كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الثالثة

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اسم المادة باللغة العربية : التحليل الرياضي

اسم المادة باللغة الإنكليزية : **Mathematical Analysis**

اسم الحاضرة االسابعة باللغة العربية: تقارب وتباعد المتتابعات

اسم المحاضرة االسابعة باللغة الإنكليزية : **Converge and diverge the Sequences**

Example:

1. $< \frac{(-1)^{n+1}}{n} > = 1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \dots$
   $|x_n| = \left|\frac{(-1)^{n+1}}{n}\right| = \frac{1}{n} \leq 1 \Rightarrow < x_n >$ is bounded
   and $M = 1$

2. $< 5 + \frac{(-1)^{n+1}}{n} > = 6, \frac{9}{2}, \frac{16}{3}, \dots$
   $< x_n \geq 5 + \frac{1}{n} \leq 5 + 1 = 6 \Rightarrow < x_n >$ is bounded
   and $M = 6$

3. $< n + (-1)^n > = \begin{cases} < n - 1 >, & if\ n\ is\ odd \\ < n + 1 >, & if\ n\ is\ even \end{cases}$

4. $|x_n| = \begin{cases} |n - 1| \geq 0 \\ |n + 1| \geq 2 \end{cases}$

Theorem: Every convergent sequence is bounded.

Proof:

Let $< x_n >$ be a convergent sequence in (X, d) and $x_n \to x$, to prove $< x_n >$ is bounded

Since $x_n \to x \Rightarrow \forall \epsilon > 0, \exists k \in N\ s.t\ d(x_n, x) < \epsilon, \forall n > k$

That $\epsilon = 1 \Rightarrow d(x_n, x) < 1, \forall n \in k.$

Let $r = \max\{1, d(x_1, x), d(x_2, x), \dots, d(x_n, x)\}$

$\Rightarrow d(x_n, x) < r$

$\therefore < x_n >$ is bounded and $M = 2r$

Remark: The convers of above theorem is not true.

Example: $< (-1)^n > = -1, 1, -1, 1, \dots$

$|x_n| = |(-1)^n| = 1 \Rightarrow < x_n >$ is bounded and $M = 1$

$< (-1)^n >$ is divergent?

Remake 3.2: If $< x_n >$ unbounded, then $< x_n >$ is divergent.

Proof:

Suppose that $< x_n >$ converged and unbounded sequence.
Since $< x_n >$ Convergent $\rightarrow < x_n >$ bounded by theorem (In metric space, every conv. Seq. is bounded) $\rightarrow$ C! ,So $< x_n >$ unbounded is $< x_n >$ is divergent

Examples:

- $< x_n > = < \sqrt{n-1} > = 0, \sqrt{1}, \sqrt{2}, \sqrt{3}, \dots$ unbounded $\Rightarrow < x_n >$ divergent

- $< x_n > = < n^2 - n > = 0, 2, 6, 11, \dots$ unbounded $\Rightarrow < x_n >$ divergent

Definitions: Let $< x_n >$ be a real sequence. Then it is called

- Non – decreasing. If $x_{n+1} \geq x_n, \forall n$
- Non – increasing. If $x_{n+1} \leq x_n, \forall n$.
- Not monotone. If it does not increasing and decreasing.

Examples:

* $< x_n > = < \frac{1}{\sqrt{n}} >$

$x_n = \frac{1}{\sqrt{n}}$, $x_{n+1} = \frac{1}{\sqrt{n+1}}$

$\forall n, n+1 > n \Rightarrow \sqrt{n+1} > \sqrt{n} \rightarrow \frac{1}{\sqrt{n+1}} \leq \frac{1}{\sqrt{2}} \rightarrow x_{n+1} \leq x_n$

$\therefore < x_n >$ is non – increasing

* $< x_n > = < \frac{n}{n+1} >$

$x_n = \frac{n}{n+1}$, $x_{n+1} = \frac{n+1}{n+2}$

$x_{n+1} - x_n = \frac{n+1}{n+2} - \frac{n}{n+1} = \frac{(n+1)-n(n+2)}{(n+1)(n+2)} = \frac{n^2+2n+1-n^2-2n}{(n+1)(n+2)} = \frac{1}{(n+1)(n+2)} > 0$

$\therefore x_{n+1} - x_n > 0 \rightarrow x_{n+1} > x_n, \forall n, \therefore < x_n >$ non – decreasing

* $< x_n > = < (-1)^n >$ not monotone
* $< x_n > = < \frac{(-1)^n}{\sin(n)} >$ not monotone.
* $< x_n > = < (-5)^n >$ not monotone.

Theorem: Every monotone bounded real seq. is convergent

Example: $< x_n > = \langle \frac{(-1)^n}{n} \rangle > 0$

$< x_n >$ Convergent seq. but not monotone.

Example: Show that $x_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}$ is convergent.

Theorem: Let (X, d) be a metric space and $S \subseteq X$:

i. If $< x_n >$ seq. in S and $x_n \rightarrow x$ then $x \in S$ or $x \in S'$

ii. If $x \in S$ *or* $x \in S'$, then there exists a sequence $< x_n >$ in S s.t $x_n \rightarrow x$

Definition: The sequence $< x_n >$ is a sub sequence of $< x_n >$, if $< m >$ is increasing sequence in N.