كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الثالثة

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اسم المادة بالغة العربية : التحليل الرياضي

اسم المادة باللغة الإنكليزية : **Mathematical Analysis**

اسم الحاضرة العاشرة باللغة العربية: بعض انواع المتسلسلات

اسم المحاضرة العاشرة باللغة الإنكليزية :**Some types of Series**

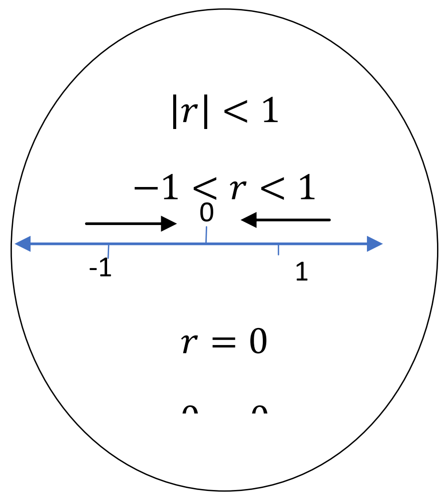
Geometric Series

$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + \cdots$
where a $> 0, r$ is called the base of Series. the sequence of partial, Sum is

$$sn = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$$

(1) if $|r| = 1$
$\therefore S_n = a + a^{-1} + a + \cdots + a = n \cdot a$
$\langle S_n \rangle = \langle n_a \rangle$ **diverg** $\Rightarrow \sum_{n=1}^{\infty} ar^{n-1}$ **diverge.**

(2) if $|r| > 1$
$S_n = a + ar + ar^2 + \cdots + ar^{n-1}$
$rS_n = ar + ar^2 + ar^3 + \cdots + ar^n$
$\rightarrow S_n - rS_n = a - ar^n$
$S_n(1 - r) = a(1 - r^n)$
$\therefore S_n = \frac{a(1-r^n)}{(1-r)}$



When $n \rightarrow \infty \Rightarrow \lim_{n\rightarrow\infty} S_n = \lim \frac{a(1-r^n)}{(1-r)}$
$= \frac{a(1-0)}{1-r} = \frac{a}{1-r}$
$\therefore \sum ar^{n-1} = \frac{a}{1-r}$ **.Converge**

(3) if $|r| > 1$
$$s_n = \frac{a(1 - r^n)}{1 - r}$$
when $n \longrightarrow \infty; r^n = \mp\infty \Rightarrow S_n \rightarrow \infty$
$\therefore$ **Sn diverge.**
$\therefore \sum_{n=1}^{\infty} ar^{n-1}$ **diverge.**

$$\sum_{n=1}^{\infty} ar^{n-1} = \begin{cases} \text{divenge} & \text{if } |r| > 1 \\ \text{Ganverge} & \text{if } |r| < 1 \end{cases}$$

$$= \sum_{n=1}^{\infty} ar^{n-1} - \frac{a}{1-r}$$

Example:

Let $\sum_{n=1}^{\infty} a_n = 1 + \frac{5}{2} + \left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^3 + \cdots$ **Geometric series.**

$$a = 1, v = \frac{5}{2} \Rightarrow |r| = \left|\frac{5}{2}\right| = \frac{5}{2} > 1$$

$\therefore \sum_{n=1}^{\infty}$ **an divenge:**

$\sum_{\Sigma=1}^{\infty} a_{n=1} = 1 - \frac{3}{4} + \frac{9}{16}\ \frac{27}{64} + \cdots$ **Geometric Series.**

$$\sum_{n=1}^{\infty} ar^{n-1} = 1 + \left(\frac{-3}{4}\right) + \left(\frac{-3}{4}\right)^2 + \left(\frac{-3}{4}\right)^3 + \cdots$$

$$a = 1, v = \frac{3}{4} \Rightarrow$$

$$|r| = \left|-\frac{3}{4}\right| = \frac{3}{4} < 1$$

$\therefore \sum_{n=1}^{\infty} a_n$ **is Converger and**

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} = \frac{15}{1+\frac{3}{4}} = \frac{1}{\frac{7}{4}} = \frac{4}{7}$$

Theorem: **If** $\sum_{n=1}^{\infty}$ **an Convergent, them** $\lim_{n\to\infty} a_n = 0$
(that is, $\forall \in > 0, \exists k \in N, s.t |a_n - 0| \langle \epsilon, y_n \rangle k$

Proof: **Suppose** $S_n = a_1 + a_2 + \cdots + a_n$
$\sum_{n=1}^{\infty}$ **an canvergent, then** $\langle S_n \rangle$ **Convergent**

