كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الثالثة

أستاذ المادة : أ.م.د. علاء محمود فرحان علي الجميلي

اسم المادة بالغة العربية : التحليل الرياضي

اسم المادة باللغة الإنكليزية : **Mathematical Analysis**

اسم الحاضرة التاسعة باللغة العربية: المتسلسلات اللانهائية

اسم المحاضرة التاسعة باللغة الإنكليزية: **Infinite Series**

Infinite Series

Definition: Let $\langle x_n \rangle$ be a real seq the series of the form, if $x_1 + x_2$ then itis Called infinite series, and it is written as $\sum_{n=1}^{\infty} x_n$.

If the series of the form $x_1 + x_2 + \cdots + x_n$, then itis Called finite Series.and written as $\sum_{k=1}^{n} x_k$

Definition: Let $\sum_{n=1}^{\infty}$ an be a finite series, the seq $\langle S_n \rangle$ is called the sequence of Partial sums of $\sum_{n=1}^{\infty} \mathrm{a_n}$

where $S_1 = a_1$

$$S_2 = a_1 + a_2$$
$$S_3 = a_1 - a_2 + a_3$$
$$\vdots$$
$$S_n = a_{1+}a_2 + \cdots + a_n$$

Definition: let $\sum_{n=1}^{\infty} a_n$ be infinite series, then it is said toble

1. Converge, if $\langle S_n \rangle$ converge
2. diverge, if $\langle s_n \rangle$ diverge.
3. If ⟨ Sn ⟩ Converge to b. then $\sum_{n=1}^{\infty} a_n = S_n$.

Example: let $a_n = 1, \forall n$, then

$\sum_{n=1}^{\infty} a_n = 1 + 1 + 1 + \cdots$

$S_1 = a_1 = 1$

$S_2 = a_1 + a_2 = 1 + 1 = 2$

$S_3 = a_1 + a_2 + a_3 = 1 + 1 + 1 = 3$

.
.
.

$S_n = a_1 + a_2 + a_3 + \cdots + a_n = 1 + 1 + 1 + \cdots + 1 = n$

The seq of partial sum is $\langle S_n \rangle = \langle n \rangle$ is divergent Since it is unbounded

$\Rightarrow \sum_{n=1}^{5}$ an is diverge.

Example: let $\sum_{n=1}^{\infty} a_n = 3 - 3 + 3 - 3 + \cdots$

$S_1 = a_1 = 3$

$S_2 = a_1 + a_2 = 3 - 3 = 0$

$S_3 = a_1 + a_2 + a_3 = 3 - 3 + 3 = 3$

$\vdots$

$$S_n = a_1 + a_2 + \cdots + a_n = \begin{cases} 3 & , if\ n\ odd \\ 0 & , if\ n\ even \end{cases}$$

The Sequence of partial Sum $\langle S_n \rangle$ is divergent

$\therefore \sum_{n=1}^{\infty} a_n$ is divergent

Example:

Let: $\sum_{n}^{\infty} a_n = 2 + 4 + 2 + 4 + 24$

$n = 1$

$S_1 = a_1 = 2$

$S_2 = a_1 + a_2 = 2 + 4 = 6$

$S_3 = a_1 + a_2 + a_3 = 2 + 4 + 2 = 8$

$Sn = a_1 + a_2 + a_3 + a_4 = 2 + 4 + 2 + 4 + \cdots = ?$

The sequence of partial sums $\langle S_n \rangle$ is unbownded, then $\langle s_n \rangle$ is diverg ent so $\sum_{n=1}^{\infty} a_n$ is diverge

Exercises

let $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{n(n+1)}$, then $\sum_{n=1}^{\infty} a_n$ is convergent.

Harmonic Series

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \text{ divergent}$$

proof:

$$S_1 = a_1 = 1$$

$$s_2 = a_1 + a_2 = 1 + \frac{1}{2}$$

$$S_3 = a + a_2 + a_3 = 1 + \frac{1}{2} + \frac{1}{3}$$

$$S_n = a_1 + a_2 + \cdots + a_{n-1} + \frac{1}{2} + \cdots + \frac{1}{n}$$

$$S_{n+1} = a_1 + a_2 + \cdots + a_n - a_{n-1} = 1 + \frac{1}{2} + \cdots + \frac{1}{n} + \frac{1}{n+1}$$

$$S_{n+n} = \frac{1}{2n} + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}$$

let $m = 2n$

$$\begin{aligned}(S_m - S_n) \quad &= \left| \left(1 + \frac{1}{2} + \cdots + \frac{1}{n} + \cdots + \frac{1}{2n}\right) - \left(1 + \frac{1}{2} + \cdots + \frac{1}{n}\right) \right| \\ &= \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n} \\ &> \frac{1}{2n} + \frac{1}{2n} + \cdots + \frac{1}{2n} \\ &= n \cdot \frac{1}{2n} = \frac{1}{2}\end{aligned}$$

If $\epsilon = \frac{1}{2}$, then $|S_m - S_n| > \epsilon$

$\therefore \langle S_n \rangle$ is not Caushy sequence $\Rightarrow \langle S_n \rangle$ is not Convergent.

So $\sum_{n=1}^{2} a_n$ is diverge.