b \in R$,**prove that:**
    1. $-(-a)=a$
    2. $(a^{-1})^{-1}=a$
    3. $(-a)+(-b)=-(a+b)$
    4. $(-a).b=-a,b$
    5. $if\ a.b=0$ **then either** $a=0\ or\ b=0$

Proof (5):

Suppose that $a\neq 0$, To prove $b=0$

Since $a\neq 0$, then $\exists a^{-1}\in R\ s.t\ a.a^{-1}=1$

$a^{-1}(a.b)=0 \implies (a^{-1}.a).b=0 \implies 1.b=0 \to b=0$

Suppose that $b\neq 0$, T.P $a=0$

Since $b\neq 0$, then $\exists b^{-1}\in R\ s.t\ b.b^{-1}=1$

$(a.b)b^{-1}=0$

$a.(b.b^{-1})=0$

$a.1=0 \to a=0$

Absolute Value:

Suppose that $a\in R$, the absolute value of a is:

$$|a|=\begin{cases} a & if\ a>0 \\ 0 & if\ a=0 \\ -a & if\ a<0 \end{cases}$$

$|a|: R\to R^{+}\cup\{0\}$ **is the function of absolute value.**

Properties of absolute value.

Theorem: let a be a real number, then

1. $|x|<a \leftrightarrow -a<x<a$
2. $|X|>a \leftrightarrow x>a\ or\ x<-a$

Corollary: let $a\in R^{+}$ and $b\in R$, then

1. $|x-b \leq a \; iff \; b-a \leq x \leq b+a$
2. $|\boldsymbol{x}-\boldsymbol{b}| \geq \boldsymbol{a} \; \boldsymbol{iff} \; \boldsymbol{x} \geq \boldsymbol{b}+\boldsymbol{a} \; \boldsymbol{or} \; \boldsymbol{x} \leq \boldsymbol{b}-\boldsymbol{a}$

Let $a, b \in R$ and k be areal number, then

1. $|a| \geq 0$
2. $|a| = 0$ **iff** $a = 0$
3. $a^2 = |a|^2$
4. $|ab| = |a|.|b|$
5. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
6. $|ka| = |k|.|a|$

Example: $\forall a \in R, \sqrt{a^2} = |a|$

Proof: **If $a > 0$ then $\sqrt{a^2} = a$. If $a < 0$ then $\sqrt{a^2} = -a$**

by def absolute value to a we have

$$|a| = \begin{cases} a = \sqrt{a^2} \;\; if \;\; a \geq 0 \\ -a = \sqrt{a^2} \;\; if \;\; a < 0 \end{cases}$$

وفي كلتا الحالتين يكون لدينا $|a| = \sqrt{a^2}$

The triangle inequality

Theorem: **if $a, b \in R$, then $|a+b| \leq |a| + |b$**

Proof: $|a+b|^2 = (a+b)^2 \leq a^2 + 2ab + b^2$

$$\leq |a|^2 + 2|ab| + |b|^2$$

$$\leq (|a| + |b|)^2$$

$\therefore |a+b| \leq |a| + |b|$

Corollary: if $a, b \in R$, then $|a-b| \geq |a| - |b|$

Definition: let $S \subset R$ **S is said to be bounded above if there is some real numbers m s.t $x \leq m \; \forall x \in S$, m is called upper bounded of S**