$\Rightarrow \langle S_n \rangle$ **canshy sequence.**
$\therefore \forall \in > 0, \exists k \in N$, **sit** $|S_m - s_n| < E, \forall n, m > k$
let $m = n + 1$
So $|S_m - S_n| < \epsilon \rightarrow |S_{n+1} - S_n| = |a_{n+1}| < \epsilon, \forall > k$
$\rightarrow |a_n| < \epsilon, \forall n > k$, So $|a_n - 0| < \epsilon, \forall n > k$.
then $\lim_{n \to \infty} a_n = 0$

Example: $\langle a_n \rangle = \left\langle \frac{1}{n} \right\rangle \rightarrow 0$

and $\lim_{n \to \infty} a_n = 0$ **bul** $\sum_{n=1}^{\infty} \mathrm{a_n} = \sum_{n=1}^{\infty} \frac{1}{n}$ **diveroe**

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Corollary:
If $\lim_{n \to \infty} a_n \neq 0$ **diverge.**

proof:
Suppose that $\sum_{n=1}^{\infty} \mathrm{a_n}$ **Convergent.**

then, $\lim_{n \to \infty} a_n = 0$, **by theorem,** $\longrightarrow$ **C!**

Example: $\sum_{n}^{\infty} a_n = \sum (\sqrt{m} - \sqrt{n-1})$

$\sum_{n=1}^{\infty} \mathrm{a_n}$ **Diverge, but** $\lim_{n \to \infty} a_n = 0$

Exercises : (1) $\sum_{n^3}^{\infty} \frac{1}{\sqrt{n}}$ **(2)** $\sum_{n=1}^{a} \sqrt{\frac{n}{3n+5}}$ **(3)** $\sum_{n=1}^{\infty} \frac{n^3+2}{2n(n+5)}$

Theorem: If $\sum_{n=1}^{\infty}$ **and** $\sum_{n=1}^{\infty}$ **bn are Convergant Series and** $k \in R$**, then**

(1) $\sum_{n=1}^{\infty}(\text{an} + b_n)$ convergent and $\sum_{n=1}^{\infty}(\text{an} + b_n) = \sum_{n=1}^{\infty}\mathrm{a_n} + \sum_{n=1}^{\infty}\mathrm{b_n}$

(2) $\sum_{n=1}^{m_n} ka_n$ convergent and $\sum_{n=1}^{\infty} ka_n = k\sum_{ns}^{\infty} a_n$

proof: (1)

let $\langle s_n \rangle$ be a sequence of partial sums of

$\sum_{n=1}^{\infty} a_n$ and

$\langle t_n \rangle$ bea seq of partipl sam of $\sum_{n=1}^{\infty} b_n$

$\sum_{n=1}^{\infty}$ an convergent, so $\exists s \in R$ s.t $\sum_{n=1} = \text{s}$

and $\langle Sn \rangle \longrightarrow \text{S} \Rightarrow \text{Lim}_{n\to\infty}\ S_n = \text{S}.$

also, $\sum_{n=1}^{\infty} b_n$ Canergent, then $\exists t = R$, s.t $\sum_{n=1}^{\infty} b_n = t$ and $\langle t_n \rangle \to t\ \to$

$\text{Lim}_{n\to\infty}\ t_n = t$

$\text{Lim}_{n\to\infty}(s_n + t_n) \to \text{S} + t$, but $\langle s_n + t_n \rangle$ is the seq of partial sum of

$\sum_{n=1}^{\infty}(\text{an} + b_n) \to \sum_{n=1}^{\infty} a_{n+} b_n = \sum_{n+1}^{\infty} a_{n+} \sum_{n=1} b_n = s + t$

(2) Suppose that $\langle s_n \rangle$ be a seq of partial sums of $\sum_{n=1}^{\infty} a_n$ but $\sum_{n=1}^{\infty} a_n$ Convergent $\Rightarrow \exists s \in \mathbb{R}$ s,t $\sum_{n=1}^{\infty} a_n = s$ and $\langle \mathrm{S_n} \rangle = \text{S}$, $\lim_{n\to\infty} s_n = S.$

$\lim_{n\to\infty} k\ s_n = k$, $\lim_{n\to\infty} s_n = kS \to\ \langle k\ S_n \rangle \to k_s$

then $\sum_{ns_1}^{2} ka_n = kS = k\sum_{n\leq 1}^{\infty}$

then $\sum_{n=1}^{\infty} \mathrm{ka_n} = k\sum_{n=1}^{\infty} \mathrm{a_n}$